# Overview of Lattice QCD Calculations of Baryons

**Ying Chen**

Institute of High Energy of Physics,
Chinese Academy of Sciences

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# Outline

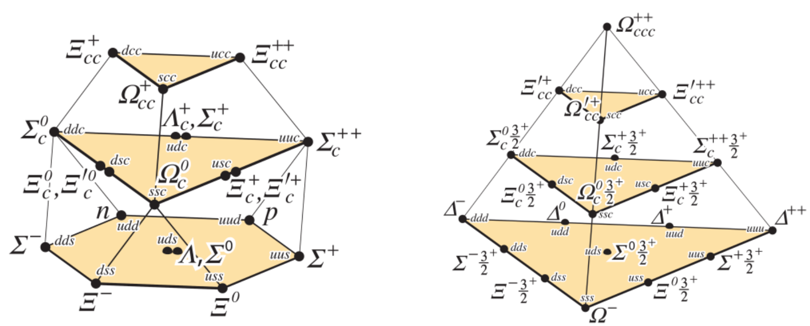
I. Baryon spectroscopy from lattice QCD
II. $N\pi$ scattering and the $\Delta$ resonance
III. Nucleon (baryon) mass decomposition and $\sigma$ term
IV. Diquarks in baryons
V. Summary

# I. Baryon spectroscopy from lattice QCD

- Constituent quark model describes baryons as three-quark bound states
- Ground state baryons that have constituent $u, d, s$ quarks

  Octet: $N, \Sigma, \Xi, \Lambda$, Decuplet: $\Delta, \Sigma^*, \Xi^*, \Omega$
- Excited states of $u, d, s$ quark baryons are described by $SU(6) \otimes O(3)$ model
- Many heavy flavored baryons were observed in experiments recently

  (For the expt. and theor. status see the review arXiv:2204.02649)




- Exotic baryon states ($P_c$)observed by LHCb in $J/\psi p$ final states

  $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4457)$

  which have the minimal quark configuration $uudc\bar{c}$.

- Since most baryons are resonances, meson-baryon interaction should be considered in lattice QCD.

# 1. Ground state baryons (octet and decuplet)

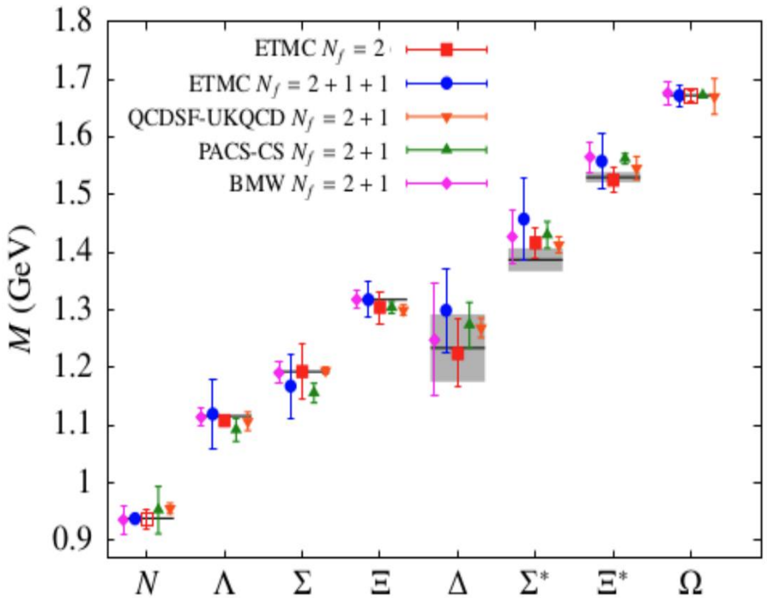
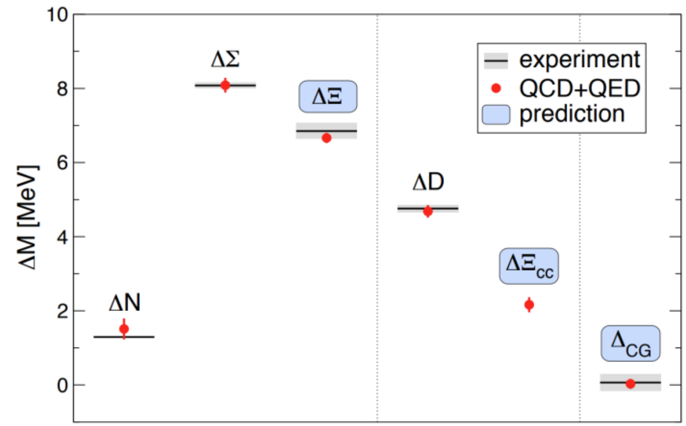



Figure taken from C. Alexandrou and C. Kallidonis, PRD96 (2017) 034511 [1704.02647]. See the Refs. therein

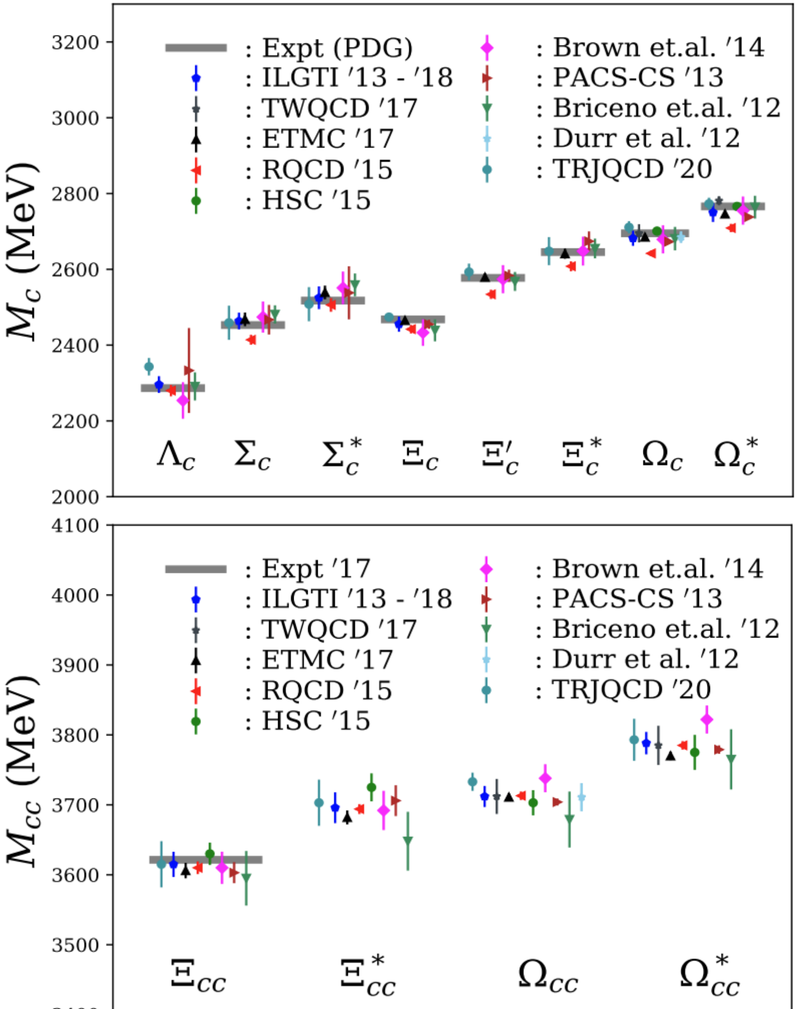



| | mass splitting [MeV] | QCD [MeV] | QED [MeV] |
|---|---|---|---|
| $\Delta N = n - p$ | 1.51(16)(23) | 2.52(17)(24) | -1.00(07)(14) |
| $\Delta\Sigma = \Sigma^- - \Sigma^+$ | 8.09(16)(11) | 8.09(16)(11) | 0 |
| $\Delta\Xi = \Xi^- - \Xi^0$ | 6.66(11)(09) | 5.53(17)(17) | 1.14(16)(09) |
| $\Delta D = D^\pm - D^0$ | 4.68(10)(13) | 2.54(08)(10) | 2.14(11)(07) |
| $\Delta\Xi_{cc} = \Xi_{cc}^{++} - \Xi_{cc}^{+}$ | 2.16(11)(17) | -2.53(11)(06) | 4.69(10)(17) |
| $\Delta_{CG} = \Delta N - \Delta\Sigma + \Delta\Xi$ | 0.00(11)(06) | -0.00(13)(05) | 0.00(06)(02) |

Sz. Borsanyi et al., Science 347 (2015) 1452-1455 [arXiv:1406.4088 ]

- Lattice QCD calculations reproduce the experimental spectrum of octet and decuplet baryons (without considering the resonance nature of decuplet baryons).
- Even the mass-splittings between isospin partners can be determined to a high precision with the QCD and EM contributions being distinguished.
- A subtle cancellation of EM effects and $m_d - m_u$ effects

## 2. Charmed Baryon Spectrum from Lattice QCD

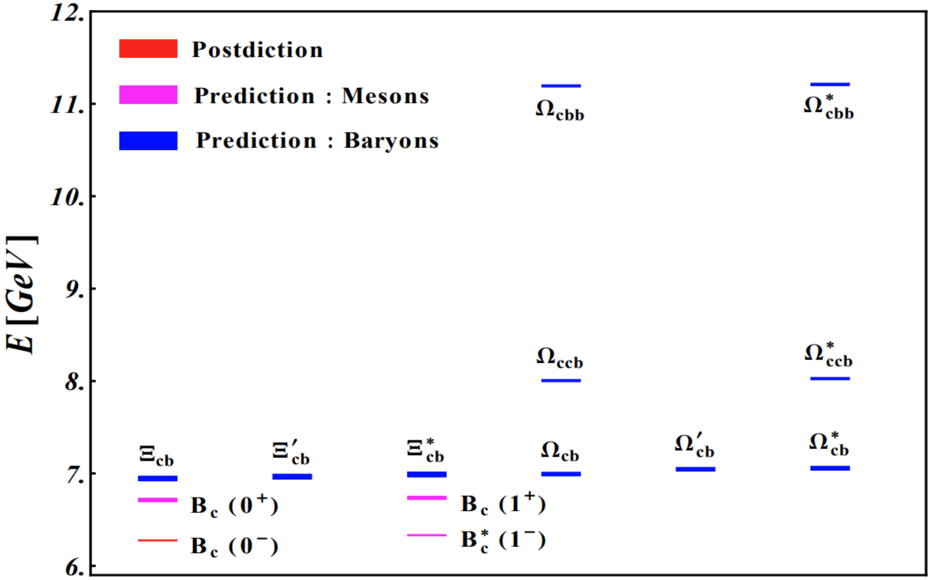



Figures taken from *PoS* CHARM2020 (2021) 014 (arXiv: 2109.04748)

ILGTI '13-'18: PoS LAT'12(2012)141, PoS LAT'13(2014)243
PoS LAT'14(2015)083, PRD99(2019)031501
(N. Mathur and M. Padmanath)
TWQCD'17: PLB767(2017)193 (Y.-C. Chen and T.-W. Chiu),
Brecino et al. '12: PRD86(2012)094504
Brown et al.'14: PRD90(2014)094507
RQCD'15: PRD92(2015)034504 (P. Perez-Rubio et al.)
Durr et al.'12: PRD86(2012)114514
TRJQCD'20: PRD102(2020)054513 (H. Bahtiyar et al.)

- Singly charmed baryons: all the lattice results are in agreement with expements (PDG 2020) served as a calibration to some extent
- Doubly charmed baryons:
  $\Xi_{cc}^{++}(3621)$ observed by LHCb (2017) (grey band)
  $\Xi_c^{+}(3519)$ observed by SELEX (2002)
  Lattice results consist with $\Xi_{cc}^{++}(3621)$, but are higher than $\Xi_c^{+}(3519)$ by roughly 100 MeV.
- $\Xi_c^{+}(dcc)$ and $\Xi_c^{++}(ucc)$ are isospin partners, their mass difference is expected to be a few MeV
- Lattice study does not confirm $\Xi_c^{+}(3519)$.
- Other doubly charmed baryons are to be discovered experimentally.

## 3. Bottom-charm Baryon Spectrum from Lattice QCD




N. Mathur et al., PRL121 (2018) 202002 [1806.04151].

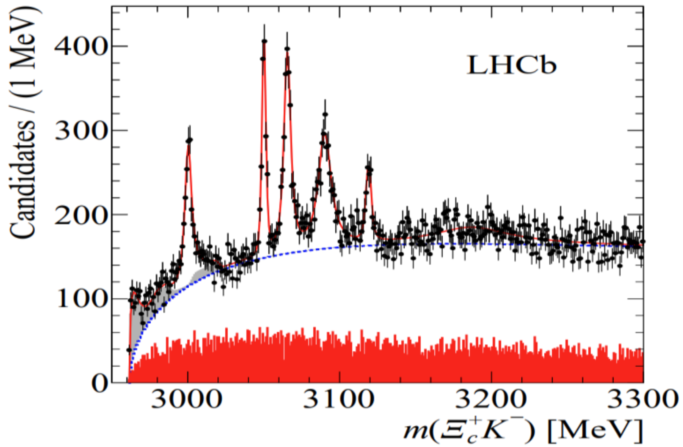
| Hadrons | Lattice | Experiment |
|---|---|---|
| $B_c(0^-)$ | 6276(3)(6) | 6274.9(8) |
| $B_c^*(1^-)$ | 6331(4)(6) | ? |
| $B_c(0^+)$ | 6712(18)(7) | ? |
| $B_c(1^+)$ | 6736(17)(7) | ? |
| $\Xi_{cb}(cbu)(1/2^+)$ | 6945(22)(14) | ? |
| $\Xi'_{cb}(cbu)(1/2^+)$ | 6966(23)(14) | ? |
| $\Xi^*_{cb}(cbu)(3/2^+)$ | 6989(24)(14) | ? |
| $\Omega_{cb}(cbs)(1/2^+)$ | 6994(15)(13) | ? |
| $\Omega'_{cb}(cbs)(1/2^+)$ | 7045(16)(13) | ? |
| $\Omega^*_{cb}(cbs)(3/2^+)$ | 7056(17)(13) | ? |
| $\Omega_{ccb}(1/2^+)$ | 8005(6)(11) | ? |
| $\Omega^*_{ccb}(3/2^+)$ | 8026(7)(11) | ? |
| $\Omega_{cbb}(1/2^+)$ | 11194(5)(12) | ? |
| $\Omega^*_{cbb}(3/2^+)$ | 11211(6)(12) | ? |

- Charmed-bottom hadron masses are determined.
- Chiral extrapolation and continuum extrapolation are performed.
- The mass of $B_c(0^-)$ serves as a calibration for the calculation.

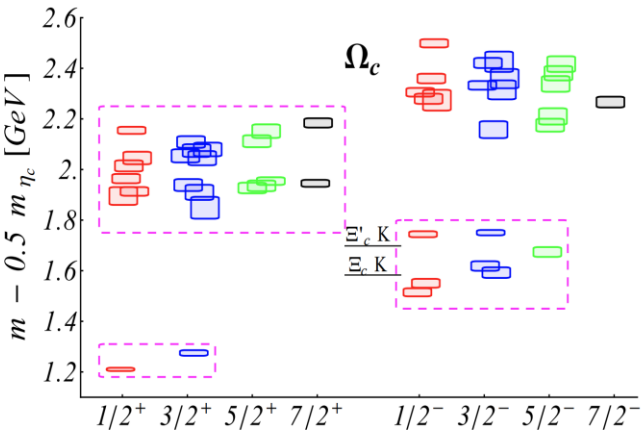
## 4. $\Omega_c^0$ and its excited states

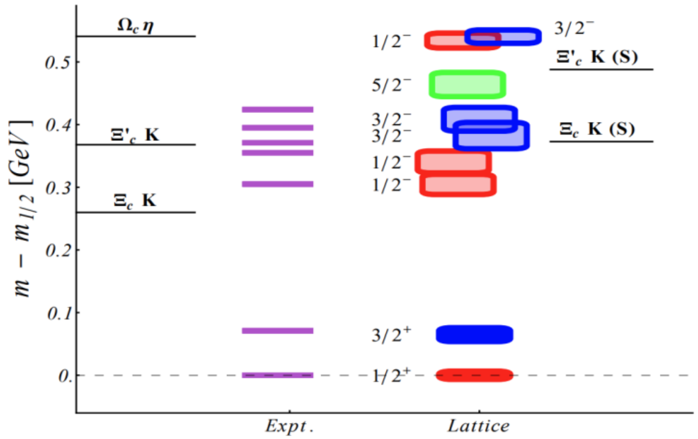
- Apart from $\Omega_c^0$ and $\Omega_c(2770)^0$, LHCb observes five new narrow structures in $\Xi_c K$ final states.

  $\Omega_c(3000)^0, \Omega_c(3050)^0, \Omega_c(3066)^0,$
  $\Omega_c(3090)^0, \Omega_c(3119)^0$
- Their $J^P$ quantum numbers have not been determined yet.




LHCb Col., PR118 (2017) 182001 [1703.04639]



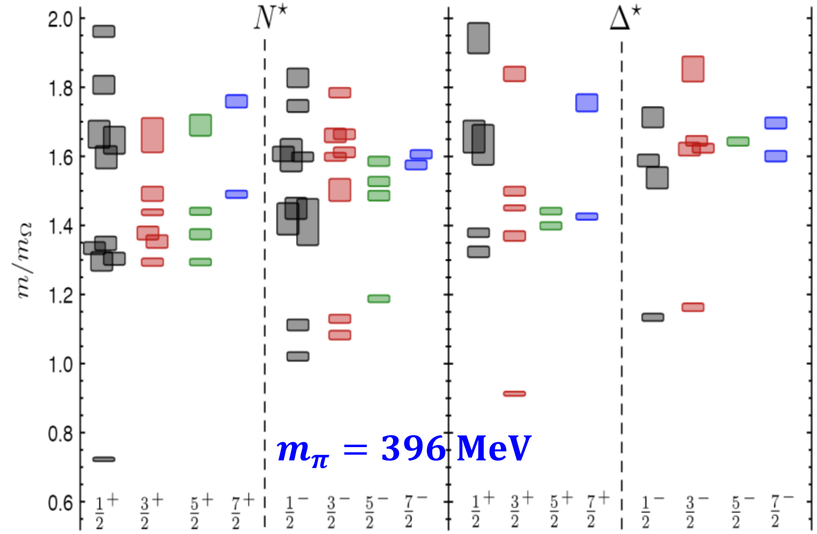




M. Padmanath and N. Mathur, PRL119 (2017) 042001 [1704.00259]

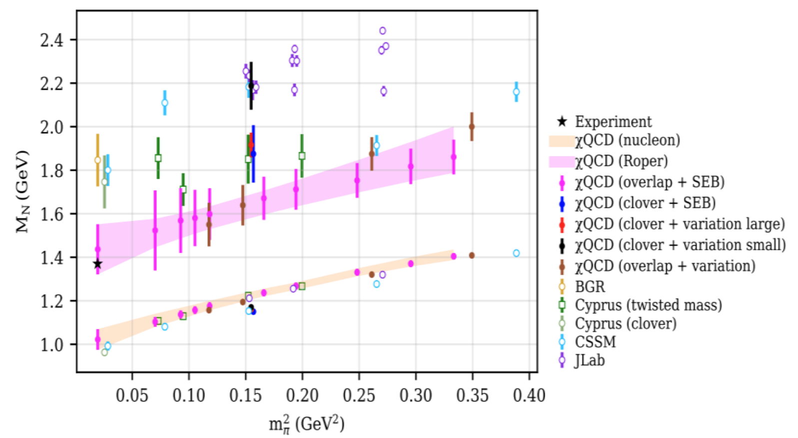
- Spin identified energy levels relevant to $\Omega_c^0$ states are determined at $m_\pi \approx 391$ MeV
- There are energy levels lying away from $\Xi_c K$ and $\Xi'_c K$ thresholds.
- There seems some correspondence between expt. states and lattice energy levels.

## 5. Excited states of nucleon and $\Delta$ baryons

- Well-separated bands of levels
- Bands appear alternatively in parity
- The spectrum pattern complies with the $SU(6) \otimes O(3)$ quark model.
- Seemingly no evidence for diquarks (the analysis of operator couplings hint the existence of $[20, 1^+]$ multiplet which is prohibited in the diquark picture, see the Reference for details).
- No evidence for the first $\frac{1}{2}^+$ excited state below the lowest $\frac{1}{2}^-$ state.
- The lattice prediction of the mass of the first $\frac{1}{2}^+$ excited state of nucleon is very diverse.
- The "Roper puzzle" has not been resolved.
- No meson-baryon operator involved yet, $m_\pi$ is still heavy.
- ***$N\pi$* scattering and even coupled channel effects should be considered.**




R.G. Edwards et al., PRD 84 (2011) 074508 [arXiv:1104.5152]




M. Sun et al., PRD101 (2020) 054511, [arXiv:1911.02635]

## 6. $P_c$ states and $\Sigma_c D(D^*)$ scatterings

H. Xing et al., arXiv:2210.08555

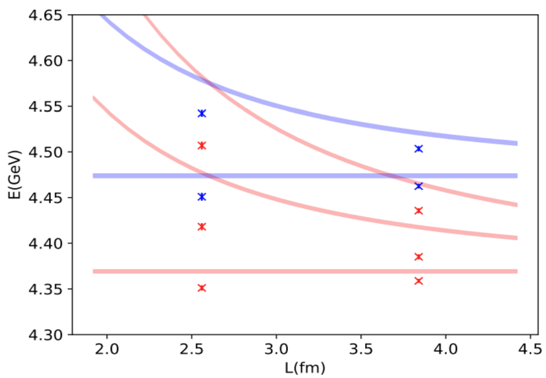
- LHCb observed several $P_c$ states in $J/\psi p$ final states

  $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4457)$

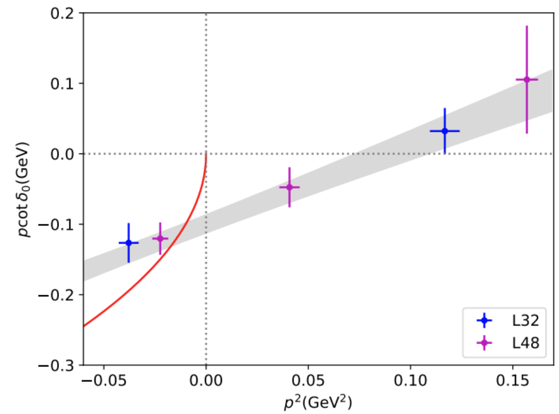
  which must have the minimal quark configuration $uudc\bar{c}$.
- The $J^P = \frac{1}{2}^-$ $\Sigma_c\bar{D}$ and $\Sigma_c\bar{D}^*$ scatterings are investigated via the Leuscher's method:

$$p\cot\delta_0(p(E)) = \frac{2}{L\sqrt{\pi}}\mathcal{Z}_{00}(1;q^2(E))$$

$$p\cot\delta_0(p) = \frac{1}{a_0} + \frac{1}{2}rp^2 \quad (ERE)$$

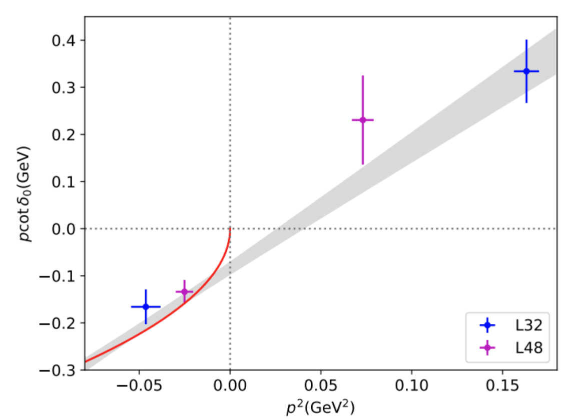



- **Points:** finite volume energies of $\Sigma_c\bar{D}$ (red) and $\Sigma_c\bar{D}^*$(blue)
- **Curves:** non-interacting energies
- of $\Sigma_c\bar{D}$ (red) and $\Sigma_c\bar{D}^*$(blue)




$\Sigma_c\bar{D}$ scattering: A bound state $P_c(4312)$?

$$a_0(\Sigma_c\bar{D}) = -2.0(3)(5)\text{fm},$$
$$r_0(\Sigma_c\bar{D}) = 0.46(6)(17)\text{fm},$$
$$E_B(\Sigma_c\bar{D}) = 6(2)(2)\text{MeV},$$




$\Sigma_c\bar{D}$ scattering: A bound state

$$a_0(\Sigma_c\bar{D}^*) = -2.3(5)(1)\text{fm},$$
$$r_0(\Sigma_c\bar{D}^*) = 1.01(8)(10)\text{fm},$$
$$E_B(\Sigma_c\bar{D}^*) = 7(3)(1)\text{MeV},$$

## II. $N\pi$ scattering and $\Delta$ resonance

- Apart from the ground state baryons, all the excited baryon states are resonance observed in hadron-hadron interaction amplitudes.
- So the resonance nature of a resonance should be investigated in the relevant scattering processes.
- For a two-body scattering system, on a finite box of a spatial size $L$, what LQCD can calculate are the energies $E$ of the system.
- The Luescher's quantization condition relates finite volume energy $E$ to the scattering amplitude $\mathcal{M}(E)$ by

$$\det\left[F^{-1}\left(E,\vec{P},L\right)-\mathcal{M}(E)\right]=0$$

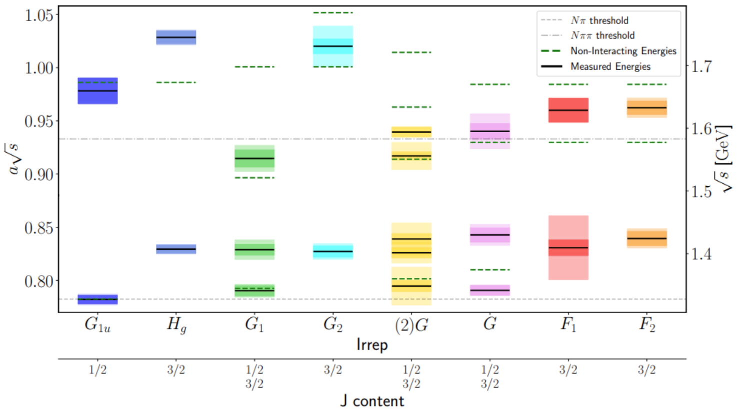
  where $\vec{P}$ is the total momentum of the system and $F^{-1}(E,\vec{P},L)$ are mathematically known functions.
- Couple channel effects should considered if there are more than one decay channels.
- The poles of $\mathcal{M}(E)$ in the complex plane of $E_{cm}$ connect the physical resonances or bound states.
- See the review paper by Brecino et al. (Rev. Mod. Phys. 90 (2018) 025001) for details.

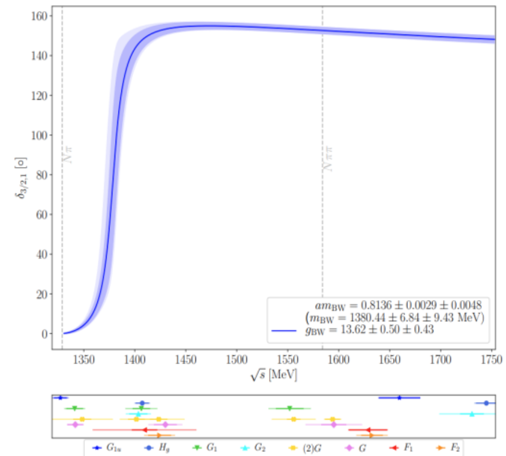
- There have been a few lattice studies on $\Delta$ resonance.

# 1. $N\pi$ scattering and the $\Delta$ resonance

G. Silvi et al., PRD103 (2021) 094508 (arXiv:2101.00689) and references therein

| | |
|---|---|
| $N_s^3 \times N_t$ | $24^3 \times 48$ |
| $\beta$ | 3.31 |
| $am_{u,d}$ | $-0.09530$ |
| $am_s$ | $-0.040$ |
| $a$ [fm] | 0.1163(4) |
| $L$ [fm] | 2.791(9) |
| $m_\pi$ [MeV] | 255.4(1.6) |
| $m_\pi L$ | 3.61(2) |
| $N_{config}$ | 600 |
| $N_{meas}$ | 9600 |




$K$-matrix rescaled: $K = \rho^{1/2}\widehat{K}\rho^{1/2}$

$K$ relates to the phase shift: $K^{Jl} = \tan\delta_{Jl}$

$$m_\Delta = (1378.3 \pm 6.6 \pm 9.0) \text{ MeV}$$
$$\Gamma_\Delta = (\ \ 16.4 \pm 1.0 \pm 1.4) \text{ MeV}$$

Breit Wigner: $\widehat{K}^{3/2,1)} = \dfrac{\sqrt{s}\Gamma(s)}{(m_{BW}^2 - s)\rho}$

$$\Gamma(s) = \frac{g_{BW}^2}{6\pi}\frac{k^3}{s}$$

$$\Gamma_{\rm EFT}^{\rm LO} = \frac{g_{\Delta\text{-}\pi N}^2}{48\pi}\frac{E_N + m_N}{E_N + E_\pi}\frac{k^3}{m_N^2}$$

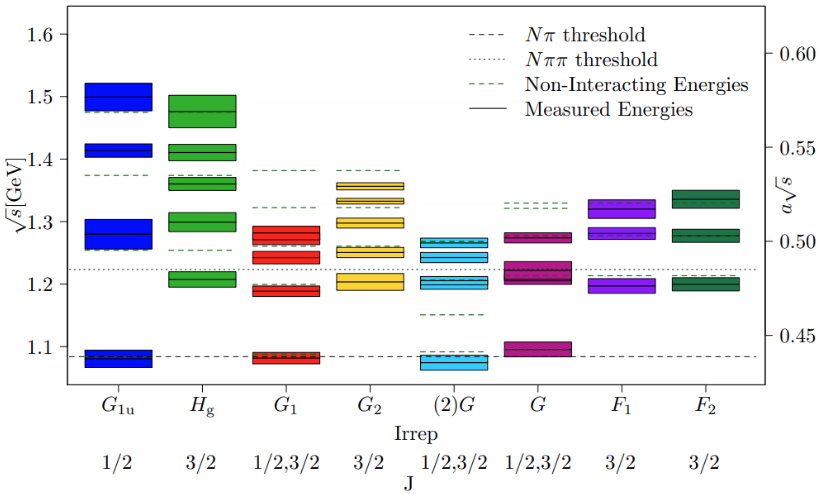
| Collaboration | $m_\pi$ [MeV] | Methodology | $m_\Delta$ [MeV] | $g_{\Delta\text{-}\pi N}$ |
|---|---|---|---|---|
| Verduci 2014 [38] | 266(3) | Distillation, Lüscher | $1396(19)_{\rm BW}$ | 19.90(83) |
| Alexandrou et al. 2013 [37] | 360 | Michael, McNeile | 1535(25) | 27.0(0.6)(1.5) |
| Alexandrou et al. 2016 [39] | 180 | Michael, McNeile | 1350(50) | 23.7(0.7)(1.1) |
| Andersen et al. 2018 [41] | 280 | Stoch. distillation, Lüscher | $1344(20)_{\rm BW}$ | 37.1(9.2) |
| Our result | 255.4(1.6) | Smeared sources, Lüscher | $1380(7)(9)_{\rm BW}$, $1378(7)(9)_{\rm pole}$ | 23.8(2.7)(0.9) |
| Physical value [5] | 139.5704(2) | phenomenology, K-matrix | $1232(1)_{\rm BW}$, $1210(1)_{\rm pole}$ | 29.4(3) [79], 28.6(3) [80] |

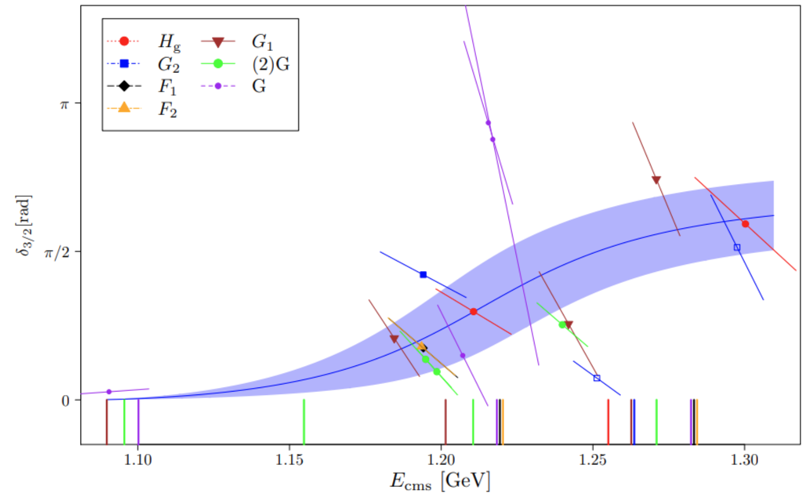
## 2. Elastic $\pi - N$ scattering in the $I = \frac{3}{2}$ channel at the physical $m_\pi$

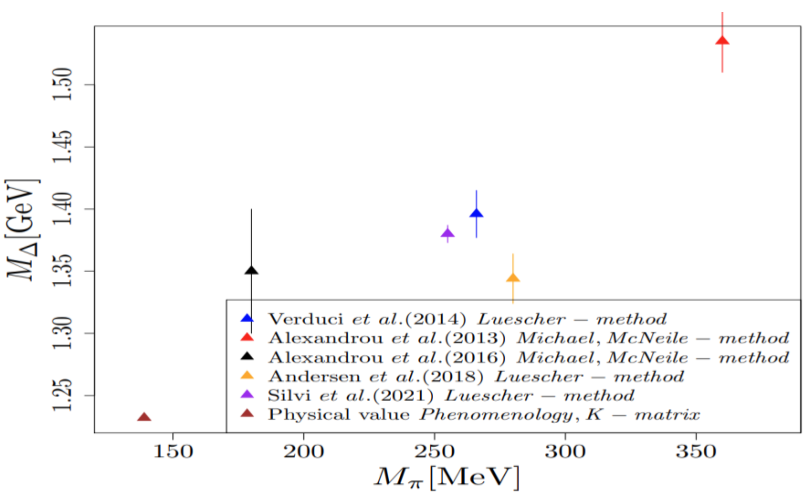
(F. Pittler et al., *PoS* LATTICE2021 (2022) 226, arXiv: 2112.04146 )

| ensemble | $M_\pi$/MeV | $M_N$/MeV | $M_\pi L$ | $a$/fm | $N_{conf}$ | $N_{src}$ |
|---|---|---|---|---|---|---|
| cB211.072.64 | 139.43(9) | 944(10) | 3.622(3) | 0.0801(2) | 388 | 64 |

ETMC Col.




- The lattice QCD procedure is similar to that in the previous page.
- Resonance parameters are close to the experimental values, but with large errors

$$m_\Delta = 1255(25)\ \text{MeV}$$
$$\Gamma_\Delta = 140(120)\ \text{MeV}$$

## 3. Progress on Meson-Baryon Scattering

(C. Morningstar et al., *PoS* LATTICE2021 (2022) 170 [arXiv:2111.07755])

- The single-channel finite volume Lüscher approach is employed.
- Single $N_f = 2 + 1$ ensemble with $m_\pi = 200$ MeV.
- Preliminary results for $I = \frac{1}{2}, \frac{3}{2}$ $N\pi$ amplitudes including the $\Delta(1232)$ resonance.

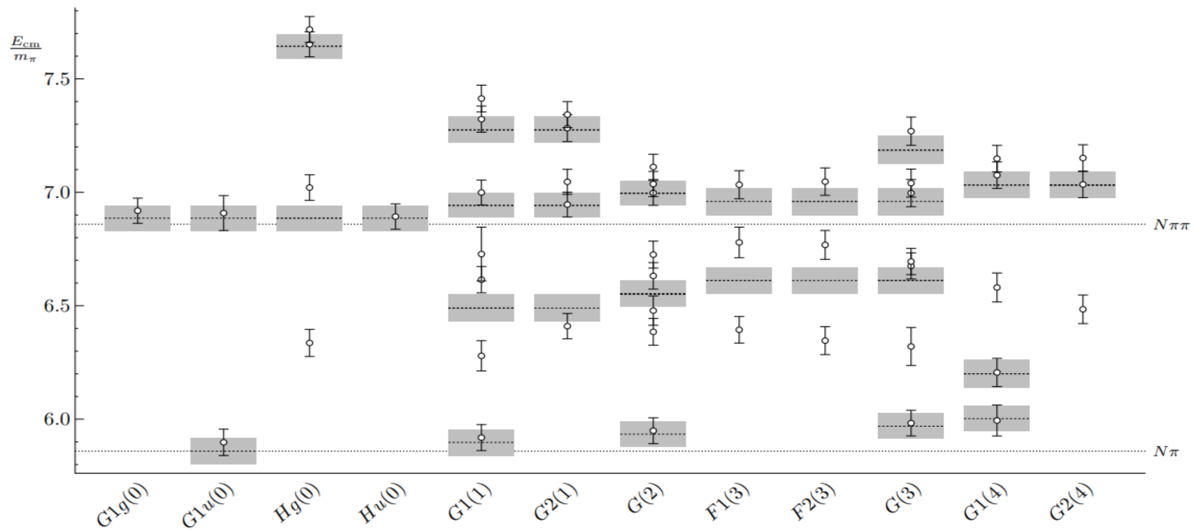



**Figure 1:** Center-of-mass energies $E_{\rm cm}$ as ratios over the pion mass $m_\pi$ in the isoquartet non-strange sector for various little group irreps. The dashed horizontal lines show the non-interacting energies of the expected free two-particle states; the errors in the non-interacting energies are indicated by the gray boxes. The integers in parentheses in the irreps indicate $\boldsymbol{d}^2$ for total momentum squared $\boldsymbol{P}^2 = (2\pi/L)^2\boldsymbol{d}^2$.

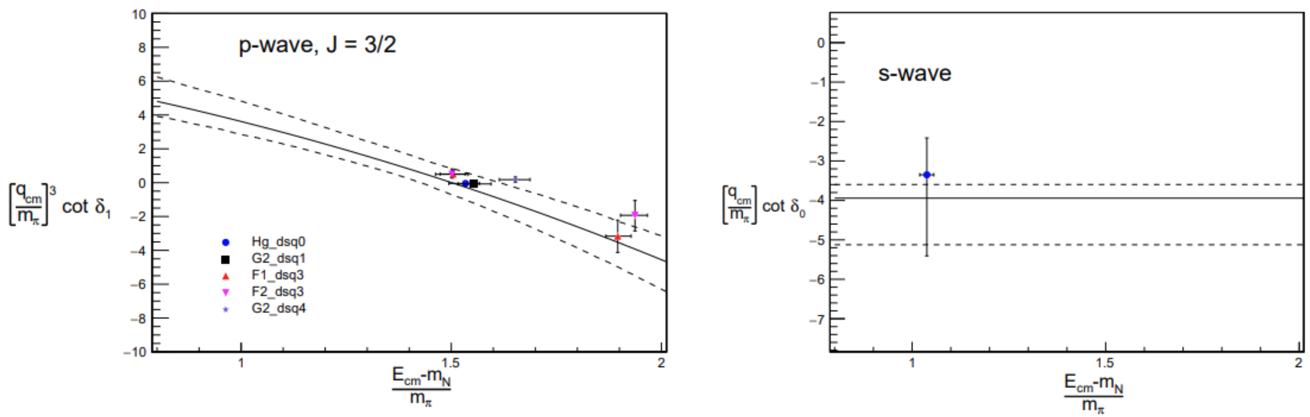



**Figure 2:** Threshold factors times cotangents of the phase shifts for the *P*-wave (left) and *S*-wave (right) for the isoquartet nonstrange channel against center-of-mass energies $E_{\rm cm}$ minus the nucleon mass $M_N$ as a ratio over the pion mass $m_\pi$. Best-fit functions are shown as solid lines with error bands shown as dashed lines.

Similar to the previous two studies, the best fit results are as follows:

$$\frac{m_\Delta}{m_\pi} = 6.380(20), \quad g_{\Delta N\pi} = 13.7(1.5), \quad \chi^2/\text{d.o.f.} = 1.74,$$

$$m_\pi a_0^{J=1/2} = -0.254(41), \quad (m_\pi a_1^{J=1/2})^{-1} = 2.61(44).$$

$g_{\Delta N\pi} = 13.62 \pm 0.50 \pm 0.43$ (PRD103 (2021) 094508)

# III. Nucleon (baryon) mass decomposition and $\sigma$ term

## 1. Proton mass decomposition from Lattice QCD

(Y.-B. Yang et al. ($\chi$QCD Col.), PRL121 (2018) 212001 [arXiv:1808.08677])

- The origin of hadron masses is still an open question

$$m_u, m_d \sim 3-5 \text{ MeV}, \qquad m_N \approx 940 \text{ MeV}$$

- Parton model sum rules—— taking the nucleon for example,

$$I \equiv \int_0^1 dx\, x\left(u+\bar{u}+d+\bar{d}+s+\bar{s}\right) = \int_0^1 dx\,[9F_2^{eN} - \frac{3}{2}F_2^{\nu N}] \approx 0.5$$

Quark only contribute 50% of the nucleon mass, others are from gluons.

- QCD energy-momentum tensor(EMT) (X. Ji, PRL74 (1995) 1071)

$$\bar{T}_{\mu\nu} = \frac{1}{4}\bar{\psi}i\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi - F_{\mu\alpha}F_{\nu}^{\alpha} + \frac{1}{4}g_{\mu\nu}F^2$$

$$\hat{T}_{\mu\nu} = \frac{1}{4}g_{\mu\nu}\left[\gamma_m m\bar{\psi}\psi + \frac{\beta(g)}{2g}F^2\right]$$

$$T^{\mu\nu} = \bar{T}^{\mu\nu} + \hat{T}^{\mu\nu}$$

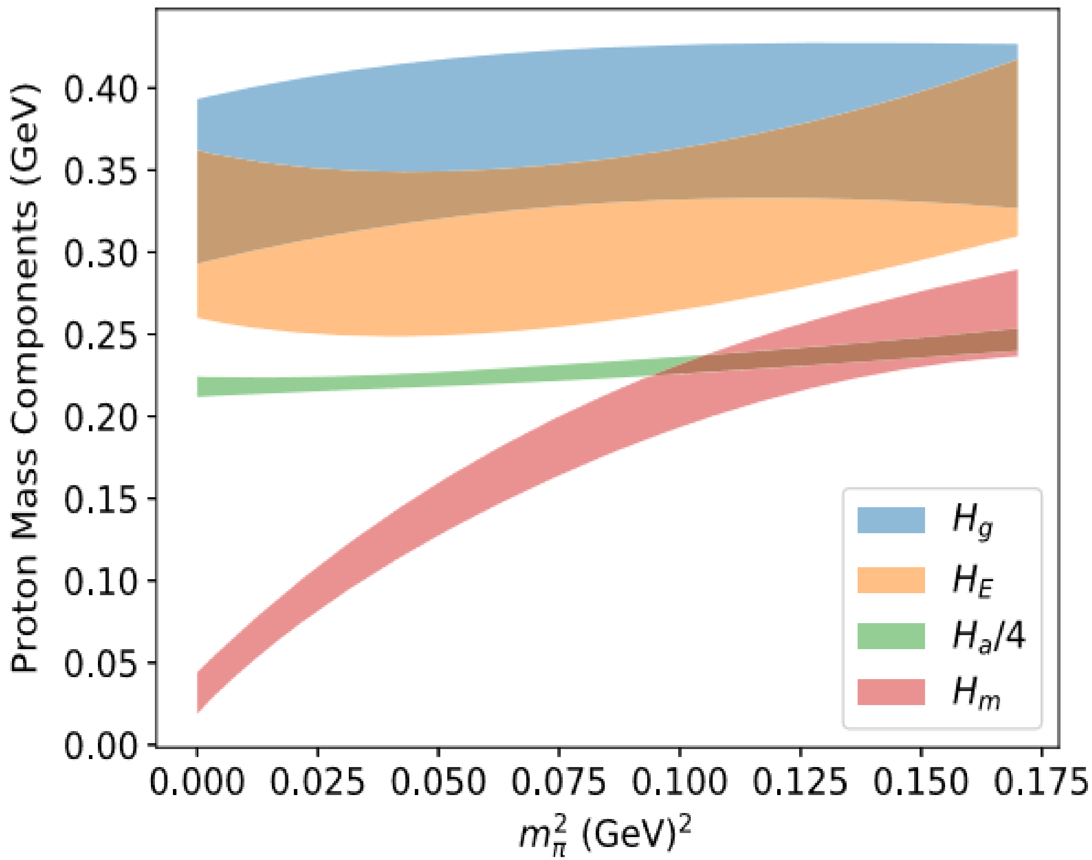
- QCD Hamiltonian decomposition

$H_m$: quark mass term
$H_E$: quark kinetic term
$H_g$: gluon contribution
$H_a$: QCD trace anomaly

$$H_{QCD} = H_m + H_E + H_g + \frac{1}{4}H_a$$

whose matrix elements between proton state give the mass decompostion

The final proton mass decomposition result in $\overline{MS}$ scheme at $\mu = 2GeV$




$$\frac{\langle H_g \rangle}{M} = 36(5)(4)\%$$

$$\frac{\langle H_E \rangle}{M} = 32(4)(4)\%$$

$$\frac{\langle H_a \rangle}{4M} = 23(1)(1)\%$$

$$\frac{\langle H_m \rangle}{M} = 9(2)(1)\%$$

$\boldsymbol{M = \langle H_m \rangle + \langle H_a \rangle}$: QCD trace anomaly contributes almost 90% of the proton mass.

$\boldsymbol{\langle H_m \rangle \approx \sigma_{\pi N} + \sigma_{Ns}}$ **:** the $\sigma$-term of $u, d, s$ quarks

$$\boldsymbol{\sigma_{\pi \mathrm{N}} = 46(7)(3)\mathrm{MeV}, \sigma_{Ns} = 40(12)(4)\mathrm{MeV}}$$

(Y.-B. Yang et al., PRD 94 (2016) 054503 [arXiv:1511.09089])

## 2. $\sigma$-terms of baryons

- The $\sigma_{\pi N}$ of nucleon: $\sigma_{\pi N} = \frac{1}{2}(m_u + m_d)\langle N|\bar{u}u + \bar{d}d|N\rangle, \quad \langle N|N\rangle = L^3$,
- $\sigma$ term of a quark flavor $q$ in a baryon $|\mathcal{B}\rangle$: $\sigma_{Bq} = m_q\langle\mathcal{B}|\bar{q}q|\mathcal{B}\rangle$
- Physical significance of nucleon $\sigma$ term:
  - Measures of explicit and spontaneously chiral symmetry breaking
  - $\pi N$ and $KN$ scattering
  - quark mass contributions to baryon masses
  - coupling with WIMP dark matter through Higgs
- Feynman-Hellmann theorem:

$$\sigma_{Bq} = \frac{\partial M_B}{\partial \ln m_q}$$

Yukawa coupling and Higgs mechanism

$$\mathcal{L}_Y \sim g_q\, \bar{q}Hq, \qquad H = h + \phi_0, \qquad \phi_0 = \langle H\rangle, \qquad m_q = g_q\phi_0$$

$$f_q^N = \frac{\sigma_{Nq}}{M_N} = \frac{\partial \ln M_N}{\partial \ln m_q} = \frac{\partial \ln M_N}{\partial \ln g_q}$$

- $f_q^N$: 1) the mass fraction of the nucleon that couples to the Higgs field via the Yukawa coupling $g_q$ of quark $q$;
  2) the fraction of the nucleon mass originating from the Higgs field via the Yukawa coupling $g_q$.

## A. Octet and decuplet baryon σ-terms and mass decompositions

(P. M. Copeland et al., arXiv: 2112.03198)

- Based on the lattice QCD results of octet and decuplet baryon masses by PACS-CS and QCDSF-UKQCD,
- $\sigma$-terms of baryons are derived through chiral SU(3) effective theory.
- The results provide constraints on the energy-momentum tensor mass decompositions of the SU(3) octet and decuplet baryons.

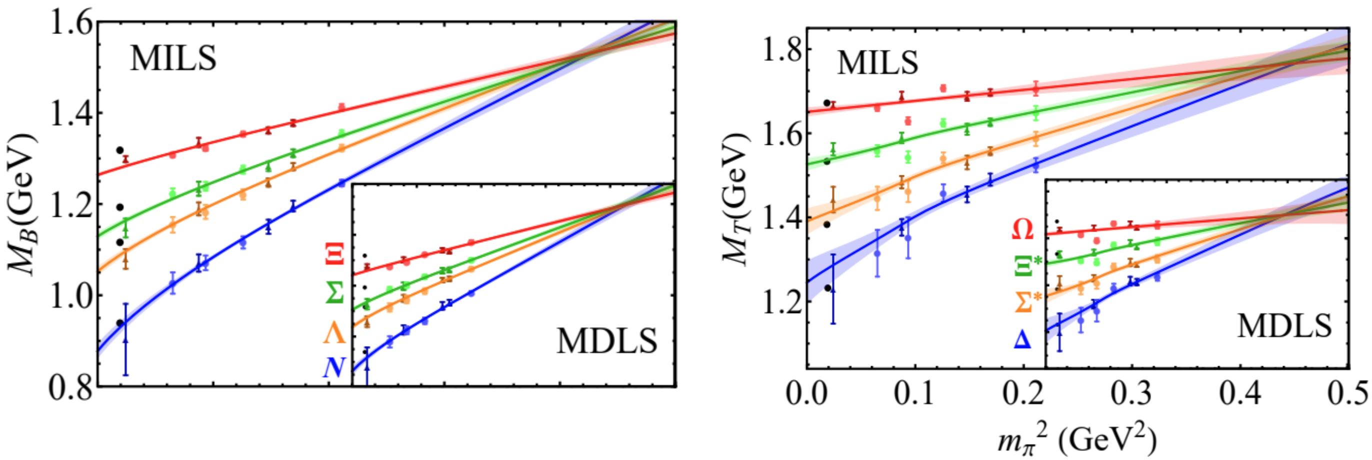
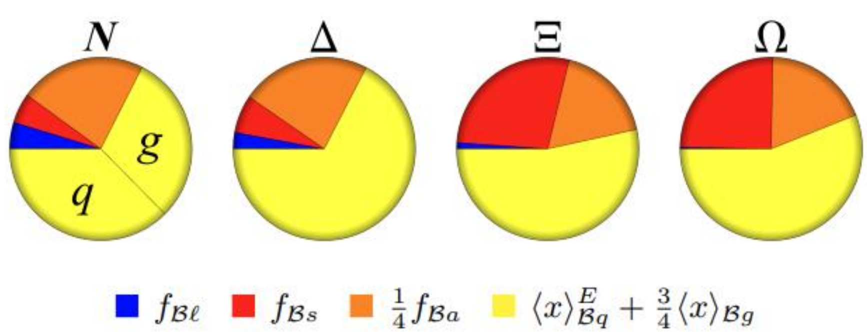


TABLE I. $\sigma$-terms ($\sigma_{\mathcal{B}q}$, $q = \ell, s$), baryon masses ($M_{\mathcal{B}}$), and their ratios ($f_{\mathcal{B}q}$), together with the trace anomaly ($f_{\mathcal{B}a}$) and the sum of the quark and gluon energy contributions $\langle x\rangle^E_{\mathcal{B}q} + \frac{3}{4}\langle x\rangle_{\mathcal{B}g}$, extracted from fits to lattice QCD data. The first uncertainty is statistical and the second is systematic from the differences between the MILS and MDLS results.

| $\mathcal{B}$ (MeV) | $\sigma_{\mathcal{B}\ell}$ (MeV) | $\sigma_{\mathcal{B}s}$ (MeV) | $M_{\mathcal{B}}$ (MeV) | $f_{\mathcal{B}\ell}$ | $f_{\mathcal{B}s}$ | $f_{\mathcal{B}a}$ | $\langle x\rangle^E_{\mathcal{B}q} + \frac{3}{4}\langle x\rangle_{\mathcal{B}g}$ |
|---|---|---|---|---|---|---|---|
| $N(939)$ | 44(3)(3) | 50(6)(1) | 920(10)(10) | 0.047(3)(3) | 0.053(6)(1) | 0.900(7)(3) | 0.675(7)(3) |
| $\Lambda(1116)$ | 31(1)(2) | 196(5)(7) | 1080(6)(10) | 0.028(1)(2) | 0.176(4)(6) | 0.796(4)(6) | 0.597(4)(7) |
| $\Sigma(1193)$ | 25(1)(1) | 256(5)(7) | 1145(5)(13) | 0.021(1)(1) | 0.215(4)(6) | 0.764(4)(6) | 0.573(4)(6) |
| $\Xi(1318)$ | 15(1)(1) | 365(5)(12) | 1269(3)(12) | 0.011(1)(1) | 0.277(4)(10) | 0.712(4)(10) | 0.534(4)(10) |
| $\Delta(1232)$ | 29(9)(3) | 67(11)(3) | 1263(28)(23) | 0.024(9)(2) | 0.054(9)(2) | 0.921(13)(3) | 0.692(13)(3) |
| $\Sigma^*(1383)$ | 18(6)(2) | 189(11)(9) | 1385(13)(22) | 0.013(4)(1) | 0.137(8)(7) | 0.850(9)(7) | 0.638(9)(7) |
| $\Xi^*(1533)$ | 10(3)(2) | 307(12)(15) | 1520(6)(21) | 0.007(2)(1) | 0.200(8)(10) | 0.793(8)(10) | 0.594(8)(10) |
| $\Omega(1672)$ | 5(1)(1) | 418(14)(20) | 1663(8)(18) | 0.003(1)(1) | 0.250(8)(12) | 0.747(8)(12) | 0.560(8)(12) |

$$M_{\mathcal{B}} = \left[\sum_q \left(\langle x\rangle^E_{\mathcal{B}q} + f_{\mathcal{B}q}\right) + \frac{3}{4}\langle x\rangle_{\mathcal{B}g} + \frac{1}{4} f_{\mathcal{B}a}\right] M_{\mathcal{B}},$$



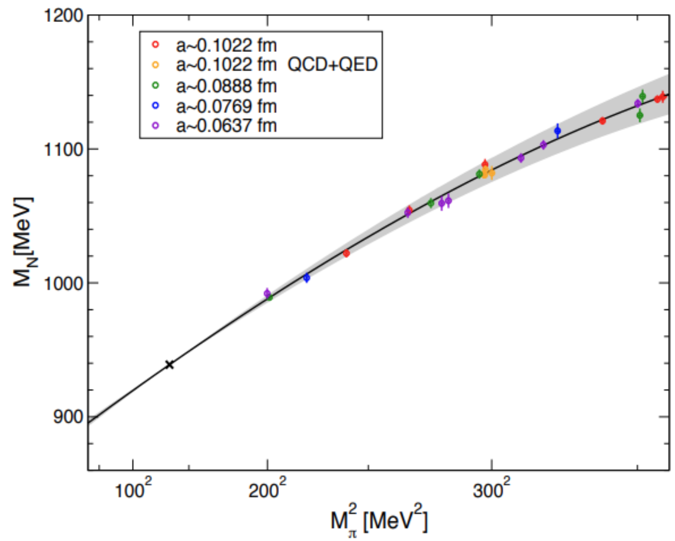

The trace anomaly and quark/gluon energies decrease for strange baryons due to their larger strange σ-terms.

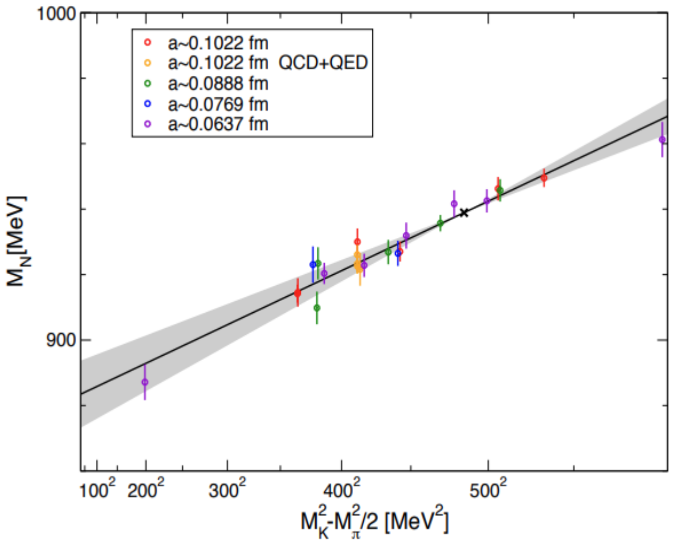
**B. $(\boldsymbol{u}, \boldsymbol{d}, \boldsymbol{s}, \boldsymbol{c}, \boldsymbol{b}, \boldsymbol{t})$ $\boldsymbol{\sigma}$ terms of neutron**

(Sz. Borsanyi et al. (BMW Col.), arXiv:2007.03319)

- $N_f = 1+1+1+1$ gauge ensembles using clover and HISQ fermion actions.
- $\sigma$ terms of $(u, d, s, c)$ quarks are derived using Feynman-Hellmann theorem

$$f_q^N = \frac{\sigma_{Nq}}{M_N} = \frac{\partial \ln M_N}{\partial \ln m_q}$$




GMO relation: $m_s \propto \frac{1}{2}(2m_K^2 - m_\pi^2)$

- HQET derivation of $f_{b,t}^N$: for a $N_f + 1$ theory with $N_f$ light flavors and one heavy flavor that to be integrated out, PQCD calculations to higher orders gives

$$f_Q^N = \sum_{n=0}^{3} \left( b_n^{N_f} - c_n^{N_f} \bar{f}_{N_f}^N \right) \alpha_s^n \left[ 1 + \mathcal{O}\left( \left( \frac{\Lambda}{m_Q} \right)^2, \dots \right) \right] \qquad \bar{f}_{N_f}^N = \sum_{i=1}^{N_f} f_{q_i}^N$$

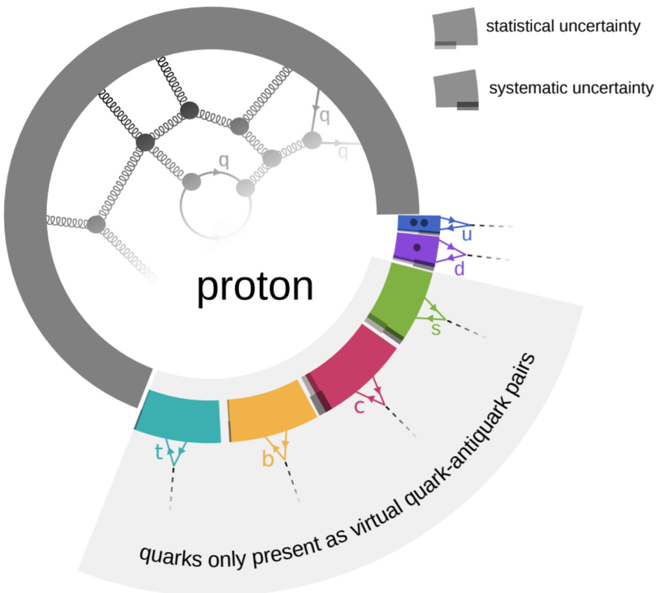
where $b_n^{N_f}$ and $c_n^{N_f}$ are coefficients appearing in PQCD calculations.
Using $m_b(m_b) = 4.18_{-3}^{+4}$ GeV, $m_t(m_t) = 160.0_{-4.3}^{-4.8}$ GeV and $\alpha_s(m_{b/t}) = 0.225(3)/0.109(1)$, one has

$$f_b^N = 0.08748(13) - 0.10129(39) \bar{f}_4^N ,$$

$$f_t^N = 0.09169(4) - 0.09840(10) \bar{f}_5^N$$

| | Nucleon | | Individual $p$ and $n$ |
|---|---|---|---|
| $f_{ud}^N$ | 0.0398(32)(44) | $f_u^p$ | 0.0142(12)(15) |
| $f_s^N$ | 0.0577(46)(33) | $f_d^p$ | 0.0242(22)(30) |
| $f_c^N$ | 0.0734(45)(55) | $f_u^n$ | 0.0117(11)(15) |
| $f_b^N$ | 0.0702(7)(9) | $f_d^n$ | 0.0294(22)(30) |
| $f_t^N$ | 0.0680(6)(7) | | |

- The sum over the scalar quark contents of the nucleon, $f_h^N = \sum_q f_q^N$ denotes the strength of the coupling of the Higgs to nucleons, in the limit of vanishing momentum transfer.




- Quark model

  $$p(uud), \qquad n(udd)$$

- $\sigma_{\pi N} \approx 37.4(5.1)$ MeV, $f_{ud}^{N} \approx 4.0(5)\%$
- $\sigma_{\pi N}$ includes $u, d$ sea quark contribution
- $\sum_{q=s,c,b,t} f_q^N \approx 26.9(9)\%$ (sea quarks)
- The rest part are from gluons
- Sea quark and gluon contribution are due to quantum fluctuation——QCD trace anomaly
- Trace anomaly contributes at least 95% of nucleon mass

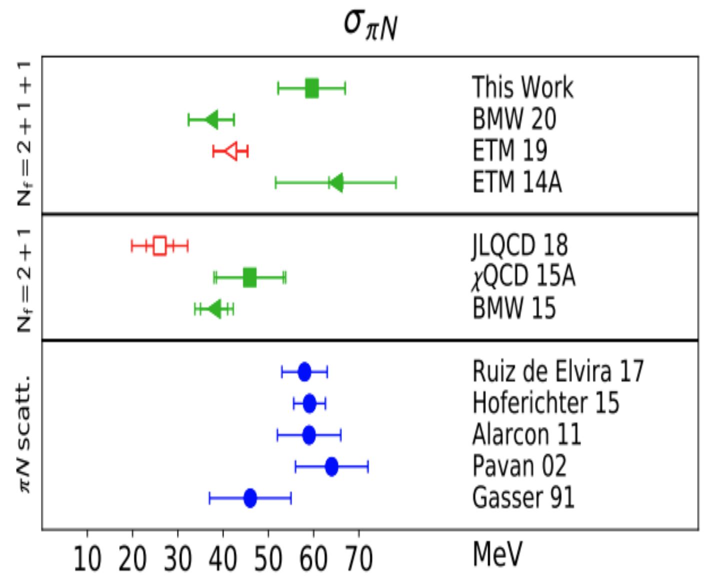
  $$M_N = \langle H_m^{u,d} \rangle + \langle H_a \rangle$$

- Consistent with that from direct mass decomposition.

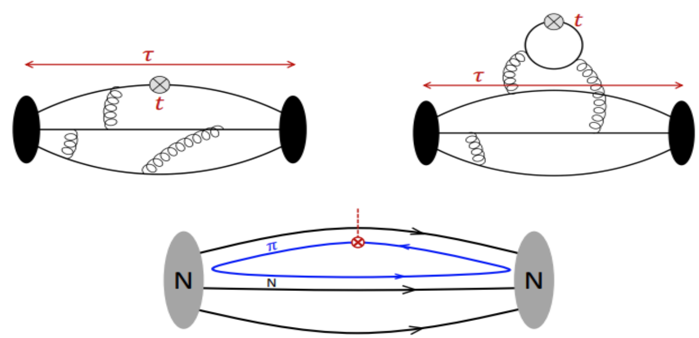
## C. $\sigma_{\pi N}$: tensions between lattice results and phenomenological values

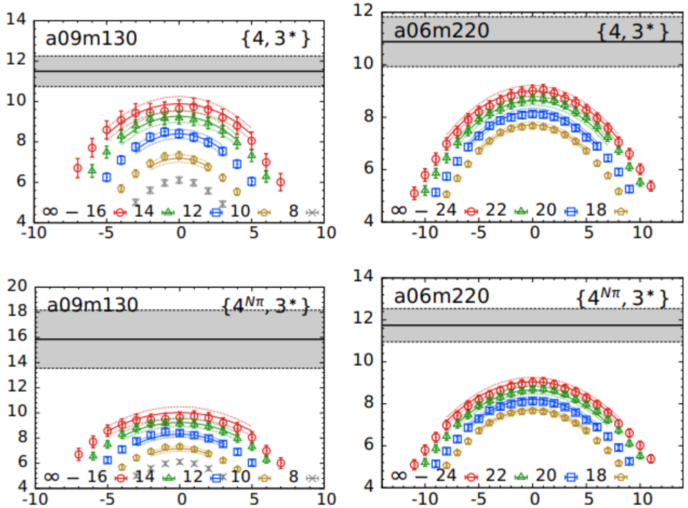
(R. Gupta et al. (BMW Col.), PRL127 (2021) 242002, [arXiv: 2105.12095])

- Low-energy theorem establishes a connection between $\sigma_{\pi N}$ and a $\pi N$ scattering amplitude.
- Previous partial wave analysis obtained a value
  $\sigma_{\pi N} \approx 45$ MeV.
- But recent phenomenological analysis favors a higher value, e.g. $\sigma_{\pi N} \approx 64(8)$MeV.
- Improved analysis combined with pionic atom data results in a value $\sigma_{\pi N} \approx 59.0(3.5)$ MeV.
- A global analysis of low-energy data leads to
  $\sigma_{\pi N} = 58(5)$ MeV
- Lattice QCD favors a lower value $\sigma_{\pi N} \approx 40$ MeV.
- The tension is attributed to not considering explicitly the $N\pi$ and $N\pi\pi$ states contamination.

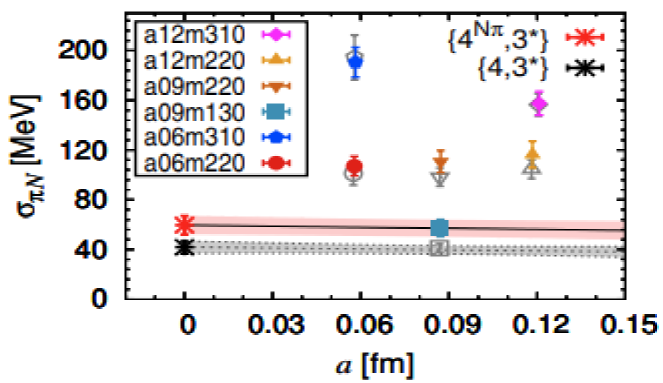


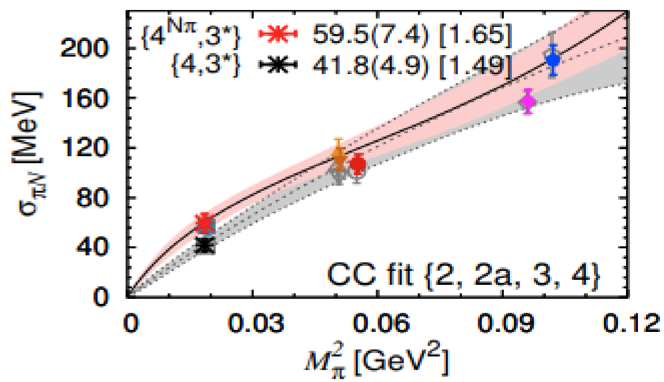

- Direct calculation of the matrix element
  $\langle N | \bar{u}u + \bar{d}d | N \rangle$
- Considering $N\pi$ states or not in the data analysis gives different results.






- Left: $m_\pi \approx 138$ MeV. Right: $m_\pi \approx 235$ MeV
- $\{4,3^*\}$: Standard analysis without considering $N\pi$ states
  $\{4^{N\pi}, 3^*\}$: Similar analysis but considering $N\pi$ states explicitly
- ME of $\{4^{N\pi}, 3^*\}$ larger than that of $\{4,3^*\}$
- This effect more pronounced for the lighter pion
- Can be understood in the framework of chiral effective theory.




- Joint continuum and chiral extrapolation of $\sigma_{\pi N}$

$$\sigma_{\pi N} \;=\; (d_2 + d_2^a a)m_\pi^2 + d_3 m_\pi^3 + d_4 m_\pi^4 + d_{4L} m_\pi^4 \ln\frac{m_\pi^2}{M_N^2}.$$

- Different results:

$$\sigma_{\pi N} = 41.9(4.9)\text{ MeV } (\{4{,}3^*\}), \quad \sigma_{\pi N} = 59.6(7.4)\text{ MeV } (\{4^{N\pi}, 3^*\})$$

# IV. A recent lattice study on diquarks in baryons

(A. Francis et al., JHEP 05 (2022) 062 [arXiv: 2106.09080])

- Diquarks may be useful building blocks for phenomenological descriptions of hadronic states, e.g., a baryon may be a quark-diquark bound state.

- The details of diquarks are omitted here.
  (a recent review: M.Yu. Barabanov et al., *PPNP*116 (2021) 103835 [arXiv: 2008.07630])

- "Good" diquark and "bad" diquark made up of $u, d$ quarks
  Good diquark: $J^P = 0^+, \quad 3^*_c, \quad 3^*_F, \quad \epsilon_{abc} u^{bT} C\gamma_5 d^c$
  bad diquark: $J^P = 1^+, \quad 3^*_c, \quad 6^*_F, \quad \epsilon_{abc} u^{bT} C\gamma_i d^c$

- Compact objects or not?
  attractive one-glue-exchange interaction

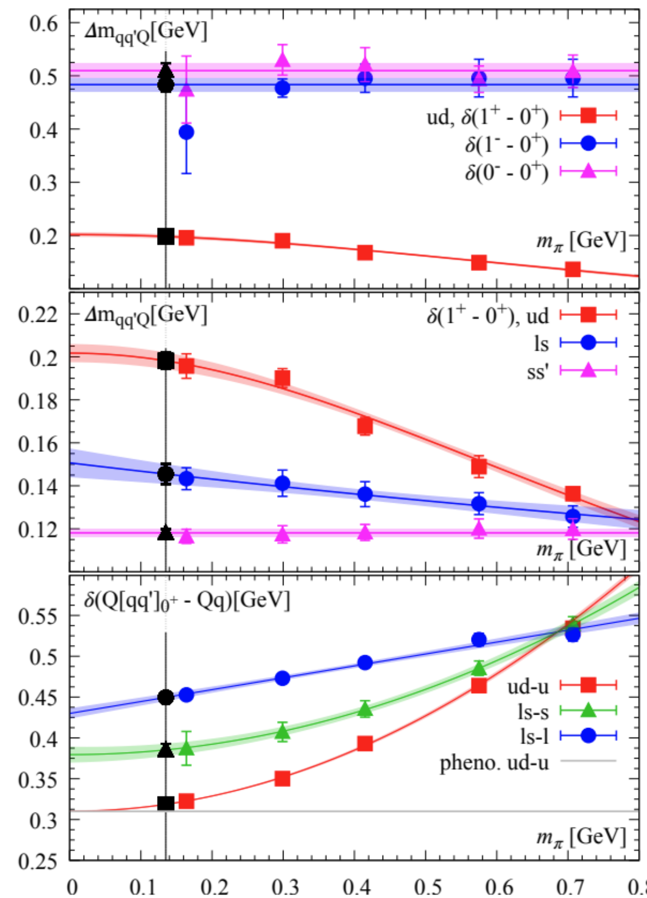
- The effective energy of diquarks in a baryon

- This study addresses these questions in a gauge invariant framework——diquark in baryons of $([qq']Q)$ configurations ($Q$ refers to a static heavy quark)

# Diquark properties from full QCD lattice simulations (arXiv: 2106.09080)

- Panel 1: mass differences of $ud$ diquarks
  $\delta(1^+ - 0^+)$ tends to saturate when $m_\pi$ approaches physical $m_\pi$.
  $ud(1^-)$ and $ud(0^-)$ are much higher
- Panel 2: mass differences of bad$(1^+)$-good $(0^+)$ $ud, ls, ss'$ diquarks
- Panel 3: mass differences of $Q[qq']$ baryon and $\bar{Q}q$ meson.
- Extrapolation to physical $m_\pi$:
  $\delta(1^+ - 0^+)_{q_1 q_2} = A/[1 + (m_\pi/B)^n]$
  $\delta(Q[qq']_{0^+} - \bar{Q}q) = C\ [1 + (m_\pi/D)^n]$

| All in [GeV] | $\delta E(m_\pi^{\text{phys}})$ | A | B |
|---|---|---|---|
| $\delta(1^+ - 0^+)_{ud}$ | 0.198(4) | 0.202(4) | 1.00(5) |
| $\delta(1^+ - 0^+)_{\ell s}$ | 0.145(5) | 0.151(7) | 3.7(15) |
| $\delta(1^+ - 0^+)_{ss'}$ | 0.118(2) | 0.118(2) | |
| | | C | D |
| $\delta(Q[ud]_{0^+} - \bar{Q}u)$ | 0.319(1) | 0.310(1) | 0.814(8) |
| $\delta(Q[\ell s]_{0^+} - \bar{Q}s)$ | 0.385(9) | 0.379(10) | 1.09(6) |
| $\delta(Q[\ell s]_{0^+} - \bar{Q}\ell)$ | 0.450(6) | 0.430(6) | 2.95(35) |

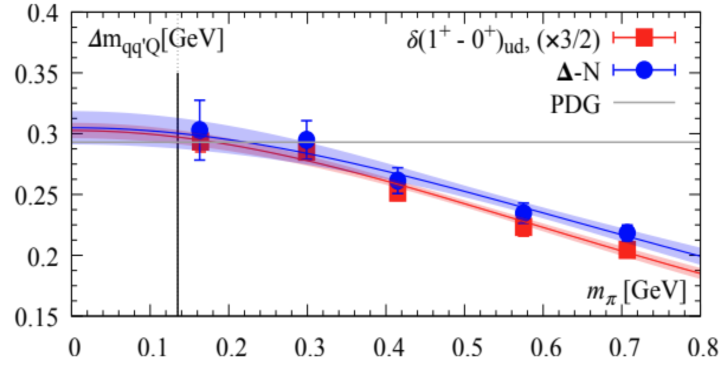



- Horizontal grey line in Panel 3: phenomenological results of
  $\delta(b[ud]_{0^+} - \bar{b}u) = 306$ MeV

- In one-gluon-exchange approximation, and chiral limit

$$\delta(\Delta - N) = \frac{3}{2}\delta(1^+ - 0^+)$$

(R. Jaffe, Exotica, Phys. Rept. 409 (2005) 1 [hep-ph/0409065])



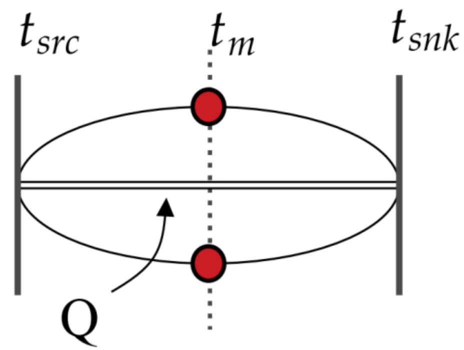

- Good diquark size

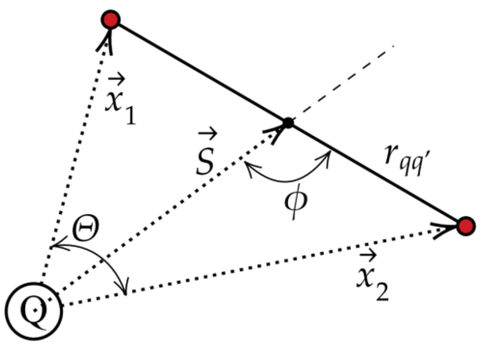
$$C_\Gamma^{ud}(\vec{x}_1, \vec{x}_2, t) = \left\langle \mathcal{O}_\Gamma\left(\vec{0}, 2t\right) \rho_u(\vec{x}_1, t)\rho_d(\vec{x}_2, t)\mathcal{O}_\Gamma^\dagger\left(\vec{0}, 0\right)\right\rangle$$

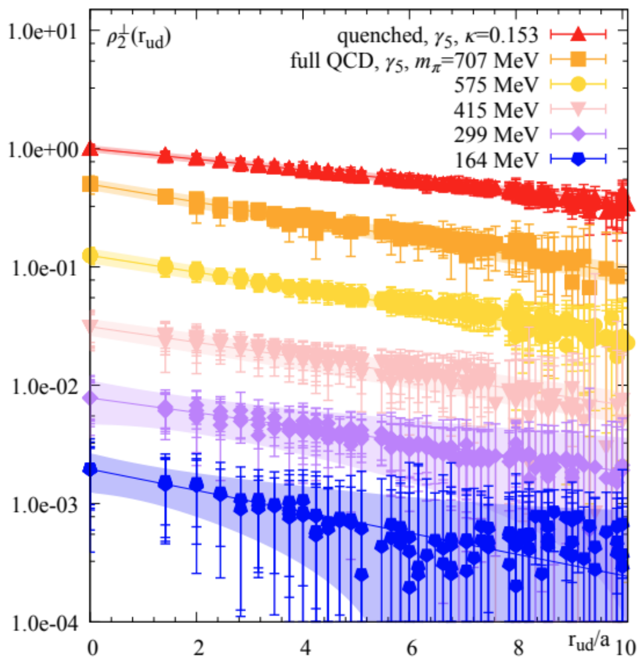
$$\rho_2(r_{ud}, S, \phi; \Gamma) \equiv C_\Gamma^{ud}(\vec{x}_1, \vec{x}_2, t_m)$$

$$\rho_2^\perp(R, \theta) \equiv \rho_2(r_{ud}, S, \pi/2) \quad R = |\vec{x}_1| = |\vec{x}_2| \text{ for } \phi = \frac{\pi}{2}$$

$$\rho_2^\parallel(r_{ud}, \theta) \equiv \rho_2(r_{ud}, S, \pi)$$

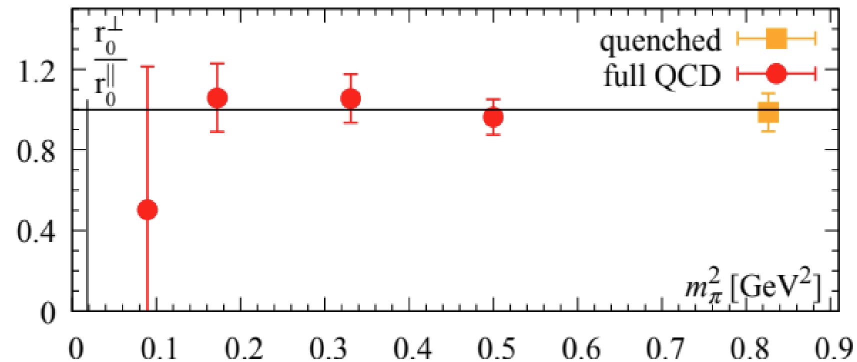





$$\rho_2^\perp(R, r_{ud}) \sim \exp\left(-\frac{r_{ud}}{r_0^\perp}\right)$$

$$\rho_2^\parallel(R, r_{ud}) \sim \exp\left(-\frac{r_{ud}}{r_0^\parallel}\right)$$

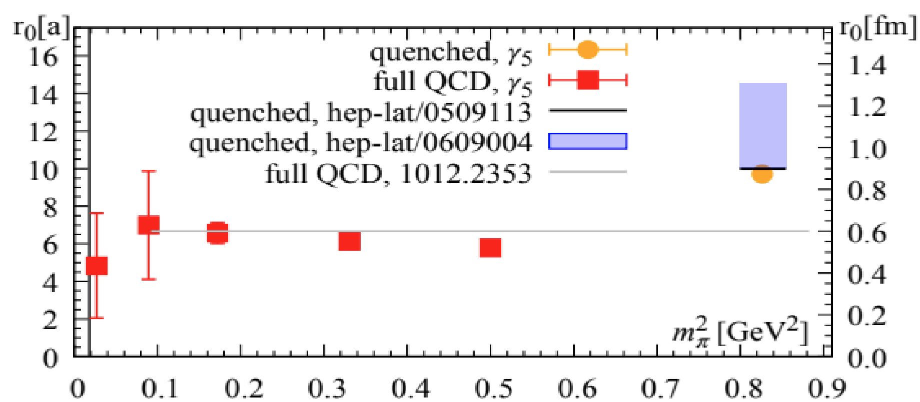



- Good diquark is spherical

  $(r_0^\perp)/(r_0^\parallel) \approx 1$
- The size of good diquark

  $r_0 \approx 0.6$ fm
- Note that charge radius of nucleon

  $r_p \approx 0.831$ fm



## V. Summary

- Lattice QCD calculation can reproduce the mass spectrum of octet and decuplet baryons (and the mass splitting within an iso-multiplet).
- Lattice QCD calculations of ground state charmed baryons comply with experimental results.
- However, for excited baryon states, meson-baryon scatterings should be taken into account.
- There have been many lattice studies on $\pi N$ scattering relevant to the $\Delta$ resonance. More works in this direction is underway.
- Nucleon $\sigma$ terms are studied extensively in lattice QCD. For $\sigma_{\pi \mathrm{N}}$, the tension between lattice and phenomenological results should be resolved.
- A new lattice study on diquarks in baryons.

# Thank you for your attention!