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Approaching Near-threshold Resonances by Analytic Maps of the S-matrix: Uniformization, and Mittag-Leffler Expansion of 2 and 3-channeled Systems

Excited states, which show up as resonances embedded in the continuum spectrum and their structure, are primary issues in the study of hadron physics. In particular, many candidates of exotic hadrons have recently been found near the threshold of hadronic channels [1, 2]. Extraction of such resonance information is an essential and challenging task. However, many of the existing analyses of the spectra assume a Breit-Wigner form [3] to a peak structure situated near the thresholds, which cannot appropriately incorporate the threshold behaviors.

We focus on the analytic structure of the multi-channel S-matrix by the introduction of uniformization[4, 5]. As a function of center-of-mass energy, the S-matrix has branch points at the corresponding channel thresholds, and thus its Riemanan Surface has a multi-sheeted structure [5]. This non-trivial structure in the vicinities of the thresholds makes it particularly challenging to extract resonance information of near-threshold resonances. The key idea of uniformization is to find a kinematic variable such that the S-matrix can be expressed single-valued. Under the uniformized-variable parameterization, the Mittag-Leffler theorem is applicable so that the S-matrix can be expressed by a simple pole expansion which we call the Uniformized Mittag-Leffler Expansion. The expansion exhibits the appropriate threshold behaviors of the multi-channel spectrum and is model-independent, simple, and suited for the extraction of the pole properties. In addition, the uniformized-variable parameterization explicitly clarifies the contribution of a pole situated on an arbitrary sheet to the observed spectrum. For the single-channel S- matrix, the center-of-mass momentum can serve as the uniformization variable (assuming the absence of left-handed cuts). Its Mittag-Leffler Expansion has already been studied in Ref. [6, 7]. Naturally, an extension of the Uniformized Mittag-Leffler program to the two-channel and three-channel S-matrix is a pivotal step forward for the analysis of multi-channel scattering.

In this presentation, we extend the Uniformized Mittag-Leffler Expansion to the two-channel case by the use of the uniformization variable introduced in Ref.[5, 8], and study the pole properties of Λ(1405) in a model-independent manner considering the $\pi\Sigma$, $\bar{K}N$ channels[9, 10]. Moreover, we report our two-channel analyses concerning Z_c(3900)[11] and point out that the lattice QCD results in Ref.[12] indicate the existence of an unusual S-matrix pole with a positive imaginary part in complex energy near the $\bar{D}D^*$ threshold, which is most likely the origin of the 'enhanced cusp'found in the near-threshold spectrum. Then, we investigate the analytic structure of the three-channel S-matrix in detail and illustrate that the three-channel S-matrix is topologically equivalent to a torus [5, 13], unlike the two-channel S-matrix which is equivalent to a Riemann Sphere. We show that an explicit form of the three-channel uniformization variable can be given by the inverse of the Jacobi elliptic function. Also, we show that the double-periodicity of the torus plays a vital role in the contributions of the S-matrix poles. This results in an explicit form of the Uniformized Mittag-Leffler expansion of the three-channel S-matrix, which is given by a series of Weierstrass zeta functions (Details regarding the formalism of the three-channel uniformization and the Mittag-Leffler expansion are shown in Ref. [14]). In addition to a demonstration with a simple non-relativistic effective field theory model with contact interaction for the $S = -2$, $I = 0$, $J^P = 0^+$, $\Lambda\Lambda$-$N\Xi$-$\Sigma\Sigma$, analyses with the three-channel Uniformized Mittag-Leffler expansion applied to the invariant-mass distributions of $\pi_L^+\Xi^-$ in the $\Xi_c^+ \to \pi^+\pi^+\Xi^-$ decay measured with the Belle detector[15], and pJ/ψ from the $\Lambda_b^0 \to K^-pJ/\psi$ decay measured by the LHCb collaboration [16] are presented.

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