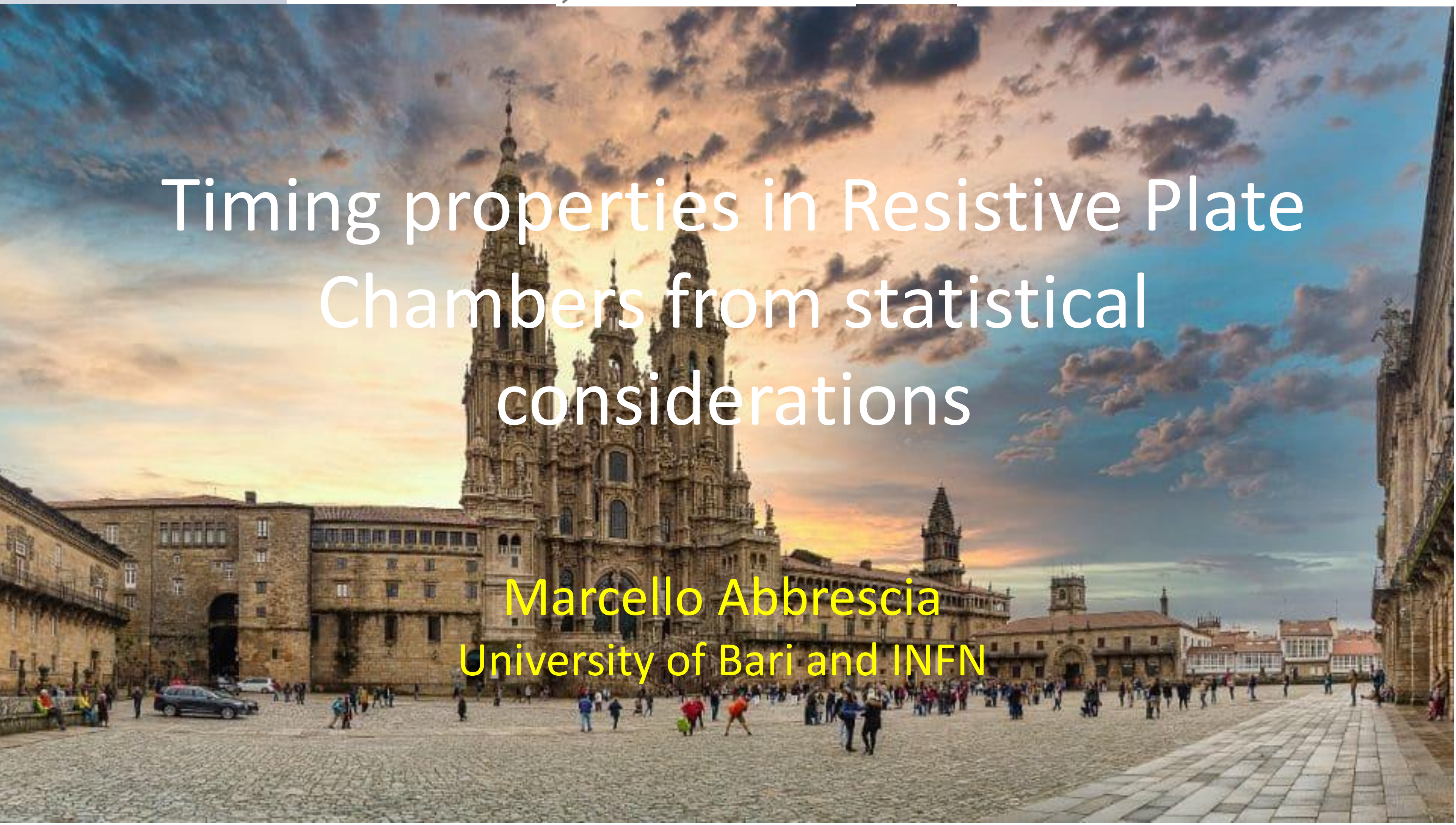


Timing properties in Resistive Plate Chambers from statistical considerations

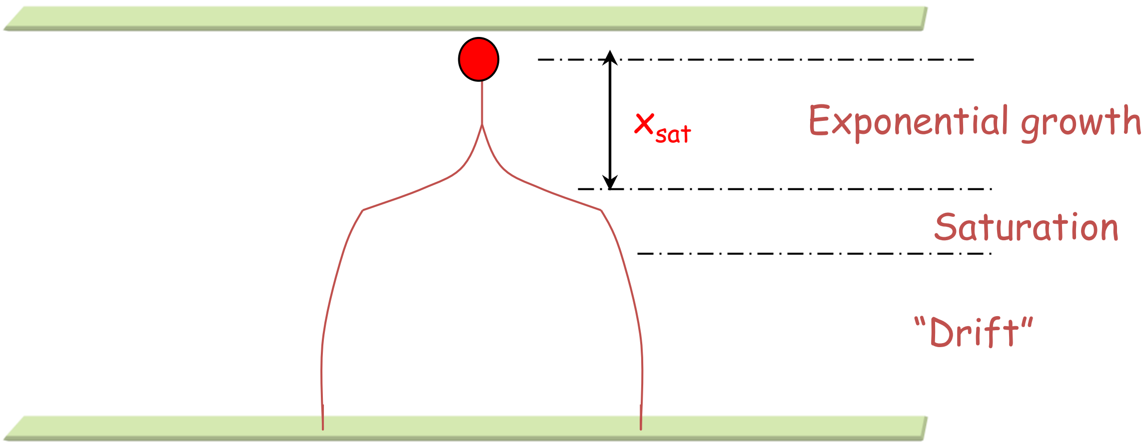
Marcello Abbrescia
University of Bari and INFN

RPC2024 - XVII Conference on Resistive Plate Chambers and related detectors

Santiago de Compostela (Spain)
Sept. 9-13 2024

Signals in Resistive Plate Chambers

Generally the growth of ONE avalanche in a Resistive Plate Chamber is schetched like in the following, containing a part of exponential growth and one of saturation because of space charge effects:



The induced charge on a planar readout electrode is usually written as:

$$q_{ind} = \underbrace{\frac{q_e}{\eta g} \Delta V_w n_0 M \left[e^{\eta(x_{sat} - x_0)} - 1 \right]}_{\text{Exponential growth}} + \underbrace{M \Delta V_w \frac{g - x_{sat}}{g} x_{sat}}_{\text{Drift}}$$

Of course the second term depens on how the saturation effects are modelled.

Signals in Resistive Plate Chambers

If one considers all clusters, in number n_{cl} , each containing n_0^j electrons, generated by the passage of one ionizing particle through the gas gap, of width g, the induced current $i_{ind}(t)$ can be usefully expressed as:

$$i_{ind}(t) = -\mathbf{v}_d \cdot \mathbf{E}_w q_e e^{\eta v_d t} \sum_{j=1}^{n_{cl}(t)} n_0^j M_j$$

Here, we neglect saturation effects (correct if we consider the initial stages of the avalanches), and:

- ✓ $\mathbf{v}_d$ is the electron drift velocity (assumed constant with time);
- ✓ $\mathbf{E}_w$ is the weighting field (assumed uniform in space);
- ✓ η is the first effective Townsend coefficient
- ✓ M_j is the avalanche fluctuation coefficient
- ✓ t is the time elapsed since the passage of the ionizing particle

Signal fluctuations in RPC

$$i_{ind}(t) = -\mathbf{v}_d \cdot \mathbf{E}_w q_e e^{\eta v_d t} \sum_{j=1}^{n_{cl}(t)} n_0^j M_j$$

ALL quantities in the red boxes are stochastic variables, namely they change from event to event; in particular:

- ✓ n_{cl} follows a Poisson distribution
- ✓ n^0_j depends strongly on the gas used, generally a $1/n^2$ distribution for small n is used, followed by a long tail toward large n-values.
- ✓ M_j is described by different distributions depending on the approach used: usually, for small avalanches, a Furry's law is used, for larger avalanches generally a Polya distribution with a suitable value of the parameter is chosen.

The fact that the induced current contains stochastic variables has the consequence that it changes from event to event.
→ These fluctuations cannot be eliminated.

Signal fluctuations in RPC

$$i_{ind}(t) = -\mathbf{v}_d \cdot \mathbf{E}_w q_e e^{\eta v_d t} \sum_{j=1}^{n_{cl}(t)} n_0^j M_j$$

Note that the number of clusters $n_{cl}(t)$ in the gas at a certain time t, is monotonically decreasing with time, due to the fact that, with time, more and more avalanches arrive onto the anode and stop.

- ✓ Due to the fact that their initial positions are, themselves, stochastic variables, this adds up a «hidden» cause for fluctuations.
- ✓ In the initial phases of the avalanches, ALL clusters are drifting (and avalanching).

Note that, at a certain time t, ALL clusters contribute at the same way to the induced current.

Vice versa, the induced charge, which is another quantity typically measured, depends mainly on the cluster closest to the cathode:

$e^{-\eta/\lambda}$ ≈ 0.02 is the average ratio between the charge induce by two consecutive clusters

Signals after readout electronics

Considering the electronics, the output of a perfect voltage amplifier can be usefully written like:

$$v_{out}(t) = \frac{1}{2} P_1 R_{in} i_{ind}(t) = -\left(\frac{1}{2} P_1 R_{in}\right) \mathbf{v}_d \cdot \mathbf{E}_w q_e e^{\eta v_d t} \sum_{j=1}^{n_{cl}(t)} n_0^j M_j$$

where:

- ✓ P_1 is the amplification factor
- ✓ R_{in} is the input impendance
- ✓ ½ takes into account that usually strips are used.

In the case of charge sensitive amplifiers:

$$v_{out}(t) = \frac{P_2}{2} q_{ind}(t) = \frac{P_2}{2} \int_0^t i_{ind}(\bar{t}) d\bar{t} =$$

$$= -\frac{P_2}{2} \frac{1}{\eta v_d} \mathbf{v}_d \cdot \mathbf{E}_w q_e \left[\left(e^{\eta v_d t} - 1\right) \sum_{j=1}^{n_{cl}(t)} n_0^j M_j + \left(e^{\eta (g - x_0^j)} - 1\right) \sum_{j=n_{cl}(t)+1}^{n_{cl}} n_0^j M_j \right]$$

where:

- ✓ P_2 is the amplifiers charge sensitivity.

Origin of timing

Generally we are interested to the time t_{thr} when $v_{out}(t)$ becomes larger than a certain electronics threshold V_{thr}. The most logical thing to do would be invert the expression:

$$v_{out}(t) = \frac{1}{2}P_1 R_{in} i_{ind}(t) = -\left(\frac{1}{2}P_1 R_{in}\right)\mathbf{v}_d \cdot \mathbf{E}_w q_e e^{\eta v_d t} \sum_{j=1}^{n_{cl}(t)} n_0^j M_j$$

- This is not possible because of the dependance of $n_{cl}(t)$ over t.

On the average, however:

$$n_{cl}(t) = \lambda(g - v_d t)$$

where λ is the primary cluster density (# cluster/mm).

Also:

- M_j has average 1 (by definition)
- n^0_j average depends on the gas

The «average» signal

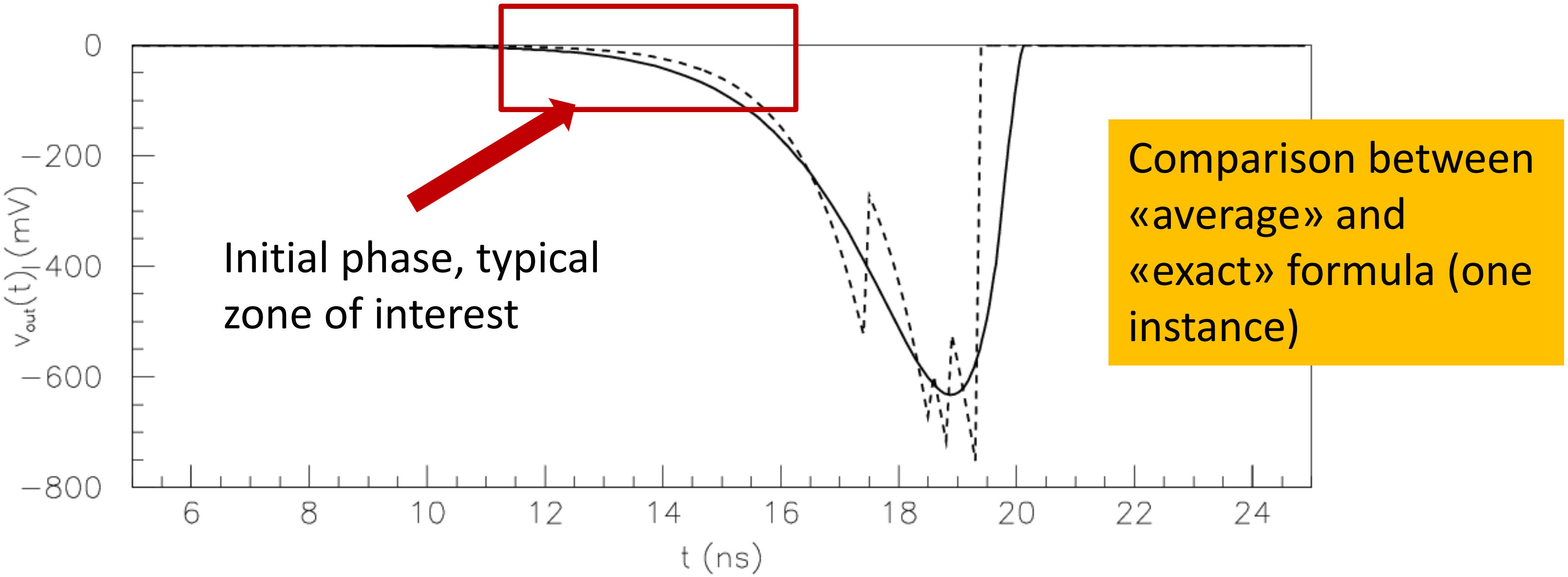
Therefore, we can re-write the previous formula for the «average» signal as:

$$v_{out}(t) = \left(\frac{1}{2}P_1 R_{in}\right) A e^{\eta v_d t} \sum_{j=1}^{n_{cl}(t)} n_0^j M_j \approx \left(\frac{1}{2}P_1 R_{in}\right) A e^{\eta v_d t} \lambda(g - v_d t) B$$

where:

$$A = -\mathbf{v}_d \cdot \mathbf{E}_w q_e$$

B is the average value of $n^0_j M_j$ (M_j has average value 1).



Inverting the «average» signal

Not even this simplified expression can be directly inverted, because of the contemporary presence of the exponential and linear terms in t:

$$v_{out}(t) = \left(\frac{1}{2}P_1 R_{in}\right) A e^{\eta v_d t} \sum_{j=1}^{n_{cl}(t)} n_0^j M_j \approx \left(\frac{1}{2}P_1 R_{in}\right) A e^{\eta v_d t} \lambda (g - v_d t) B$$

However, we can take the logarithm of both terms, and expand the logarithm at the right of the equal sign (stopping at the second power):

$$\log\left(\frac{v_{out}}{RAB\lambda g}\right) = \eta v_d t + \log\left(1 - \frac{v_d}{gt}\right) \approx \eta v_d t - \frac{v_d}{g}t - \frac{1}{2}\left(\frac{v_d t}{g}\right)^2$$

where $R = ½ P_1 R_{in}$

This is equivalent to approximate, in the initial stages of the avalanche as:

$$v_{out}(t) \approx RAB\lambda g e^{\left[\left(\eta - \frac{1}{g}\right) v_d t - \frac{1}{2}\left(\frac{v_d t}{g}\right)^2\right]}$$

Inverting the «average» signal 2

THIS expression can be easily inverted, to obtain, for $v(t)=v_{thr}$:

$$t_{thr} \approx \frac{1}{\left(\eta - \frac{1}{g}\right) v_d} \log \frac{v_{thr}}{RAB\lambda g}$$

in the linear approximation, and, for the quadratic approximation:

$$t_{thr} \approx \left(\frac{g}{v_d}\right)^2 \left[-\left(\eta - \frac{1}{g}\right) v_d - \sqrt{v_d^2 \left(\eta - \frac{1}{g}\right)^2 - 2\left(\frac{v_d}{g}\right)^2 \log \frac{v_{thr}}{RAB\lambda g}} \right]$$

The time t_{thr} at which the signal overcomes the threshold is inversely proportional to v_d, inversely proportional to $\eta - 1/g$, and only logarithmically dependant on λ, g and v_{thr} (and R, A and B).

Timing fluctuations in RPCs

Moreover, from the expression (modified exploiting the fact that «n_{cl}»=λg):

$$t_{thr} = \frac{1}{v_d} \frac{1}{\eta - \frac{1}{g}} \log \left[\frac{v_{thr}}{RABn_{cl}} \right]$$

one can compute the partial derivatives with respect to M_j , n^0_j (included in B) and n_{cl}, which are the stochastic variables contained therein.
These are, within the approximations done, the contributions to the fluctuations of t_{thr}:

$$\sigma_{n_{cl}} = \frac{1}{v_d} \frac{1}{\eta - \frac{1}{g}} \frac{\sigma_{n_{cl}}}{n_{cl}}$$

$$\sigma_{n_0} = \frac{1}{v_d} \frac{1}{\eta - \frac{1}{g}} \frac{\sigma_{n_0}}{n_0}$$

$$\sigma_M = \frac{1}{v_d} \frac{1}{\eta - \frac{1}{g}} \frac{\sigma_M}{M}$$

Since they appear in the same term, the numerical factors in front of the relative fluctuations on n_{cl}, M_j and n^0_j are the same, which simplifies the conclusions to be drawn.

Time resolution of RPC

An estimation of the RPC resolution is therefore:

$$\sigma_t = \sqrt{\sigma_{n_{cl}}^2 + \sigma_{n_0}^2 + \sigma_M^2} = \frac{1}{v_d} \frac{1}{\eta - \frac{1}{g}} \sqrt{\left(\frac{\sigma_{n_{cl}}}{n_{cl}}\right)^2 + \left(\frac{\sigma_{n_0}}{n_0}\right)^2 + \left(\frac{\sigma_M}{M}\right)^2}$$

Time resolution is inversely proportional to v_d (hence the need for gas mixtures characterized by a large drift velocity), and inversely proportional to $\eta - 1/g$.

- Note that, keeping the same electronics, the η g product, which is related to the gain, is roughly constant in «narrow» or «wide» gap RPCs

→ Narrow gap RPCs are characterized by a good time resolution simply because η is larger in this case.

Of course, there are other sources of signal fluctuations (electronics, diffusion), but these above derive directly from the statistics of electron-ion pairs formation and avalanche growth: they cannot be eliminated.

In this sense, they represent a lower limit of the time resolution for an RPC (within the hypothesys done).

Time resolution of RPC

To quantify the time resolution using the previous formula, the various terms need to be quantified, at least approximately.

$$\frac{\sigma_{n_{cl}}}{n_{cl}} = \frac{1}{n_{cl}} = \frac{1}{\sqrt{\lambda g}}$$

easy, denser gases provide better time resolution

$$\frac{\sigma_M}{M} \approx 0.8$$

Actually depends on the preferred model for avalanche fluctuations; this is an approximate value for a Polya distribution with θ = 1.5

$$\frac{\sigma_{n_0}}{n_0}$$

Strongly depends on the gas (mixture) used

For a 2 mm RPC, $\eta = 9\ \text{mm}^{-1}$, $v_d = 120\ \mu\text{m/ns}$ → $\sigma_t \approx 1$ ns

For a 300 µm RPC, $\eta = 110\ \text{mm}^{-1}$, $v_d = 200\ \mu\text{m/ns}$ → $\sigma_t \approx 50$ ps

Both in the right ballpark.

Time resolution of RPC

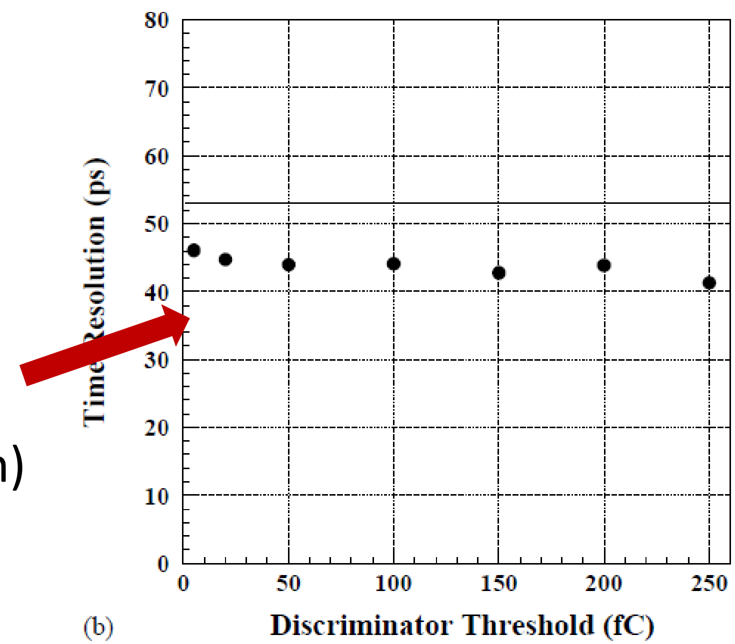
Following a different approach, and other simplifying assumptions

- one cluster only
- exponentially distribution of the amplitudes

Riegler, Lippman and Veenhof found out a similar expression:

$$\sigma_t \approx \frac{1.28}{\eta v_d}$$

W. Riegler et al., Nuclear Instruments and Methods in Physics Research A 500 (2003) 144–162, doi:10.1016/S0168-9002(03)00337-1



Comparison between the above formula (line) and a full Monte Carlo simulation (dots), for a timing (300 µm) RPC

Weak dependance of time resolution on v_{thr}, predicted by both approaches, confirmed by Monte Carlo.

Conclusions

In some communities of gaseous detectors, there are discussions about timing properties of RPCs; this presentation stemmed from one of them.
→ «narrow» gap RPCs are characterized by a better time resolution with respect to «wide» gap RPCs because of the larger values of η and v_d.

Statistical considerations can help a lot in understanding the physical reasons of signal fluctuations and derive approximate expressions for time resolution.

Even if approximate, these expressions give results in the right ballpark and, above all, provide limits beyond which is hard (impossible?) to go.