# Next-to-eikonal quark propagator in the CGC and applications

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in collaboration with
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# Outline of the talk

1. Introduction
2. Quark propagator through a shockwave at the next-to-eikonal accuracy
    - Eikonal limit: quark propagator in a pure $\mathcal{A}^-$ background
    - Subeikonal corrections: random walk in a pure $\mathcal{A}^-$ background
    - Subeikonal corrections: single $\mathcal{A}_\perp$ interaction
    - Subeikonal corrections: instantaneous double $\mathcal{A}_\perp$ interaction
    - Full result
3. Quark-nucleus scattering
    - Unpolarized cross section
    - Quark helicity asymmetry
4. Summary and conclusions

# Eikonal High-Energy scattering

**Pillars of the CGC approach to dense-dilute high-energy scattering:**

- **Semi-classical approx. :**
  Dense target replaced by a strong classical gluon field $\mathcal{A}^\mu(x)$
  $\Rightarrow$ Averaging over $\mathcal{A}^\mu(x)$ to do at cross section level

- **Eikonal approx. :**
  Keep only leading power in the high-energy limit

The Eikonal approx. can be understood as the limit of **infinite boost** of $\mathcal{A}^\mu(x)$:

- Under a boost of parameter $\gamma$ along the "$-$" direction, $\mathcal{A}^-$ is enhanced and $\mathcal{A}^+$ is suppressed
  $\Rightarrow$ Strong hierarchy of components of $\mathcal{A}^\mu$:
  $$\mathcal{A}^- = O(\gamma) \;\gg\; \mathcal{A}_\perp = O(1) \;\gg\; \mathcal{A}^+ = O(1/\gamma)$$

- Lorentz contraction of $\mathcal{A}^\mu(x)$ (shockwave limit)
  $\Rightarrow$ Partons from the projectile interact instantly with the target

- $\mathcal{A}^\mu(x)$ independent on $x^-$ due to Lorentz time dilation
  $\Rightarrow$ No $p^+$ transfer from the target

Background field in the eikonal limit: $\mathcal{A}^\mu(x^+, x^-, \mathbf{x}) \approx \delta^{\mu -}\mathcal{A}^-(x^+, \mathbf{x}) \propto \delta(x^+)$

# Beyond the Eikonal approximation

There are several motivations to go beyond the Eikonal approximation :

- **Theory :** Study power-suppressed corrections to better understand the foundations of the CGC and its domain of validity
- **Pheno :** Not so high energies at EIC and RHIC $\Rightarrow$ Power-suppressed corrections can be sizable
- **Spin :** Eikonal scattering is blind to spin $\Rightarrow$ QCD Spin physics at low $x$ driven by non-eikonal contributions

**Beyond eikonal approximation, at Next-to-Eikonal accuracy (NEik):**

* Target with finite width $\Rightarrow$ transverse motion of the parton within the medium
* Interactions with $\mathcal{A}_\perp$ field taken into account, not only $\mathcal{A}^-$

High-energy expansion with small parameters $\sim \frac{L^+}{k^+} Q^2$ ($L^+$ : target width, $k^+$ : parton momentum, $Q^2$ : any transverse scale in the problem)

In this study, $x^-$ dependence of $\mathcal{A}^\mu(x)$ still neglected : expected effects at NNEik accuracy only

# Quark propagator - basics

**Full quark Feynman propagator in background field $\mathcal{A}^\mu(x)$**

$$S_F(x,y)_{\alpha\beta} = S_{0,F}(x,y)_{\alpha\beta} + \delta S_F(x,y)_{\alpha\beta}$$

free propagator + corrections due to interactions with the background field

Free quark Feynman propagator:

$$S_{0,F}(x,y)_{\alpha\beta} = (\mathbf{1})_{\alpha\beta} \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\, \frac{i(\not{k}+m)}{[k^2-m^2+i\epsilon]}$$

**Corrections:**

- **at the eikonal order**

$$\delta S_F\Big|^{\text{Eik}} \equiv \delta S_F\Big|^{\text{Eik}}_{\text{pure } \mathcal{A}^-}$$

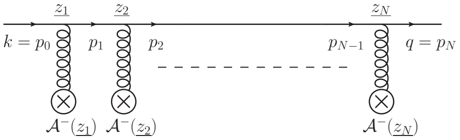
- **at the next-to-eikonal order**

$$\delta S_F\Big|^{\text{NEik}} \equiv \delta S_F\Big|^{\text{NEik}}_{\text{pure } \mathcal{A}^-} + \delta S_F\Big|^{\text{NEik}}_{\text{single } \mathcal{A}_\perp} + \delta S_F\Big|^{\text{NEik}}_{\text{double } \mathcal{A}_\perp}$$

# Quark propagator in the eikonal limit

$$S_F(x,y)_{\alpha\beta}\Big|^{\text{Eik}} = S_{0,F}(x,y)_{\alpha\beta} + \delta S_F(x,y)_{\alpha\beta}\Big|^{\text{Eik}}_{\text{pure }\mathcal{A}^-}$$

**In eikonal limit, the quark already interacts with arbitrarily many $\mathcal{A}^-$ fields**




**Eikonal interactions with the medium resummed into the Wilson lines:**

$$\mathcal{U}_F(x^+, y^+; \mathbf{z}) \equiv \mathbf{1} + \sum_{N=1}^{+\infty} \frac{1}{N!}\, \mathcal{P}_+ \left[-ig \int_{y^+}^{x^+} dz^+\, t\cdot\mathcal{A}^-(z^+, \mathbf{z})\right]^N$$

# Quark propagator in the eikonal limit

$$S_F(x,y)_{\alpha\beta}\Big|^{\text{Eik}} = S_{0,F}(x,y)_{\alpha\beta} + \delta S_F(x,y)_{\alpha\beta}\Big|^{\text{Eik}}_{\text{pure } \mathcal{A}^-}$$

**In eikonal limit, the quark already interacts with arbitrarily many $\mathcal{A}^-$ fields**
For generic $x$ and $y$, with notations $\underline{k} \equiv (k^+, \mathbf{k})$, and $\check{k}$ on-shell version of $k$:

$$\begin{aligned} S_F(x,y)_{\alpha\beta}\Big|^{\text{Eik}} = \; & \mathbf{1}_{\alpha\beta}\, \delta^{(3)}(\underline{x}-\underline{y})\, \text{sgn}(x^- - y^-)\, \frac{\gamma^+}{4} \\ & + \int \frac{d^3\underline{q}}{(2\pi)^3} \int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+ - k^+) e^{-ix\cdot\check{q}+iy\cdot\check{k}}\, \frac{(\check{\not{q}}+m)\gamma^+(\check{\not{k}}+m)}{(2k^+)^2} \\ & \times \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})} \Big\{ \theta(k^+)\theta(x^+ - y^+)\, \mathcal{U}_F(x^+, y^+; \mathbf{z})_{\alpha\beta} \\ & \qquad - \theta(-k^+)\theta(y^+ - x^+)\, \mathcal{U}_F^\dagger(y^+, x^+; \mathbf{z})_{\alpha\beta} \Big\} \end{aligned}$$

**Eikonal interactions with the medium resummed into the Wilson lines:**

$$\mathcal{U}_F(x^+, y^+; \mathbf{z}) \equiv \mathbf{1} + \sum_{N=1}^{+\infty} \frac{1}{N!}\, \mathcal{P}_+ \left[ -ig \int_{y^+}^{x^+} dz^+\, t\cdot\mathcal{A}^-(z^+, \mathbf{z}) \right]^N$$

- background field $\mathcal{A}_a^-(z^+, \mathbf{z})$ has a finite support $[-L^+/2, L^+/2]$ - this is where the non-trivial medium contributions come from in the interval $[y^+, x^+]$
- if there is no support the propagator reduces to the Feynman propagator in vacuum

**Full next-to-eikonal quark propagator:**

$$S_F\Big|^{\text{NEik}} = \underbrace{S_F\Big|^{\text{Eik}} + \delta S_F\Big|^{\text{NEik}}_{\text{pure } \mathcal{A}^-}}_{S_F\big|_{\text{pure } \mathcal{A}^-}} + \delta S_F\Big|^{\text{NEik}}_{\text{single } \mathcal{A}_\perp} + \delta S_F\Big|^{\text{NEik}}_{\text{double } \mathcal{A}_\perp}$$

From now on, always $x^+ > L^+/2$ and $y^+ < -L^+/2$ : quark propagating through the whole target

**Quark propagator in pure $\mathcal{A}^-$ background field up to next-to-eikonal order for positive energy:**

$$\begin{aligned}
S_F(x,y)_{\alpha\beta}\Big|_{\text{pure } \mathcal{A}^-} = & \int \frac{d^3\underline{q}}{(2\pi)^3} \int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+ - k^+)\, \frac{\theta(k^+)}{(2k^+)^2}\, e^{-ix\cdot\check{q}+iy\cdot\check{k}}\, (\not{\check{q}} + m)\gamma^+(\not{\check{k}} + m) \\
& \times \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})} \Bigg\{ \mathcal{U}_F\left(\frac{L^+}{2}, -\frac{L^+}{2}; \mathbf{z}\right) \\
& - \frac{(\mathbf{q}^j + \mathbf{k}^j)}{4k^+} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[ \mathcal{U}_F\left(\frac{L^+}{2}, z^+; \mathbf{z}\right) \overleftrightarrow{\partial_{\mathbf{z}^j}}\, \mathcal{U}_F\left(z^+, -\frac{L^+}{2}; \mathbf{z}\right) \right] \\
& - \frac{i}{2k^+} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[ \mathcal{U}_F\left(\frac{L^+}{2}, z^+; \mathbf{z}\right) \overleftarrow{\partial_{\mathbf{z}^j}}\, \overrightarrow{\partial_{\mathbf{z}^j}}\, \mathcal{U}_F\left(z^+, -\frac{L^+}{2}; \mathbf{z}\right) \right] \Bigg\}
\end{aligned}$$

**NEik corrections: transverse drift term + transverse Brownian motion term**

Analog to earlier results on the gluon propagator with subeikonal corrections:

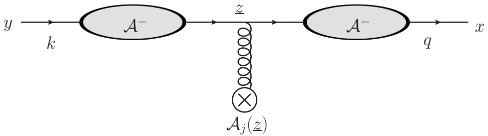
Altinoluk, Armesto, Beuf, Martinez, Salgado, JHEP **1407**, 068 (2014)
Altinoluk, Armesto, Beuf, Moscoso, JHEP **1601**, 114 (2016)

**Full next-to-eikonal quark propagator:**

$$S_F\Big|^{\text{NEik}} = S_F\Big|^{\text{Eik}} + \delta S_F\Big|^{\text{NEik}}_{\text{pure } \mathcal{A}^-} + \delta S_F\Big|^{\text{NEik}}_{\text{single } \mathcal{A}_\perp} + \delta S_F\Big|^{\text{NEik}}_{\text{double } \mathcal{A}_\perp}$$

Replace one $\gamma^+ \mathcal{A}_a^-$ interaction by $\gamma^j \mathcal{A}_j^a$

$$\delta S_F(x,y)\Big|^{\text{NEik}}_{\text{single } \mathcal{A}_\perp} = \int d^4z \; S_F(x,z)\Big|^{\text{Eik}} \left[-ig\, \gamma^j\, t^a\right] \mathcal{A}_j^a(\underline{z}) \quad S_F(z,y)\Big|^{\text{Eik}}$$



# Subeikonal corrections: single $\mathcal{A}_\perp$ insertion

**Full next-to-eikonal quark propagator:**

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Replace one $\gamma^+ \mathcal{A}_a^-$ interaction by $\gamma^j \mathcal{A}_j^a$

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**Subeikonal correction due to an interaction with $\mathcal{A}_\perp$:**

$$\begin{aligned}\delta S_F(x,y)\Big|^{\text{NEik}}_{\text{single } \mathcal{A}_\perp} = &\int \frac{d^3\underline{q}}{(2\pi)^3} \int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+ - k^+)\, \frac{\theta(k^+)}{(2k^+)^3}\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}} \\ &\times (\check{\not{q}} + m)\gamma^j\gamma^+\gamma^i\,(\check{\not{k}} + m) \int d^3\underline{z} \left[e^{-i\mathbf{z}\cdot\mathbf{q}}\, \mathcal{U}_F\left(\frac{L^+}{2}, z^+; \mathbf{z}\right)\right] \\ &\times \left[\overleftarrow{\partial_{\mathbf{z}^j}}\left[gt\cdot\mathcal{A}_i(\underline{z})\right] - \left[gt\cdot\mathcal{A}_j(\underline{z})\right]\overrightarrow{\partial_{\mathbf{z}^i}}\right] \left[\mathcal{U}_F\left(z^+, -\frac{L^+}{2}; \mathbf{z}\right) e^{i\mathbf{z}\cdot\mathbf{k}}\right]\end{aligned}$$

Reminder: $x^+ > L^+/2$ and $y^+ < -L^+/2$ :
quark propagating through the whole medium

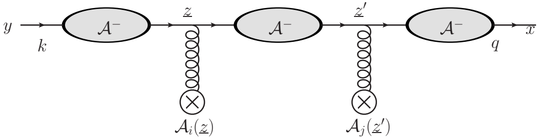
# Subeikonal corrections: double $\mathcal{A}_\perp$ insertion

**Full next-to-eikonal quark propagator:**

$$S_F\Big|^{\text{NEik}} = S_F\Big|^{\text{Eik}} + \delta S_F\Big|^{\text{NEik}}_{\text{pure }\mathcal{A}^-} + \delta S_F\Big|^{\text{NEik}}_{\text{single }\mathcal{A}_\perp} + \delta S_F\Big|^{\text{NEik}}_{\text{double }\mathcal{A}_\perp}$$

Replace two $\gamma^+\mathcal{A}_a^-$ interactions by $\gamma^j\mathcal{A}_j^b$ and $\gamma^i\mathcal{A}_i^a$

$$\delta S_F(x,y)\Big|^{\text{NEik}}_{\text{double }\mathcal{A}_\perp} = \int d^4z \int d^4z'\; S_F(x,z')\Big|^{\text{Eik}} \left[-ig\,\gamma^j\, t^b\right] \mathcal{A}_j^b(\underline{z}') \times S_F(z',z)\Big|^{\text{Eik}} \left[-ig\,\gamma^i\, t^a\right] \mathcal{A}_i^a(\underline{z}) \quad S_F(z,y)\Big|^{\text{Eik}}$$




Naively of NNEik order, but instantaneous contribution in the middle Eikonal propagator produces a NEik contribution

**Full next-to-eikonal quark propagator:**

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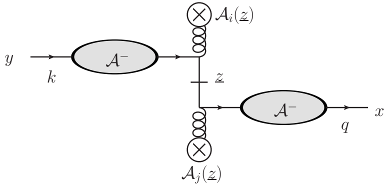


Instantaneous double $\mathcal{A}_\perp$ insertion at NEik

# Subeikonal corrections: double $\mathcal{A}_\perp$ insertion

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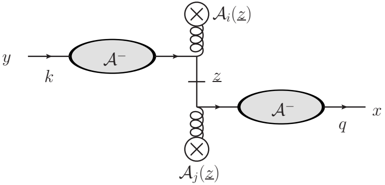



$$\delta S_F(x,y)\Big|^{\text{NEik}}_{\text{double }\mathcal{A}_\perp} = \int \frac{d^3\underline{q}}{(2\pi)^3} \int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+ - k^+)\, \frac{\theta(k^+)}{(2k^+)^3}\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}$$

$$\times\, (\check{\not{q}} + m)\gamma^j\gamma^+\gamma^i\, (\check{\not{k}} + m) \int d^3\underline{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}$$

$$\times\, (-i)\ \ \mathcal{U}_F\left(\frac{L^+}{2}, z^+; \mathbf{z}\right) \left[gt\cdot\mathcal{A}_j(\underline{z})\right]\left[gt\cdot\mathcal{A}_i(\underline{z})\right] \mathcal{U}_F\left(z^+, -\frac{L^+}{2}; \mathbf{z}\right)$$

# Next-to-eikonal quark propagator - spinor structure

**Spinor structure:**

- $\mathcal{A}^-$ field associated with $(\check{\not{q}}+m)\gamma^+(\check{\not{k}}+m)$
- $\mathcal{A}_\perp$ field associated with $(\check{\not{q}}+m)\gamma^j\gamma^+\gamma^i\,(\check{\not{k}}+m)$

- separate symmetric and anti-symmetric parts:

$$\gamma^j\gamma^+\gamma^i = \delta^{ij}\,\gamma^+ + \gamma^+\frac{[\gamma^i,\gamma^j]}{2}$$

(helicity independent + helicity dependent)

**Helicity dependence:**

$$[\gamma^i,\gamma^j] = -4i\epsilon^{ij}S^3$$

$S^3$ - helicity operator, acting on spinors as:

$$\begin{aligned} S^3 u(\check{k},h) &= h u(\check{k},h) \\ S^3 v(\check{k},h) &= -h v(\check{k},h) \end{aligned}$$

**Quark propagator**

$$S_F(x,y) = S_F(x,y)\Big|_{\text{unpol.}} + S_F(x,y)\Big|_{\text{h. dep.}}$$

# Next-to-eikonal quark propagator - full result

**Unpolarized part**

$$S_F(x,y)\Big|_{\text{unpol.}} = \int \frac{d^3\underline{q}}{(2\pi)^3}\int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+-k^+)\,\frac{\theta(k^+)}{(2k^+)^2}\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}\,(\check{\not q}+m)\gamma^+(\check{\not k}+m)$$

$$\times \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}\Big\{\mathcal{U}_F\Big(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{z}\Big)$$

$$-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{4k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\,\Big[\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\overleftrightarrow{\mathcal{D}_{\mathbf{z}^j}}\,\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)\Big]$$

$$-\frac{i}{2k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\,\Big[\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\overleftarrow{\mathcal{D}_{\mathbf{z}^j}}\,\overrightarrow{\mathcal{D}_{\mathbf{z}^j}}\,\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)\Big]\Big\}$$

**Helicity-dependent part**

$$S_F(x,y)\Big|_{\text{h. dep.}} = \int \frac{d^3\underline{q}}{(2\pi)^3}\int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+-k^+)\,\frac{\theta(k^+)}{(2k^+)^3}\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}$$

$$\times(\check{\not q}+m)\gamma^+\frac{[\gamma^i,\gamma^j]}{4}(\check{\not k}+m)\int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}$$

$$\times\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\,\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\, gt\cdot\mathcal{F}_{ij}(\underline{z})\,\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)$$

**The final result fully gauge invariant (due to covariant derivatives):**

$$\overrightarrow{\mathcal{D}_{z^\mu}} \equiv \overrightarrow{\partial_{z^\mu}} + igt\cdot\mathcal{A}_\mu(\underline{z});\quad \overleftarrow{\mathcal{D}_{z^\mu}} \equiv \overrightarrow{\mathcal{D}_{z^\mu}}^{\dagger};\quad \overleftrightarrow{\mathcal{D}_{z^\mu}} \equiv \overrightarrow{\mathcal{D}_{z^\mu}} - \overleftarrow{\mathcal{D}_{z^\mu}}$$

**Longitudinal chromo-magnetic field of the target associated with helicity:**

$$\mathcal{F}_{ij}^a(\underline{z}) \equiv \partial_{\mathbf{z}^i}\mathcal{A}_j^a(\underline{z}) - \partial_{\mathbf{z}^j}\mathcal{A}_i^a(\underline{z}) - gf^{abc}\mathcal{A}_i^b(\underline{z})\mathcal{A}_j^c(\underline{z})$$

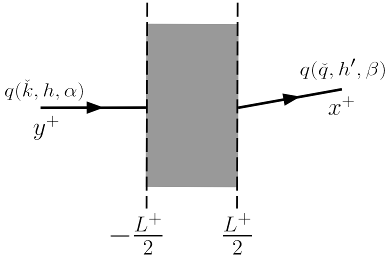
# Scattering amplitude from quark propagator

- Simplest observable where NEik correction matter: quark-target cross section
- Need for the relevant scattering amplitude

**$S$-matrix element**

**Formal definition:**
$$S_{q(\check{q},h',\beta)\leftarrow q(\check{k},h,\alpha)} = \langle 0|\hat{b}_{\text{out}}(\check{q},h,\beta)\hat{b}^{\dagger}_{\text{in}}(\check{k},h,\alpha)|0\rangle$$
$$= (2k^+)2\pi\delta(q^+ - k^+)i\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})$$

**LSZ reduction:**
$$S_{q(\check{q},h',\beta)\leftarrow q(\check{k},h,\alpha)} = \lim_{x^+\to\infty}\lim_{y^+\to-\infty}\int d^2\mathbf{x}\int dx^-\int d^2\mathbf{y}\int dy^-$$
$$\times\, e^{ix\cdot\check{q}-iy\cdot\check{k}}\,\bar{u}(\check{q},h')\gamma^+ S_F(x,y)_{\alpha\beta}\gamma^+ u(\check{k},h)$$



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**LSZ reduction:**
$$S_{q(\check{q},h',\beta)\leftarrow q(\check{k},h,\alpha)} = \lim_{x^+\to\infty}\lim_{y^+\to-\infty}\int d^2\mathbf{x}\int dx^-\int d^2\mathbf{y}\int dy^- \times e^{ix\cdot\check{q}-iy\cdot\check{k}}\,\bar{u}(\check{q},h')\gamma^+ S_F(x,y)_{\alpha\beta}\gamma^+ u(\check{k},h)$$

**Quark-target scattering amplitude**

$$\begin{aligned} i\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q}) = \delta_{hh'}\int d^2\mathbf{z}\,e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}\Bigg\{&\mathcal{U}_F\left(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{z}\right)\\ &+\frac{\epsilon^{ij}h}{2k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}}dz^+\left[\mathcal{U}_F\left(\frac{L^+}{2},z^+;\mathbf{z}\right)\Big(-igt\cdot\mathcal{F}_{ij}(\underline{z})\Big)\mathcal{U}_F\left(z^+,-\frac{L^+}{2};\mathbf{z}\right)\right]\\ &-\frac{i}{2k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}}dz^+\left[\mathcal{U}_F\left(\frac{L^+}{2},z^+;\mathbf{z}\right)\overleftarrow{\mathcal{D}_{\mathbf{z}j}}\,\overrightarrow{\mathcal{D}_{\mathbf{z}j}}\mathcal{U}_F\left(z^+,-\frac{L^+}{2};\mathbf{z}\right)\right]\\ &-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{4k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}}dz^+\left[\mathcal{U}_F\left(\frac{L^+}{2},z^+;\mathbf{z}\right)\overleftrightarrow{\mathcal{D}_{\mathbf{z}j}}\,\mathcal{U}_F\left(z^+,-\frac{L^+}{2};\mathbf{z}\right)\right]\Bigg\}_{\alpha\beta}\end{aligned}$$

# Unpolarized cross section

Differential cross section for quark scattering on the target is

$$\frac{d^2\sigma^{qA\to q+X}}{d^2\mathbf{q}} = \frac{1}{(2\pi)^2}\,\frac{1}{2N_c}\sum_{h,h'}\sum_{\alpha,\beta}\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})^\dagger\,\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})\Big|_{q^+=k^+}$$

**Cross section averaged over the target** $(\mathbf{z}-\mathbf{z}')\equiv\mathbf{r}$ and $(\mathbf{z}+\mathbf{z}')\equiv 2\mathbf{b}$

$$\left\langle\frac{d^2\sigma^{qA\to q+X}}{d^2\mathbf{q}}\right\rangle_A = \frac{1}{(2\pi)^2}\int d^2\mathbf{r}\; e^{-i(\mathbf{q}-\mathbf{k})\cdot\mathbf{r}}\Big\{1-\bar{P}(\mathbf{r}) + \Big(\bar{O}(\mathbf{r}) - \frac{(\mathbf{q}^j+\mathbf{k}^j)}{4k^+}\Big[\mathcal{O}^j_{(1)}(\mathbf{r})-\mathcal{O}^{\dagger j}_{(1)}(\mathbf{r})\Big] - \frac{i}{2k^+}\Big[\mathcal{O}_{(2)}(\mathbf{r})-\mathcal{O}^\dagger_{(2)}(\mathbf{r})\Big]\Big)\Big\}$$

**Dipole operator:**

$$d_F(\mathbf{r}) = 1-\bar{P}(\mathbf{r})+\bar{O}(\mathbf{r})$$

$$d_F(\mathbf{r}) = \frac{1}{N_c}\int d^2\mathbf{b}\Big\langle \mathrm{tr}\Big[\mathcal{U}_F^\dagger\Big(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{b}-\frac{\mathbf{r}}{2}\Big)\mathcal{U}_F\Big(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\Big\rangle_A$$

**Decorated dipole operators:**

$$\mathcal{O}^j_{(1)}(\mathbf{r}) = \frac{1}{N_c}\int d^2\mathbf{b}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\,\Big\langle \mathrm{tr}\Big[\mathcal{U}_F^\dagger\Big(\mathbf{b}-\frac{\mathbf{r}}{2}\Big) \times\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\overleftrightarrow{\mathcal{D}}_{\mathbf{b}^j+\frac{\mathbf{r}^j}{2}}\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\Big\rangle_A$$

]

$$\mathcal{O}_{(2)}(\mathbf{r}) = \frac{1}{N_c}\int d^2\mathbf{b}\int_{-L^+/2}^{L^+/2} dz^+\,\Big\langle \mathrm{tr}\Big[\mathcal{U}_F^\dagger\Big(\mathbf{b}-\frac{\mathbf{r}}{2}\Big) \times\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\overleftarrow{\mathcal{D}}_{\mathbf{b}^j+\frac{\mathbf{r}^j}{2}}\overrightarrow{\mathcal{D}}_{\mathbf{b}^j+\frac{\mathbf{r}^j}{2}}\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\Big\rangle_A .$$

# Unpolarized cross section : antiquark and symmetries

**Anti-quark–target differential cross section:**

$$\left\langle \frac{d^2\sigma^{\bar{q}A\to\bar{q}+X}}{d^2\mathbf{q}} \right\rangle_A = \frac{1}{(2\pi)^2}\int d^2\mathbf{r}\; e^{-i(\mathbf{q}-\mathbf{k})\cdot\mathbf{r}}\Big\{1-\bar{P}(\mathbf{r})$$
$$-\Big(\bar{O}(\mathbf{r}) - \frac{(\mathbf{q}^j+\mathbf{k}^j)}{4k^+}\Big[\mathcal{O}^j_{(1)}(\mathbf{r}) - \mathcal{O}^{\dagger j}_{(1)}(\mathbf{r})\Big] - \frac{i}{2k^+}\Big[\mathcal{O}_{(2)}(\mathbf{r}) - \mathcal{O}^{\dagger}_{(2)}(\mathbf{r})\Big]\Big)\Big\}$$

- By definition : $d_F(\mathbf{r})^\dagger = d_F(-\mathbf{r})$
  $\Rightarrow$ Decomposition $d_F(\mathbf{r}) = 1 - \bar{P}(\mathbf{r}) + \bar{O}(\mathbf{r})$ with
  - Real part $1-\bar{P}(\mathbf{r})$ even in $\mathbf{r}$
  - Imaginary term $\bar{O}(\mathbf{r})$ odd in $\mathbf{r}$

- **Signature transformation ($\mathcal{U}\to\mathcal{U}^\dagger$) and charge conjugation ($q\to\bar{q}$):**

  $\bar{P}(\mathbf{r})$: Pomeron is **even** under both transformations
  $\bar{O}(\mathbf{r})$: Odderon is **odd** under both transformations

  * **Eikonal terms contain both Pomeron and Odderon**
  * **Next-to-eikonal corrections are of Odderon-type**

# Quark helicity asymmetry

The difference between the cross sections for a quark of positive and negative helicity scattering on the nucleus target is:

$$\frac{d^2\Delta\sigma^{qA\to q+X}}{d^2\mathbf{q}} \equiv \frac{1}{(2\pi)^2}\,\frac{1}{2N_c}\sum_{h,h'}\sum_{\alpha,\beta}(2h)\,\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})^\dagger\,\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})\Big|_{q^+=k^+}$$

**Quark helicity asymmetry averaged over the target**

$$\left\langle\frac{d^2\Delta\sigma^{qA\to q+X}}{d^2\mathbf{q}}\right\rangle_A = \frac{1}{(2\pi)^2}\int d^2\mathbf{r}\, e^{-i(\mathbf{q}-\mathbf{k})\cdot\mathbf{r}}\,\frac{(-i)}{4k^+}\left[O_{(3)}(\mathbf{r}) - O^\dagger_{(3)}(\mathbf{r})\right]$$

**New decorated dipole operator:**

$$\begin{aligned} O_{(3)}(\mathbf{r}) \;=\; & \frac{1}{N_c}\int d^2\mathbf{b}\int_{-L^+/2}^{L^+/2} dz^+\Big\langle \mathrm{Tr}\Big[\mathcal{U}_F^\dagger\Big(\mathbf{b}-\frac{\mathbf{r}}{2}\Big) \\ & \times\,\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big\{\epsilon^{ij}\Big[gt\cdot\mathcal{F}_{ij}\Big(z^+,\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\Big\}\,\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\Big\rangle_A \end{aligned}$$

**The anti-quark helicity asymmetry is identical!**

Remarks:

- $O_{(3)}(\mathbf{r})$ behaves neither like Pomeron nor like Odderon (odd under signature transformation but even under charge conjugation)
- $\mathcal{F}_{ij}$ and $O_{(3)}(\mathbf{r})$ vanish for unpolarized target, but not for longitudinally polarized target $\Rightarrow$ Double longitudinal spin asymmetry $A_{LL}$

# Summary and final remarks

- Full next-to-eikonal (NEik) expression for the quark propagator through the background field derived
  - Corrections due to transverse motion (drift and Brownian) of the quark while crossing the Lorentz contracted target
  - Corrections due to interaction with the $\mathcal{A}_\perp$ components of the background field
  - However, effects of $x^-$ dependence of the $\mathcal{A}_\mu$ and of the $\mathcal{A}^+$ component should appear only at NNEik accuracy
- Gauge covariant expression: covariant derivatives and field strength insertions in Wilson lines
- Eikonal limit blind to spin, whereas one NEik term is proportional to helicity
- Helicity piece consistent with results of Kovchegov *et al.* (2016-2020)
- Detailed comparison with results of Chirilli (2019-2021) underway
- $qA$ (and $\bar{q}A$) cross section (unpolarized and helicity asymmetry) studied at NEik
- The NEik quark propagator is a building block for scattering processes at NEik (calculation of DIS di-jet production in progress, and other observables in project)