CESGA

Infraestructuras Científicas y Técnicas Singulares

Quantum Computing

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Schedule

- Lecture 1: Introduction to Quantum Computing.
  - My First Quantum Program.
- Lecture 2: Programming Quantum Algorithms
  - My first Quantum Program with ProjectQ
- Lecture 3: Basic Quantum algorithms
- Lecture 4: Advanced algorithms

Lecture 1

- A brief history of QC and needs. Types of quantum computers.
- Basic concepts: qubit, tensors, multiqubit, quantum gates, measurement, amplitudes
- My first quantum program.
- Quantum Circuits. Width, Depth, Quantum Volume.

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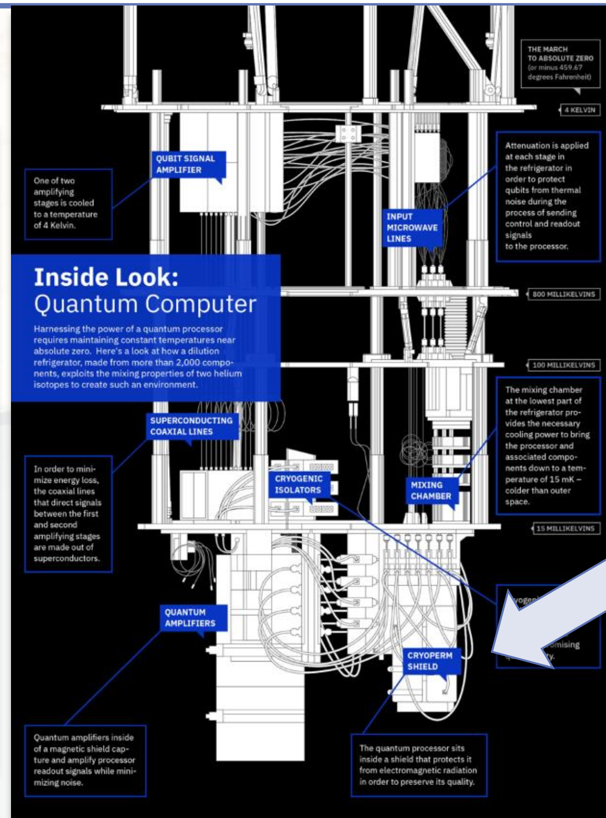
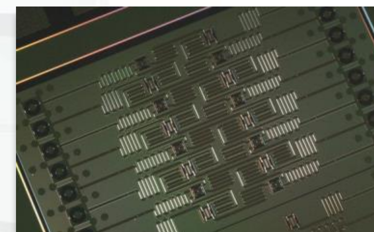
Welcome to a Dream!

Yuri Manin (1980) and Richard Feynman (1981) proposed independently the concept of Quantum Computer



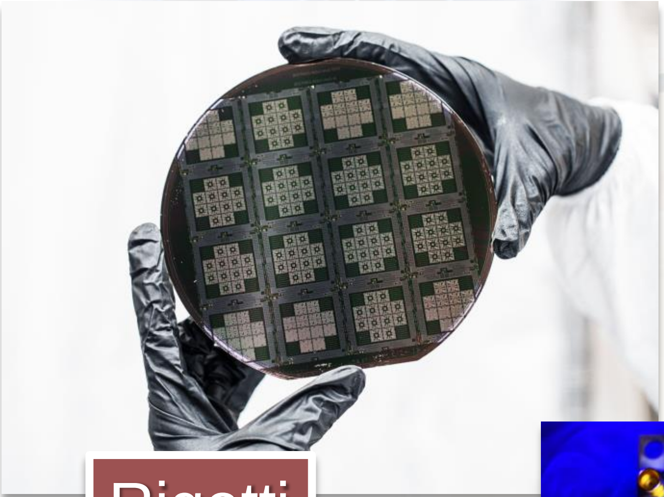


I'm here very "hot"!!
-273ºC

Source: IBM

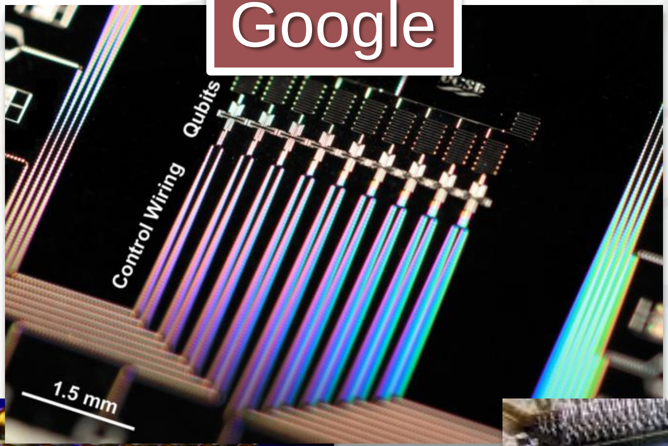
https://en.wikipedia.org/wiki/Timeline_of_quantum_computing

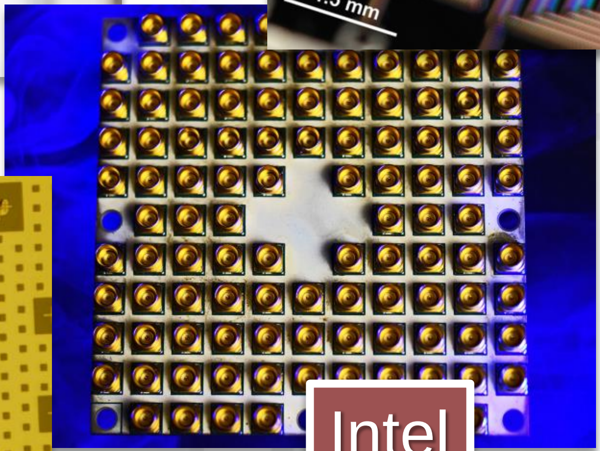
Welcome to a Dream!



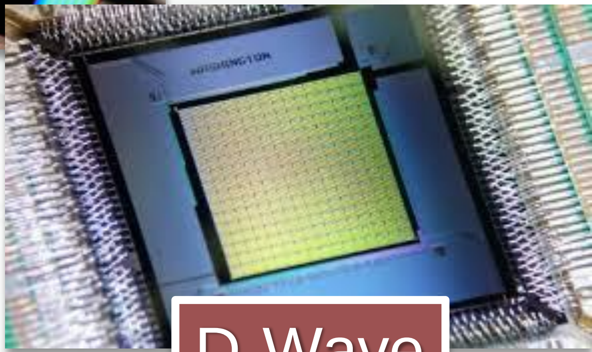
Rigetti

Google





Intel



D-Wave

Qilimanjaro (Spain)

And more in Europe, China, Australia, etc…..

Welcome to (my) Nightmare!(*)

Superposition and Entanglement

$$|\boldsymbol{\Phi}^{\pm} > = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\left(|\mathbf{00} > \pm |\mathbf{11} > \right)$$

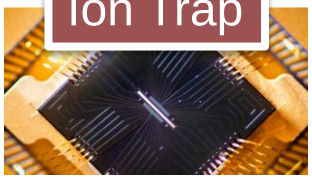
$$|\boldsymbol{\Psi}^{\pm} > = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\left(|\mathbf{01} > \pm |\mathbf{10} > \right)$$

Bell States

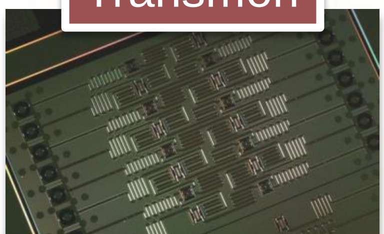
(*) When I was a student long time ago!

Quantum Technologies

Ion Trap

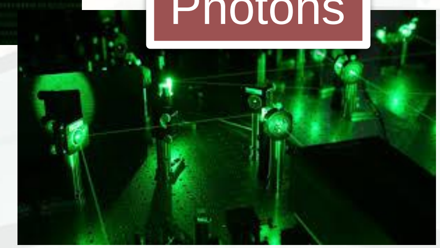


Transmon

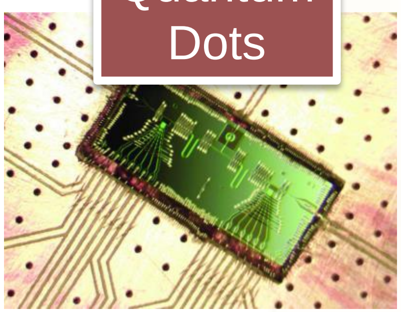


NV-Defect Diamond

Photons



Quantum Dots

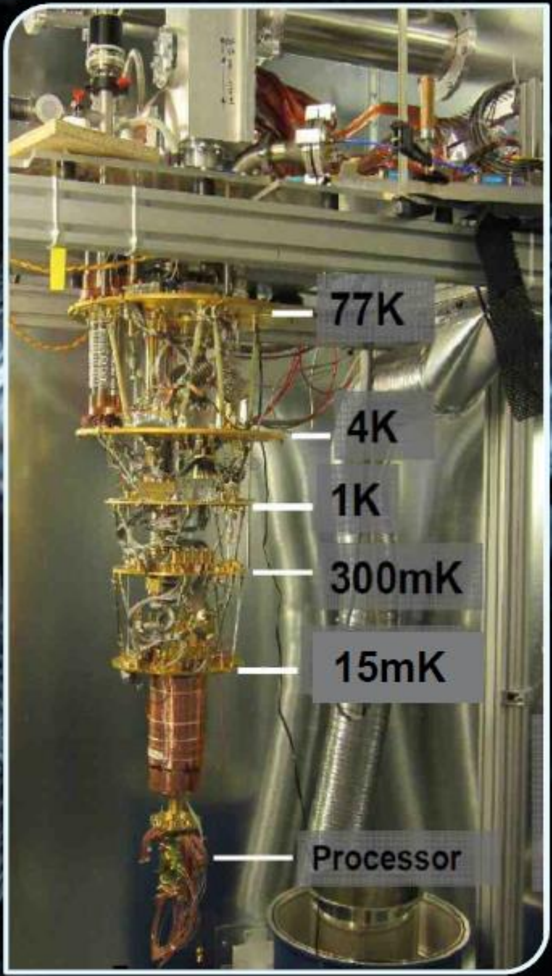


Majorama

And more in the future….

Processor Environment

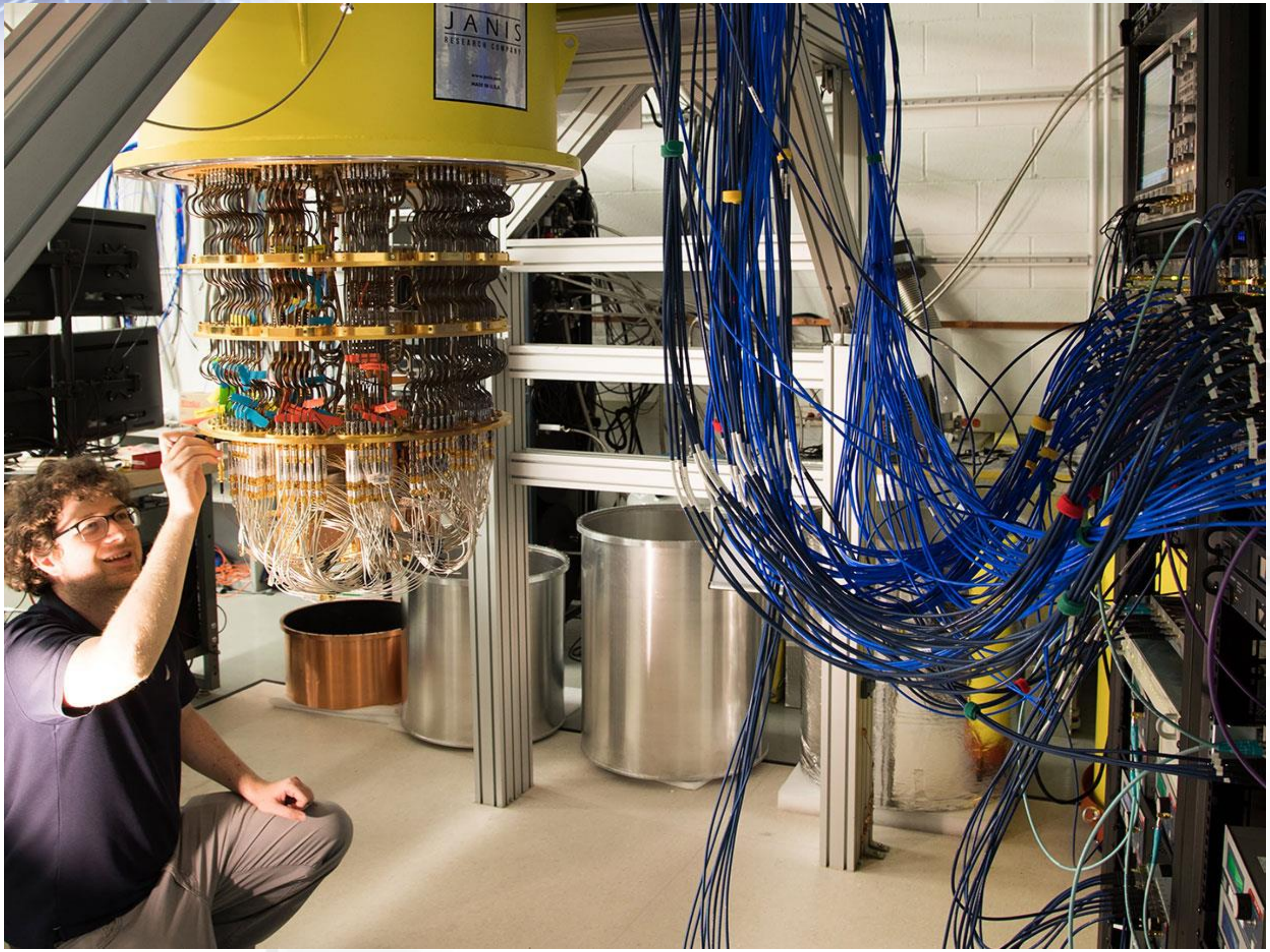
- Cooled to 0.015 Kelvin, 175x colder than interstellar space
- Shielded to 50,000 × less than Earth's magnetic field
- In a high vacuum: pressure is 10 billion times lower than atmospheric pressure
- On low vibration floor
- <25 kW total power consumption – for the next few generations



D-Wave Container –Faraday Cage - No RF Interference

System Shielding

- 16 Layers between the quantum chip and the outside world
- Shielding preserves the quantum calculation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RESEARCH COMPANY

Quantum Computer

- Quantum simulator [1]. Simulate a quantum system using another one, maybe simpler, that can be controlled by the experimenter.

- Adiabatic Quantum Computer [2]. Prepares a known and easy Hamiltonian and lets it evolve to solution.

- Topological Quantum Computer[4]. Uses topological properties.

- Continuous Variable Quantum Computer [5].

- **Universal Quantum Computer [3].**

[1] Reviewed in Georgescu, I. M., Ashhab, S., & Nori, F. (2014). Quantum simulation. *Reviews of Modern Physics*, *86*(1), 153–185. http://doi.org/10.1103/RevModPhys.86.153 **arXiv:1308.6253**
[2] Reviewed in Albash, T., & Lidar, D. A. (2016). Adiabatic Quantum Computing. arxiv:1611.04471
[3] Proposed in Deutsch, D. (1985). http://doi.org/10.1098/rspa.1985.0070 and Deutsch, D. (1989). http://doi.org/10.1098/rspa.1989.0099
[4] Lahtinen V., Pachos J.K.. SciPost Phys. 3, 021 (2017) **arXiv:1705.04103**
[5] Lloyd S. & Braunstein, A.L. Phys.Rev.Lett. 82 (1999) 1784-1787. arXiv:quant-ph/9810082

Adiabatic Quantum Computer

H_B = Initial Hamiltonian, which ground state is easy to find

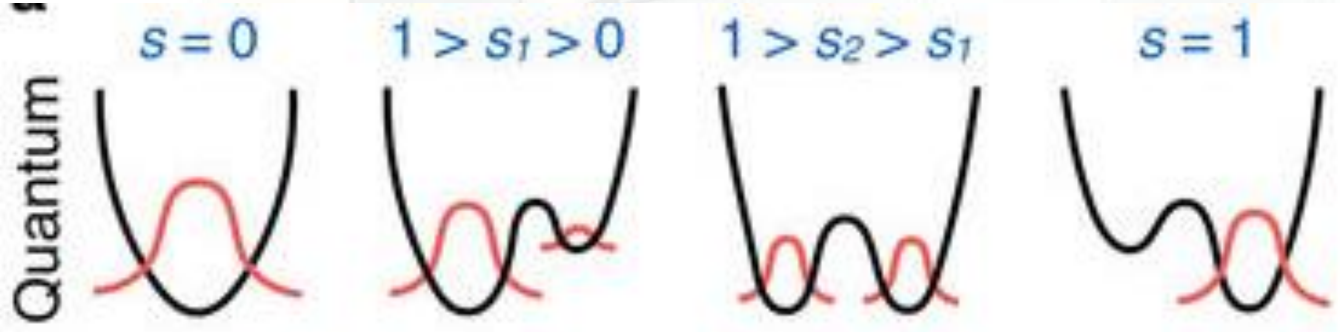
H_P = Problem Hamiltonian, whose ground state encodes the solution to the problem

H(s) = Combined Hamiltonial to evolve slowly:

A(s) decrease smoothly and monotonically

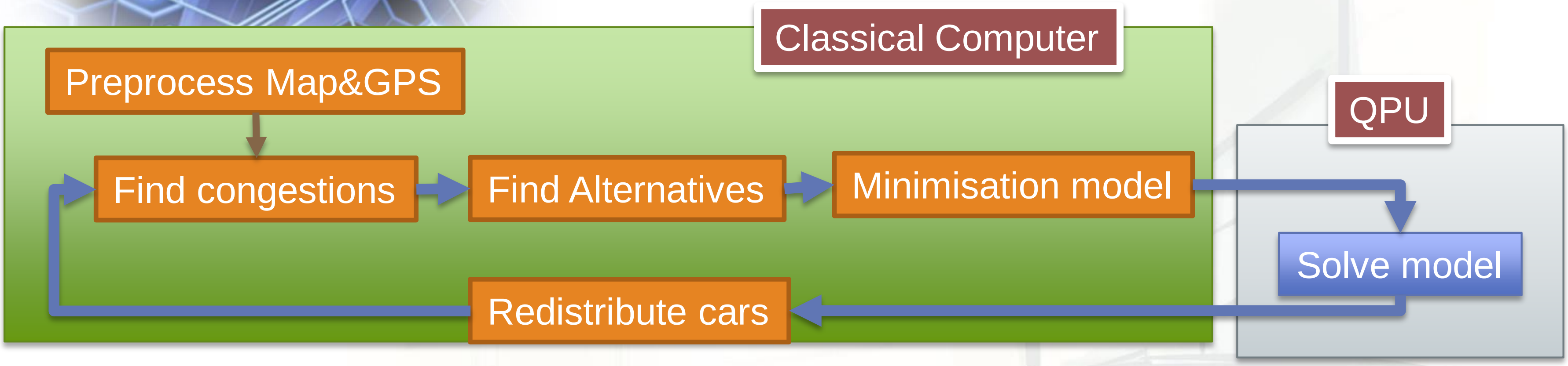
B(s) increase smothly and monotonically

$$H(s) = A(s)H_B + B(s)H_P$$



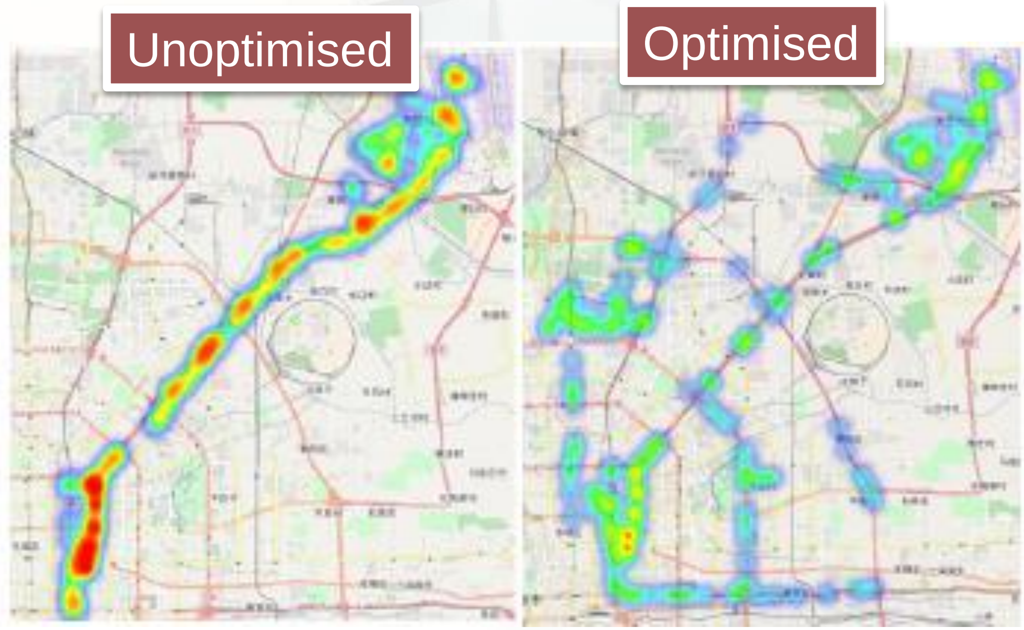
Li, R. Y., Felice, R. Di, Rohs, R., & Lidar, D. A. (2018). Quantum annealing versus classical machine learning applied to a simplified computational biology problem. Npj Quantum Information 2018 4:1, 4(1), 14. http://doi.org/10.1038/s41534-018-0060-8

A real example: Traffic Flow Optimisation



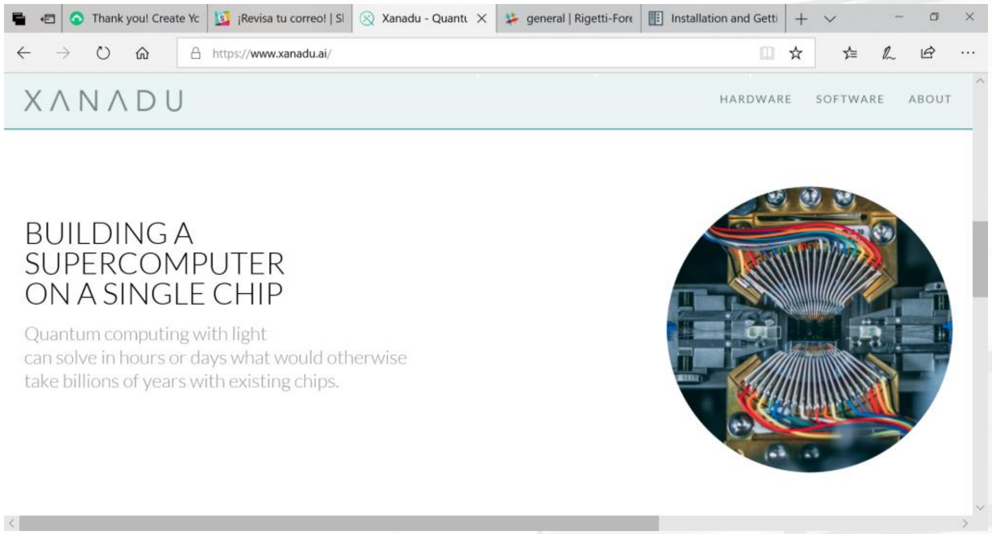
- **D-Wave Adiabatic Computer**
- Optimisation
- Classical Computer + QPU

DATA:LAB MUNICH / FUELING FUTURE BUSINESS



Neukart, F., Dollen, D. Von, Compostella, G., Seidel, C., Yarkoni, S., & Parney, B. (2017). Traffic flow optimization using a quantum annealer. arXiv:1708.01625v2

Xanadu. Continuous Variable



- Language: Strawberry Fields
- Cloud service for Research

https://www.xanadu.ai/

European Quantum Flagship

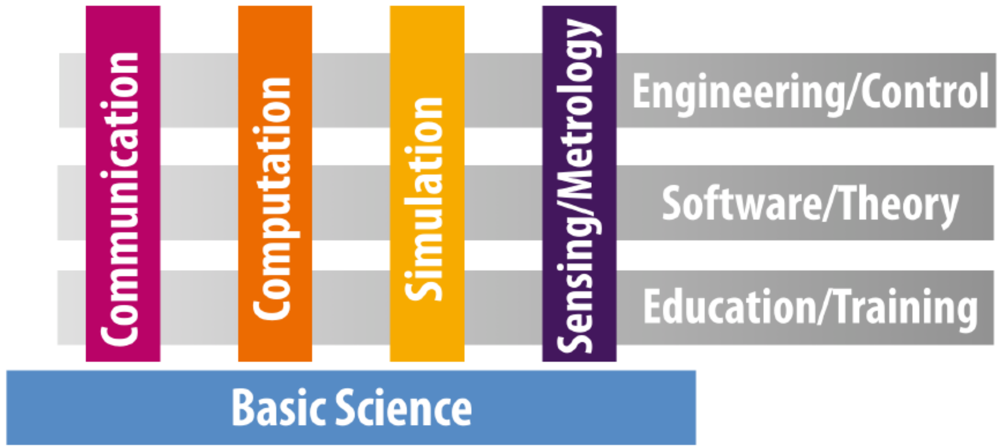


Figure: Structure of the Strategic Research Agenda

- AQTION : Trapped Ions
- OpenSuperQ : Superconducting

- SQUARE: Scalable Rare Earth Ion Quantum Computing Nodes
- MicroQC: Microwave driven ion trap quantum computing

http://qt.eu

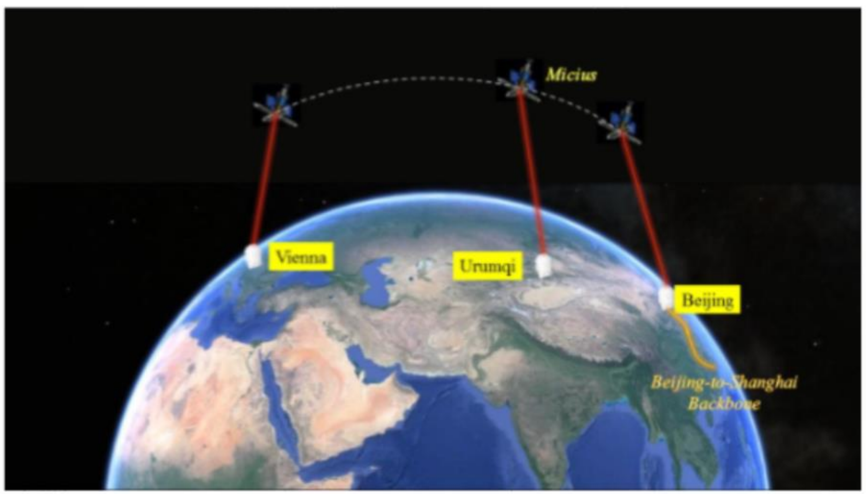
Quantum Networks

China Builds World's First Space-ground Integrated Quantum Communication Network

Sep 29, 2017 Email Print Text Size A A Share

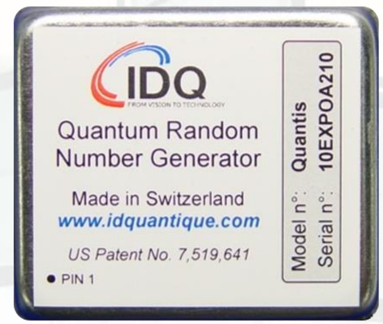
Using Micius for a quantum-safe intercontinental video conference between China and Austria.

The first quantum-safe video conference was held between President BAI Chunli of the Chinese Academy of Sciences in Beijing and President Anton Zeilinger of the Austria Academy of Sciences in Vienna, as the first real-world demonstration of intercontinental quantum communication on September 29th.



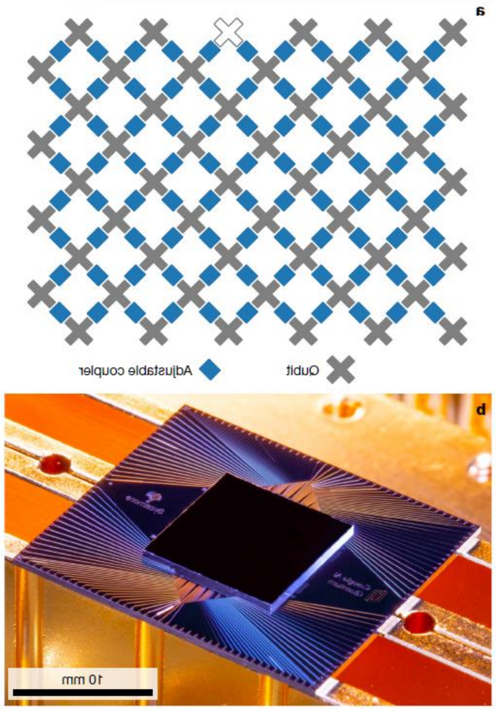
Message sending from Vienna to Beijing through space-ground integrated quantum network. (Image by PAN Jianwei's team)

Private and secure communications are fundamental human needs. In particular, with the exponential growth of Internet use and e-commerce, it is of paramount importance to establish a secure network with global protection of data. Traditional public



http://english.cas.cn/newsroom/news/201709/t20170928_183577.shtml

Google Quantum “Supremacy”



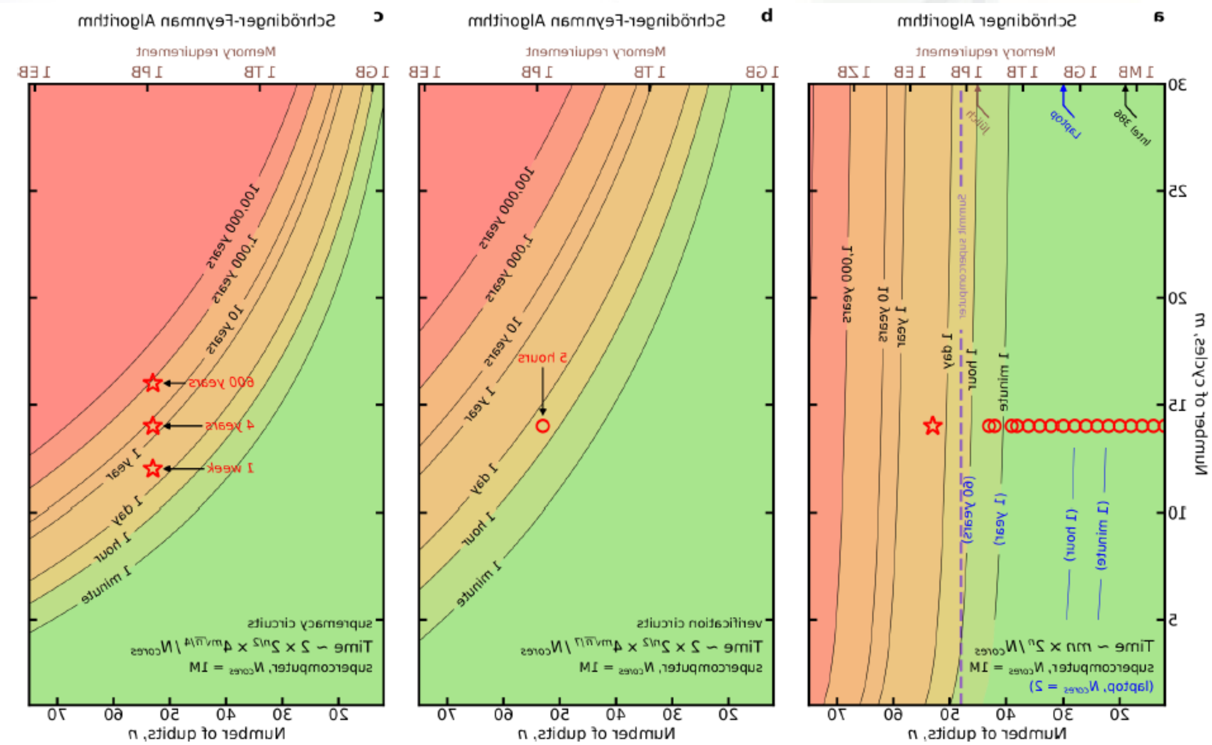


FIG. S40. Scaling of the computational cost of XEB using SA and SFA. a, For a Schrödinger algorithm, the limitation is RAM size, shown as vertical dashed line for the Summit supercomputer. Circles indicate full circuits with $n = 12$ to 43 qubits that are benchmarked in Fig. 4a of the main paper. 53 qubits would exceed the RAM of any current supercomputer, and shown as a star. b, For the hybrid Schrödinger-Feynman algorithm, which is more memory efficient, the computation time scales exponentially in depth. XEB on full verifiable circuits was done at depth $m = 14$ (circle). c, XEB on full supremacy circuits is out of reach within reasonable time resources for $m = 12, 14, 16$ (stars), and beyond. XEB on patch and elided supremacy circuits was done at $m = 14, 16, 18$, and 20.

Arute F, Arya K, Babbush R, Bacon D, Bardin JC, Barends R, et al. Quantum supremacy using a programmable superconducting processor. Nature. 2019;574:505.

Google Quantum “Supremacy”

qubits	cycles	$\mathcal{F}_{\mathrm{XEB}}$ (%)	N_s	nodes	runtime	PFlop/s* peak	PFlop/s* sust.	efficiency (%) peak	efficiency (%) sust.	power (MW)	energy (MWh)
53	12	0.5	1M	4550	1.29 hours	235.2	111.7	57.4	27.3	5.73	8.21
		1.4	0.5M		1.81 hours**						11.2**
		1.4	3M		10.8 hours**						62.7**
	14	2.22×10^{-6}	1M		0.72 hours	347.5	252.3	84.8	61.6	7.25	6.11
		0.5	1M		67.7 days**						1.18×10^4**
		1.0	0.5M		67.7 days**						1.18×10^4**
		1.0	3M		1.11 years**						7.07×10^4**

TABLE VI. **Runtimes, efficiency and energy consumption for the simulation of random circuit sampling of N_s bitstrings from Sycamore with fidelity $\mathcal{F}$ using qFlex on Summit.** Simulations used 4550 nodes out of 4608, which represents about 99% of Summit. Single batches of 64 amplitudes were computed on each MPI task using a socket with three GPUs (two sockets per node); given that one of the 9100 MPI tasks acts as master, 9099 batches of amplitudes were computed. For the circuit with 12 cycles, 144/256 paths for these batches were computed in 1.29 hours, which leads to the sampling of about 1M bitstrings with fidelity $\mathcal{F} \approx 0.5\%$ (see Ref. [50] for details on the sampling procedure); runtimes and energy consumption for other sample sizes and fidelities are extrapolated linearly in N_s and $\mathcal{F}$ from this run. At 14 cycles, 128/524288 paths were computed in 0.72 hours, which leads to the sampling of about 1M bitstrings with fidelity 2.22×10^{-6}. In this case, one would need to consider 288101 paths on all 9099 batches in order to sample about 1M (0.5M) bitstrings with fidelity $\mathcal{F} \approx 0.5\%$ (1.0%). By extrapolation, we estimate that such computations would take 1625 hours (68 days). For N_s =3M bitstrings and $\mathcal{F} \approx 1.0\%$, extrapolation gives us an estimated runtime of 1.1 years. Performance is higher for the simulation with 14 cycles, due to higher arithmetic intensity tensor contractions. Power consumption is also larger in this case. Job, MPI, and TAL-SH library initialization and shutdown times, as well as initial and final IO times are not considered in the runtime, but they are in the total energy consumption. *Single precision. **Extrapolated from the simulation with a fractional fidelity.

Lecture 1

- A brief history of QC and needs. Types of quantum computers.
- **Basic concepts: qubit, tensors, multiqubit, quantum gates, measurement, amplitudes**
- My first quantum program.
- Quantum Circuits. Width, Depth, Quantum Volume.

DiVincenzo's Criteria

1. A scalable physical system with well characterized qubits.
2. The ability to initialize the state of the qubits to a simple fiducial state, such as |000….000>
3. Long relevant decoherence times, much longer than the gate operation time.
4. A "universal" set of quantum gates.
5. A qubit-specific measurement capability.

D. DiVincenzo (2000). "The Physical Implementation of Quantum Computation", arXiv:quant-ph/0002077

What do you need (today)?

- Complex numbers
- Matrix multiplication
- Understand TENSOR products
- Understand measurement and probabilities
- Imagination

Classical Computer Business Card

BIT: A “classical” physical system with TWO states

↑ ↓

0 OR **1**

What 0 or 1 means is a convention
Information is codified as a list of BITs
BIT can be transformed from 0 to 1 and vice versa
BITs can be operated with logical gates (OR,XOR,AND…)
One BIT can be cloned
BITs can be stored
BITs can have a long life
BITs move through logical gates

Quantum Computer Business Card

QuBIT: A "Quantum" physical system which yields one of TWO states when is measured

0 OR **1**

What 0 or 1 means is a convention*

Information is codified in several ways

QuBIT can be transformed from 0 to 1 and vice versa

QuBITs can be operated with UNITARY gates

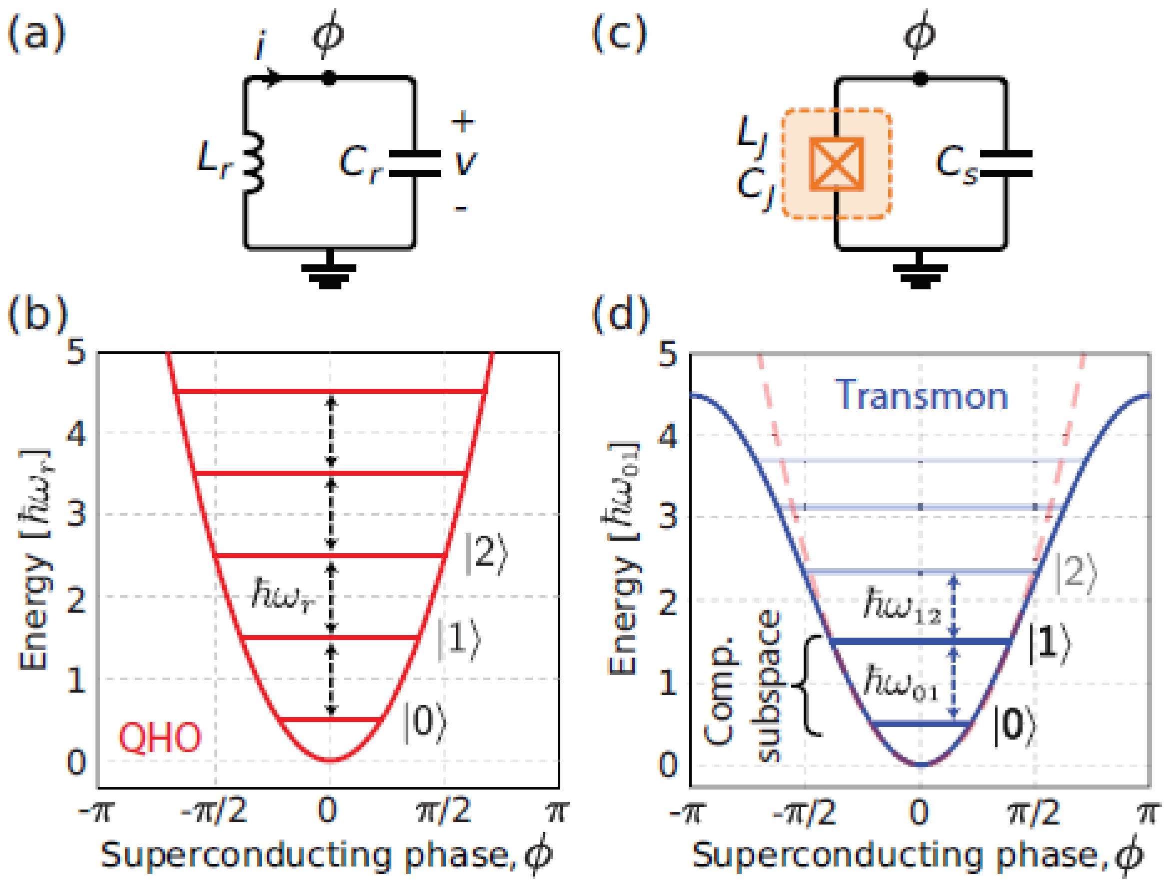
QuBITs cannot be cloned (no-clone theorem)

QuBITs cannot be stored (yet)

QuBITs cannot have a long life (yet)

Usually, QuBITs are quiet

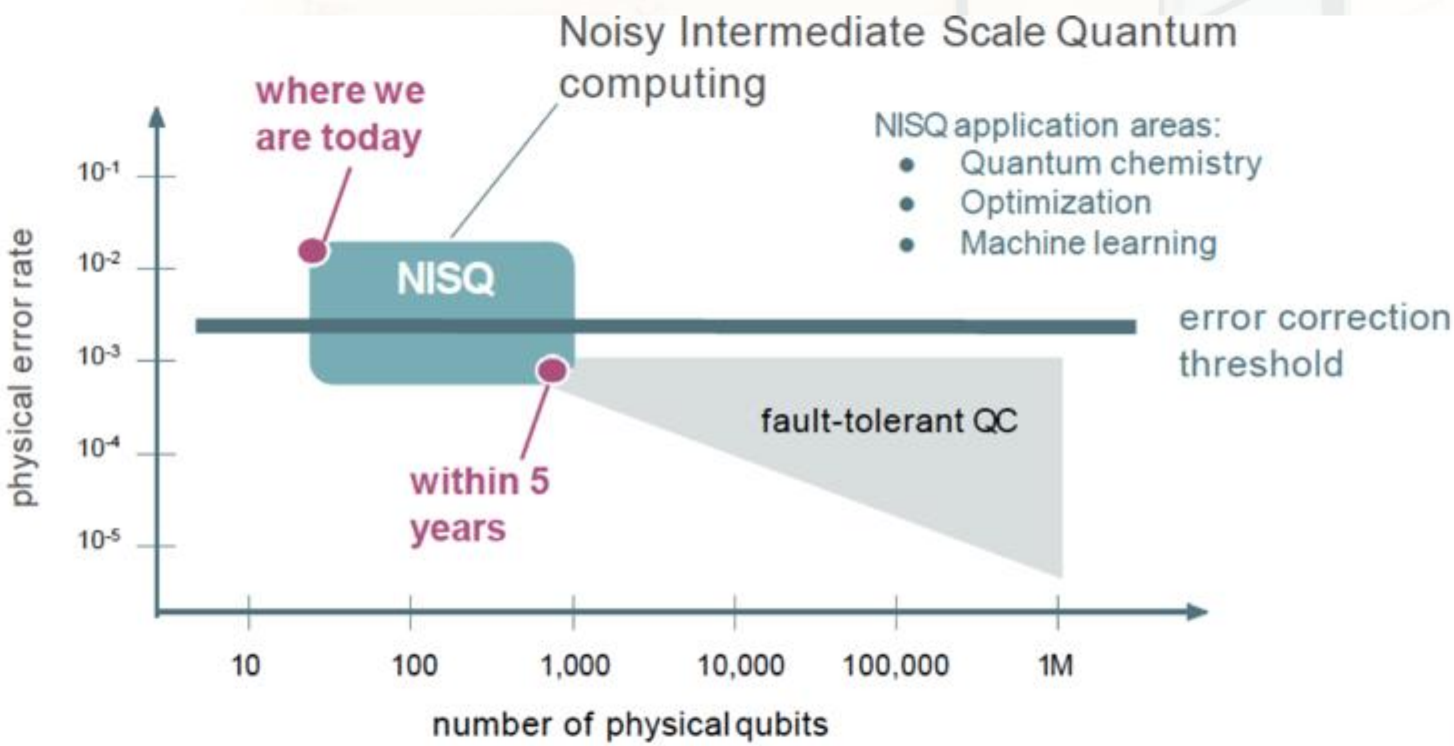
Quantum Technologies



Krantz P, Kjaergaard M, Yan F, Orlando TP, Gustavsson S, Oliver WD. A Quantum Engineer's Guide to Superconducting Qubits. Arxiv: 1904.06560

Our current nightmare!

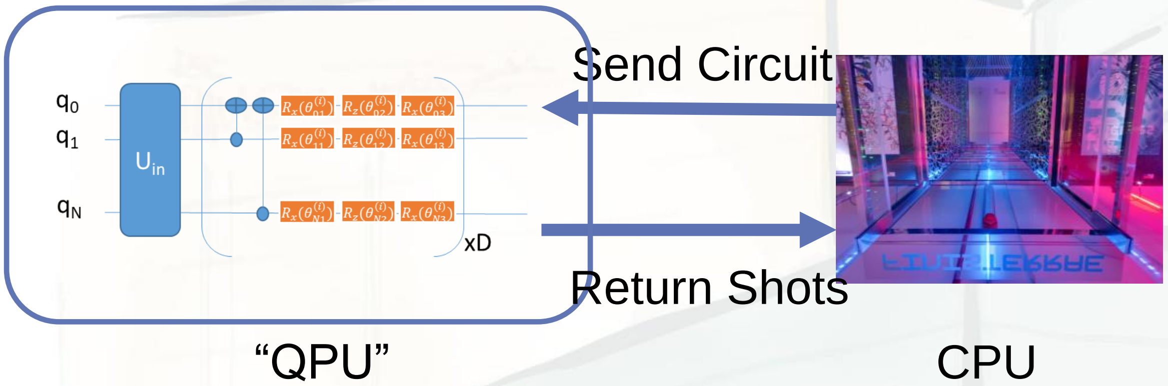
NOISE

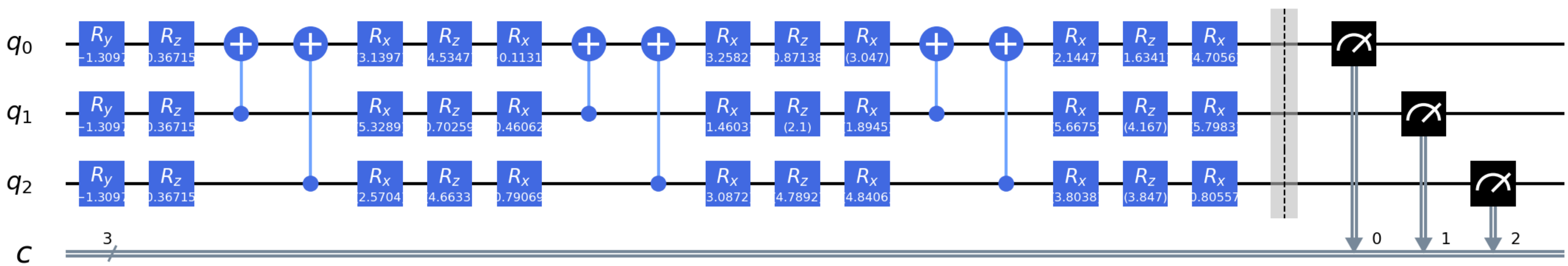


"Quantum computing in the NISQ era and beyond" Preskill, 2018 https://arxiv.org/abs/1801.00862

https://medium.com/@pchojecki/quantum-advantage-b3458646bd9

Parametric Quantum Circuit Learning

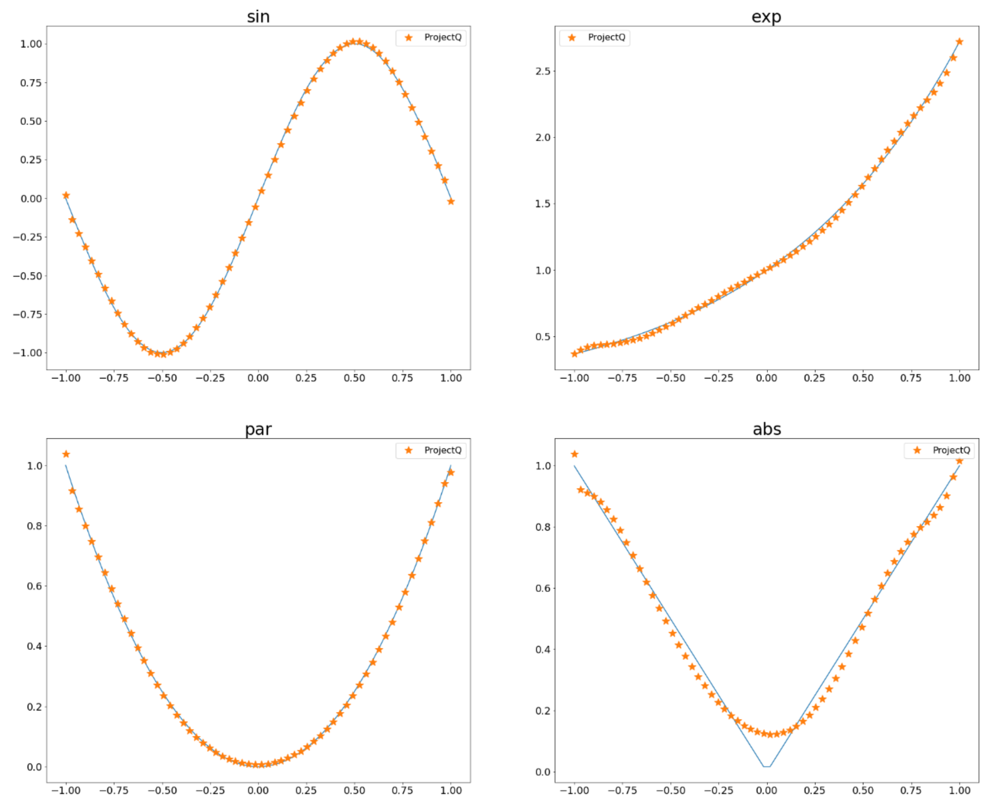




Parametric Quantum Circuit Learning

ProjectQ Results

ProjectQ Results



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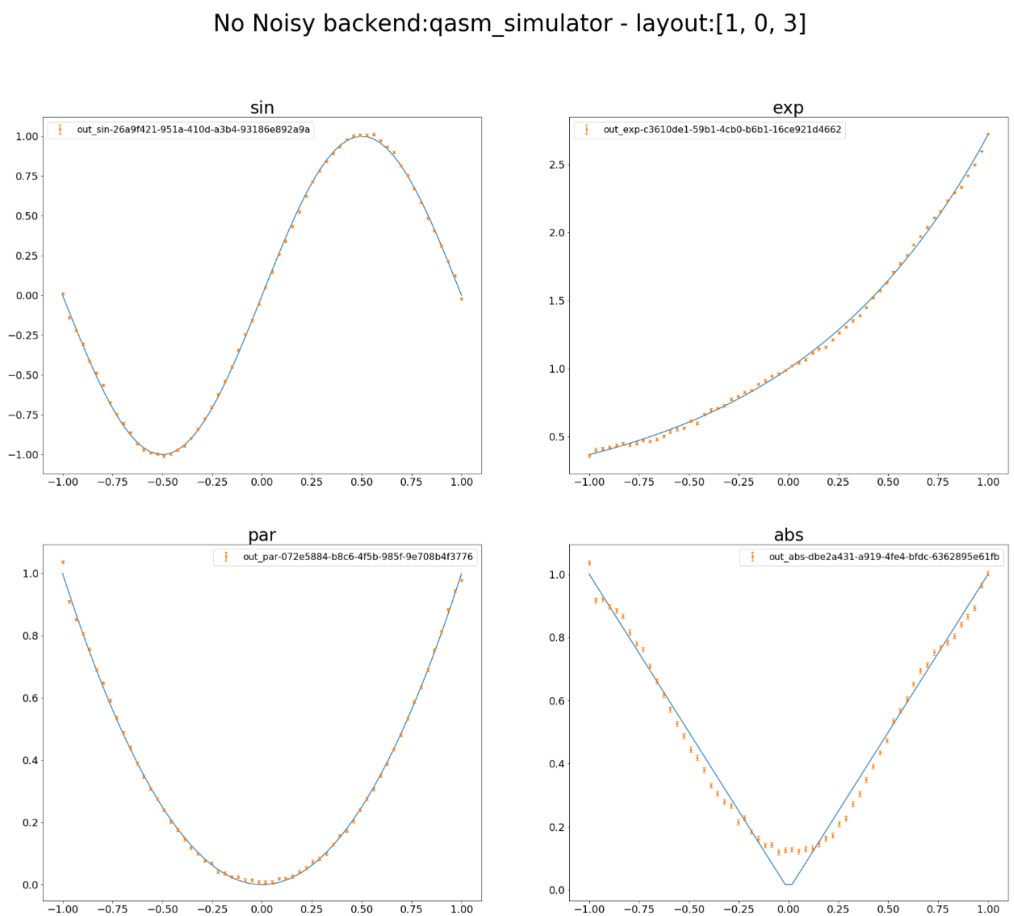
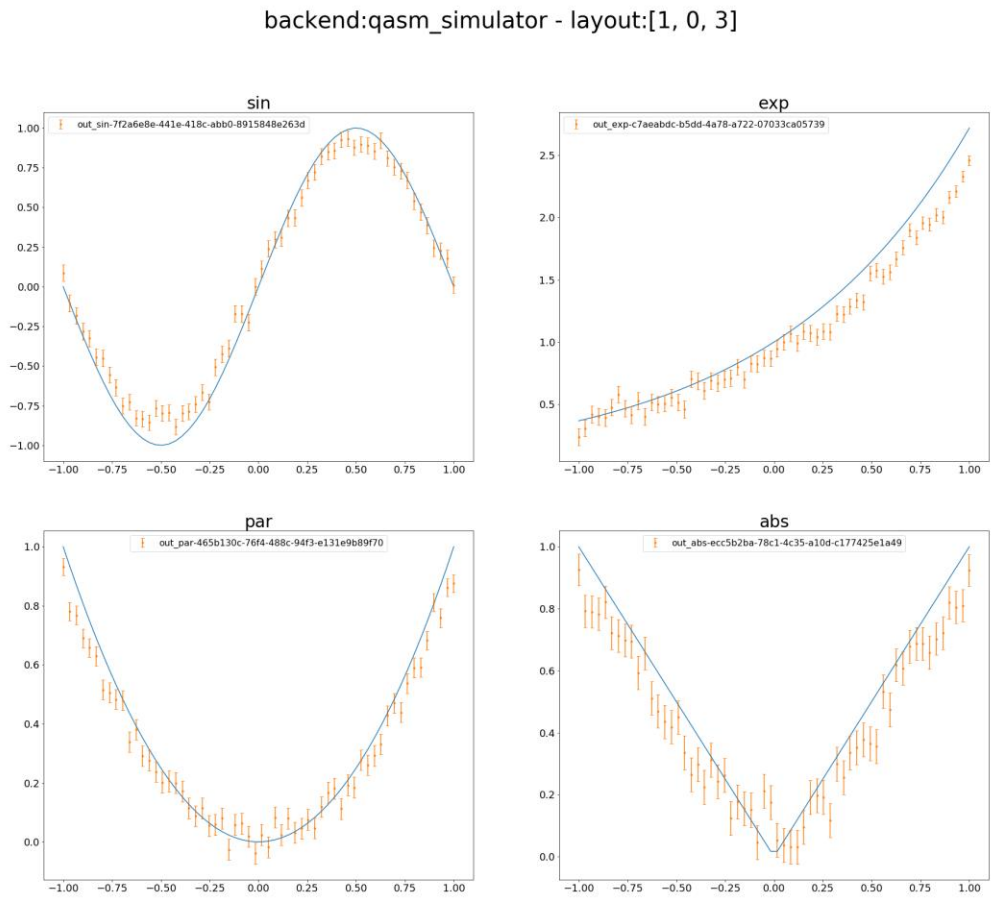
Parametric Quantum Circuit Learning



Parametric Quantum Circuit Learning

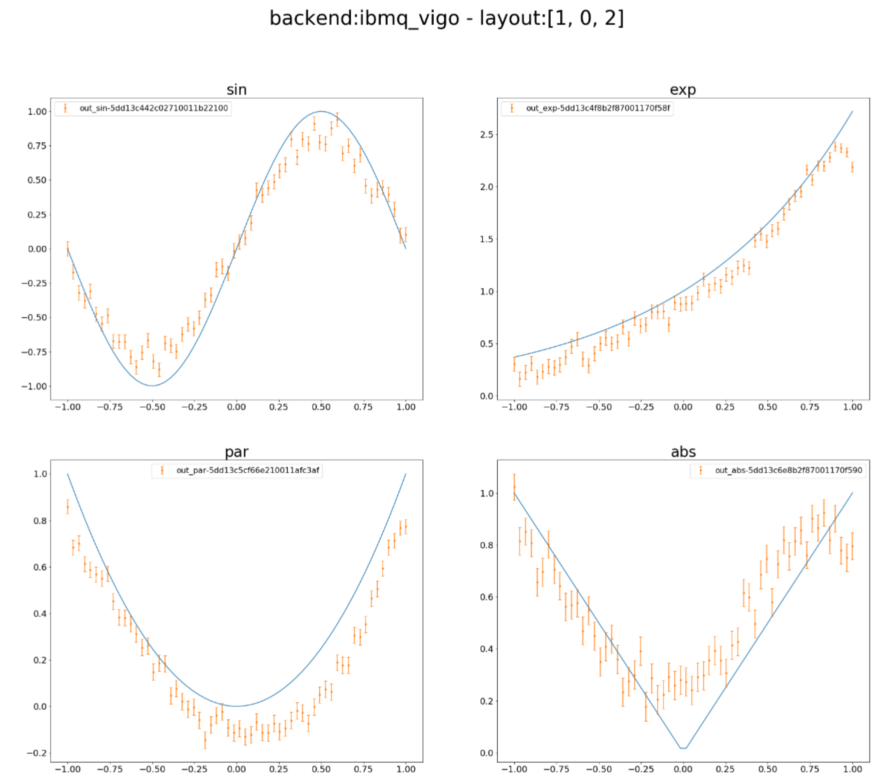
SIMULATING WITH NOISE FEW SHOTS



Parametric Quantum Circuit Learning

IBMQ_VIGO
Nov. 17th, 2019

Source: IBM© Nov. 24th, 2019



N=3, D=3

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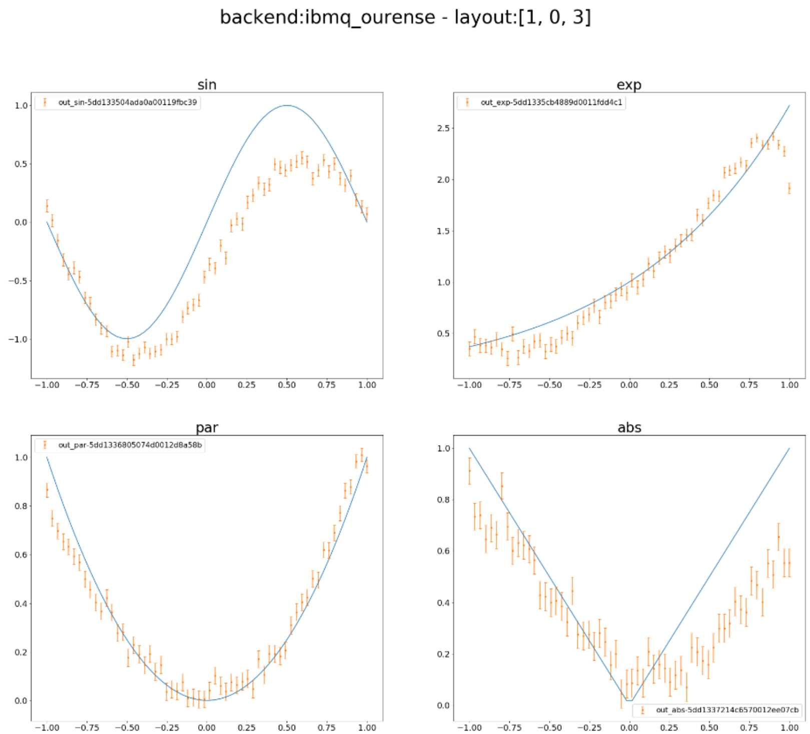
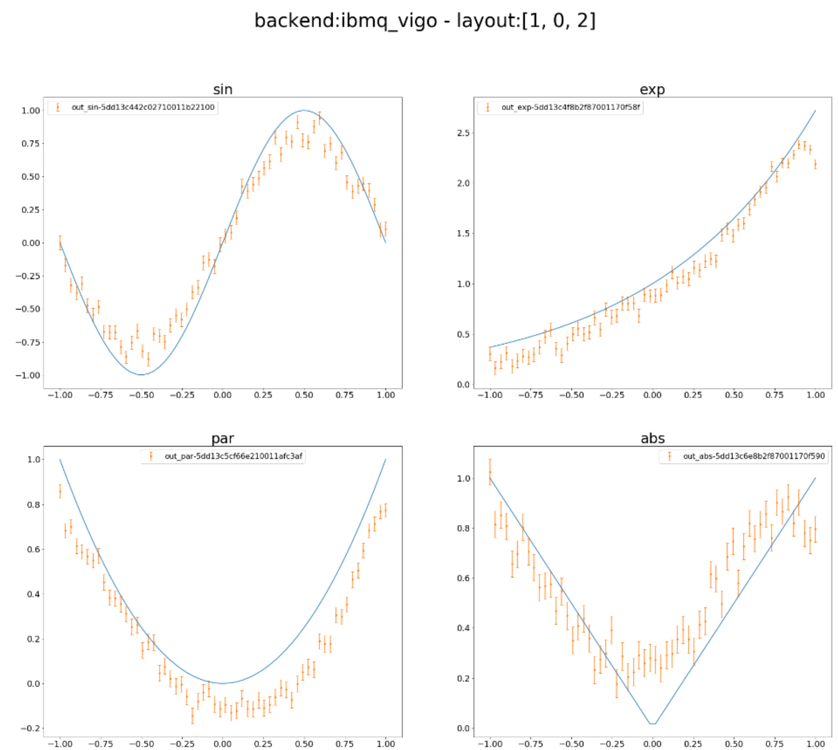
Parametric Quantum Circuit Learning





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Algorithms with shallow circuits

- **QVE**: Quantum Variational Eigensolver: https://arxiv.org/abs/1304.3061

- **QAOA**: Quantum Approximate Optimization Algorithm. http://arxiv.org/abs/1411.4028

- **Variational Quantum Factoring**: https://arxiv.org/abs/1808.08927

- **Quantum Machine Learning**:
  - Quantum Support Vector Machine
  - Quantum Principal Component Analysis
  - Quantum Variational Autoencoder,
  - Etc.

Quantum Variational Eigensolver

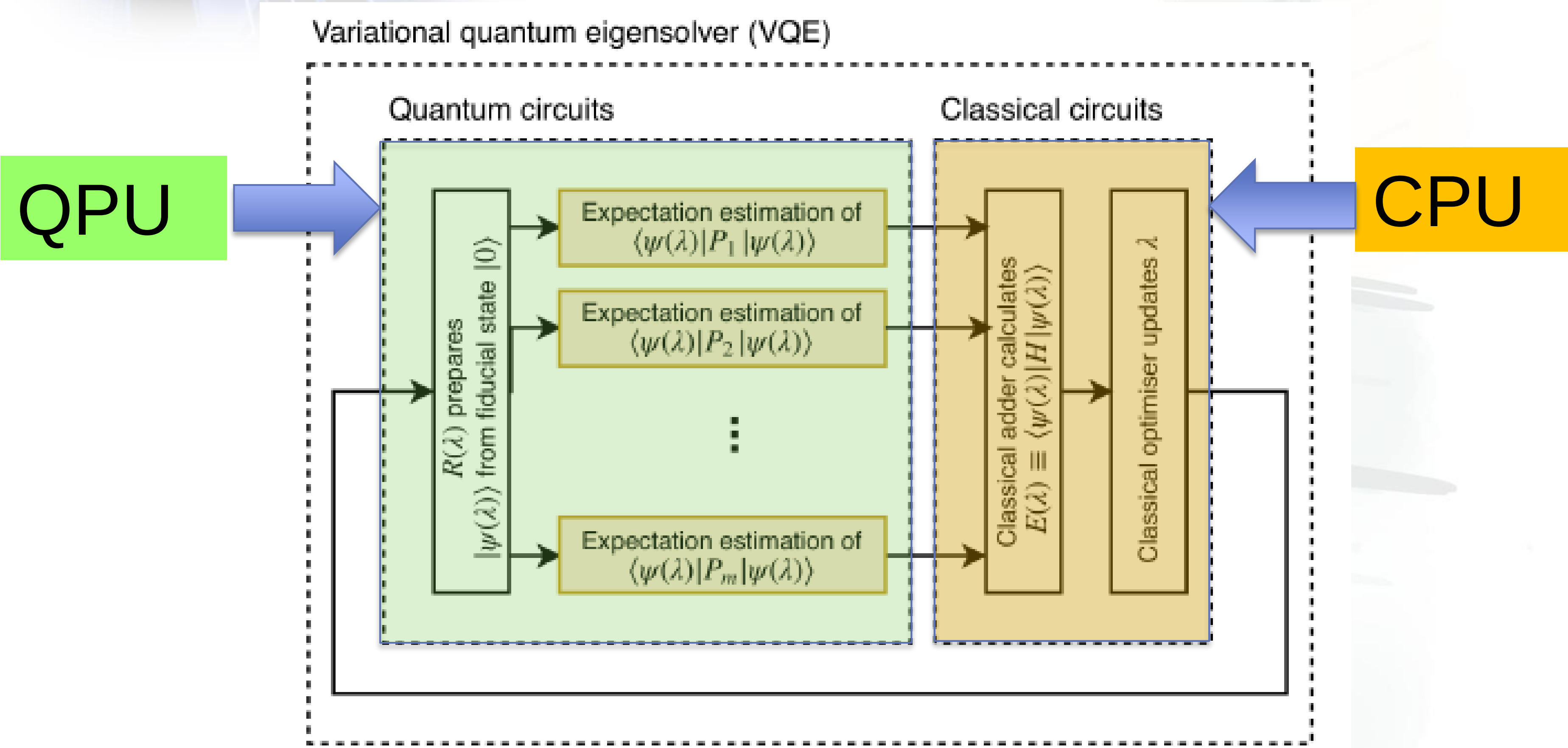

Figure source: Wang, D., Higgott, O., & Brierley, S. (n.d.). A Generalised Variational Quantum Eigensolver.

Quantum Machine Learning?

"Despite a number of promising results, the theoretical evidence presented in the current literature does not yet allow us to conclude that quantum techniques can obtain an exponential advantage in a realistic learning setting"

Ciliberto et.al. "Quantum machine learning: a classical perspective"
http://dx.doi.org/10.1098/rspa.2017.0551

So:

A lot of research to do!!!

Complex Numbers

If $i^2 = -1$, a complex number is defined by:

c=$a + b * i$, with $a, b \in \mathbb{R}, c \in \mathbb{C}$

Complex conjugate: $\bar{c} = a - b * i$

Modulus: $|c|^2 = c\bar{c} = (a + b * i)(a - b * i) = a^2 + b^2$

Polar form: c = $|c|\, cos\theta + |c|\, sin\theta\, i = |c|\, e^{i\theta}$

QUBIT

$$|0> = \begin{bmatrix}1\\0\end{bmatrix}$$

$$|1> = \begin{bmatrix}0\\1\end{bmatrix}$$

Superposition

Complex numbers

$$|\phi> = \alpha\,|0> + \beta\,|1>$$

$$|\alpha|^2 + |\beta|^2 = 1$$

$$Amplitude\ \alpha = |\alpha|\, e^{i\varphi}$$

$|\alpha|^2$ *Probability density*

φ *Phase*

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QUBIT

$$|\phi> = \alpha\,|0> + \beta\,|1> = \begin{bmatrix}\alpha \\ \beta\end{bmatrix}$$

$$|\psi> = \gamma\,|0> + \delta\,|1> = \begin{bmatrix}\gamma \\ \delta\end{bmatrix}$$

$$<\phi| = \bar{\alpha} < 0| + \bar{\beta} < 1| = \begin{bmatrix}\bar{\alpha}\ \bar{\beta}\end{bmatrix}$$

$$<\phi|\phi> = |\alpha|^2 + |\beta|^2$$

$$<\psi|\phi> = \alpha\,\bar{\gamma} + \beta\,\bar{\delta}$$

Measurement of $|\phi>$ in standard basis (|0>, |1>) :

|0> with probability $|\alpha|^2$. State after measurement |0>

or

|1> with probability $|\beta|^2$. State after measurement |1>

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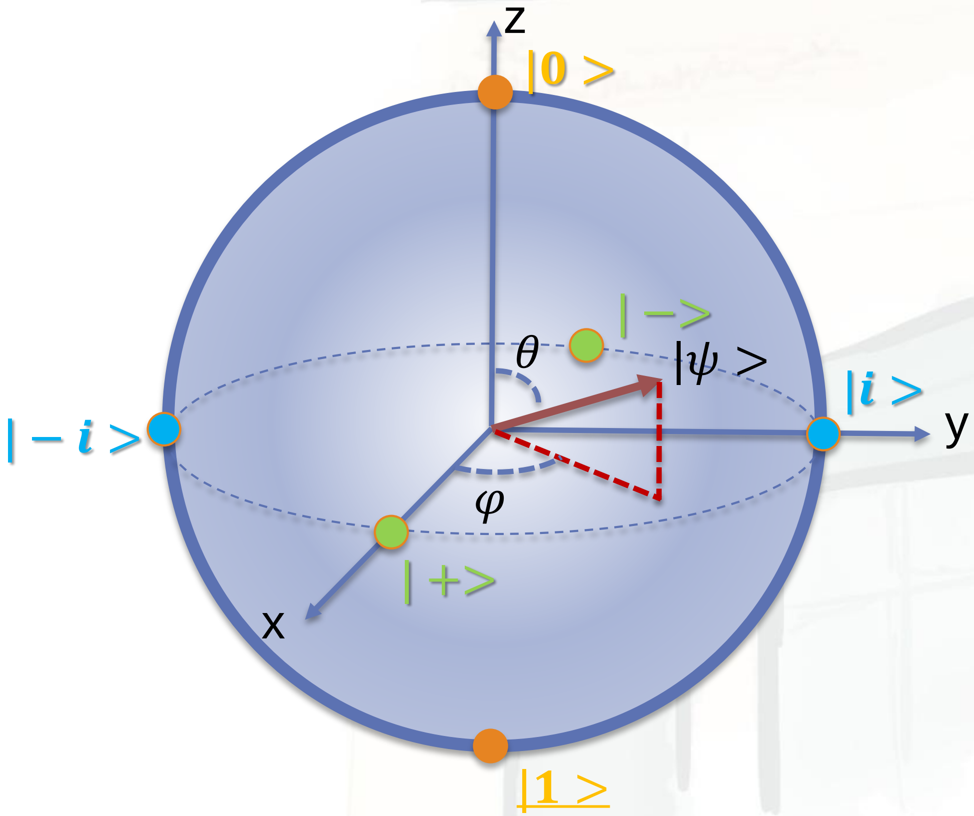
Bloch's Sphere

$$|\psi> = \cos(\frac{\theta}{2})|0> \; + \sin(\frac{\theta}{2})\, e^{i\varphi}|1>$$



$$\theta = \frac{\pi}{2}, \varphi = 0, |+> = \frac{|0> + |1>}{\sqrt{2}}$$

$$\theta = \frac{\pi}{2}, \varphi = \pi, |-> = \frac{|0> - |1>}{\sqrt{2}}$$

$$\theta = \frac{\pi}{2}, \varphi = \frac{\pi}{2}, |i> = \frac{|0> + i|1>}{\sqrt{2}}$$

$$\theta = \frac{\pi}{2}, \varphi = \frac{3\pi}{2}, |-i> = \frac{|0> - i|1>}{\sqrt{2}}$$

Hint: $e^{i\varphi}$ = cos(φ)+i sin(φ)

One-Qubit Transformations

$$|\phi> = \alpha\,|0> + \beta\,|1> = \begin{bmatrix}\alpha \\ \beta\end{bmatrix}$$

- Transform vector space in itself $|\phi'>$ = U $|\phi> = \begin{bmatrix}a_{11} & a_{12} \\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}\alpha \\ \beta\end{bmatrix}$
- Unit lengh vectors must go to unit lenght vectors: $<\psi\left|U^{\dagger}U\right|\phi> = <\psi|\phi> \Rightarrow U^{\dagger}U$=I
- **Reversible**
- Geometrically, they are rotations of the complex vector space associated to $|\phi>$

$Note$: $U^{\dagger} = \overline{U^{T}}$

One-Qubit Transformations

- Phase shift $K(\delta) = e^{i\delta} I$
- Rotation, $R(\beta) = \begin{bmatrix} \cos(\beta) & \sin(\beta) \\ -\sin(\beta) & cos(\beta) \end{bmatrix}$
- Phase rotation, $T(\alpha) = \begin{bmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{bmatrix}$
- Any other Qubit unitary transformation can be written as:

$$K(\delta)T(\alpha)R(\beta)T(\gamma) = \begin{bmatrix} e^{i(\delta+\alpha+\gamma)}\cos(\beta) & e^{i(\delta+\alpha-\gamma)}\sin(\beta) \\ -e^{i(\delta-\alpha+\gamma)}\sin(\beta) & e^{i(\delta-\alpha-\gamma)}\cos(\beta) \end{bmatrix}$$

Source: Eleanor G. Rieffel.Quantum Computing: A Gentle Introduction

One-Qubit Transformations

- Phase shift $K(\delta) = e^{i\delta} I$
- Rotation around x, $R_x(\theta) \equiv e^{-i\theta X} = \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) & -i\sin\left(\frac{\theta}{2}\right) \\ -i\sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{bmatrix} = \cos\left(\frac{\theta}{2}\right) I - i\, sin\left(\frac{\theta}{2}\right) X$
- Rotation around y, $R_y(\theta) \equiv e^{-i\theta Y} = \begin{bmatrix} \cos(\frac{\theta}{2}) & -\sin(\frac{\theta}{2}) \\ \sin(\frac{\theta}{2}) & cos(\frac{\theta}{2}) \end{bmatrix}$
- Phase rotation, Rotation around z, $R_z(\theta) \equiv e^{-i\theta Z} = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix}$
- Any other QuBit unitary transformation can be written as:

$$U = K(\delta) Rz(\gamma) R_y(\beta) R_z(\alpha)$$

Source: Nielsen & Chuang, Quantum Computation And Quantum Information

One-Qubit Transformations

- Pauli Gates
  - $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, bit-flip or NOT.
  - $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
  - $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

- Cliffort group
  - $Hadamard,\ H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
  - $Phase, S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
  - $\frac{\pi}{8}, T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{bmatrix}$

Hint: $U^{\dagger} = \overline{U^T}$

- IBM group

$$U_1(\lambda) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\lambda} \end{bmatrix}$$

$$U_2(\phi, \lambda) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -e^{i\lambda} \\ e^{i\phi} & e^{i(\phi+\lambda)} \end{bmatrix}$$

$$U_3(\theta, \phi, \lambda) = \begin{bmatrix} \cos(\frac{\theta}{2}) & -\sin(\frac{\theta}{2})e^{i\lambda} \\ \sin(\frac{\theta}{2})e^{i\phi} & \cos(\frac{\theta}{2})e^{i(\phi+\lambda)} \end{bmatrix}$$

Expectation Value of U

$$< U > \equiv < \varphi | U | \varphi >$$

Example:

$$< 0|Z|0 >= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1$$

$$< 1|Z|1 >= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -1$$

$$|\varphi > = a|0 > + b|1 >$$

$$< \varphi|Z|\varphi > = \left(\bar{a} < 0\right| + \bar{b} < 1\left|\right) Z(a|0 > + b|1 >) = |a|^2 < 0|Z|0 > + |b|^2 < 1|Z|1 >$$

Exercise with 1 QuBit

OPEN QUIRK.HTML

Multi-Qubits

TENSOR PRODUCT

$$|a> = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$|b> = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \otimes \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} a_1 \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \\ a_2 \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} a_1 b_1 \\ a_1 b_2 \\ a_2 b_1 \\ a_2 b_2 \end{bmatrix}$$

Multi-Qubits

TENSOR PRODUCT

$$|0> = \begin{bmatrix}1\\0\end{bmatrix}$$

$$|1> = \begin{bmatrix}0\\1\end{bmatrix}$$

$$|0> \otimes|0>= |00> = \begin{bmatrix}1\\0\\0\\0\end{bmatrix} = |0>$$

$$|1> \otimes|0>= |10> = \begin{bmatrix}0\\0\\1\\0\end{bmatrix} = |2>$$

$$|0> \otimes|1>= |01> = \begin{bmatrix}0\\1\\0\\0\end{bmatrix} = |1>$$

$$|1> \otimes|1>= |11> = \begin{bmatrix}0\\0\\0\\1\end{bmatrix} = |3>$$

Superposition Multi-Qubits

For 2 QuBits:

$$|\boldsymbol{\psi}> = \boldsymbol{\alpha}|\mathbf{00}> + \boldsymbol{\beta}|\mathbf{01}> + \boldsymbol{\gamma}|\mathbf{10}> + \boldsymbol{\delta}|\mathbf{11}>$$

$$|\boldsymbol{\psi}> = \boldsymbol{\alpha}|\mathbf{0}> + \boldsymbol{\beta}|\mathbf{1}> + \boldsymbol{\gamma}|\mathbf{2}> + \boldsymbol{\delta}|\mathbf{3}>$$

For N QuBits:

$$|\boldsymbol{\psi}> = \sum_{\boldsymbol{i}=\mathbf{0}}^{\mathbf{2}^{\boldsymbol{N}}-\mathbf{1}} \boldsymbol{\lambda}_{\boldsymbol{i}}|\boldsymbol{i}>$$

Pay Attention. You can map classical information to:

- $|i >$, example Shor's algorithm

  and/or

- λ_i, example HHL algorithm

Entanglement Multi-Qubits

When you cannot write a state as a product of single states

$$|\Phi^{\pm}> = \frac{1}{\sqrt{2}}\,(\,|00> \,\pm\, |11>) \neq (\alpha\,|0> + \beta|1>) \otimes (\gamma\,|0> + \delta|1>)$$

AND NOW, YOU HAVE WONDERFUL THINGS AS TELEPORTATION!

Multi-Qubit Transformations

$Let\ U_1 = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{21} \end{bmatrix}$ on qubit 1

$Let\ V_2 = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{21} \end{bmatrix}$ on qubit 2

$$U_1 \otimes V_2 = \begin{bmatrix} U_{11}\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{21} \end{bmatrix} & U_{12}\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{21} \end{bmatrix} \\ U_{21}\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{21} \end{bmatrix} & U_{22}\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{21} \end{bmatrix} \end{bmatrix}$$

$$U_1 \otimes V_2 = \begin{bmatrix} U_{11}V_{11} & U_{11}V_{12} & U_{12}V_{11} & U_{12}V_{12} \\ U_{11}V_{21} & U_{11}V_{22} & U_{12}V_{21} & U_{12}V_{22} \\ U_{21}V_{11} & U_{21}V_{12} & U_{22}V_{11} & U_{22}V_{12} \\ U_{21}V_{21} & U_{21}V_{22} & U_{22}V_{21} & U_{22}V_{22} \end{bmatrix}$$

Multi-Qubit Transformations

Example: Apply X gate on second qubii. Let first qubit unchanged

$$I \otimes X = \begin{bmatrix} 1 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} & 0 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ 0 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} & 1 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \end{bmatrix}$$

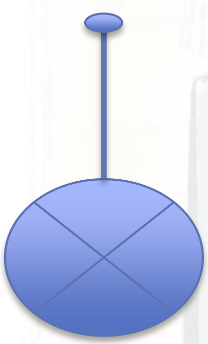
$$I \otimes X = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Controled Gates

Apply one gate on one qubit, depending on the values of other qubits

$$CNOT = |0><0| \otimes I + |1><1| \otimes X$$

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



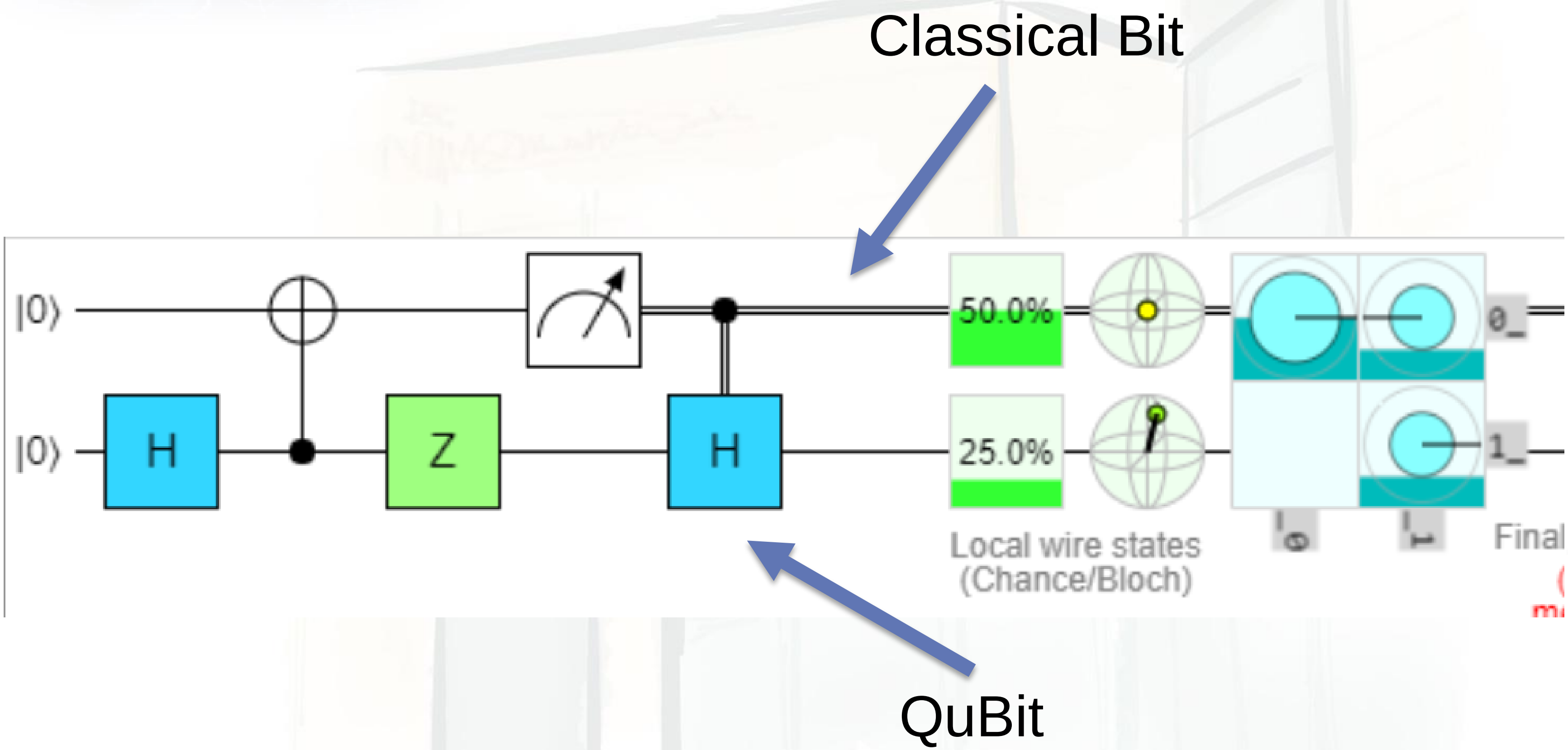
$$CNOT|00> = |00>$$

$$CNOT|01> = |01>$$

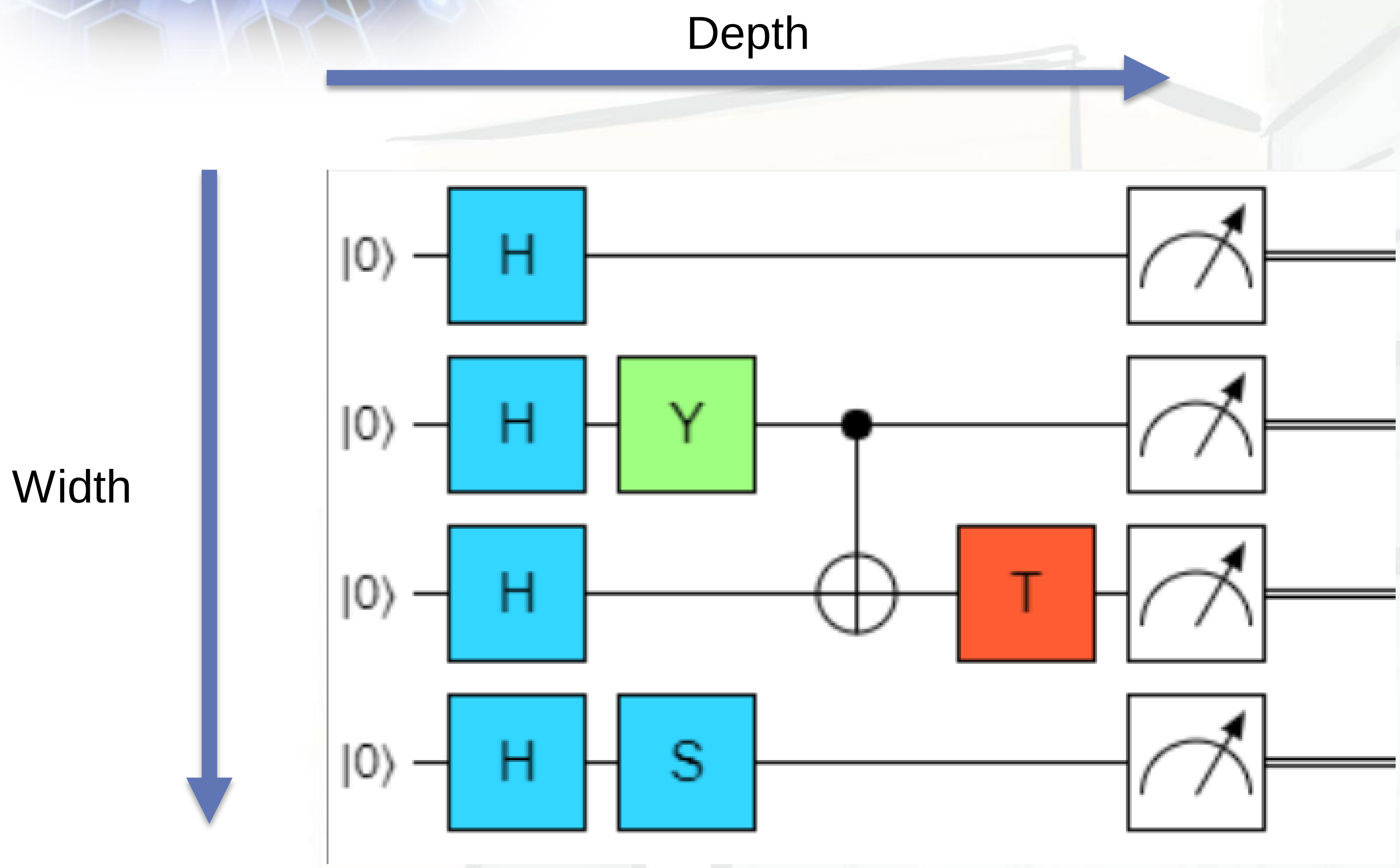
$$CNOT|10> = |11>$$

$$CNOT|11> = |10>$$

Measurement



Quantum Circuit



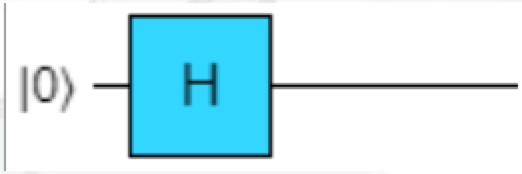
MY FIRST QUANTUM PROGRAM: Superdense Coding

OPEN QUIRK.HTML

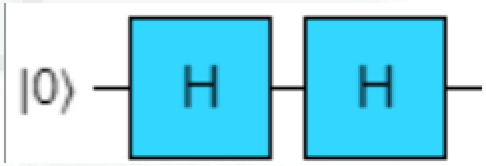
My First Quantum Program

- Using Quirk. Launch quirk.html. **QUIRK does not need measurement. Remember to add it in your real circuit.**

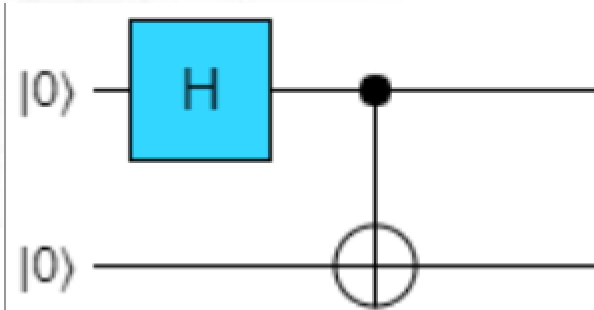
- Apply a Hadamard Gate (H) on the first qubit



- Apply a second H to the same qubit. Result?



- Remove Second H and apply a CNOT on a second qubit.
  - Result: an entangled system (Bell's)



Superdense Coding

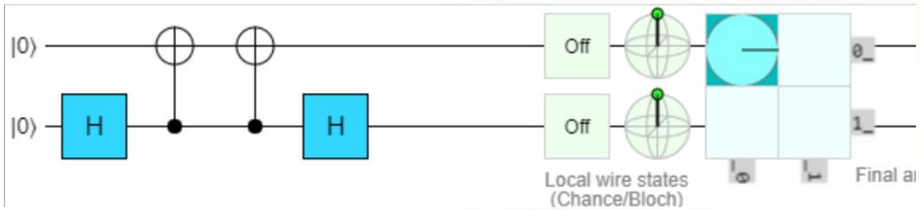
- Transmit two classical bits with a single qubit

A. **Bobs generates a Bell's state**
B. **Bob sends one qubit to Alice. Bob keeps the second.**
C. **Alice applies a single-qubit gate to her qubit to encode 2 bits:**
   - **01 -> X**
   - **10 -> Z**
   - **11 -> Y**
   - **00 -> I**

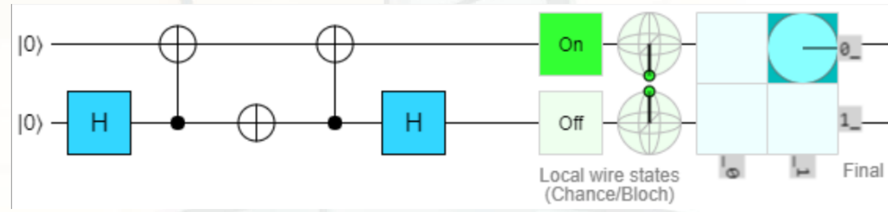
D. **Alice returns her qubit to Bob.**
E. **Bob uncomputes entanglement (applies the gates in reverse order)**
F. **Bob measures both qubits.**

Superdense Coding

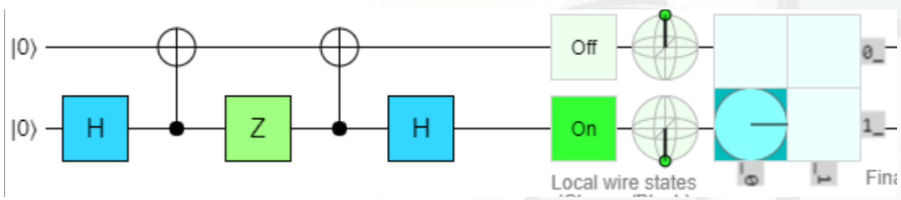
|00>



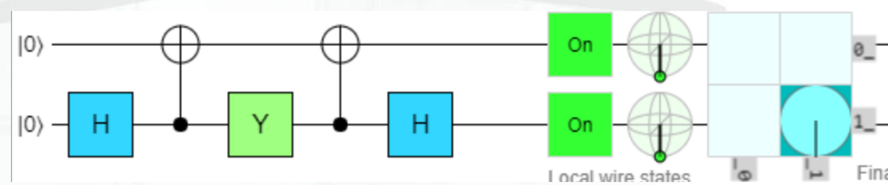
|01>



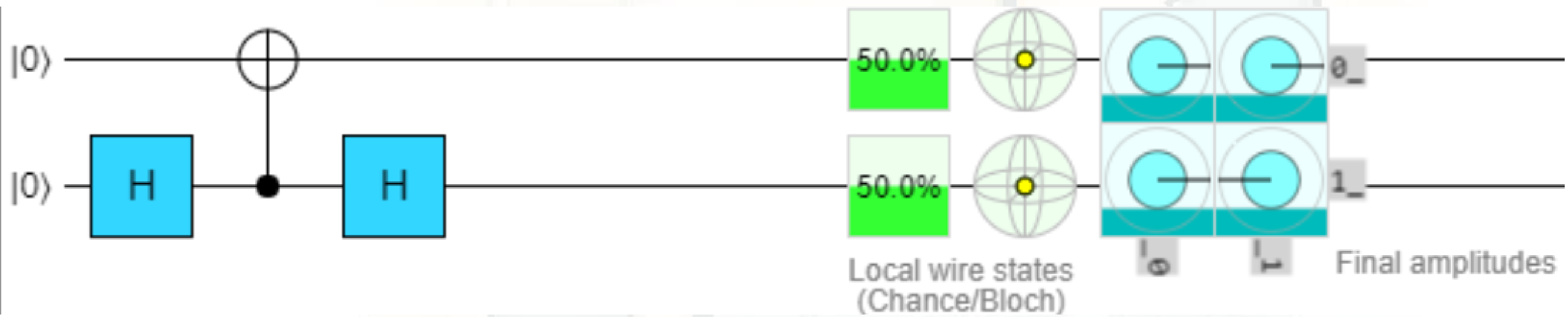
|10>



|11>



Caution!!!

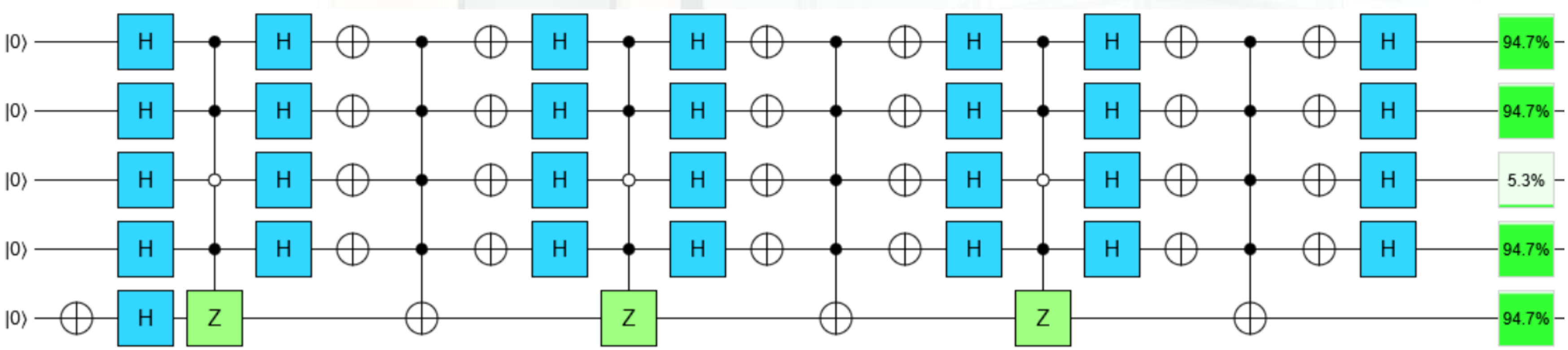

|0⟩
|0⟩
H
H
50.0%
50.0%
0_
1_
Local wire states
(Chance/Bloch)
Final amplitudes

Exercise 2: IBM Quantum Experience

CONNECT TO: HTTPS://QUANTUM-COMPUTING.IBM.COM/

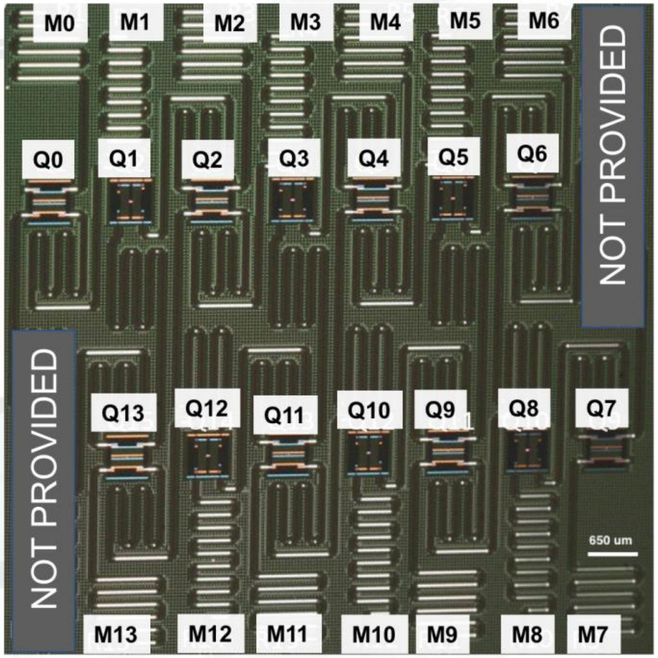
Quantum Volume

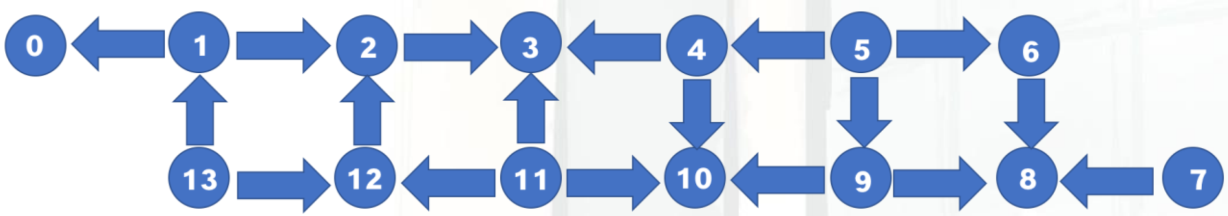
- Width: The number of physical qubits;
- Depth: The number of gates that can be applied before errors make the device behave essentially classically;
- **Topology**: The connectivity of the device;
- Gate Parallelism: The number of operations that can be run in parallel



TOPOLOGY

https://medium.com/rigetti/the-rigetti-128-qubit-chip-and-what-it-means-for-quantum-df757d1b71ea





https://github.com/Qiskit/ibmq-device-information/blob/master/backends/melbourne

Quantum Volume

- Effective error rate ε_{eff}: specifying how well a device can implement arbitrary pairwise interactions between qubits

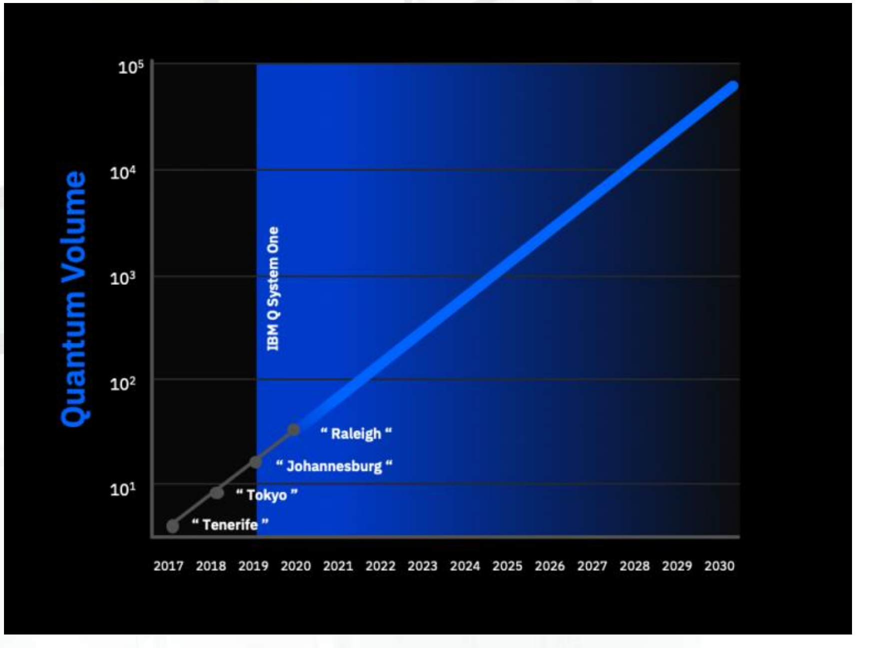
- n is the number of qubits of the Computer

- n' number of qubits used by the algorithm

- Depth $d \simeq \dfrac{1}{n\,\varepsilon_{eff}}$

- Quantum Volume

$$V_Q = \max_{n'<n} min\left[n', \frac{1}{n\prime\,\varepsilon_{eff}(n')}\right]^2$$



Source: IBM, 2019

Classical Resources

<table>
<tr><th>1 qubit</th><th>2 qubits</th><th>3 qubits</th><th>N qubits</th></tr>
<tr><td>|0></td><td>|00> = |0></td><td>|000> = |0></td><td>|0….0> = |0></td></tr>
<tr><td>|1></td><td>|01> = |1></td><td>|001> = |1></td><td>|0….1> = |1></td></tr>
<tr><td></td><td>|10> = |2></td><td>|010> = |2></td><td></td></tr>
<tr><td></td><td>|11> = |3></td><td>|011> = |3></td><td></td></tr>
<tr><td></td><td></td><td>|100> = |4></td><td></td></tr>
<tr><td></td><td></td><td>|101> = |5></td><td></td></tr>
<tr><td></td><td></td><td>|110> = |6></td><td></td></tr>
<tr><td></td><td></td><td>|111> = |7></td><td></td></tr>
<tr><td></td><td></td><td></td><td>|1…..1> = |2^N-1></td></tr>
<tr><td>2</td><td>4</td><td>8</td><td>2^N</td></tr>
<tr><td>$\alpha\ |0 > \ + \beta\ |1 >$</td><td>$\sum_{n=0}^{3} \alpha_n |n >$</td><td>$\sum_{n=0}^{7} \alpha_n |n >$</td><td>$\sum_{n=0}^{2^N-1} \alpha_n |n >$</td></tr>
<tr><td>2 * complex= 2x2x8=32 bytes</td><td>4*2*8=64 bytes</td><td>8*2*8=128 bytes</td><td>$2^N * 2^4 = 2^{N+4}$</td></tr>
</table>

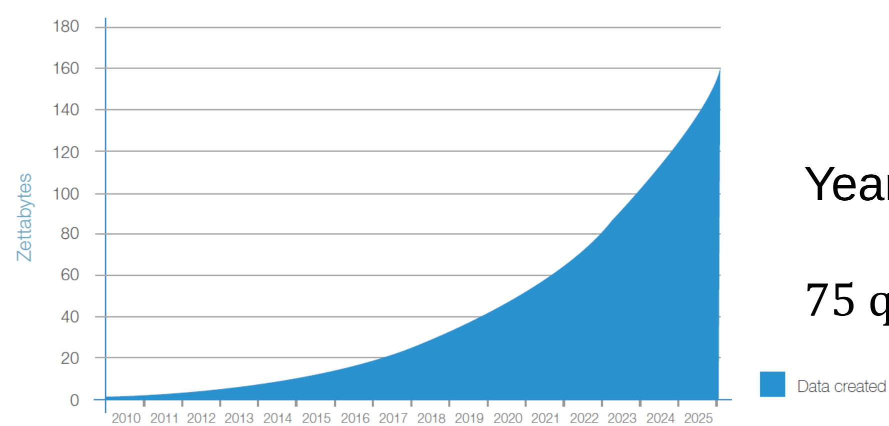
How many states?

Asimov calculated the number of nucleons+electrons in the Universe as $\sim 10^{79}$

$\sim$ 10.000.000.000.000.000.000.000.000.000.000.000.000.000.000.000.000.000.000.000.000.000.000.000.000.000.000.000

Having a QPU with 270 qubits, one can store in the amplitudes: $\sim 10^{81}$ FPs.

Figure 2. | Annual Size of the Global Datasphere



Source: IDC's Data Age 2025 study, sponsored by Seagate, April 2017

Year 2025: $\sim 170\text{ZB/year} \sim 10^{23}$ bytes/year

75 qubits: $\sim 3 \cdot 10^{23}$ FPs $= \sim 24$ years!!!!

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Classical Resources

qubits	RAM
1	32 bytes + memory for gates
2	64 bytes + memory for gates
3	128 bytes + memory for gates
4	256 bytes + memory for gates
8	4 kbytes + memory for gates
16	1 Mbytes + memory for gates
32	64 Gbytes + memory for gates
36	1TB + …..
38	**4TB (Limit CESGA FT2 FAT node ….)**
45	0,5PB [1]
64	512 ExaBytes!!!

THIS IS ONLY TRUE IF YOU NEED ALL POSSIBLE STATES!

[1] Häner, T., & Steiger, D. S. (2017). 0.5 Petabyte Simulation of a 45-Qubit Quantum Circuit. Arxiv:1704.01127

End of Lecture 1
Questions?