

# **Classical and Quantum Integrable Systems - Alessandro Torrielli**

**Monday, September 16, 2019 - Friday, September 20, 2019**

**Instituto Galego de Física de Altas Enerxías (IGFAE)**

## **Scientific Program**

## Classical and Quantum Integrable Systems - Program

### • Session 1 (2 hours) Classical integrability part 1:

- Liouville theorem, examples of classically integrable dynamical systems, superintegrability (1 hour) [1];
- Lax pair, monodromy and transfer matrix, example of the Non-Linear Schrödinger Equation (NLS) (1 hour) [1, 2].

*Prerequisites: Classical Mechanics*

### • Session 2 (2 hours) Classical integrability part 2:

- Classical r-matrices, Belavin-Drinfeld theorems, non-local charges, example of the Principal Chiral Model (1 hour) [1, 3];
- Classical inverse scattering method, solitons (1 hour) [1, 3].

*Prerequisites: Lie algebras*

### • Session 3 (2 hours) Quantum integrability part 1:

- Hopf algebras and universal R-matrix, quantum groups, example of  $U_q(\mathfrak{su}(2))$  (1 hour) [4];
- RTT relations, Algebraic Bethe Ansatz, example of the NLS (1 hour) [5–7].

*Prerequisites: Quantum Mechanics*

### • Session 4 (2 hours) Quantum integrability part 2:

- Exact S-matrices, bound states, perturbation theory of the NLS (1 hour) [8, 9];
- Coordinate Bethe ansatz and Thermodynamic Bethe ansatz (1 hour) [10].

*Prerequisites: Classical and Quantum Statistical Mechanics*

### • Session 5 (2 hours) Quantum integrability part 3:

- massless scattering and massless flows, example of the Tricritical to Critical Ising Model (1 hour) [11];
- Quantisation of the Kadomtsev-Petviashvili equation (1 hour) [12];

*Prerequisites: Quantum Field Theory*

## References

- [1] Alessandro Torrielli. Lectures on Classical Integrability. *J. Phys.*, A49(32):323001, 2016.
- [2] E. K. Sklyanin. Quantum version of the method of inverse scattering problem. *J. Sov. Math.*, 19:1546–1596, 1982. [*Zap. Nauchn. Semin.*95,55(1980)].
- [3] Olivier Babelon, Denis Bernard, and Michel Talon. *Introduction to Classical Integrable Systems*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2003.
- [4] C. Kassel. *Quantum groups*. 1995.
- [5] L. D. Faddeev. How algebraic Bethe ansatz works for integrable model. In *Relativistic gravitation and gravitational radiation*. Proceedings, School of Physics, Les Houches, France, September 26-October 6, 1995, pages pp. 149–219, 1996.
- [6] Fedor Levkovich-Maslyuk. The Bethe ansatz. *J. Phys.*, A49(32):323004, 2016.
- [7] H. B. Thacker. Exact Integrability in Quantum Field Theory. In *4th Workshop on Current Problems in High-Energy Particle Theory* Bad Honnef, Germany, June 2-4, 1980, page 0179, 1980.
- [8] P. Dorey. Exact S matrices. In *Conformal field theories and integrable models*. Proceedings, Eotvos Graduate

Course, Budapest, Hungary, August 13-18, 1996, pages 85–125, 1996.

[9] Diego Bombardelli. S-matrices and integrability. *J. Phys.*, A49(32):323003, 2016.

[10] Stijn J. van Tongeren. Introduction to the thermodynamic Bethe ansatz. 2016. [*J. Phys.*A49,no.32,323005(2016)].

[11] P. Fendley and H. Saleur. Massless integrable quantum field theories and massless scattering in

(1+1)-dimensions. In *Proceedings, Summer School in High-energy physics and cosmology: Trieste, Italy, June*

14-July 30, 1993, pages 301–332, 1993. [,87(1993)].

[12] Karol K Kozlowski, Evgeny Sklyanin, and Alessandro Torrielli. Quantization of the KadomtsevPetviashvili

equation. *Theor. Math. Phys.*, 192(2):1162–1183, 2017.