

# **Classical and Quantum Integrable Systems - Alessandro Torrielli**

**Monday 16 September 2019 - Friday 20 September 2019**

**Instituto Galego de Física de Altas Enerxías (IGFAE)**

## **Scientific Programme**

## Classical and Quantum Integrable Systems - Program

- **Session 1** (2 hours) **Classical integrability part 1:**

- Liouville theorem, examples of classically integrable dynamical systems, superintegrability (1 hour) [1];
- Lax pair, monodromy and transfer matrix, example of the Non-Linear Schrödinger Equation (NLS) (1 hour) [1, 2].

*Prerequisites: Classical Mechanics*

- **Session 2** (2 hours) **Classical integrability part 2:**

- Classical r-matrices, Belavin-Drinfeld theorems, non-local charges, example of the Principal Chiral Model (1 hour) [1, 3];
- Classical inverse scattering method, solitons (1 hour) [1, 3].

*Prerequisites: Lie algebras*

- **Session 3** (2 hours) **Quantum integrability part 1:**

- Hopf algebras and universal R-matrix, quantum groups, example of  $U_q(\text{su}(2))$  (1 hour) [4];
- RTT relations, Algebraic Bethe Ansatz, example of the NLS (1 hour) [5–7].

*Prerequisites: Quantum Mechanics*

- **Session 4** (2 hours) **Quantum integrability part 2:**

- Exact S-matrices, bound states, perturbation theory of the NLS (1 hour) [8, 9];
- Coordinate Bethe ansatz and Thermodynamic Bethe ansatz (1 hour) [10].

*Prerequisites: Classical and Quantum Statistical Mechanics*

- **Session 5** (2 hours) **Quantum integrability part 3:**

- massless scattering and massless flows, example of the Tricritical to Critical Ising Model (1 hour) [11];
- Quantisation of the Kadomtsev-Petviashvili equation (1 hour) [12];

*Prerequisites: Quantum Field Theory*

## References

- [1] Alessandro Torrielli. Lectures on Classical Integrability. *J. Phys.*, A49(32):323001, 2016.
- [2] E. K. Sklyanin. Quantum version of the method of inverse scattering problem. *J. Sov. Math.*, 19:1546–1596, 1982. [*Zap. Nauchn. Semin.* 95, 55(1980)].
- [3] Olivier Babelon, Denis Bernard, and Michel Talon. *Introduction to Classical Integrable Systems*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2003.
- [4] C. Kassel. *Quantum groups*. 1995.
- [5] L. D. Faddeev. How algebraic Bethe ansatz works for integrable model. In *Relativistic gravitation and gravitational radiation*. Proceedings, School of Physics, Les Houches, France, September 26–October 6, 1995, pages pp. 149–219, 1996.
- [6] Fedor Levkovich-Maslyuk. The Bethe ansatz. *J. Phys.*, A49(32):323004, 2016.
- [7] H. B. Thacker. Exact Integrability in Quantum Field Theory. In *4th Workshop on Current Problems in High-Energy Particle Theory* Bad Honnef, Germany, June 2–4, 1980, page 0179, 1980.
- [8] P. Dorey. Exact S matrices. In *Conformal field theories and integrable models*. Proceedings, Eotvos Graduate

- Course, Budapest, Hungary, August 13-18, 1996, pages 85–125, 1996.
- [9] Diego Bombardelli. S-matrices and integrability. *J. Phys.*, A49(32):323003, 2016.
- [10] Stijn J. van Tongeren. Introduction to the thermodynamic Bethe ansatz. 2016. [*J. Phys.*A49,no.32,323005(2016)].
- [11] P. Fendley and H. Saleur. Massless integrable quantum field theories and massless scattering in (1+1)-dimensions. In Proceedings, Summer School in High-energy physics and cosmology: Trieste, Italy, June 14-July 30, 1993, pages 301–332, 1993. [,87(1993)].
- [12] Karol K Kozlowski, Evgeny Sklyanin, and Alessandro Torrielli. Quantization of the KadomtsevPetviashvili equation. *Theor. Math. Phys.*, 192(2):1162–1183, 2017.