Integrability in gauge and string theory

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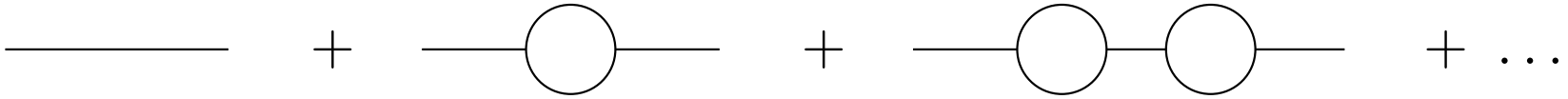
Plan

- Maximally supersymmetric **gauge theory in 4D**
- **String theory** on $AdS_5 \times S^5$
- **Deformations** preserving integrability

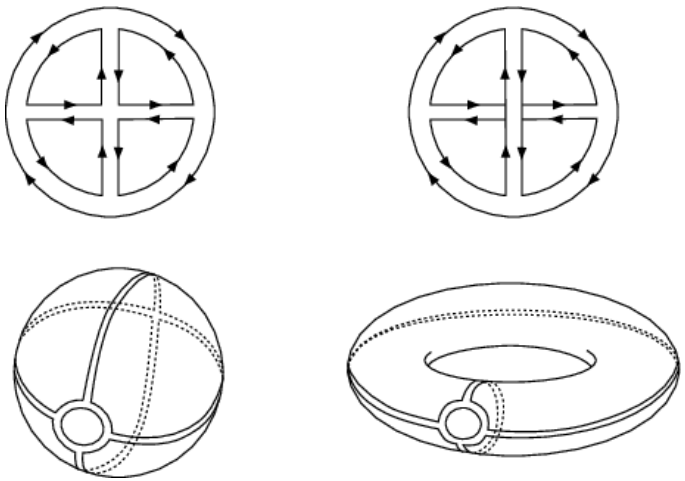
Gauge theory



$\mathcal{N}$=4 super Yang-Mills

gauge group $SU(N)$, coupling constant g_{YM}
scalar+fermions+gauge bosons with **maximal susy in 4D**

Conformal: $\beta(g_{\text{YM}}) = 0$



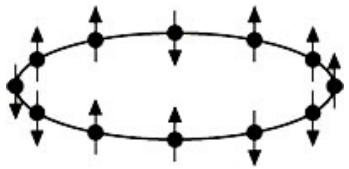
Planar limit:
$g_{\text{YM}} \to 0,\ N \to \infty$
while 't Hooft coupling
$\lambda \equiv N g_{\text{YM}}^2$ is fixed

picture stolen from Alfonso's review [arXiv:1310.4319]

The spin chain

$\mathfrak{su}(2)$ sector $\supset$ scalar fields $\underset{\downarrow}{\mathbf{\Phi}}, \underset{\uparrow}{\bar{\mathbf{\Phi}}}$ of $\mathcal{N} = 4$ SYM

$$\mathcal{O}(x) = \mathrm{Tr}[\mathbf{\Phi\Phi\bar{\Phi}\bar{\Phi}\Phi\bar{\Phi}\Phi} \dots \mathbf{\Phi\Phi\bar{\Phi}\Phi\Phi}]$$



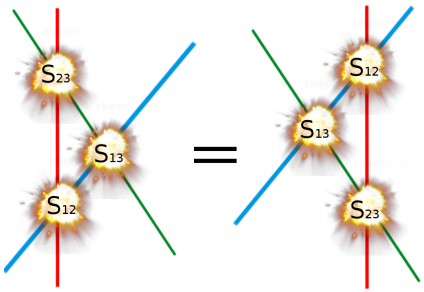
[Minahan, Zarembo 02]

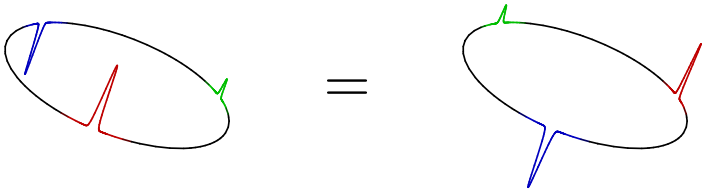
Anomalous dimension at 1-loop: operators mix and the mixing matrix is the Hamiltonian of **Heisenberg's XXX spin chain**!

Higher-loop corrections $\Longrightarrow$ **long-range** interactions

For similar methods applied to QCD see [arXiv:1012.4000]

Magnon excitations interact with **factorised S-matrix**





$$e^{ip_k L} \prod_{j \neq k} S(p_k, p_j) = 1$$

Bethe eqs

S-matrix fixed at **all loops** from supersymmetry and analyticity

Exact spectrum in λ and L (size of the chain) from "Thermodynamic Bethe Ansatz" or "Quantum Spectral Curve"

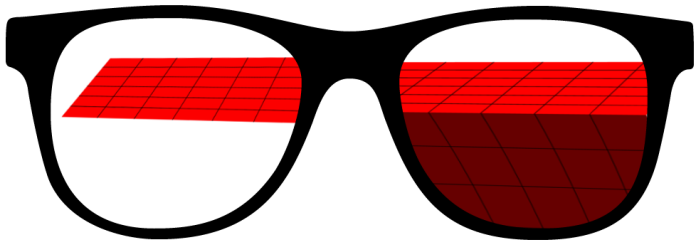
String theory

[Maldacena 97]

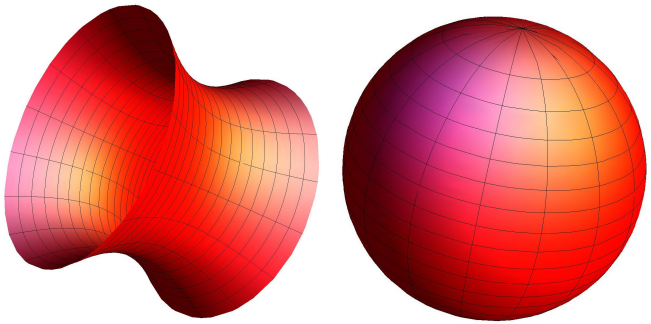
$\mathcal{N} = 4$ SYM

Conformal dimension in a CFT

Energy of a string



$\mathrm{AdS}_5 \times \mathrm{S}^5$



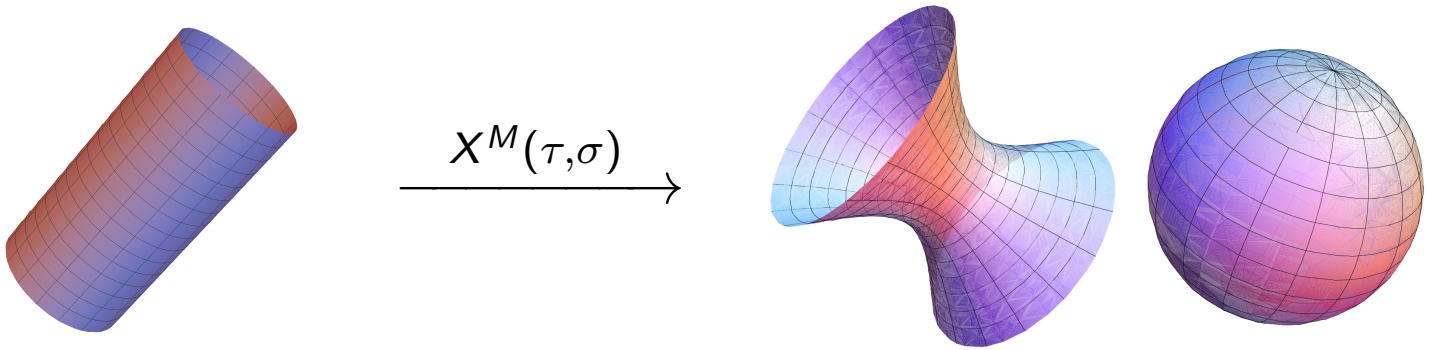
$$-X_0^2 + \sum_{i=1}^{4} X_i^2 - X_5^2 = -1$$

$$\sum_{i=1}^{6} Y_i^2 = 1$$

$\lambda \ll 1$ weakly-coupled gauge theory / $\lambda \gg 1$ classical string

$$S = -\frac{\sqrt{\lambda}}{4\pi}\int \mathrm{d}\tau\mathrm{d}\sigma\ \gamma^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N G_{MN} \ + \ \text{fermions}$$

$$\mathrm{d}s^2 = G_{MN}\ \mathrm{d}X^M \mathrm{d}X^N = \mathrm{d}s^2_{\mathrm{AdS}_5} + \mathrm{d}s^2_{\mathrm{S}^5}$$



Hamiltonian in light-cone gauge for 8 bosons + 8 fermions

$$\mathbf{H} = \mathbf{H}_2 + \frac{1}{\lambda}\mathbf{H}_4 + \frac{1}{\lambda^2}\mathbf{H}_6 + \dots$$

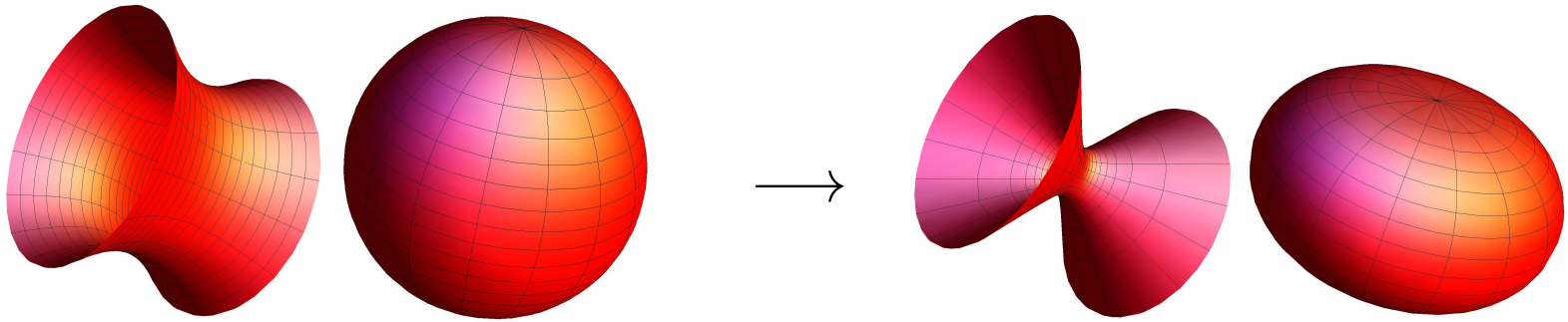
Same S-matrix of spin-chain but expanded at $\lambda \sim \infty$

(Classical integrability)

Deformations

Integrability **beyond** spectrum of AdS_5/CFT_4

- **Higher point-functions**
- **Lower dimensional** dualities
  e.g. AdS_4/CFT_3, AdS_3/CFT_2
- **Deformations** of AdS_5/CFT_4



Break isometries of target space of string

Some deformations $\sim$ twisted boundary conditions for the string

Deformations of the gauge theory?

On the gauge theory we can break e.g. **supersymmetry**, **conformal invariance**

In certain cases, deformations correspond to **non-commutative** gauge theories

Extension of the integrability methods to the deformed models?

Classical integrability: **Lax connection** $L_\alpha(z,\tau,\sigma),\ \alpha=\tau,\sigma$

Flatness condition

$$\boxed{\partial_\alpha L_\beta - \partial_\beta L_\alpha + [L_\alpha, L_\beta] = 0 \iff \text{EOM}\ \ \frac{\delta S}{\delta X^M} = 0}$$

Monodromy matrix:

$$\boxed{T(z) = Pexp \int d\sigma L_\sigma(z)}$$

Generating function of conserved quantities