Introduction to GW from compact binary coalescence (CBC)

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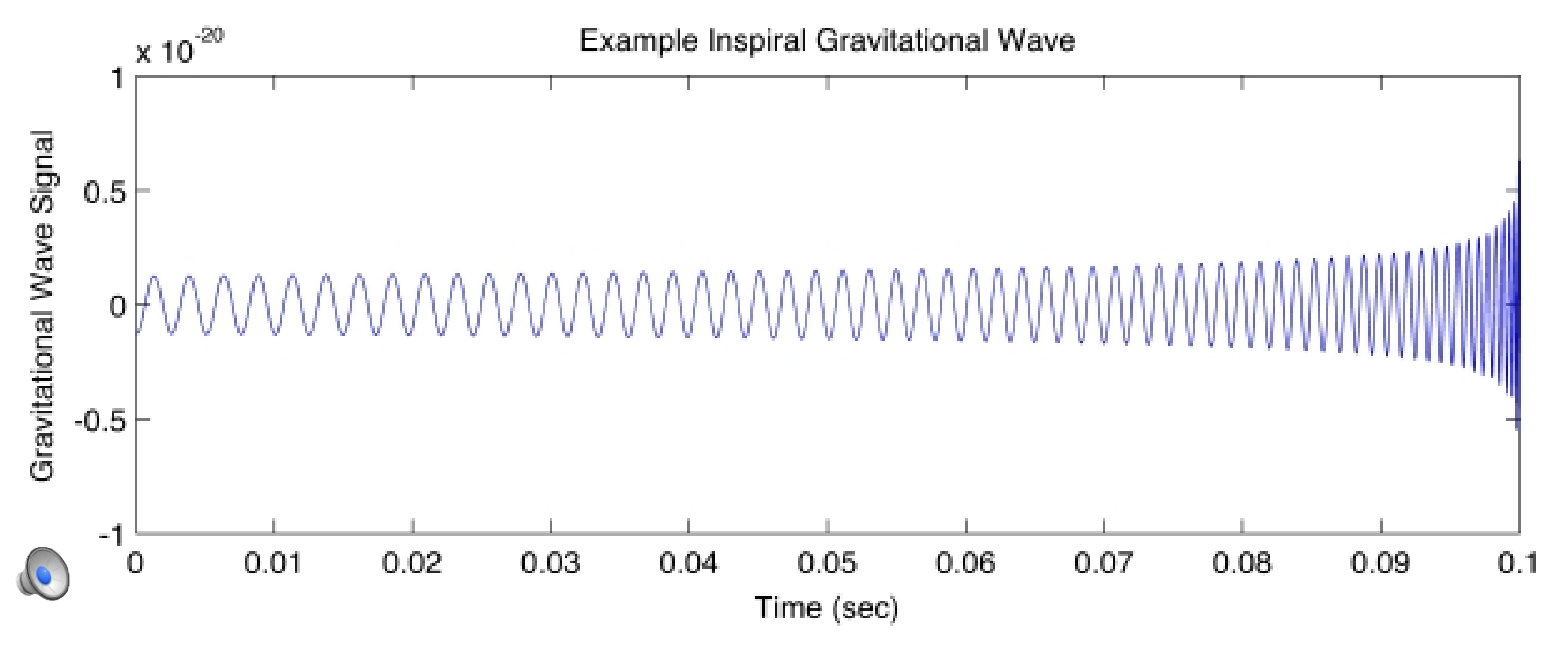
Galician Gravitational Wave Week 2019
Lecture 2

Plan of lecture

- Matched filter detection : why and how
- Form of CBC signals and detectability in LIGO-Virgo
- Basic physics of GW150914 – the first detection
  - How do we know the signal was from a compact binary ?
  - How do we know the (approximate) source parameters?
  - How do we know binary objects were black holes ?
    (or something that behaves very like them)

A binary inspiral chirp

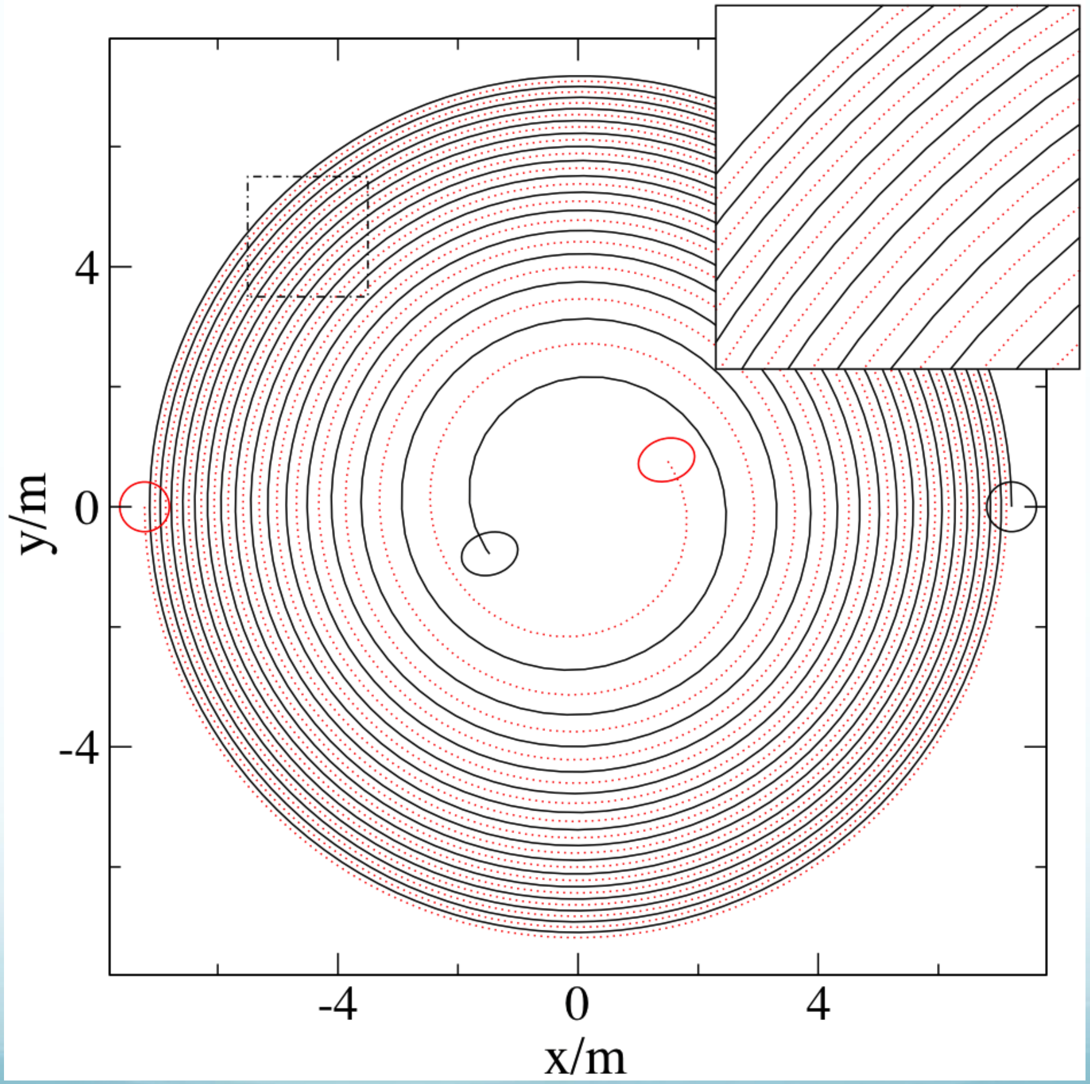


- Highest GW power in last few hundred cycles
- In LIGO frequency band if

  $m \sim$ few $M_\odot$ up to (few×10) $M_\odot$

Image: A. Stuver, LIGO

Binary inspiral orbit



Filtering for inspiral signals

- Want some sort of time dependent filter

  − follow frequency of source as it evolves, exclude noise at other frequencies

- Formal method : ‘matched filtering’

- General idea : transform one time series $s(t)$ into another $w(t)$, then search for peaks in w

$$w(t) = \int_{-\infty}^{\infty} \mathrm{d}t' \, K(t - t')s(t')$$

  - *K* : kernel
  - eg ‘high pass’, ‘low pass’, ‘band pass’ ...

S/N for filter output

- Compare filtered signal with mean square noise fluctuation

$$\left(\frac{S}{N}\right)^2(t) = \frac{|\int \mathrm{d}t'\, K(t-t')h(t')|^2}{\langle | \int \mathrm{d}t'\, K(t-t')n(t')|^2 \rangle}$$

  - Want kernel K that optimizes S/N for known signal $h(t)$ at given output time t (set =0 for simplicity)

Proceed by going to frequency domain ..

$$\frac{S}{N}(t=0) = \frac{\int \mathrm{d}f\, K^*(f)h(f)}{\sqrt{\int \mathrm{d}f\, |K(f)|^2 \frac{1}{2} S_n(f)}}$$

NB power spectral density $S_n(f)$!

Filtering as 'inner product'

- Clever way to find kernel $K(f)$ that maximizes S/N : rewrite SNR as *inner product*
- For data streams *a*(*t*) *b*(*t*)

$$\langle a|b\rangle = \mathrm{Re}\int_{-\infty}^{\infty} df\, \frac{a^*(f)b(f)}{\frac{1}{2}S_n(f)}$$

$$\langle a|b\rangle = \langle b|a\rangle \qquad \langle a|a\rangle \geq 0$$

Now $$\frac{S}{N} = \frac{\langle u|h\rangle}{\sqrt{\langle u|u\rangle}} \quad \text{where} \quad u(f) \equiv \frac{1}{2}S_n(f)K(f)$$

Note : $\dfrac{u(t)}{\sqrt{\langle u|u\rangle}}$ is 'unit vector' in the space of all possible signals

The optimal matched filter

- Unit vector that maximizes inner product with $h(f)$ is proportional to h itself !

$$u(f) = \text{const} \times h(f) \qquad \Rightarrow K(f) = \text{const} \times \frac{h(f)}{S_n(f)}$$

  - Optimal matched filter is the signal, *inverse weighted by the noise spectrum*

- Optimal SNR is just $\sqrt{\langle h|h\rangle}$

- Finally define *normalized* (variance 1) matched filter output for data $s(t)$

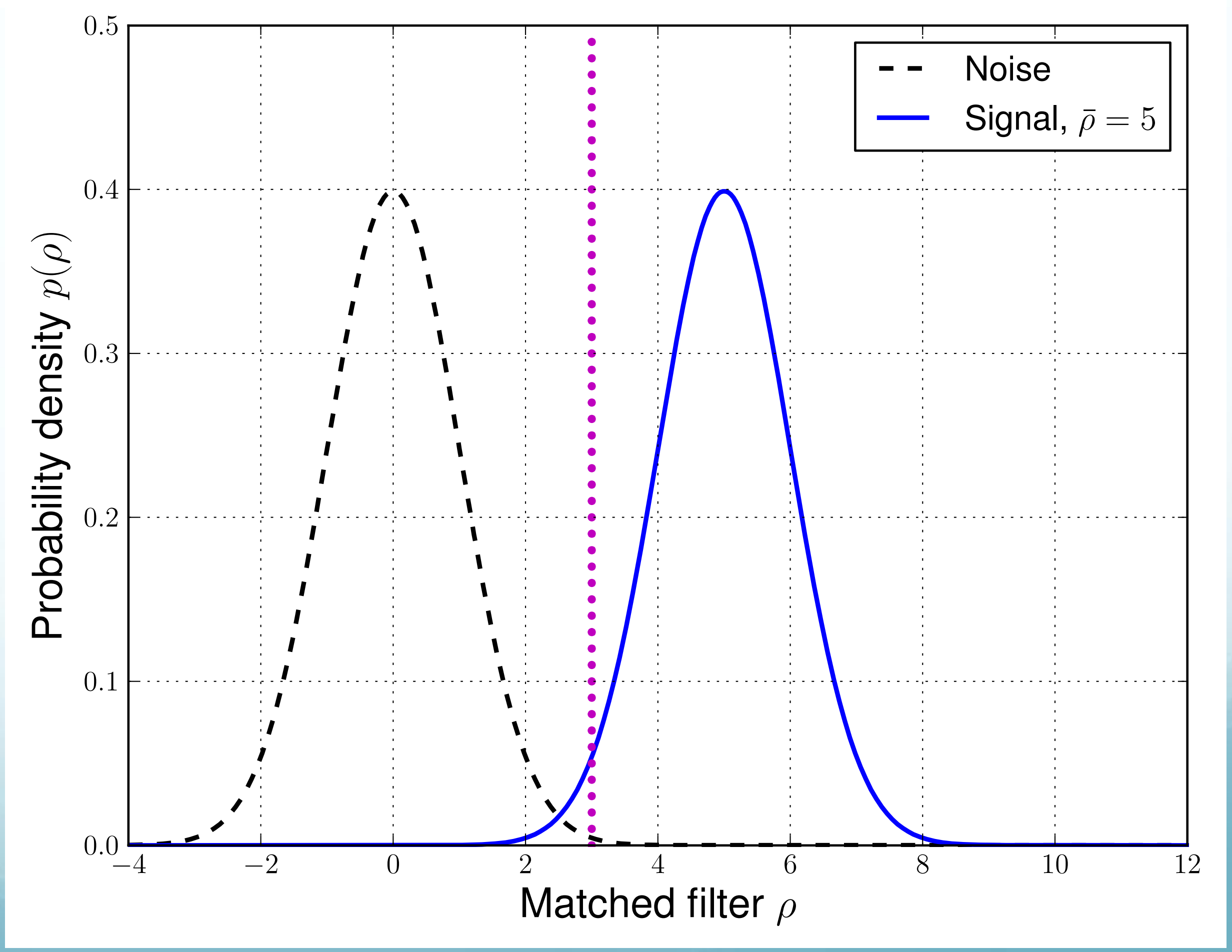
$$\rho = \frac{\langle s|h\rangle}{\sqrt{\langle h|h\rangle}}$$

Matched filter output statistics

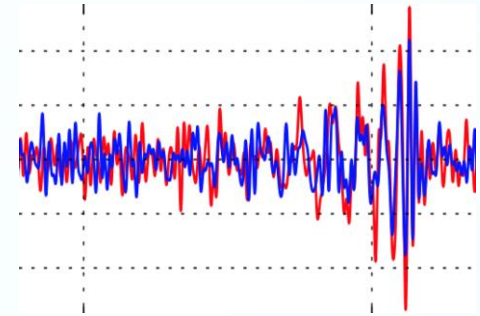
$$p(\rho|\bar{\rho})\,d\rho = \frac{1}{\sqrt{2\pi}} e^{-(\rho-\bar{\rho})^2/2}\,d\rho$$

Expected value in presence of signal *h*

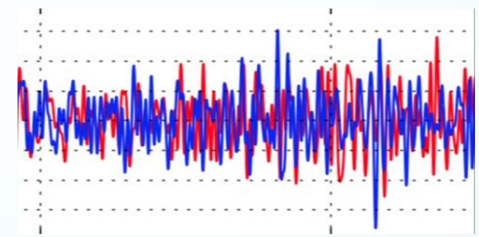
$$\bar{\rho} = \sqrt{\langle h|h \rangle}$$



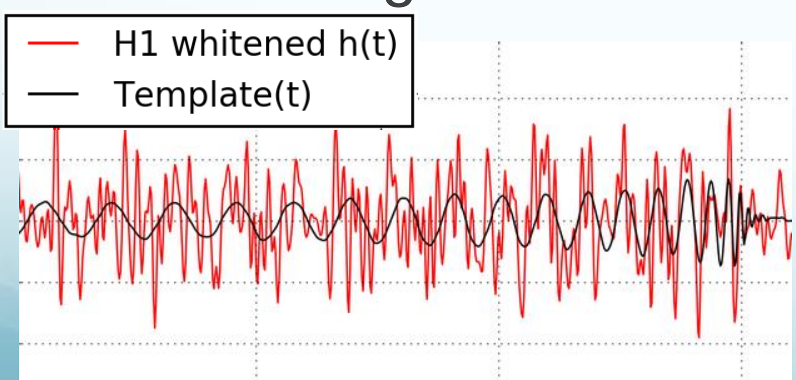
Modelled binary merger search



- GW150914 ‘easily’ visible in detector output

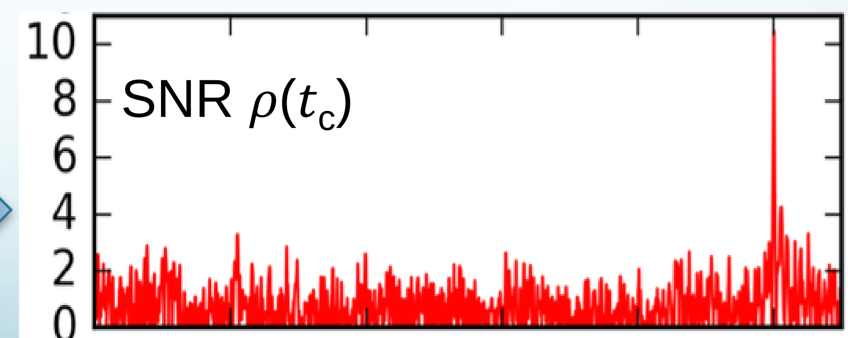


- 2 out of 3 candidates in O1 were not
- GW151226 detected *only* by matched filtering



time t

$\Sigma\, \mathrm{d}f$



merger time t_c

Chirp in frequency domain

- Fourier transform of $h_+(t)$ [*not entirely straightforward*]

$$\tilde{h}_+(f) \propto e^{i\Psi_+(f)} \frac{1}{f^{7/6}}$$

- GW phase in frequency domain:

$$\Psi_+(f) = 2\pi f (t_c + r/c) - \Phi_0 - \frac{\pi}{4} + \frac{3}{4}\left(\frac{GM_c}{c^3} \cdot 8\pi f\right)^{-5/3} + \cdots$$

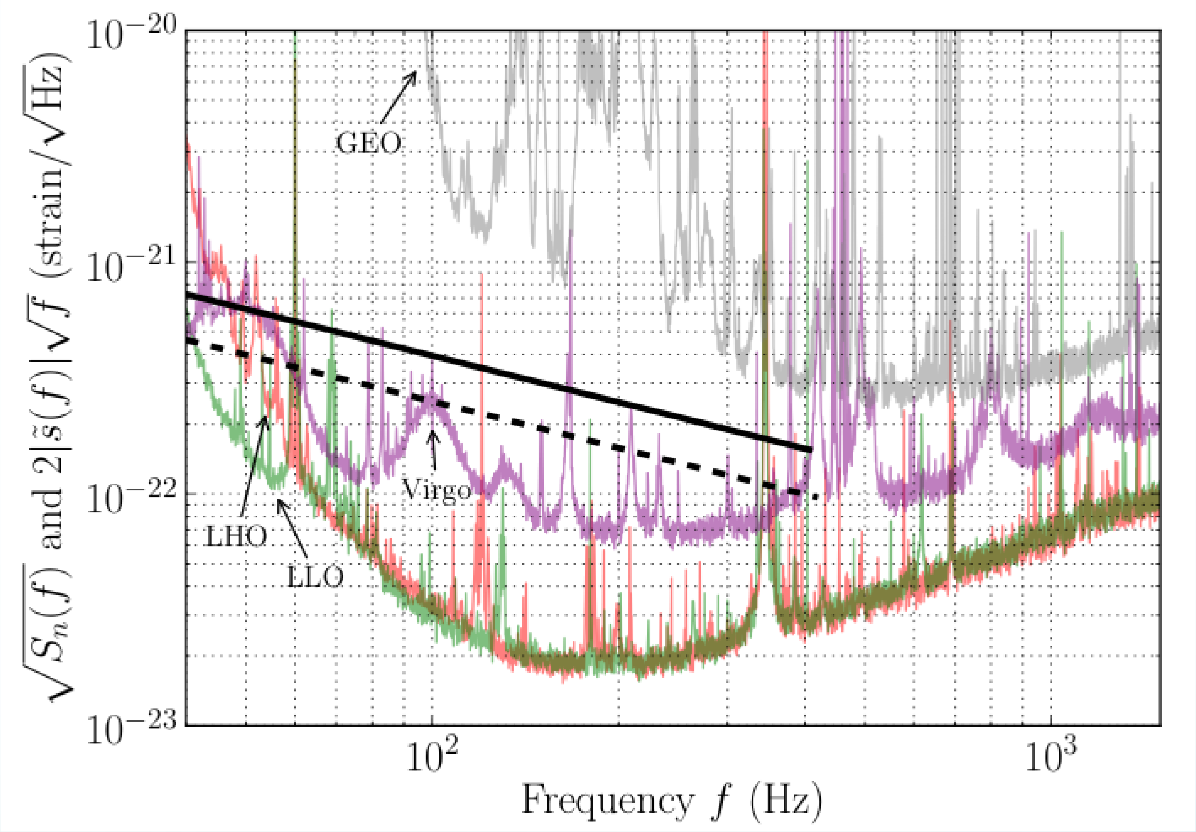
- Higher terms in $f \propto v/c$: 'Post-Newtonian' theory
  - need to go beyond linear/low-velocity approximations

Frequency dependence

Frequency domain chirp

$|h(f)| \sim f^{-7/6}$

as f increases PN corrections get bigger

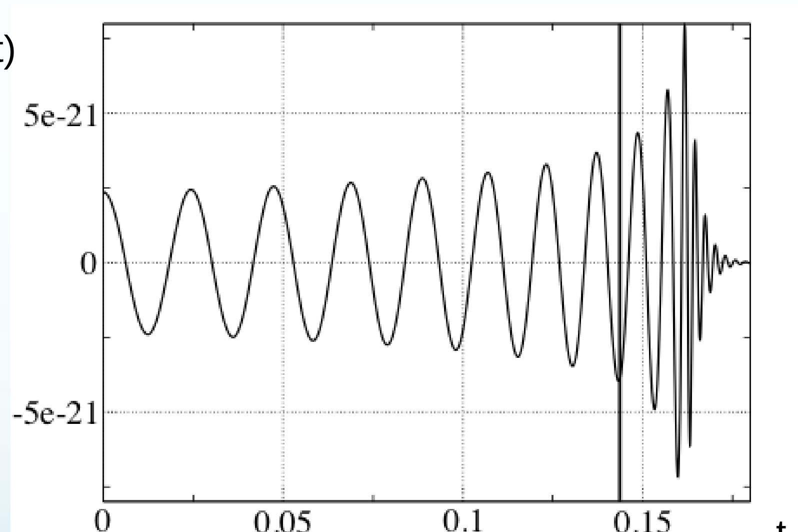


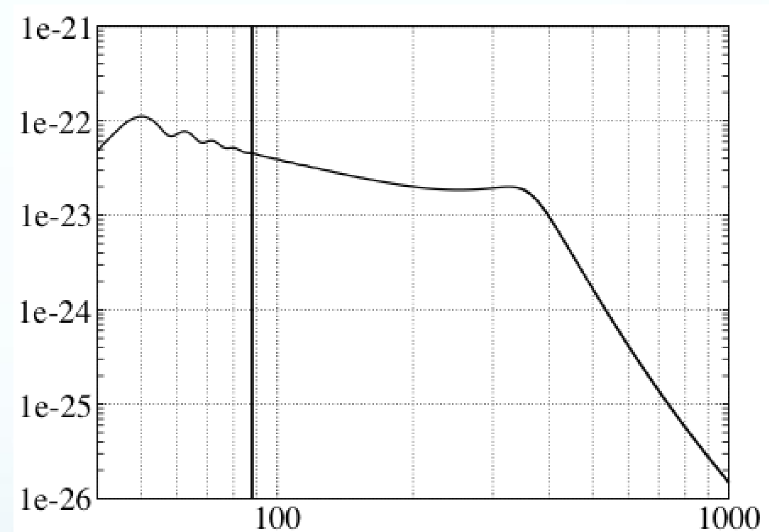
(5,6)$M_\odot$ BBH inspirals vs. detector noises
"Blind hardware injection"

http://www.ligo.org/science/GW100916/

Waveforms with merger/ringdown

- Highly nonlinear & difficult problem
- Combine numerical (‘NR’) and analytic techniques



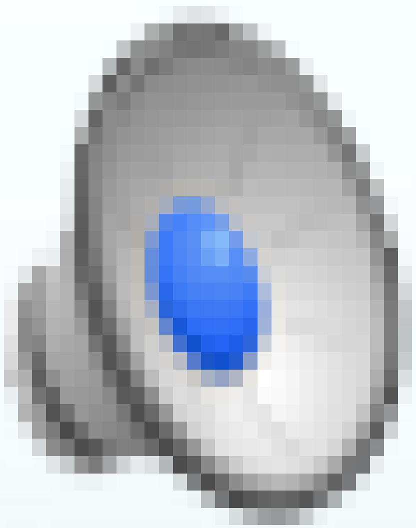


Abadie et al. arXiv:1102.3781

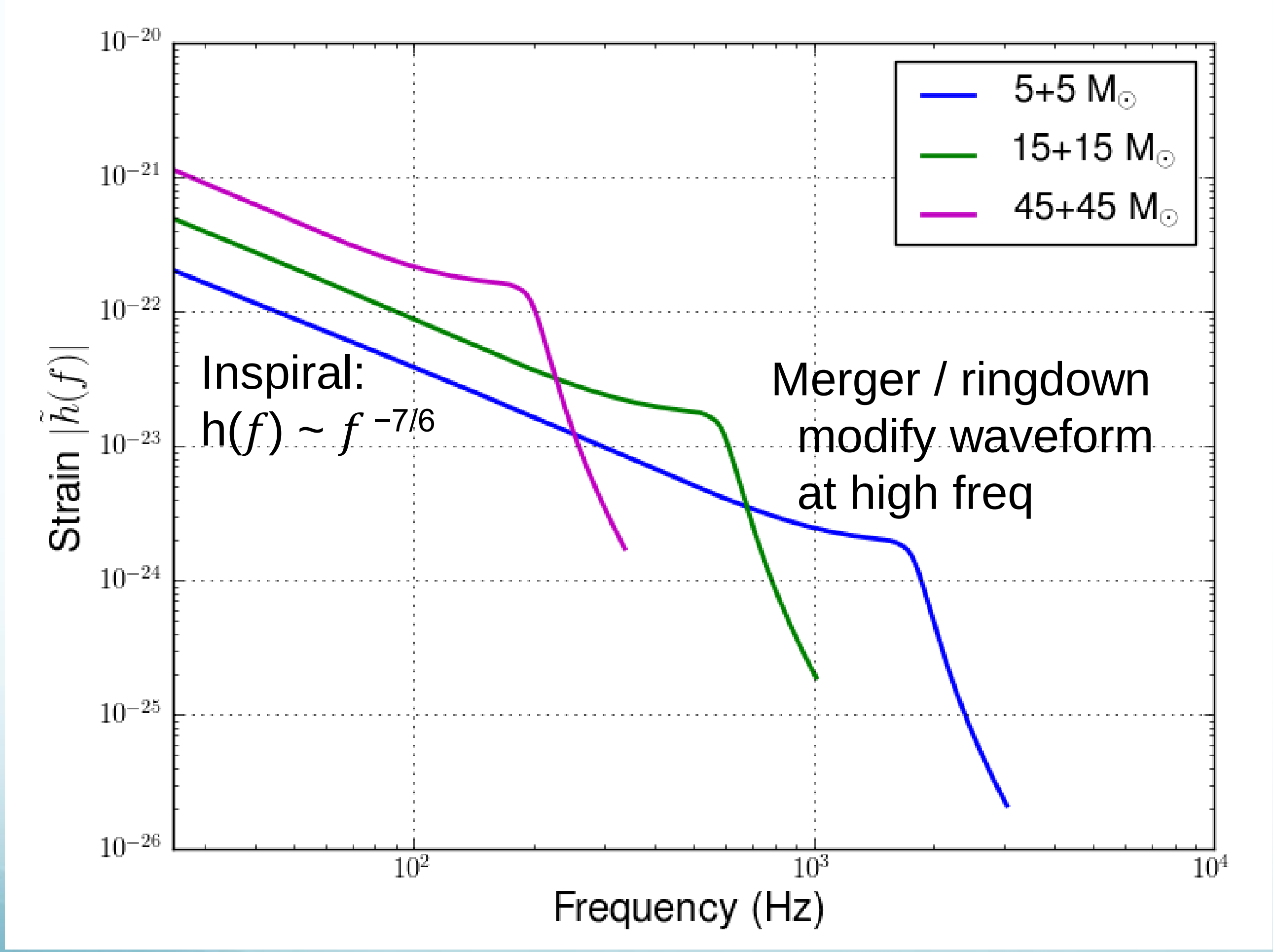
25+25 $M_\odot$ “EOBNR” waveform

Used in search for binaries with black hole(s) : $m_1+m_2 > 4\ M_\odot$

Visualizing an NR solution



Signal in frequency domain



GR has no intrinsic scale

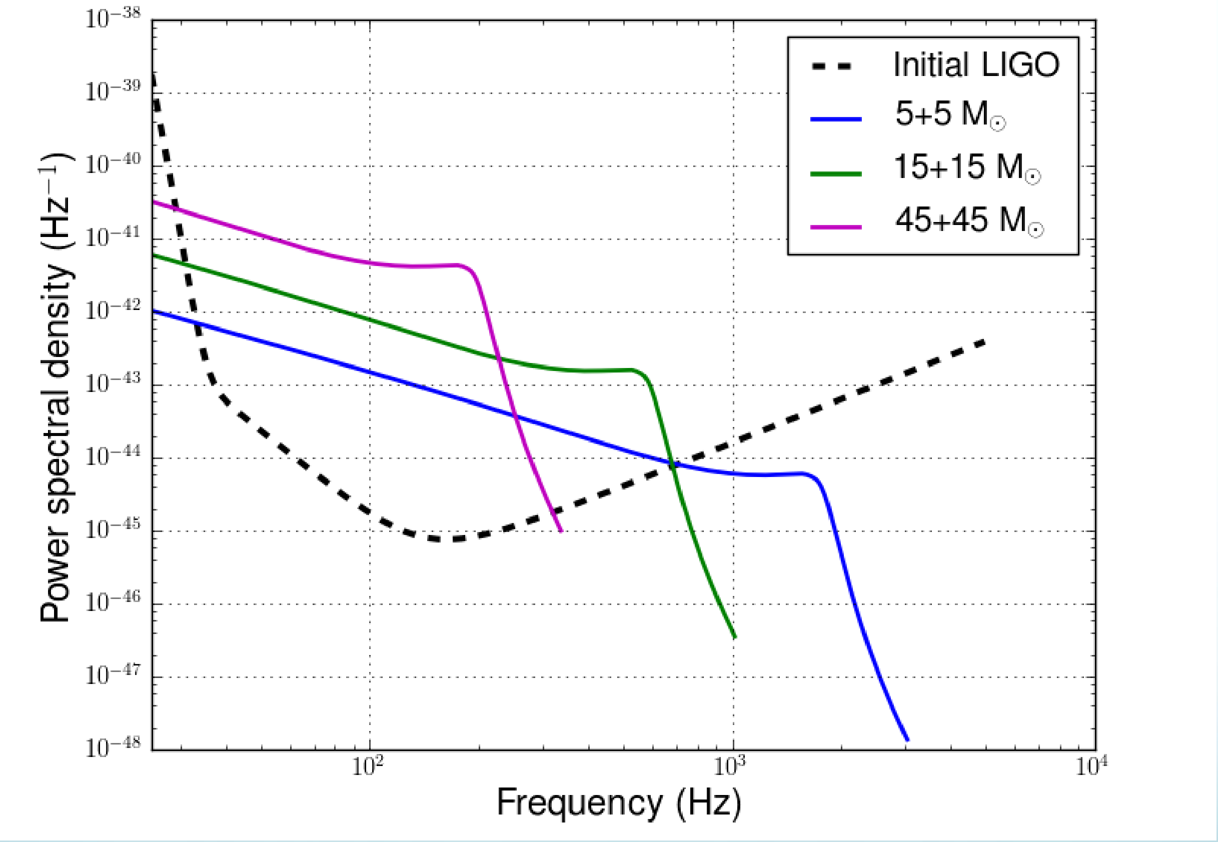
⇒ can freely rescale solutions

As M increases

- |h(f)| at fixed distance grows
- maximum GW frequency decreases

Signal vs. noise in freq domain

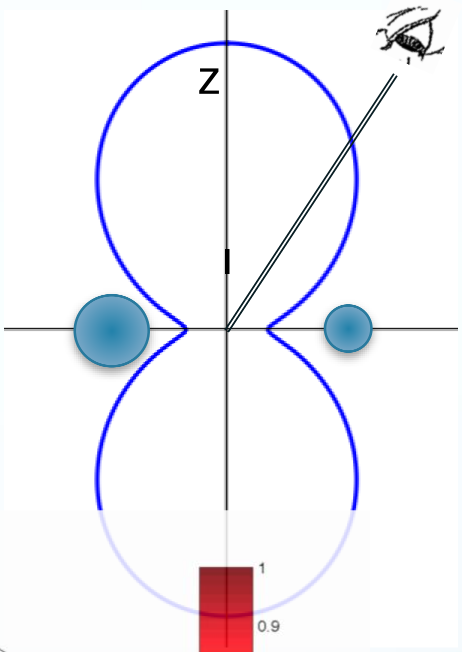
$|h(f)|^2 \times f$ for optimally aligned & located signals at 30 Mpc



Angular dependence : two peanuts

- GW emission preferentially along rotation axis

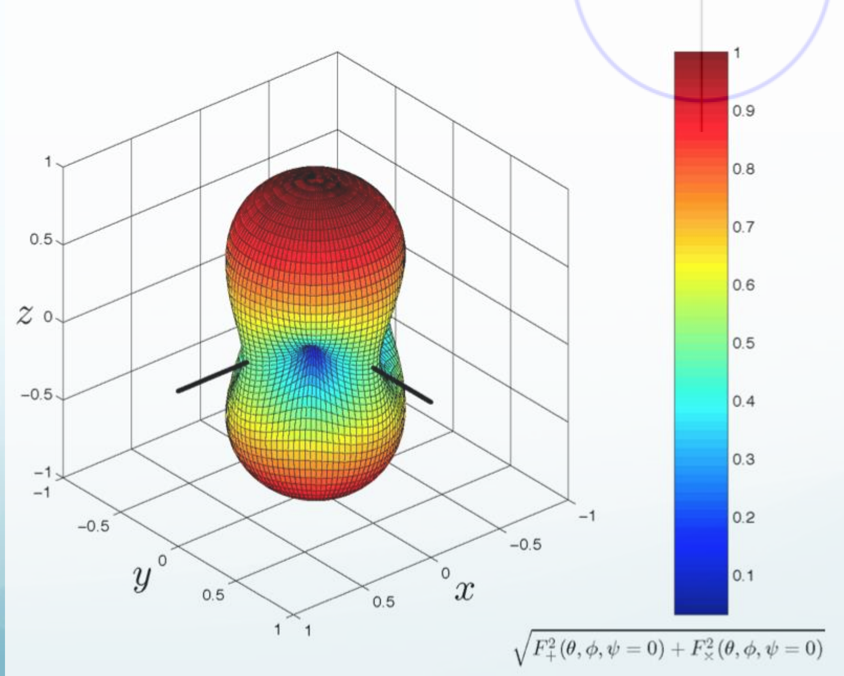
$$\frac{dP}{d\Omega} = \frac{2G\mu^2 R^4 \omega_2^6}{\pi c^5}\left[\left(\frac{1+\cos^2\iota}{2}\right)^2 + \cos^2\iota\right]$$



- Strain at the detector

$$h(t) = F_+ h_+(t) + F_\times h_\times(t)$$

(take ι = 0 i.e. 'face on' binary)



Binary signal seen in 1 detector

- Combine $F_+\cos(\Phi(t))$, $F_\times\sin(\Phi(t))$ components into a single sinusoid:

$$h(t) = \frac{A(t)}{\mathcal{D}_{\text{eff}}} \cos\left(\Phi(t) - \theta\right)$$

$$A(t) = -\frac{2G\mu}{c^4}\left[\pi G M f(t)\right]^{\frac{2}{3}}$$

- Effective distance (nb : $\mathcal{D}_{\text{eff}} \geq r$)

$$\mathcal{D}_{\text{eff}} = \frac{r}{\sqrt{F_+^2(1+\cos^2\iota)^2/4 + F_\times^2\cos^2\iota}}$$

- Phase shift

$$\tan\theta = \frac{F_\times 2\cos\iota}{F_+(1+\cos^2\iota)}$$

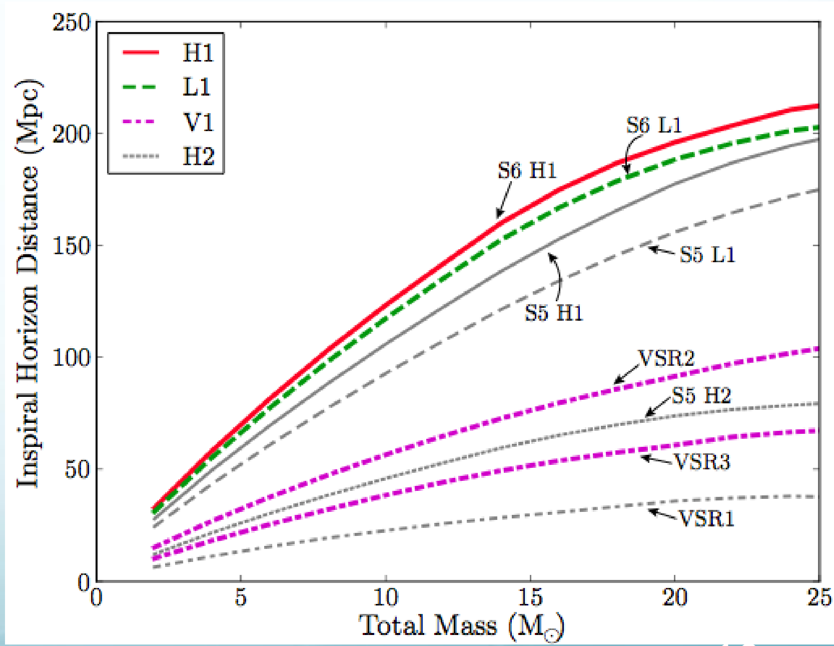
Horizon distance

- Farthest distance *D* where a merger could produce a given expected SNR ρ (e.g. =8)

$$h(f) = \frac{1\,\mathrm{Mpc}}{D_{\mathrm{eff}}} \mathcal{A}_{1\mathrm{Mpc}} f^{-7/6} \exp(i\Psi(f;\mathcal{M},M))$$

$$\left(\frac{S}{N}\right)^2 = 4\int_0^\infty \mathrm{d}f \frac{|\tilde{h}(f)|^2}{S_n(f)}$$

D depends on binary masses & detector noise spectrum

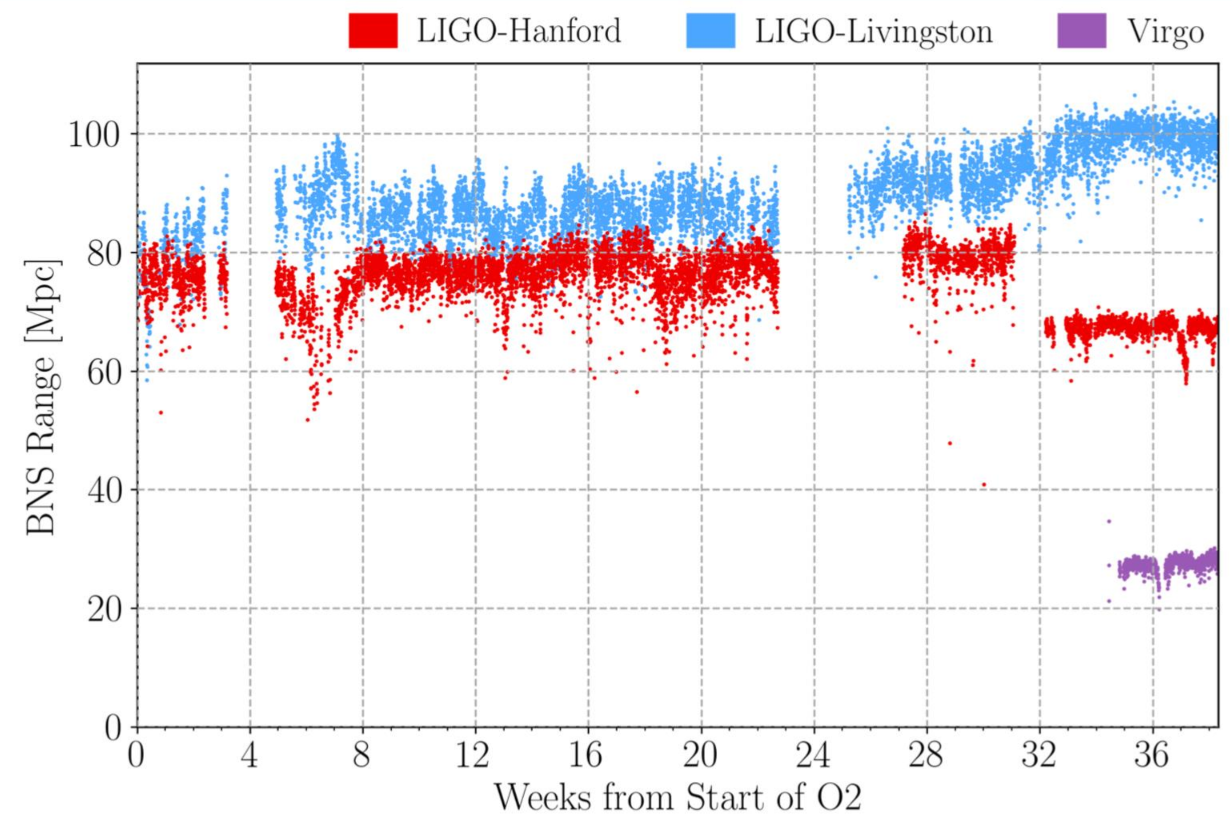


J. Abadie *et al.*, arXiv:1111.7314

BNS range as figure of merit

- 'Range' : distance at which $(1.4,1.4)M_\odot$ source is detectable, averaged over ι and sky location
- Sensitive volume

$$V_{\rm BNS} \sim \frac{4\pi R_{\rm BNS}^3}{3}$$



The basic physics of the binary black hole merger GW150914

Abstract

The first direct gravitational-wave detection was made by the Advanced Laser Interferometer Gravitational Wave Observatory on September 14, 2015.

The GW150914 signal was strong enough to be apparent, without using any waveform model, in the filtered detector strain data. Here those features of the signal visible in these data are used, along with only such concepts from Newtonian and General Relativity as are accessible to anyone with a general physics background.

The simple analysis presented here is consistent with the fully general-relativistic analyses published elsewhere, in showing that the signal was produced by the inspiral and subsequent merger of two black holes.
The black holes were each of approximately 35 Msun, still orbited each other as close as 350 km apart and subsequently merged to form a single black hole.

Similar reasoning, directly from the data, is used to roughly estimate how far these black holes were from the Earth, and the energy that they radiated in gravitational waves.

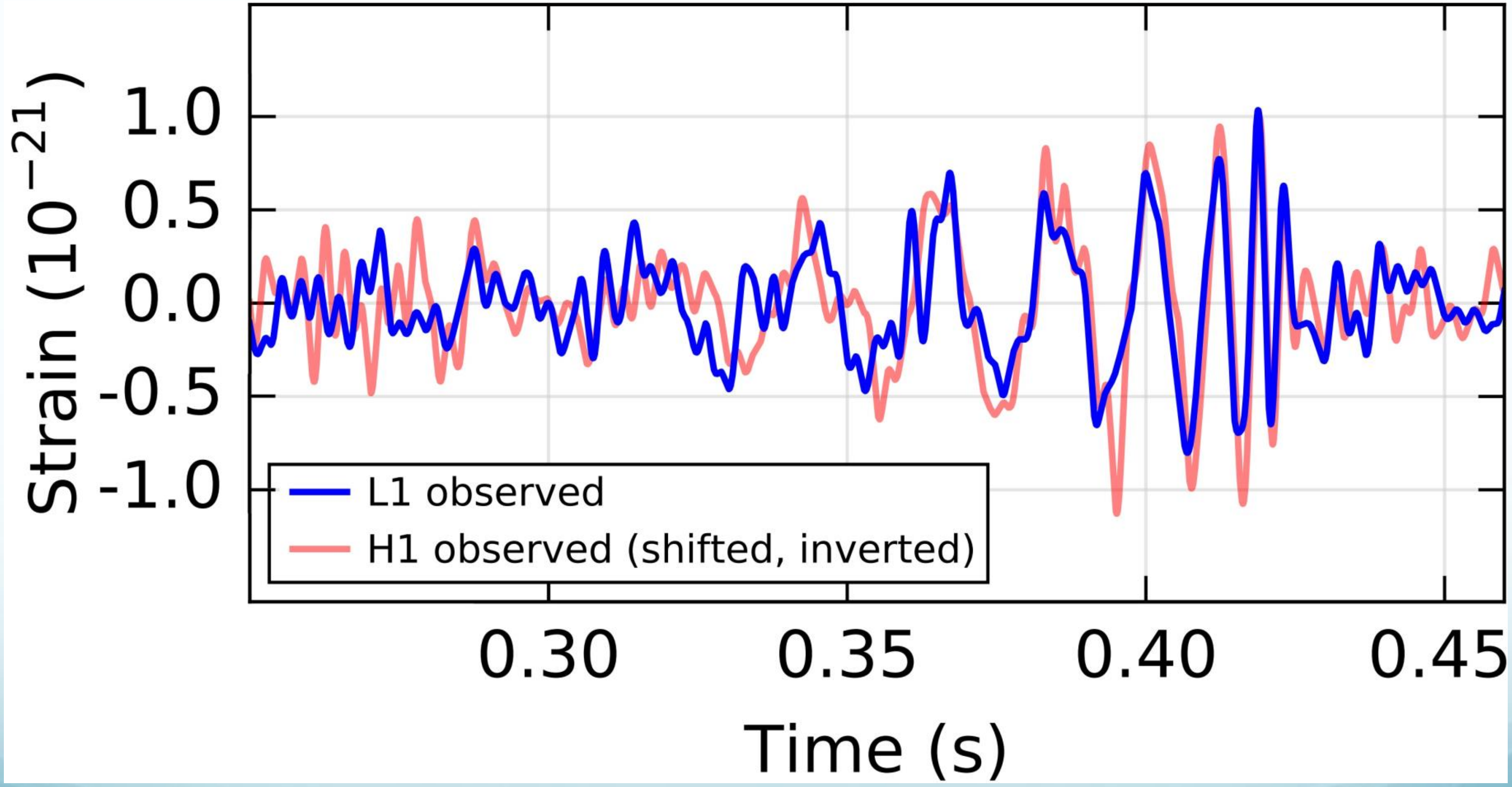
LSC and Virgo Collaborations, *Annalen der Physik,* 2016

Abstract (shorter)

- First direct gravitational-wave detection : September 14, 2015
- Signal was apparent in the filtered detector strain data.
- Features in these data show that the signal was produced by the inspiral and subsequent merger of two black holes.
- The black holes were each approximately 35 $M_{\odot}$, still orbited each other as close as 350 km apart and subsequently merged to form a single black hole.
- Similar reasoning used to estimate how far these black holes were from the Earth and the energy they radiated in gravitational waves.

Annalen der Physik, 2016

The data



Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
0.30
0.35
0.40
0.45
Time (s)
L1 observed
H1 observed (shifted, inverted)

How these lines were made

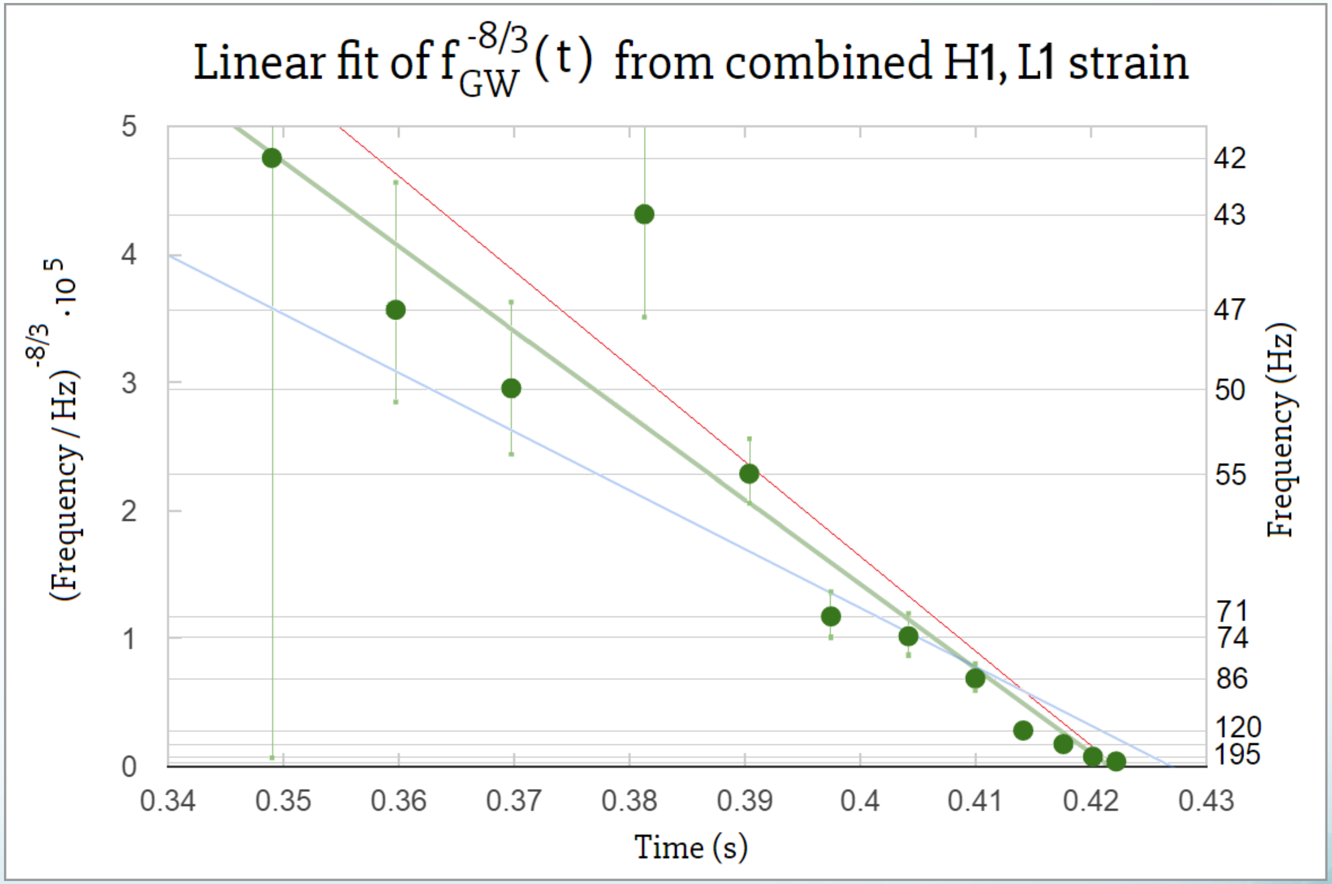
1. Build Advanced LIGO (2 observatories)
2. Align & lock while is GW passing, record strain h(t)
3. Remove excess noise at low and high frequencies (<35Hz and >350Hz), center at zero
4. Phase shift H1 data by 180° (relative orientation) – i.e. sign flip
5. Time shift H1 data by 6.9 ms (time delay)

Argument for a compact binary

- GW signal shows several oscillations of massive body/bodies **increasing in frequency & amplitude**
- Not a perturbed system returning to equilibrium (damped sinusoid)
- Only physically plausible configuration is rotating (orbiting) binary
- Binary masses and orbital radius imply compact objects, i.e. radius comparable to Schwarzschild

Reading off the chirp mass

- Estimate $f(t)$ from zero-crossings
- $f^{-8/3}$ is $\propto (t_c - t)$
- Fit shows chirp mass $M_c \sim 30\ M_\odot$



$$f_{\rm gw} = 130\,{\rm Hz} \left(\frac{1.2\,M_\odot}{M_c}\right)^{5/8} \left(\frac{1\,{\rm s}}{\tau}\right)^{3/8}$$

Measures of compactness

- Schwarzschild radius for object of mass *m* :

$$r_S = 2Gm/c^2 \simeq 3\text{km} \times (m/M_\odot)$$

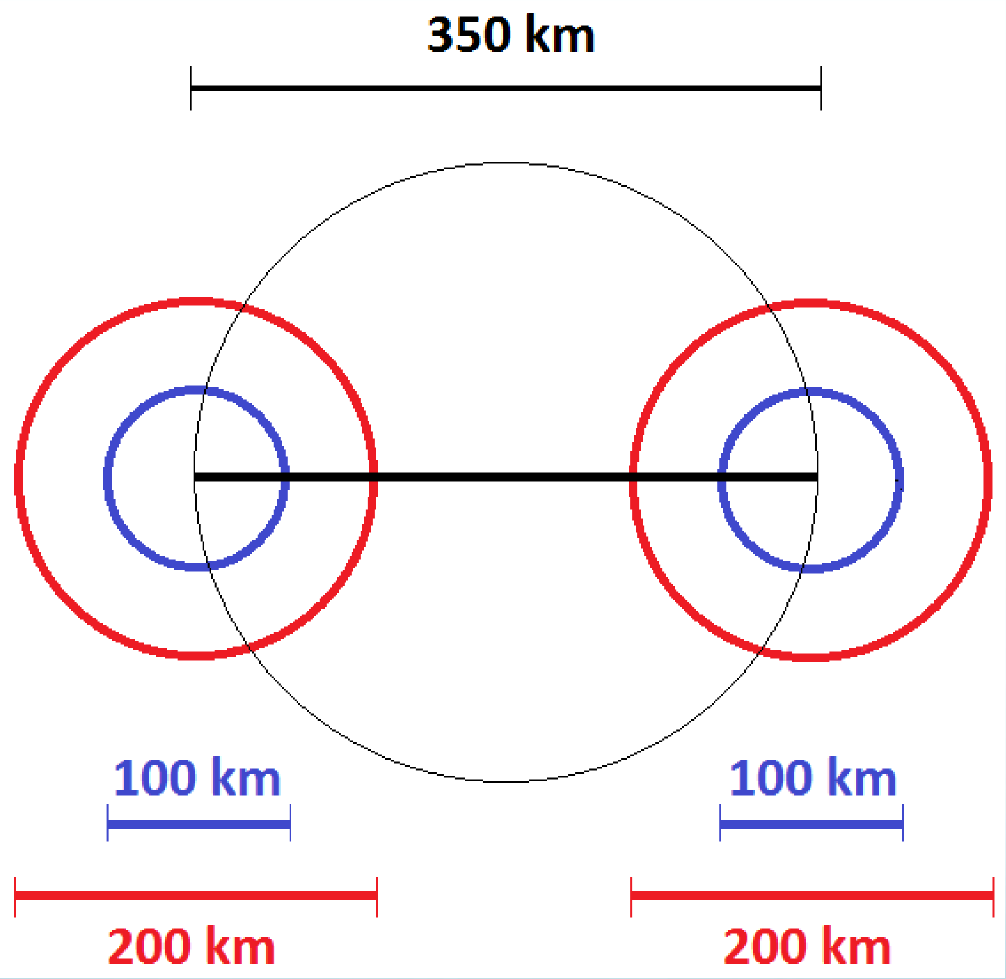
- Anything that fits within a radius ~ r_S either is a black hole or will be one soon

- For a binary with Keplerian orbital separation r_{sep} define 'compactness ratio'

$$\mathcal{R} = r_{sep} \,/\, (r_S\,(m_1) + r_S\,(m_2))$$

$\mathcal{R} = 1$ means even the compactest possible objects would be 'touching'

Zeroth approximation to BBH



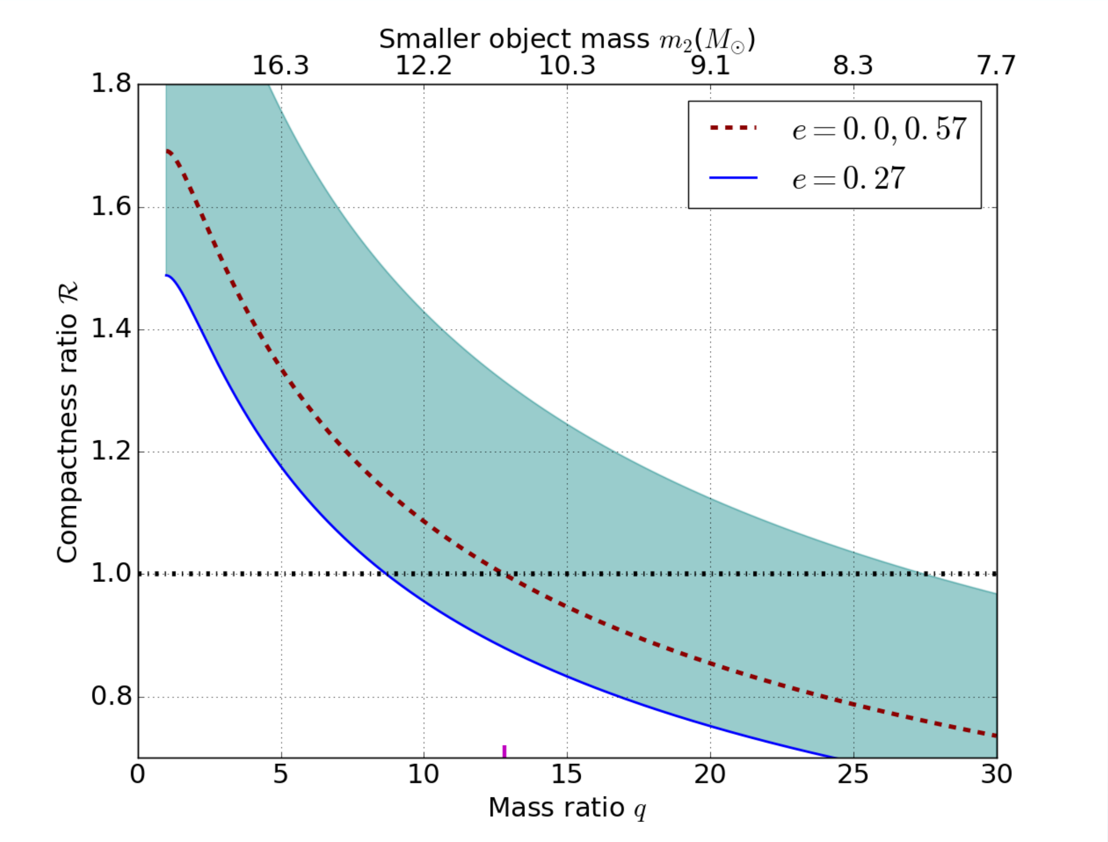
Equal mass case

- At peak amplitude GW freq is ~150Hz :
  Keplerian orbital angular freq is $\omega_{\mathrm{Kep}} = 2\pi f_{\mathrm{GW}}/2$
  Keplerian separation is $R^3 = GM / \omega_{\mathrm{Kep}}^2$
- For equal masses $M_c = m_{1,2} / 2^{0.2}$
  ⇒ **Component masses are ~35 $M_\odot$**
  ⇒ **Orbital separation is ~350 km**
- Compactness ratio close to peak is $\mathcal{R} \sim 1.7$
  - for maximally spinning BH have $r_S \rightarrow r_S /2$, $\mathcal{R} \sim 3.4$
- Non-compact objects would have collided/merged well before this

Unequal masses

- Mass ratio $q = m_1 / m_2$
- Component masses
  $m_1 = M_c (1 + q)^{1/5} q^{2/5}$, $m_2 = M_c (1 + q)^{1/5} q^{-3/5}$
  Total mass
  $M = M_c (1 + q)^{6/5} q^{-3/5}$
- Compactness ratio $\mathcal{R} \propto M^{1/3}/(m_1 + m_2) \propto M^{-2/3}$
  $\propto M_c^{-2/3} q^{2/5} (1 + q)^{-4/5}$
- For constant M_c compactness **decreases** as q↑

Compactness vs. mass ratio



- Beyond q ~ 13 the binary system is within its own Schwarzschild radius : bounds $m_2 \geq 11\ M_\odot$

Why the system is not an IMRI

- Newtonian dynamics is pretty inaccurate close to black holes
- Suppose the system was a heavy BH (mass M) with a much lighter companion, can we bound max GW frequency?
- Can't orbit faster than light!
  Light ring radius is $\geq GM/c^2$, GW frequency emitted is at most $c^3/(2\pi GM) = 32(M_\odot/M)$ kHz
- So M can be at most ~200 $M_\odot$

GW luminosity and distance

- So far have not used the strain amplitude $h_{max} \sim 10^{-21}$
- Recall the formula $$h_+(t;\theta,\phi) = \frac{1}{r}\frac{4G\mu\omega_s^2 R^2}{c^4} \times \ldots$$

  Set masses to 35 $M_\odot$, R~350km, ω_s~150Hz ...
- Get r ~ $(2GM_\odot/c^2) \times 35 \times (\pi\times150\text{Hz})^2 \times (350\text{ km})^2 \div (10^{-21}\, c^2)$

  ~ 3.2×10^{22} km (!) ~ 1.1 Gpc
- NB this is *maximum* distance (optimal position, etc.)

The real answers!

Primary black hole mass	$36^{+5}_{-4} M_\odot$
Secondary black hole mass	$29^{+4}_{-4} M_\odot$
Final black hole mass	$62^{+4}_{-4} M_\odot$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$

Bonus : Energy radiated

- Orbital energy was $$E_{\text{orbital}} = E_K + E_P = -\frac{Gm_1m_2}{2R}$$
- At very large R this is ~0
- At peak GW emission R ~ 350 km
- Recall $2GM_\odot \simeq 3$ km × c^2
- Get ~ 2.6 $M_\odot c^2$ ...

The estimated total energy radiated in gravitational waves is $3.0^{+0.5}_{-0.5} M_\odot c^2$.