Loop Quantum Cosmology and the trans-Planckian problem

Mercedes Martín-Benito

Based on: Phys. Rev. D 109, 123534 (2024)

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Adapted from a presentation given by Rita Neves

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JUGAR II, IEM-CSIC, Madrid

UNIVERSIDAD COMPLUTENSE MADRID

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Quantum Fields & Gravity Group

$\langle G | \hbar \rangle$

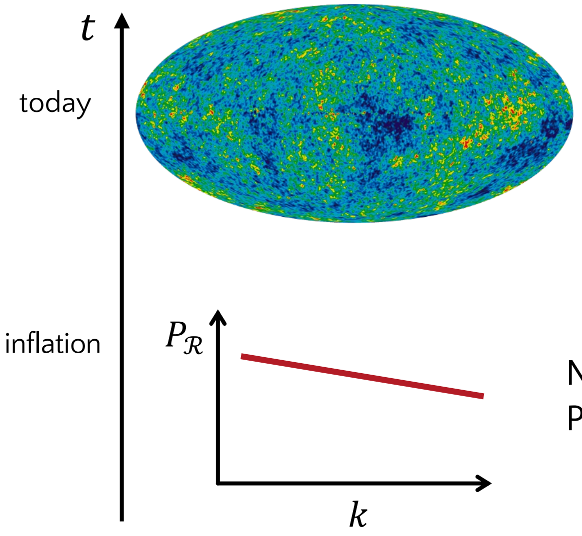
1

The trans-Planckian problem

Standard cosmology

Overview

Inflation + perturbations:

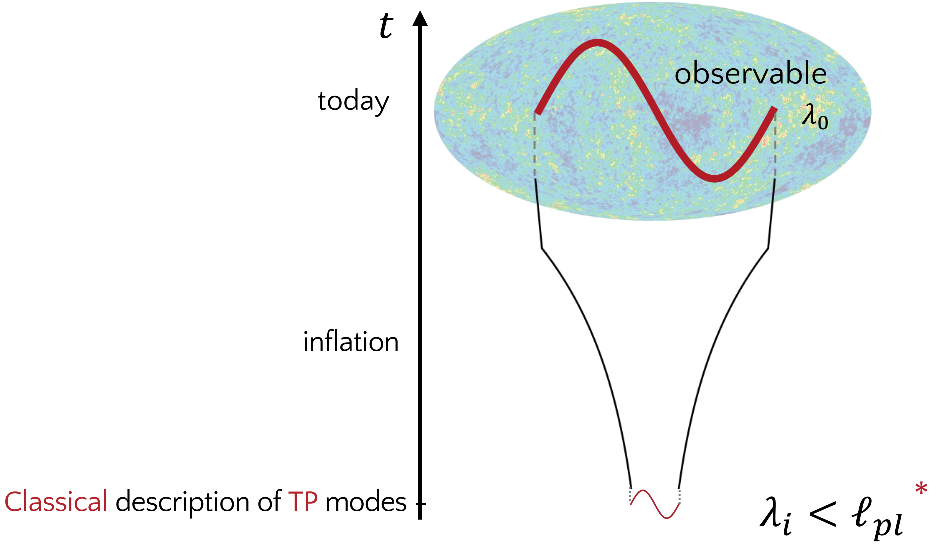


✓

Near-scale-invariant
Primordial power spectrum

Overview

Scales that are observable today were trans-Planckian at the onset of inflation



* Depends on total # e-folds N

$$\lambda_i = \frac{a_i}{a_0}\,\lambda_0 = e^{-N}\,\lambda_0$$

$\lambda_0 \sim 2\,\text{Mpc}$ (most UV scale obs. by Planck)

$\lambda_i \leq \ell_{pl} \Rightarrow N \gtrsim 130$

Cosmological perturbations within Inflation

- Study of gauge-invariant scalar modes v_k :

$$v_k'' + [k^2 + s(\eta)]v_k = 0 \xrightarrow{\text{UV}} k^2 \gg s(\eta): \quad v_k'' + k^2 v_k \simeq 0$$

Fix initial conditions s.t. UV modes behave as free waves = **Bunch-Davies (BD) vacuum**

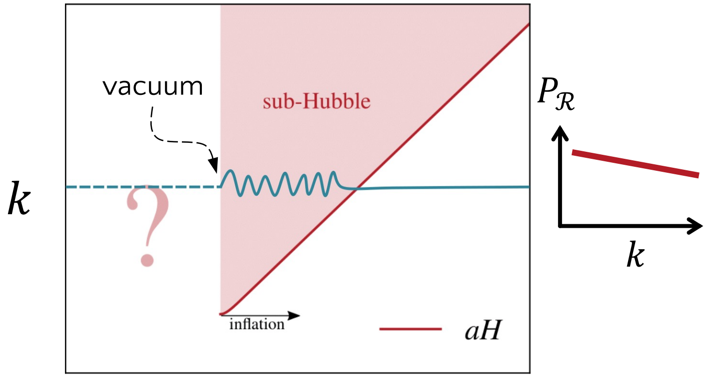
Sub-Hubble: $k \gg aH \Leftrightarrow \lambda \ll R_H$

$\rightarrow v_k\ (\eta)$ oscillatory

Super-Hubble: $k \ll aH \Leftrightarrow \lambda \gg R_H$

$\rightarrow v_k\ (\eta) \sim$ constant

R_H = Hubble radius



The problem

What if trans-Planckian behaviour is different?

Strategies:

- Ideally:

  Obtain trans-Planckian physics

  from fundamental theory

  → Quantum Gravity?

- Alternatively:

  Robustness of predictions:

  Consider modified disp. relations

  → Simulate effects from QG

  → Linear for $\kappa = k/a \ll \kappa_c$

The problem

What if trans-Planckian behaviour is different?

Trans-Planckian corrections

Physical wavenumber $\kappa = k/a$

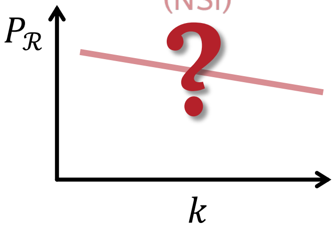
F linear for $\kappa \ll \kappa_c$:

$F(\kappa) = \kappa + \mathcal{O}(\kappa^2) \;\Rightarrow aF = k + \ldots$

$$v_k'' + \omega_k^2(\eta) v_k = 0 \qquad \omega_k^2(\eta) = [a(\eta)F(\kappa)]^2 + s(\eta)$$

$k^2 \gg s(\eta)$ → free waves ? → BD vacuum ? →

Near scale invariance (NSI)



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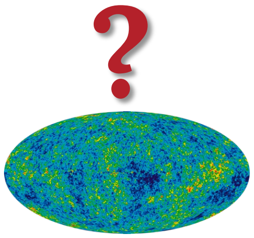
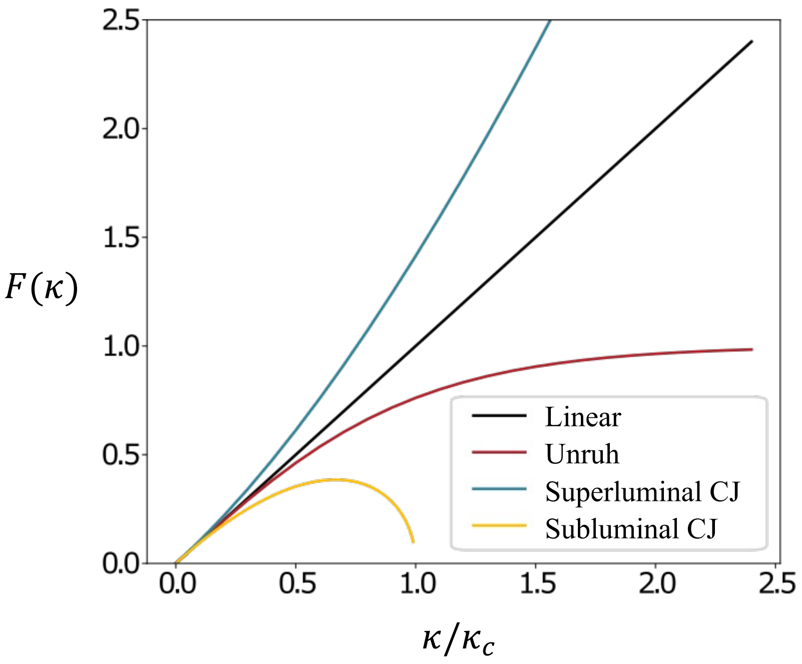


Are predictions robust under trans-Planckian corrections?

Modified dispersion relations



$$\omega_k^2(\eta) = [a(\eta)F(\kappa)]^2 + s(\eta)$$

Physical wavenumber $\kappa = k/a$

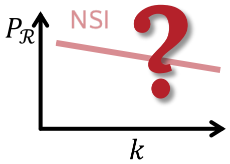
F linear for $\kappa \ll \kappa_c$:

$F(\kappa) = \kappa + \mathcal{O}(\kappa^2) \;\Rightarrow aF = k + \ldots$

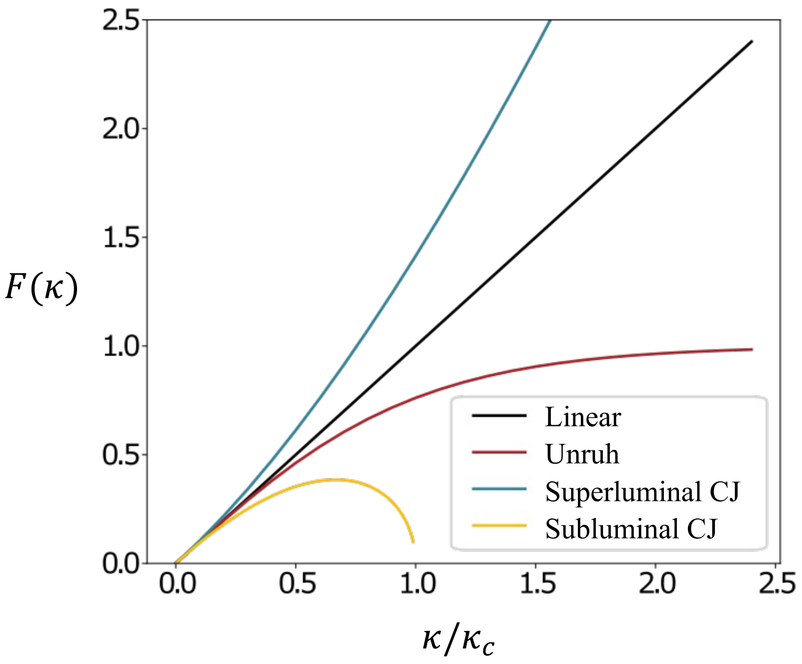
a) Brute force

Each $F(\kappa)$: compute $P_{\mathcal{R}}$

Check if NSI is lost



Modified dispersion relations



$$\omega_k^2(\eta) = [a(\eta)F(\kappa)]^2 + s(\eta)$$

b) Adiabaticity analysis [of TP modes]

$$\epsilon(k,\eta) = \left|\frac{\omega_k'}{\omega_k^2}\right|$$

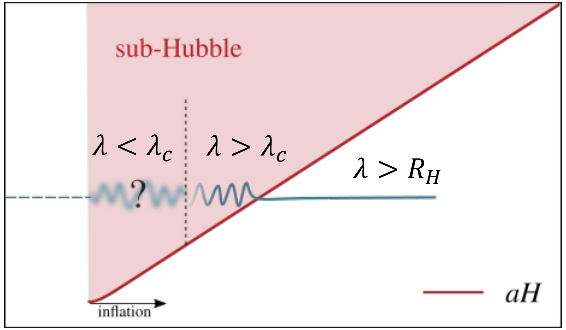
$\epsilon \ll 1 \rightarrow$ power spectrum ~ NSI ✓

$\epsilon \gtrsim 1 \rightarrow$ departure from NSI ≠

Conclusions for standard cosmology [monotonic $F(\kappa)$]

Scale separation

$$\frac{H}{\kappa_c} \ll 1 \Leftrightarrow \lambda_c \ll R_H \Rightarrow \epsilon \ll 1 \Rightarrow \text{NSI preserved} \quad ✓$$



[1] Martin, Brandenberger, Phys. Rev. D 63, 1 (2001).
The Trans-Planckian problem of inflationary cosmology.

[2] Niemeyer, Parentani, Phys. Rev. D 64, 101301 (2001).
Trans-Planckian dispersion and scale-invariance of inflationary perturbations.

Conclusions for standard cosmology [monotonic $F(\kappa)$]

- Scale separation

$$\frac{H}{\kappa_c} \ll 1 \Leftrightarrow \lambda_c \ll R_H \Rightarrow \epsilon \ll 1 \Rightarrow \text{NSI preserved} \quad \checkmark$$

↪ UV modes (affected by corrections) have enough time to adapt to standard vacuum solution

- Satisfied during slow-roll as long as $\kappa_c \sim$ Planckian

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2 Loop Quantum Cosmology

Pre-inflationary dynamics

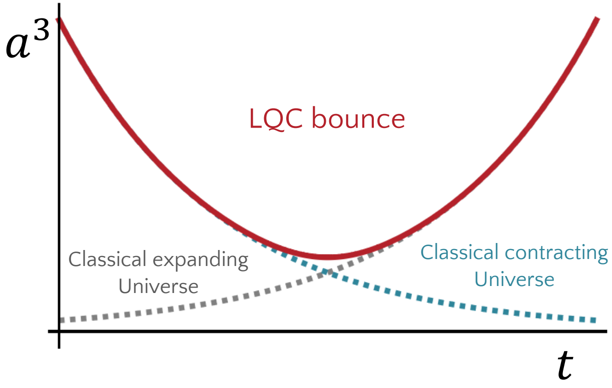
- LQC: big-bang → bounce

  [defines characteristic scale k_{LQC}]

  ⇒ well-defined pre-inflationary dynamics

- Bounce dynamics (potential negligible)

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_{LQC}}\right)$$



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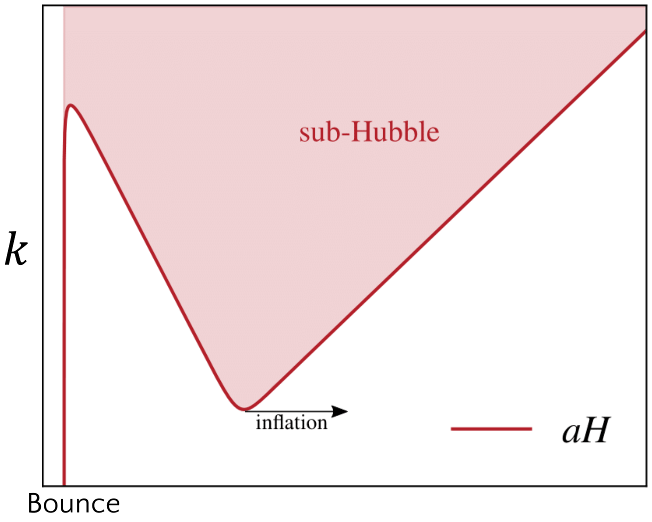

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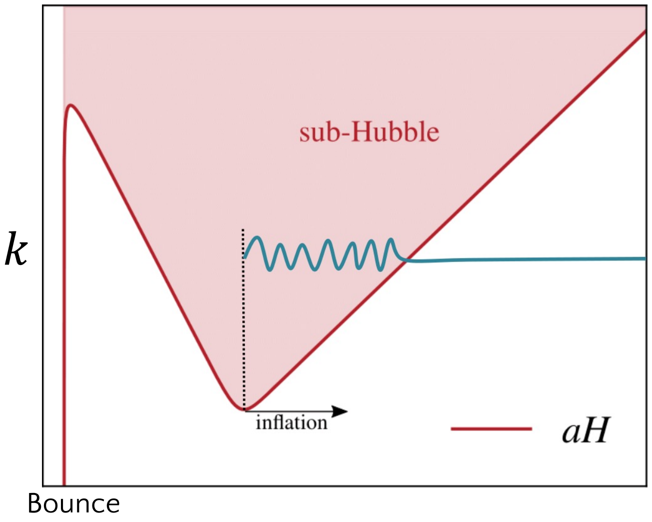

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➢ No longer start description at onset of inflation

  [no obvious natural vacuum]

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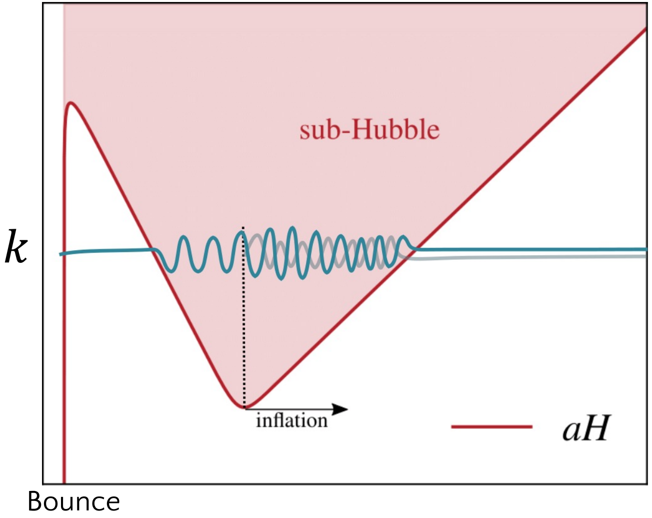


➢ No longer start description at onset of inflation

  [no obvious natural vacuum]

Perturbations

- Different approaches:*
  - Both take background + perturbations

quantisation Loops QFT

- Corrected equations of motion:

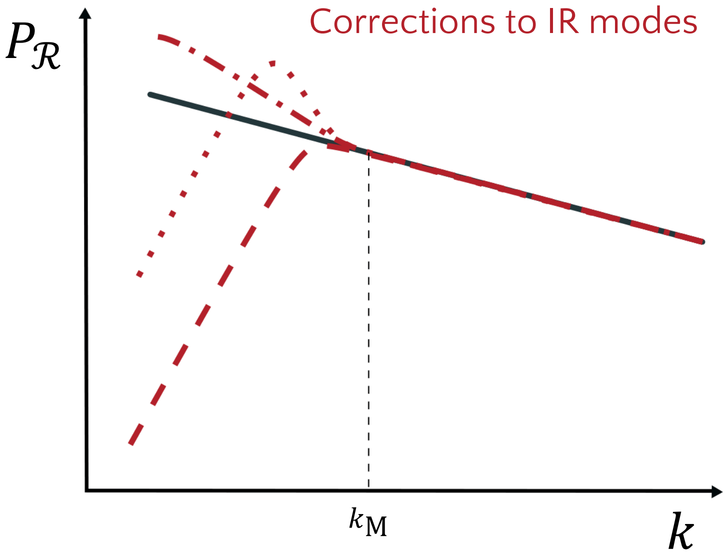
$$v_k'' + \omega_k^2(\eta) v_k = 0 \qquad \omega_k^2(\eta) = k^2 + s_{LQC}(\eta)$$

* Hybrid: $s_{LQC}(\eta) > 0$ always, $s_{LQC}(\eta = B) \simeq \kappa_{LQC}{}^2/3$

Dressed metric: $s_{LQC}(\eta = B) \simeq -\kappa_{LQC}{}^2 < 0$
$\Rightarrow \omega_k^2(\eta) < 0$ for some scales at the bounce

Primordial power spectrum

- Depends on vacuum



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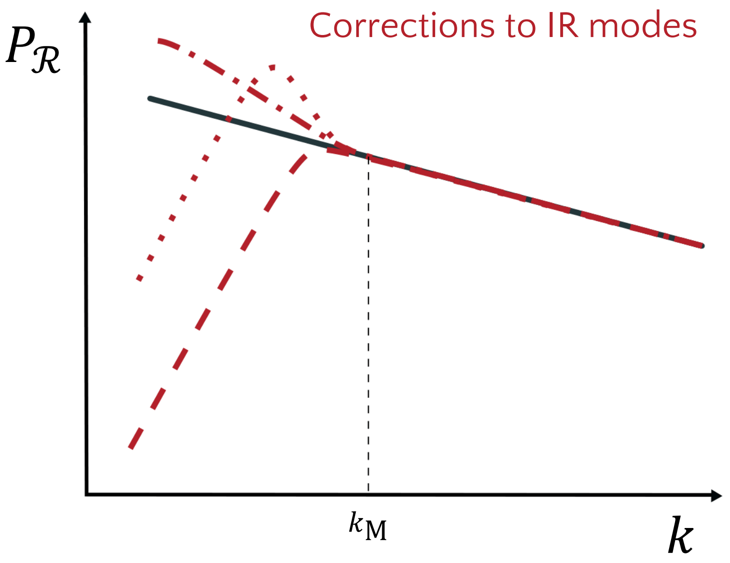
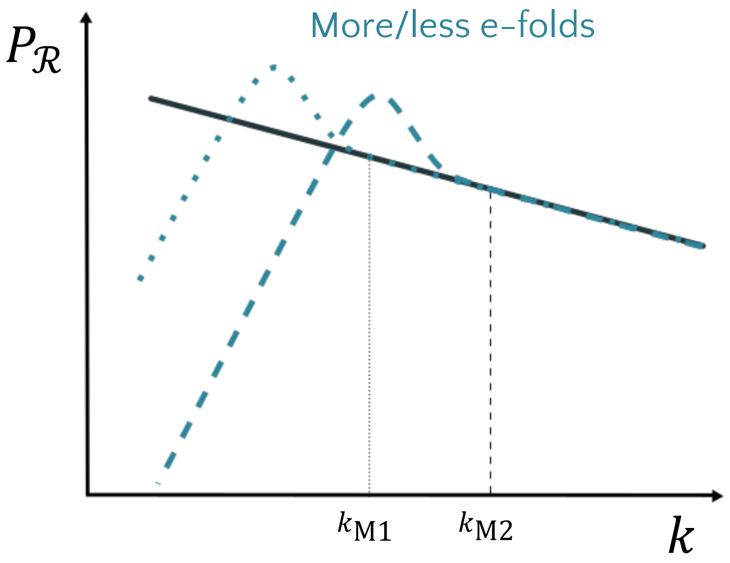


- + parameters of background



3

The trans-Planckian problem

in LQC

Models – background (Starobinski inflation in LQC)

Model	Number of e-folds		
	Pre-inflation	Inflation	Total until today
A [3]	4.9	72	141
B [4]	4.9	61	132

- Both compatible with observations (for different vacua)
- Will stretch physical scales differently

[3] Ashtekar, Gupt, Class. Quant. Grav. 34, 014002 (2017).
Quantum Gravity in the Sky: Interplay between fundamental theory and observations.

[4] Martín-Benito, Neves, Olmedo, Phys. Rev. D 108, 103508 (2023).
Alleviation of anomalies from the nonoscillatory vacuum in loop quantum cosmology.

→ From dressed metric (background parameters)

Strategy

1. Track observable window.
2. For each modified dispersion relation:

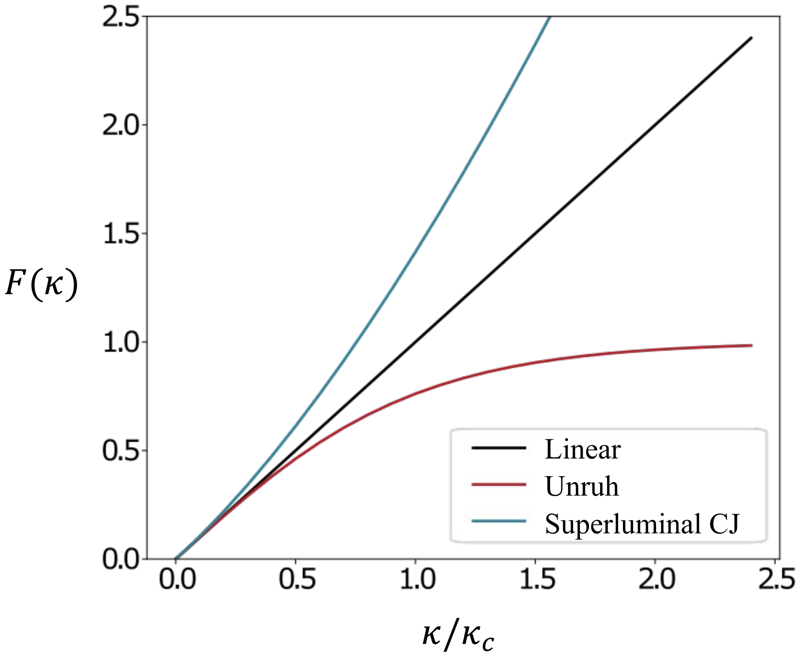
$$v_k'' + \omega_k^2(\eta) v_k = 0 \qquad \omega_k^2(\eta) = [a(\eta) F(\kappa)]^2 + s_{LQC}(\eta)$$

Study the adiabaticity coefficient: $\epsilon(k,\eta) = \left|\frac{\omega_k'}{\omega_k^2}\right|$

(instead of computing the PPS as done in [5], without really understanding the origin of possible modifications)

[5] K. Martineau, A. Barrau, and J. Grain, Int. J. Mod. Phys. D 27, 1850067 (2018).
A first step towards the inflationary trans-planckian problem treatment in Loop Quantum Cosmology.

Strategy – dispersion relations

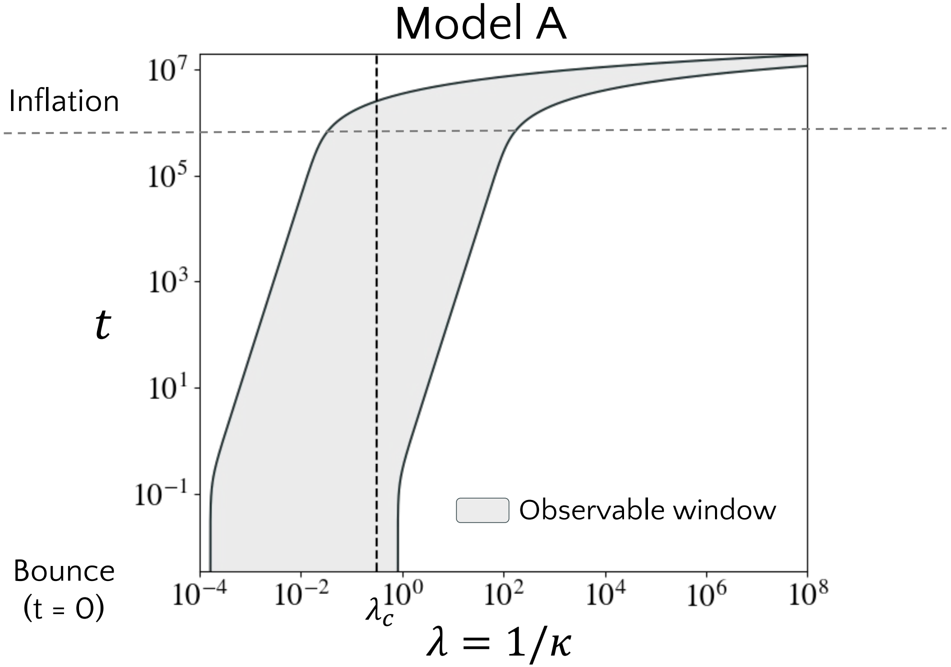


- Monotonous $F(\kappa)$
- Choose $\kappa_c = \kappa_{LQC} = 3.21\, m_{pl}$

  $\lambda_c = 0.31\, \ell_{pl}$

  (bounce scale)

1 Are observable modes trans-Planckian?



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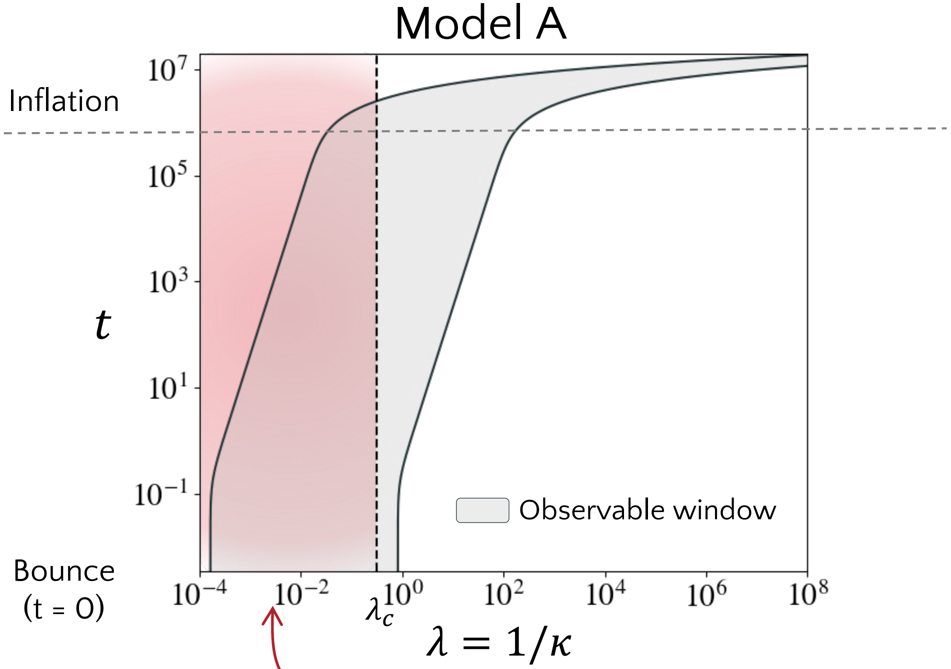

Yes

more e-folds of inflation

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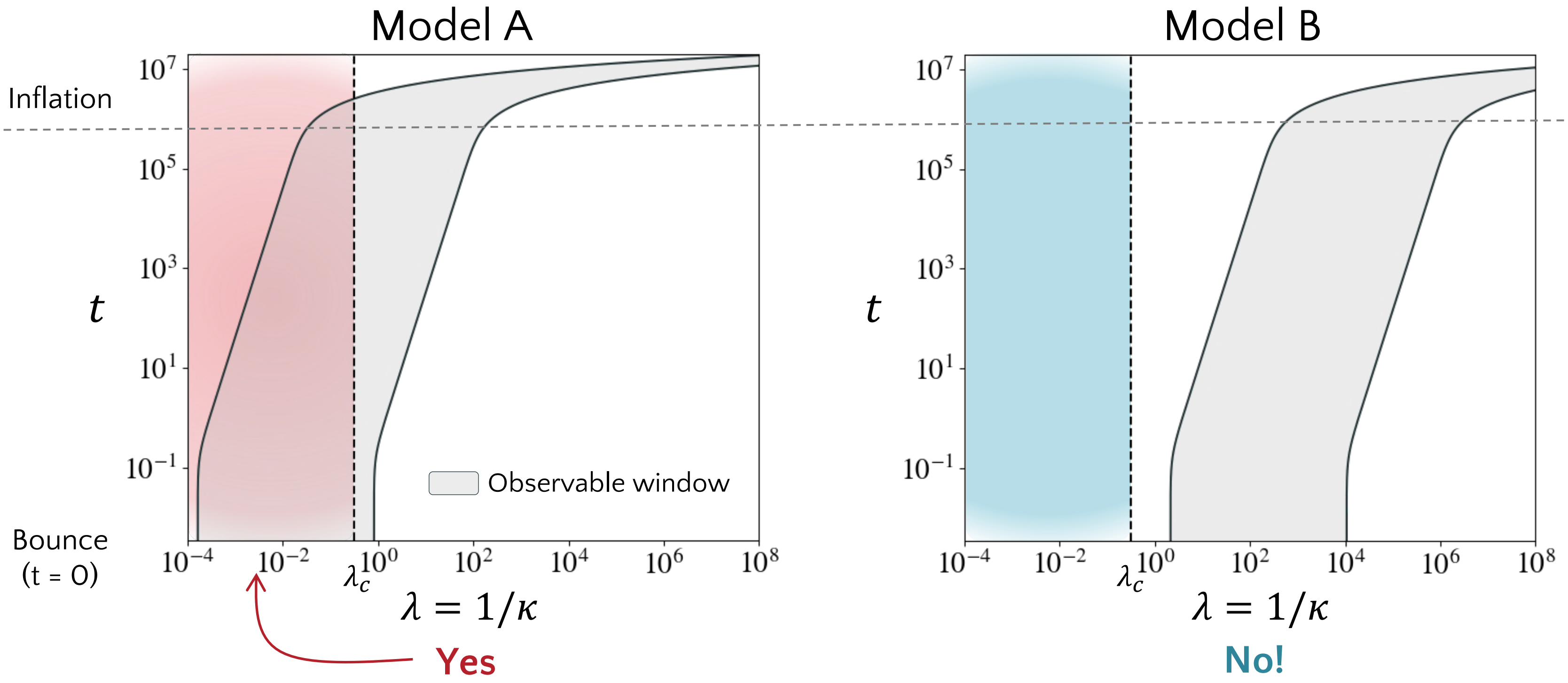
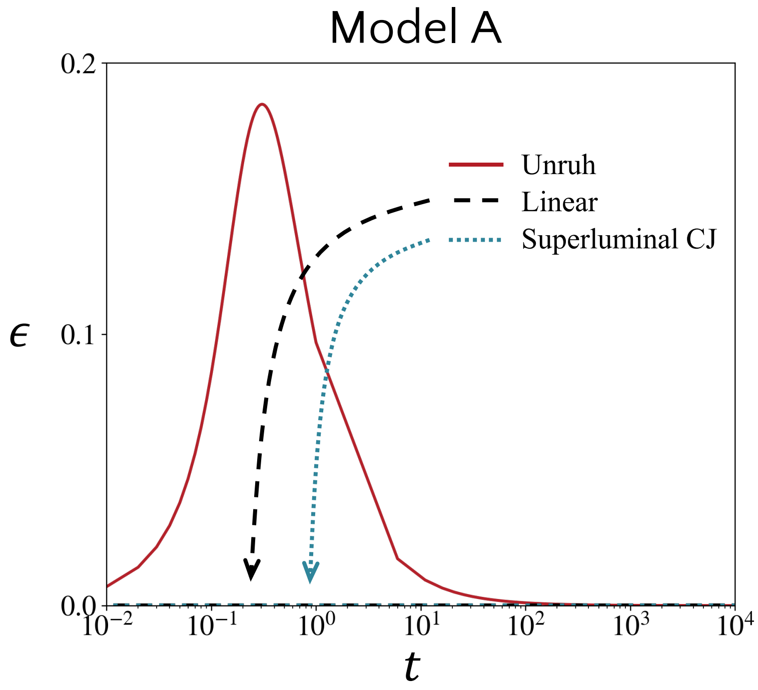


more e-folds of inflation

less e-folds of inflation

2 Are they adiabatic?



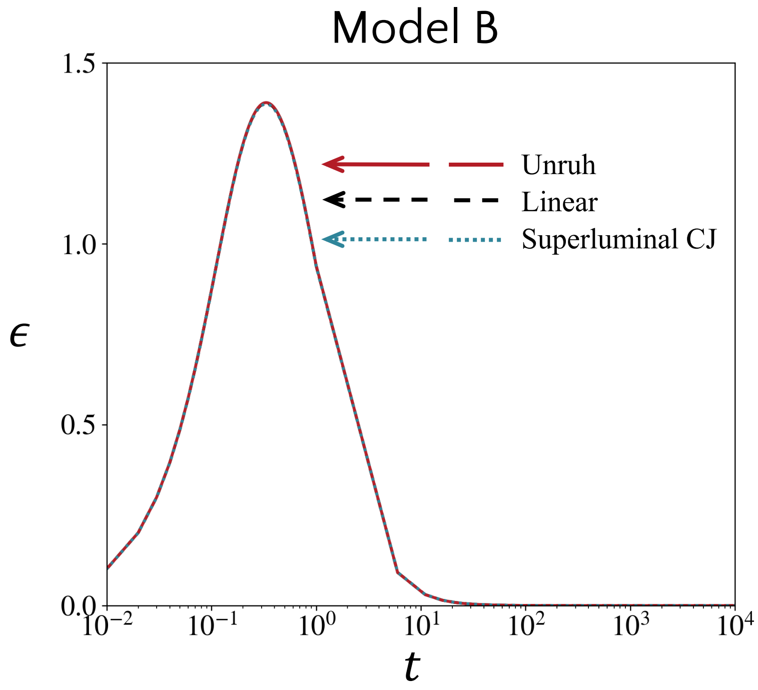
(most UV scale obs. by Planck)

- $\epsilon \sim 0.2 \rightarrow$ small* but non negligible
- Depends heavily on $F(\kappa)$
- While modes are TP

$\Rightarrow$ (soft?) Trans-Planckian problem

* In hybrid LQC.

2 Are they adiabatic?



(most UV scale obs. by Planck)

- Definitely not: $\epsilon > 1$
- But not trans-Planckian!

[that is why there is no dependence on $F(\kappa)$

$\Rightarrow$ no effect on observable power spectrum]

$\Rightarrow$ Avoids the trans-Planckian problem

4 Conclusions

Conclusions

- LQC models do not ***necessarily*** present a trans-Planckian problem.
    - Model A: more e-folds lead to modes that are trans-Planckian and (slightly) non-adiabatic,
    - Model B: less e-folds, avoids the issue.

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- Our work complements [4], where the PPS with modified dispersion relations were computed in different LQC approaches, but no insight was placed in understanding the origin of the modifications to those PPS.

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- Caveat:
  - Trans-Planckian modifications directly rooted at the quantum nature of geometry?

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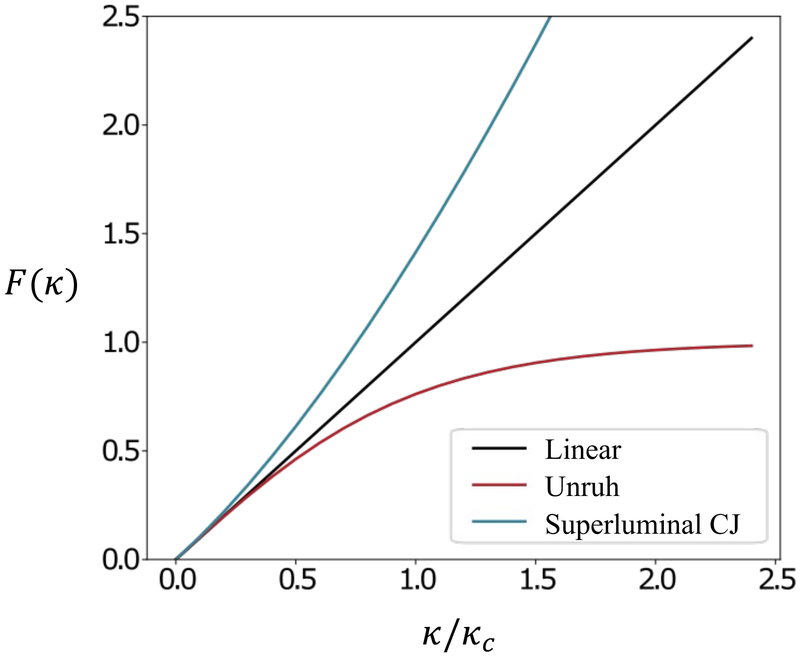
Thank you for your attention!

Backup slides

Modified dispersion relations



- Unruh

$$F(\kappa) = \kappa_c \tanh\left(\frac{\kappa}{\kappa_c}\right)$$

- Superluminal Corley–Jacobson

$$F(\kappa) = \kappa\sqrt{1 + \left(\frac{\kappa}{\kappa_c}\right)^2}$$

Hybrid LQC – scalar perturbations

$$v_k'' + \omega_k^2(\eta) v_k = 0 \qquad \omega_k^2(\eta) = k^2 + s(\eta)$$

$$s(\eta) = -\frac{4\pi G}{3} a^2(\rho - 3p) + a^2 U$$

$$U = V''(\phi) + 48\pi G\, V(\phi)\left(1 - \frac{V(\phi)}{\rho}\right) + 6\frac{a'\phi'}{a^3\rho} V'(\phi)$$

Inflaton potential

Models A and B

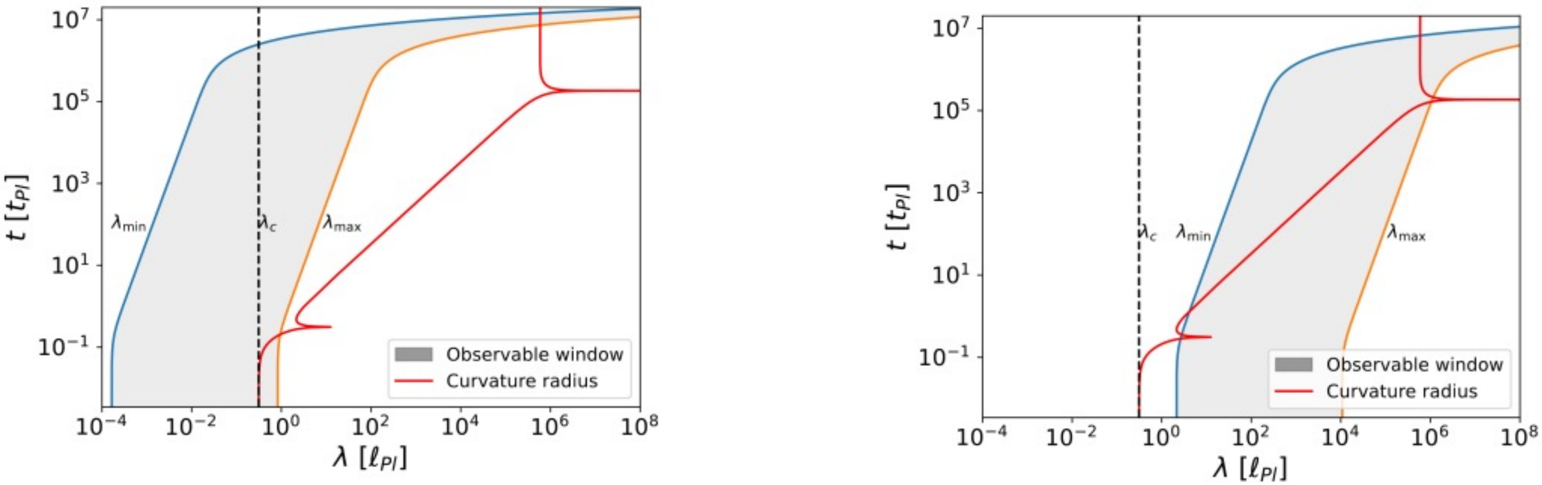


FIG. 4. Evolution of the observable window (shaded region) from the bounce to inflation. Upper panel: Model A. A big part of the window is trans-Planckian during the evolution (even at the onset of inflation) before exiting the horizon. Lower panel: Model B. The whole window is always above the ultraviolet scale λ_c.

Model B

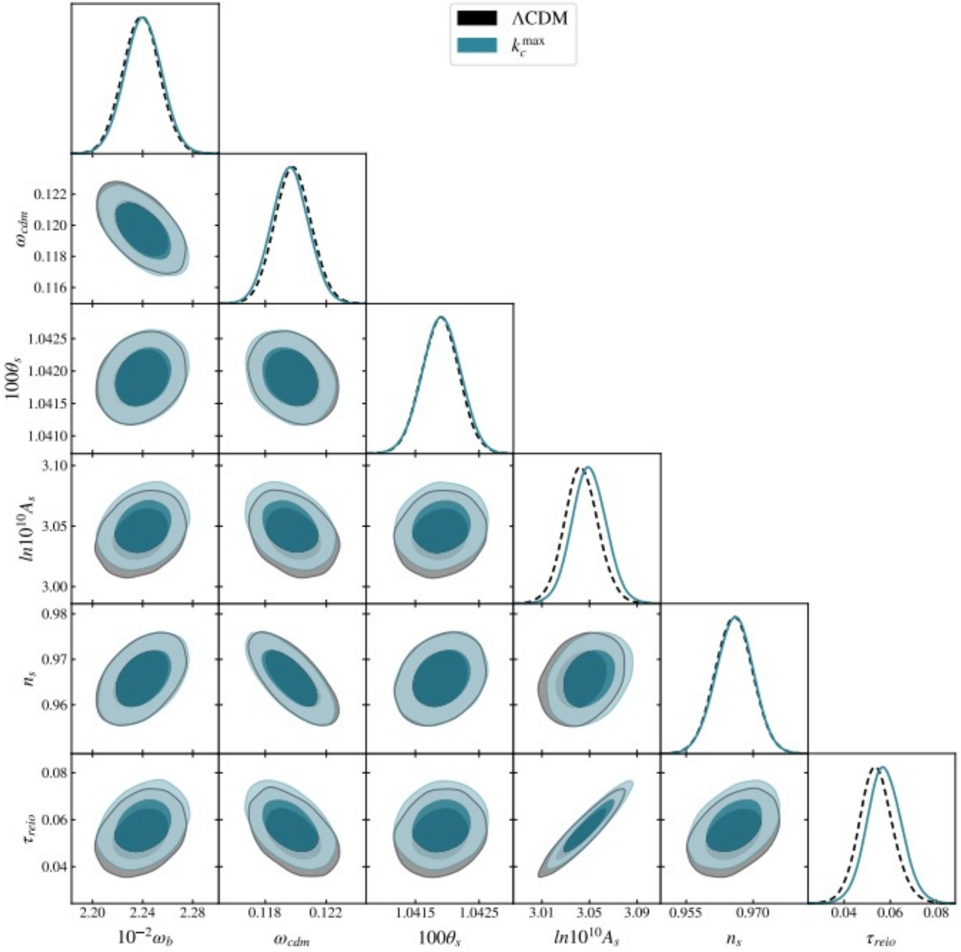


FIG. 3. 1 and 2-σ C.L. 2D contours for the 6 ΛCDM parameters, plus 1D marginalized posteriors, for ΛCDM (black) and LQCNO with k_c^{max} (blue).