Regular black holes and their relationship to polymerised models

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Motivation

Investigate quantum black hole models interesting for quantum gravity

Many people contribute:

[Ashtekar, Alonso-Bardaji, Bojowald, Brizuela, Modesto, Cartin, Khanna, Boehmer, Vandersloot, Chiou, Campiglia,, Gambini, Pullin, Sabharwal, Brannlund, Kloster, De Benedictis, Olmedo, Dadhich, Joe, Singh, Haggard, Rovelli, Vidotto, Corichi, Saini, Cortez, Cuervo, Morales-Técotl, Ruelas, Pawlowski, Bianchi,, Christodoulo, D'Ambrosio, Alesci, Bahrami, Pranzetti, Husain, Kelly, Santacruz, Wilson-Ewing, Lewandowski, Zhang, Ma, Song, Bodendorfer, Mele, Münch, Navascués, Mena Marugán, García-Quismondo, Perez, Speziale, Viollet, Han, K.G.,Liu, Li, Vera, Weigl,,...]

Recent reviews: [Gambini,Olmedo,Pullin '22], [Ashtekar, Olmedo, Singh '23]

Dynamical formulation of gravitational collapse: consider spherically spherically symmetric models with dust

LTB models, Oppenheimer-Snyder collapse also special case of vacuum solution

Here we will consider effective models to formulate such models which involve (LQG inspired) quantum corrections

Aim: Develop formalism that allows to investigate a broad class of effective models [seminal work by Bojowald, Harada, Reyes, Tibrewala '08 '09]

I. Classical LTB models and vacuum solutions

I. Classical LTB models

LTB: Spherically symmetric solution with dust

We consider Ashtekar-Barbero variables for spherical symmetry $\left(A_a^j, E_j^a\right)$

After implementing the Gauss constraint: [Bojowald Kastrup '00], [Bojowald, Swiderski'03]

$$(x, \theta, \phi)$$

$$A_a^j \tau_j \, \mathrm{d}X^a = 2\beta K_x(x)\tau_1 \, \mathrm{d}x + \left(\beta K_\phi(x)\tau_2 + \frac{\partial_x E^x(x)}{2E^\phi(x)}\tau_3\right)\mathrm{d}\theta$$

$$+ \left(\beta K_\phi(x)\tau_3 - \frac{\partial_x E^x(x)}{2E^\phi(x)}\tau_2\right)\sin(\theta)\mathrm{d}\phi + \cos(\theta)\tau_1 \, \mathrm{d}\phi$$

$$E_j^a \tau^j \frac{\partial}{\partial X^a} = E^x(x)\sin(\theta)\tau_1\partial_x + \left(E^\phi(x)\tau_2\right)\sin(\theta)\partial_\theta + \left(E^\phi(x)\tau_3\right)\partial_\phi,$$

Reduced phase space including dust:

$$\{K_x(x), E^x(y)\} = G\delta(x,y) \quad \left\{K_\phi(x), E^\phi(y)\right\} = G\delta(x,y) \quad \{T(x), P_T(y)\} = \delta(x,y)$$

I. Classical LTB models: LTB condition

General spherically symmetric metric

$$\mathrm{d}s^2 = -N(x,t)^2 dt^2 + \frac{\left(E^\phi\right)^2}{|E^x|}\left(dx + N^x dt\right)^2 + |E^x|\, d\Omega^2$$

Consider the form of the LTB metric [Lemaître '33], [Tolman '34], [Bondi '47]

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \frac{\left((E^x)'\right)^2}{4\,|E^x|\,(1+\mathcal{E}(x))}\mathrm{d}x^2 + |E^x|\,\mathrm{d}\Omega^2$$

To match both metric we need [Bojowald, Harada, Reyes, Tibrewala '08 '09]

$$N = 1 \quad N^x = 0 \quad G_x(x) = \frac{E^{x\prime}}{2E^\phi}(x) - \sqrt{1+\mathcal{E}(x)} = 0$$

marginally bound case $\mathcal{E}(x) = 0$ shells decouple classically

$$\partial_t E^x(x) = -2\sqrt{E^x}(x) K_\phi(x), \quad \partial_t K_\phi(x) = \frac{\left(K_\phi\right)^2}{2\sqrt{E^x}}(x)$$

I. Classical Vacuum Solutions

Gravitational contribution to the scalar constraint, conserved quantity $M(x)$

$$C(x)|_{LTB} = \partial_x \widetilde{H}(x), \quad -M(x) = \widetilde{H}(x) := -\frac{1}{2G}\sqrt{E^x(x)}\,(K_\phi)^2\,(x)$$

We can rewrite EOMs in form of Friedmann equation $R(x) = \sqrt{E^x(x)}$

$$\frac{\dot{R}^2}{R^2}(x) = \frac{8\pi G}{3}\rho(x) \quad \text{with} \quad \rho(x) = \frac{3}{4\pi}\frac{M}{R^3}(x).$$

Classical vacuum solution: choose dust mass profile $M(x) = m = \text{const}$
using that M is conserved: $s(x)$ integration constant

$$K_\phi(x) = \frac{\sqrt{2Gm}}{(E^x)^{\frac{1}{4}}(x)} \longrightarrow \partial_t E^x(x) = -2\sqrt{2Gm}\,(E^x)^{\frac{1}{4}}(x) \longrightarrow E^x = \left[\frac{3}{2}\sqrt{2Gm}(s(x)-t)\right]^{\frac{4}{3}} \longrightarrow E^\phi$$

Transforming to Schwarzschild coordinates $r = \sqrt{E^x}, \quad r_s := 2Gm$

$$\mathrm{d}s^2 = -\left(1-\mathcal{G}(r)^2\right)\mathrm{d}\tau^2 + \frac{1}{(1-\mathcal{G}(r)^2)}\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2 \qquad \mathcal{G}(r)^2 := \frac{(\partial_x E^x)^2}{4E^x} = \frac{r_s}{r},$$

II. Effective models

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1. Can we construct polymerised vacuum solutions in a similar way?
2. Given a Schwarzschild-like metric can we reconstruct the associated polymerised model?

II. How to construct solutions here?

Class II: Compatible LTB condition exists + conserved energy density restricts possible polymerisation functions and form of LTB condition

Dynamics of effective model:

EOM: $\partial_t E^x = -2\sqrt{E^x} f^{(2)}, \quad \partial_t K_\phi = \frac{1}{2\sqrt{E^x}} \left(f^{(1)} - g^2_{(\alpha)} \left(2h_2 + 4E^x \partial_{E^x} h_2 - h_1 \right) + h_1 \right)$

Cons. energy dens. $\widetilde{H}^{(\alpha)}(x) := \frac{1}{2G} \left[\frac{\sqrt{E^x}}{g_{(\alpha)}} \left(-F + h_2 \left(g^2_{(\alpha)} - 1 \right) \right) \right](x), \quad C^{(\alpha)}(x)\Big|_{LTB} = \partial_x \widetilde{H}^{(\alpha)}(x),$

Requirements for polarisation and inverse triad corrections in class II:

$\partial_{K_\phi} F\left(K_\phi, E^x\right) = 2f^{(2)}\left(K_\phi, E^x\right)$ and $\frac{h_1 - 2E^x \partial_{E^x} h_2}{h_2} = \frac{-2E^x \partial_{E^x} F + f^{(1)}}{F}$

General effective LTB condition: $C^{(\alpha)}_{LTB}(x) := \left[\frac{E^{x\prime}}{2E^\phi} - \widetilde{g}_{(\alpha)}\left(K_\phi, E^x, \Xi\right) \right](x) \quad \Xi(x) := \sqrt{1 + \mathcal{E}(x)}$

Here class II marginally bound: $g_{(\alpha)} = g_{(\alpha)}\left(E^x\right)$ classical case: $g_{(\alpha)} = 1$

II.Can we still construct a solution for E^{x} ?

We need the inverse of F:

$$\widetilde{H}^{(\alpha)}(x) := \frac{1}{2G}\left[\frac{\sqrt{E^x}}{g_{(\alpha)}}\left(-F + h_2\left(g_{(\alpha)}^2 - 1\right)\right)\right](x)$$

Given this we can solve for $K_\phi(t,x)$

$$K_\phi(t,x) = F_{(i)}^{-1}\left[h_2\left(g_{(\alpha)}^2 - 1\right) - 2GM(x)\frac{g_{(\alpha)}}{\sqrt{E^x}}, E^x\right] =: F_{(i),M(x)}^{-1}\left[E^X\right]$$

Class II important: LTB condition only depends on E^x and likewise h_2

Why label (i) ?

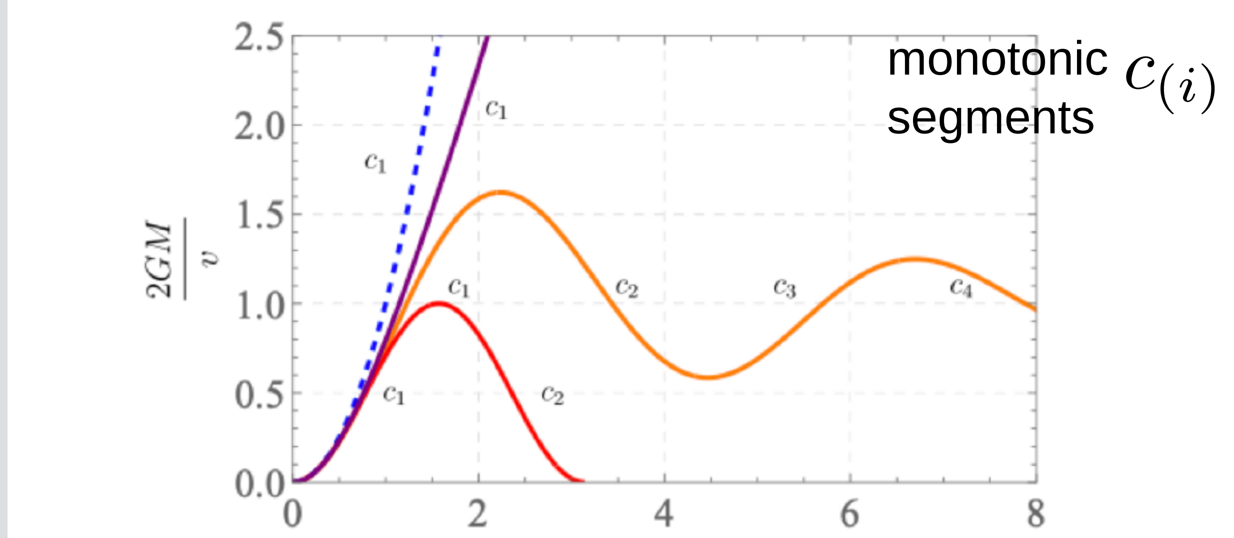
Specialise to models where classical LTB condition is compatible + no inverse triad corrections (mimetic model exists)

Phase space trajectories: level sets

$$c\left(K_\phi, E^x\right) := \widetilde{H}^{(\alpha)}(x) = -M(x)$$

$$c(b,v) \qquad b := \frac{K_\phi}{\sqrt{E^x}}, v := (E^x)^{\frac{3}{2}}$$

$$\widetilde{H}^\alpha(x) = -M(x) = -\frac{1}{2G}v\tilde{F}(b), \quad F = E^x\tilde{F}(b)$$



II. Now we can construct the solution

Given the inverse F^{-1} we obtain for the EOM and its solution:

$$\frac{\partial_t E^x}{2\sqrt{E^x}}(t,x) = -f^{(2)}\left[F^{-1}_{(i),M(x)}\left[E^X\right], E^X\right]$$

$$E^x_{(i)}(t,x) = \mathcal{F}^{-1}_{(i)}(s(x)-t)$$

with

$$\mathcal{F}_{(i)} = \int \frac{dE^x}{2\sqrt{E^x} f^{(2)}\left[F^{-1}_{(i),M(x)}\left[E^x\right], E^x\right]}$$

and $E^x = E^x_{(1)}(t,x) \circ E^x_{(2)}(t,x) \circ \cdots$ continuous by suitable choice of s(x)

Form of the solution in Schwarzschild-like coordinates

$$\mathrm{d}s^2 = -\left(1-\mathcal{G}_{(i)}(r)^2\right)\mathrm{d}\tau^2 + \frac{1}{g_{(\alpha)}(r)^2\left(1-\mathcal{G}_{(i)}(r)^2\right)}\mathrm{d}r^2 + r^2\ \mathrm{d}\Omega^2$$

Trafo $r=\sqrt{E^x}$ only defined for each piecewise segment

with

$$\mathcal{G}_{(i)}(r)^2 := \frac{\left(\partial_x E^x_{(i)}\right)^2}{4g^2_{(\alpha)}E^x(i)} = \frac{\left(f^{(2)}\left[F^{-1}_{(i),m}\left[E^x\right], E^x\right]\right)^2}{g^2_{(\alpha)}}(r)$$

In general can be different $\mathcal{G}_{(i)}$ for each i

example later

II. Reconstruction algorithm

So far we chose some polymerisation determined F^{-1} and from solution $\mathcal{G}_{(i)}$

Can we also obtain the polymerised model given some choice of $\mathcal{G}$?

In particular interesting for regular black holes like Bardeen and Hayward

Bardeen $\mathcal{G}(r)^2 = \frac{r_s r^2}{(r^2+l^2)^{\frac{3}{2}}}$ Hayward $\mathcal{G}(r)^2 = \frac{r_s r^2}{r^3 + l^2 r_s}$

Reconstruction: special case of class II with classical compatible LTB condition+no inverse triad corrections, then factorisation in M

$$-M(x) = \widetilde{H}^{(\alpha)}(x) = -\frac{1}{2G}[v\tilde{F}(b)](x), \quad v := (E^x)^{\frac{3}{2}}$$

Using the result for $\mathcal{G}_{(i)}$ from before we obtain it as a function of $\frac{2Gm}{v} = \frac{r_s}{r^3}$

$$\mathcal{G}(r) = \frac{r}{2}\tilde{F}'\left[\tilde{F}^{-1}\left[\frac{r_s}{r^3}\right]\right] = -\frac{3r_s}{2r^3\frac{\mathrm{d}}{\mathrm{d}r}\left(\tilde{F}^{-1}\left[\frac{r_s}{r^3}\right]\right)} \longrightarrow \tilde{F}^{-1}\left[\frac{r_s}{r^3}\right] = -\int \mathrm{d}r\frac{3r_s}{2r^3\mathcal{G}(r)} = \int \mathrm{d}\left(\frac{r_s}{r^3}\right)\frac{r}{2\mathcal{G}(r)}$$

From this we can derive the polymerisations of Bardeen and Hayward metrics

III. Examples

(i). Standard LQC polymerisation

Polymerised vacuum solutions with a symmetric bounce: (no inverse triad corr.)

$$\tilde{F}(b) = \frac{\sin^2(\alpha_\Delta b)}{\alpha_\Delta^2}, \quad \alpha_\Delta := \gamma\sqrt{\Delta}, \quad \Delta := 4\pi l_P$$ LTB condition $g_{(\alpha)} = g_\Delta = 1$

Then we obtain in Schwarzschild-like coordinates

$$\mathcal{G}(r)^2 = \frac{r_s}{r} - \frac{\alpha_\Delta^2 r_s^2}{r^4}$$

Since $\mathcal{G}(r)$ is real, minimal radius $r_{\min} = \left(r_s \alpha_\Delta^2\right)^{\frac{1}{3}}$

[J. G. Kelly, R. Santacruz, and E. Wilson-Ewing '20, Lewandowski, Y. Ma, J. Yang, and C. Zhang '22]

This coordinate is only defined for $b \in \left(0, \frac{\pi}{2}\right)$ or $b \in \left(\frac{\pi}{2}, \pi\right)$ two monotone segments have same

Taking $\mathcal{G}(r)$ as starting point one can also obtain by reconstruction $\tilde{F}(b) = \frac{\sin^2(\alpha_\Delta b)}{\alpha_\Delta^2}$

Next: underlying effective spherically symmetric model

$$\tilde{f}^{(1)}(b) = \frac{3\sin^2(\alpha_\Delta b) - \alpha_\Delta b \sin(2\alpha b)}{\alpha_\Delta^2}, \quad \tilde{f}^{(2)}(b) = \frac{\sin(2\alpha_\Delta b)}{2\alpha_\Delta}$$

mimetic Lagrangian

Effective Hamiltonian:

$$C^\Delta = -\frac{E^\phi\sqrt{E^x}}{2G}\left[\frac{3\sin^2(\alpha_\Delta b)}{\alpha_\Delta^2} + \left(\frac{2\sqrt{E^x}K_x}{E^\phi} - b\right)\frac{\sin(\alpha_\Delta b)}{\alpha_\Delta} + \frac{1-\left(\frac{E^{x\prime}}{2E^\phi}\right)^2}{E^x} - \frac{2}{E^\phi}\left(\frac{E^{x\prime}}{2E^\phi}\right)'\right].$$

(i). Standard LQC polymerisation

Modified Friedmann equation gives the following polymerised vacuum solution:

$$R(t,x) = (2Gm)^{\frac{1}{3}} \left(\frac{9}{4}(s(x)-t)^2 + \alpha_\Delta^2 \right)^{\frac{1}{3}}$$

bouncing solution with minimal radius

$$r_{\min} = \left(2Gm\alpha_\Delta^2\right)^{\frac{1}{3}}$$

We can further extend the solution to the marginally bound case LTB solution:

$$R(t,x) = (2GM(x))^{\frac{1}{3}} \left(\alpha_\Delta^2 + \frac{9}{4} z^2 \right)^{\frac{1}{3}}, \quad z = s(x) - t$$

minimal radius

$$r_{\min} = \left(2GM(x)\alpha_\Delta^2\right)^{\frac{1}{3}}$$

Curvature scalars for generic dust profile M(x)

$$\mathcal{R} = \frac{\mathcal{A}}{\left(9z^2 + 4\alpha_\Delta^2\right)^2 \mathcal{S}}, \quad \mathcal{K} = \frac{\mathcal{B}}{\left(9z^2 + 4\alpha_\Delta^2\right)^4 \mathcal{S}^2},$$

$$\mathcal{S} = M'(x)\left(9z^2 + 4\alpha_\Delta^2\right) + 18M(x)s'(x)z$$

at the bounce z=0, no dependence on M

$$M'(x) \neq 0: \quad \mathcal{R}|_{z=0} = \frac{9}{\alpha_\Delta^2}, \quad \mathcal{K}|_{z=0} = \frac{27}{\alpha_\Delta^4}$$

Central singularity is resolved, shell crossing if $9M(x)^2 s'(x)^2 - 4\alpha_\Delta^2 M'(x)^2 \geq 0$ (real roots of $\mathcal{S}(x)$)

(ii). Non-standard LQC polymerisation

[Dapor-Liegener '17], [Ying, Ding, Ma '09]

Polymerised vacuum solutions with an asymmetric bounce: (no inverse triad corr.)

$$\tilde{F}(b) = \frac{\sin^2(\alpha_\Delta b\gamma)\left(1-\left(\gamma^2+1\right)\sin^2(\alpha_\Delta b\gamma)\right)}{(\alpha_\Delta\gamma)^2}$$

LTB condition $g_{(\alpha)} = g_\Delta = 1$

Then we obtain in Schwarzschild-like coordinates

$$\dot{R}(t,x) = \mathcal{G}_\pm(r)|_{r=R(t,x)} = \frac{R(t,x)\sqrt{\frac{1}{2}-\gamma^2 x_0}\sqrt{x_0 \pm \sqrt{1-2\gamma^2 x_0}+1}}{\alpha_\Delta\left(\gamma^2+1\right)}$$

Here example of two different $\mathcal{G}(r)$:

Taking $\mathcal{G}_\pm(r)$ as starting point one reconstruct $\tilde{F}(b)$

Next: underlying effective spherically symmetric model (+mimetic Lagrangian)

$$\tilde{f}^{(1)}(b) = \frac{\sin(\alpha_\Delta b\gamma)\left(\sin(\alpha_\Delta b\gamma)\left(\left(\gamma^2+1\right)\left(2\alpha_\Delta b\gamma\sin(2\alpha_\Delta b\gamma)-3\sin^2(\alpha_\Delta b\gamma)\right)+3\right)-2\alpha_\Delta b\gamma\cos(\alpha_\Delta b\gamma)\right)}{\alpha_\Delta^2\gamma^2}$$

$$\tilde{f}^{(2)}(b) = \frac{\sin(2\alpha_\Delta b\gamma)\left(\left(\gamma^2+1\right)\cos(2\alpha_\Delta b\gamma)-\gamma^2\right)}{2\alpha_\Delta\gamma}$$

$\longrightarrow$ effective Hamiltonian

(ii). Non-standard LQC polymerisation

[Dapor-Liegener '17], [Ying, Ding, Ma '09]

In this case the marginally bound LTB solution reads:

$$R(t,x) = \sqrt[3]{\frac{2GM(x)\left(4\alpha_\Delta^2\gamma^2 + 9\eta^2\right)^2}{18\eta^2 - 8\alpha_\Delta^2\gamma^4}}, \quad s(x) - t = \eta - \frac{2}{3}\alpha_\Delta\left(\gamma^2+1\right)\tanh^{-1}\left(\frac{2\alpha_\Delta\gamma^2}{3\eta}\right)$$

Bouncing solutions with bounce at η_0 and minimal radius $r_{\min} = 2^{2/3}\sqrt[3]{a^2\gamma^2\left(\gamma^2+1\right)M(x)}$

Modified FRW eqn defined for each segments $\mathcal{C}_\pm$

$\eta \geq \eta_0 \;\; \mathcal{C}_-$ and $\frac{2}{3}\alpha_\Delta\gamma^2 < \eta \leq \eta_0 \quad \mathcal{C}_+$

Curvature scalars

$$\mathcal{R} = \frac{\mathcal{A}}{\left(9\eta^2 + 4\alpha_\Delta^2\gamma^2\right)^4\mathcal{S}}, \quad \mathcal{K} = \frac{\mathcal{B}}{\left(9\eta^2 + 4\alpha_\Delta^2\gamma^2\right)^8\mathcal{S}^2}$$

for vacuum case

$$\mathcal{K} \sim \frac{81\gamma^2}{16\alpha_\Delta^2\left(\gamma^2+1\right)^2\left(2\gamma^2+1\right)\left(\eta-\eta_0\right)^2}$$

$$\mathcal{S} = M'(x)\left(9\eta^2 + 4\alpha_\Delta^2\gamma^2\right)^2 + 18M(x)s'(x)\eta\left(4\alpha_\Delta^2\gamma^2\left(2\gamma^2+1\right) - 9\eta^2\right)$$

Central singularity is resolved, shell crossing singularity still present

(iii). Bardeen and Hayward

Polymerised vacuum solutions with a symmetric bounce: (no inverse triad corr.)

	Bardeen	Hayward
metric	$\mathcal{G}(r)^2 = \dfrac{r_s r^2}{\left(r^2 + \alpha^{\frac{4}{3}} r_s^{\frac{2}{3}}\right)^{\frac{3}{2}}}$	$\mathcal{G}(r)^2 = \frac{r_s r^2}{(r^3+\alpha^2 r_s)}$
Polymerization function	$\tilde{F}^{-1} = \dfrac{\alpha r_s}{2r^3} {}_2F_1\left(-\dfrac{3}{2}, -\dfrac{3}{4}, -\dfrac{1}{2}, -\left(\alpha^2 \dfrac{r_s}{r^3}\right)^{-\frac{2}{3}}\right) + \dfrac{\sqrt{\pi}\Gamma\left(\frac{3}{4}\right)}{\alpha\Gamma\left(-\frac{3}{4}\right)}$	$\tilde{F}^{-1} = \frac{2\eta\alpha + \sinh(2\alpha\eta)}{4\alpha}$, $\alpha\eta = \sinh^{-1}\sqrt{\frac{\alpha^2 r_s}{r^3}}$
Marginally bound solution	$R(t,x) = (2GM(x))^{\frac{1}{3}}\sqrt{\eta^{\frac{4}{3}} - \alpha^{\frac{4}{3}}}, \quad \eta \geq \alpha$ $s(x) - t = \dfrac{2}{3}\eta + \alpha\tan^{-1}\left(\eta^{\frac{1}{3}}\alpha^{-\frac{1}{3}}\right) - \alpha\,\mathrm{Re}\tanh^{-1}\left(\eta^{\frac{1}{3}}\alpha^{-\frac{1}{3}}\right)$	$R(t,x) = \left(\dfrac{2GM(x)\alpha^2}{\sinh^2(\alpha\eta)}\right)^{\frac{1}{3}}, \quad \eta \geq 0$ $s(x) - t = \dfrac{2}{3}\alpha(\coth(\alpha\eta) - \alpha\eta)$
Curvature scalars (see App. B)	$\mathcal{R} = \dfrac{\mathcal{A}}{\eta^{14/3}\mathcal{S}}, \quad \mathcal{K} = \dfrac{\mathcal{B}}{\eta^{28/3}\mathcal{S}^2}$ $\mathcal{S} = M'(x)\eta + 3M(x)s'(x)$	$\mathcal{R} = \dfrac{\mathcal{A}}{4\alpha^3\mathcal{S}}, \quad \mathcal{K} = \dfrac{\mathcal{B}}{16\alpha^6\mathcal{S}^2}$ $\mathcal{S} = M'(x) + 3M(x)s'(x)\dfrac{\tanh(\alpha\eta)}{r}$

shell crossing singularities: situation like in GR: can be avoided by choosing suitable dust profile M(x) and s(x), e.g. $M'(x) \geq 0$ and $s'(x) \geq 0$

IV. Summary & Conclusions

Formalism allows to investigate a broad class of effective models with different kind of polymerisation

The formalism can be used in different ways:

1.) start with a given polymerisation in the LTB sector determine the underlying gauge-unfixed spherically symmetric model

2.) For regular black holes: start with a modified Schwarzschild-metric and derive the corresponding effective model or vice versa.

Here we considered examples for bounded and unbounded polymerisations

Saw that gauge fixing and/or coordinate choice is more subtle in effective models in general underlying covariant mimetic model helpful.

Next steps:

Investigate non-marginally bound case in a similar manner

Investigate more in detail shock solutions and whether we can construct polymerised model without the presence of shell-crossing singularities

Thank you for your time!

Underlying covariant Lagrangian

Extended mimetic gravity

$$S\left[g_{\mu\nu},\phi,\lambda\right]=\frac{1}{8\pi G}\int_{M_4}d^4x\sqrt{-g}\left[\frac{1}{2}R^{(4)}+L_\phi\left(\phi,\chi_1,\cdots,\chi_p\right)+\frac{1}{2}\lambda(\phi_\mu\phi^\mu+1)\right]$$

mimetic field ϕ, λ is a Lagrange multiplier for the mimetic condition, 2+1 dof

$$\chi_n\equiv\sum\phi^{\mu_2}_{\mu_1}\phi^{\mu_3}_{\mu_2}\cdots\phi^{\mu_n}_{\mu_{n-1}}\phi^{\mu_1}_{\mu_n},\quad \phi_\mu=\nabla_\mu\phi,\quad \phi_{\mu\nu}=\nabla_\mu\nabla_\nu\phi.$$

Spherically symmetric model: Sufficient to have $L_\phi\left(\chi_1,\chi_2\right)$, $\psi=ln(E^x)$ 2D action
[[Achour, Lamy, Liu, Noui '18], [Han, Liu '22]

$$S_2=\frac{1}{4G}\int_{\mathcal{M}_2}\mathrm{d}^2x\,\det(e)e^{2\psi}\left\{\mathcal{R}+L_\phi(X,Y)+\frac{\lambda}{2}\left(\phi_{,j}\phi^{,j}+1\right)\right\}$$

(Smooth) mimetic field defines foliation into spacelike hyper surfaces $\phi=\text{const}$
Generalised Einstein's equation

$$G^\Delta_{\mu\nu}:=G_{\mu\nu}-T^\phi_{\mu\nu}=-\lambda\partial_\mu\phi\partial_\nu\phi,\quad \partial_\mu\phi\partial^\mu\phi=-1$$

Underlying covariant Lagrangian

Now for models with have no inverse triad corrections + compatible with

$\overline{\mu}$-scheme we can relate the choice of the mimetic potential to specific choices of polymerisation function

[Achour, Lamy, Liu, Noui '18], [Han, Liu '22]
]

$$S_2 = \frac{1}{4G}\int_{\mathcal{M}_2} \mathrm{d}^2x \det(e) e^{2\psi}\left\{\mathcal{R} + L_\phi(X,Y) + \frac{\lambda}{2}\left(\phi_{,j}\phi^{,j}+1\right)\right\}$$

Higher derivative couplings can be expressed in terms of X,Y and relate to extrinsic curvature

$$X = -\Box_h\phi + Y = \frac{\partial_t E^\phi}{E^\phi}, \quad Y = -h^{ij}\partial_i\psi\partial_j\phi = \frac{\partial_t E^x}{2E^x} = \frac{\sin(2\alpha b)}{2\alpha}$$

Underlying covariant model allows to gauge-unfix temporal gauge with respect to mimetic field and consider coordinate taros in (t,x)

Interpretation: effective model has different clock than classical model