Optica Iónica & Espectrómetros

1era Clase: 21/01/2025, 09:30 - 10:30
Definiciones; Formalismo; Principales elementos de óptica iónica

2da Clase: 28/01/2025, 09:30 - 10:30
Higher Orders ; Ejemplos

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Bibliografía

• Optics of Charged Particles, Hermann Wollnik, Academic Press, Orlando, 1987.

• The Optics of Charged Particle Beams, David.C Carey, Harwood Academic Publishers, New York 1987.

• A First- and Second-Order Matrix Theory for the Design of Beam Transport Systems and Charged Particle Spectrometers. Karl L. Brown. SLAC Report-75. June 1982 https://indico.fnal.gov/event/23242/attachments/44759/53900/KarlBrown_transport_model.pdf

Computing Codes:

• COSY INFINITY 10.2 Beam Physics Manual https://www.bmtdynamics.org/cosy/manual/COSYBeamMan102.pdf

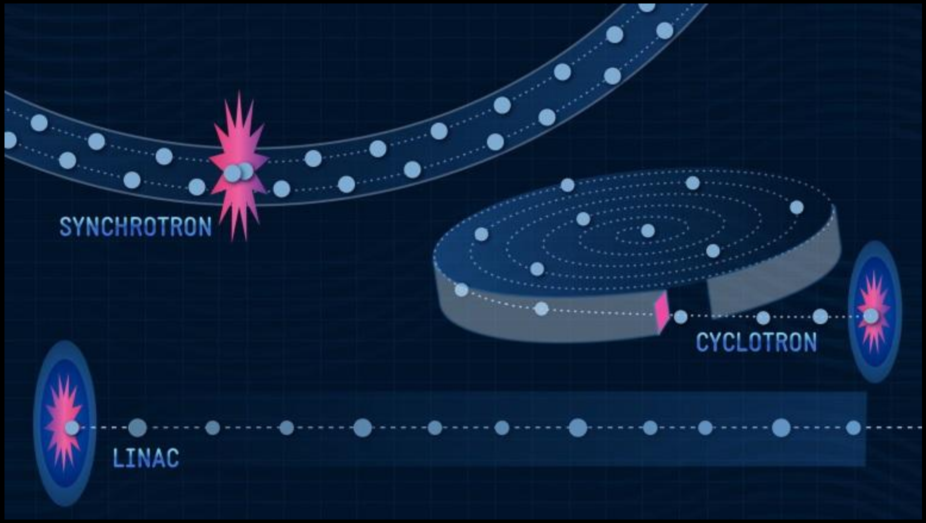
• GICOSY Manual https://web-docs.gsi.de/~weick/gicosy/

• TRANSPORT http://aea.web.psi.ch/Urs_Rohrer/MyWeb/trans.htm

Thanks B. Jacquot and Franck Varenne/ GANIL and H. Wieck / GSI for some material used in this course

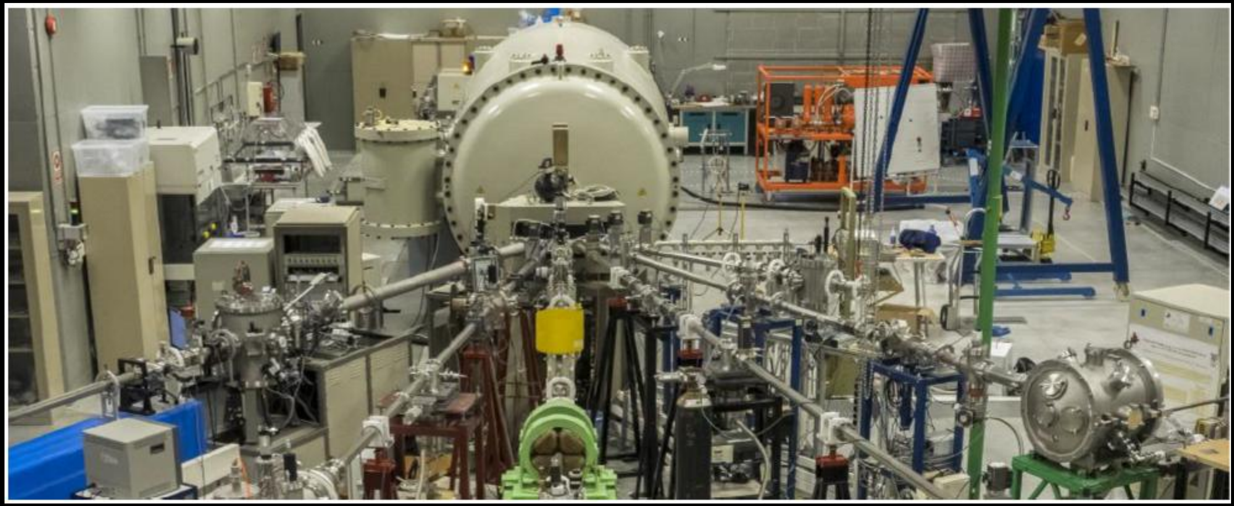
What is an ion Accelerator?

Device that uses electromagnetic fields to accelerate charged particles to high velocity



CMAM Madrid

CERN Accelerator complex



Electrostatic accelerators :

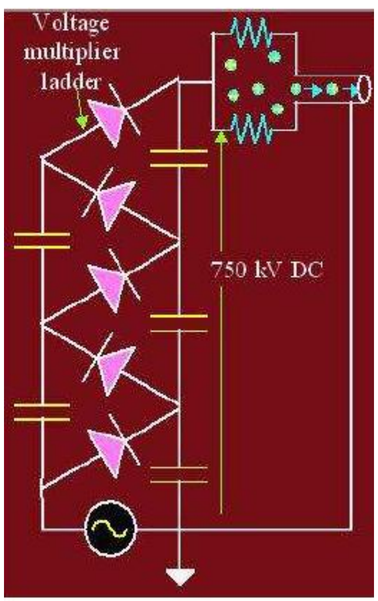
The kinetic energy of a charged particle q passing through an electrical potential V, is ***Ek = qV*.**

1932 Primera Reacción con protones:

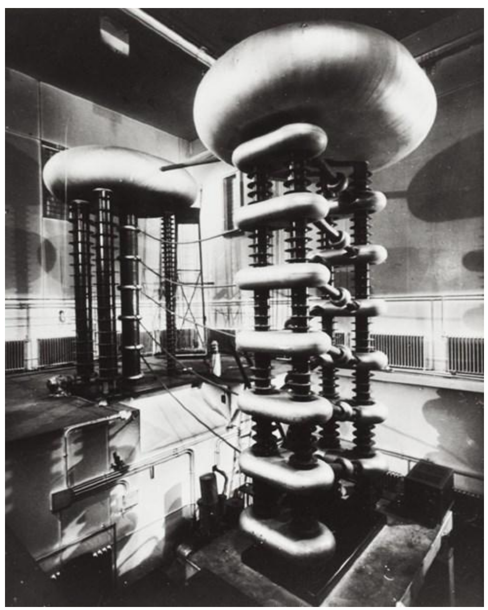
$$p + {}^{7}Li \rightarrow {}^{4}He + {}^{4}He$$

Li

Crockoft & Walton (PN 1951) Construyeron (1930) el primer acelerador para explorar el núcleo



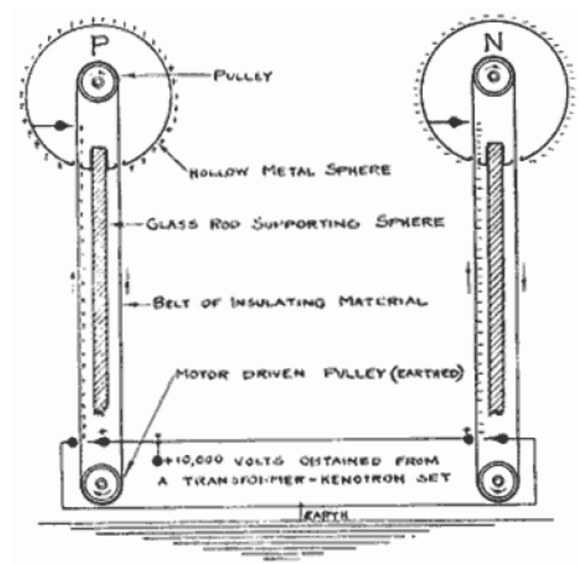
Concept



The Cockcroft-Walton generator at the University of Edinburgh.

1929 **Robert J. Van de Graaff** (postdoc Princeton)

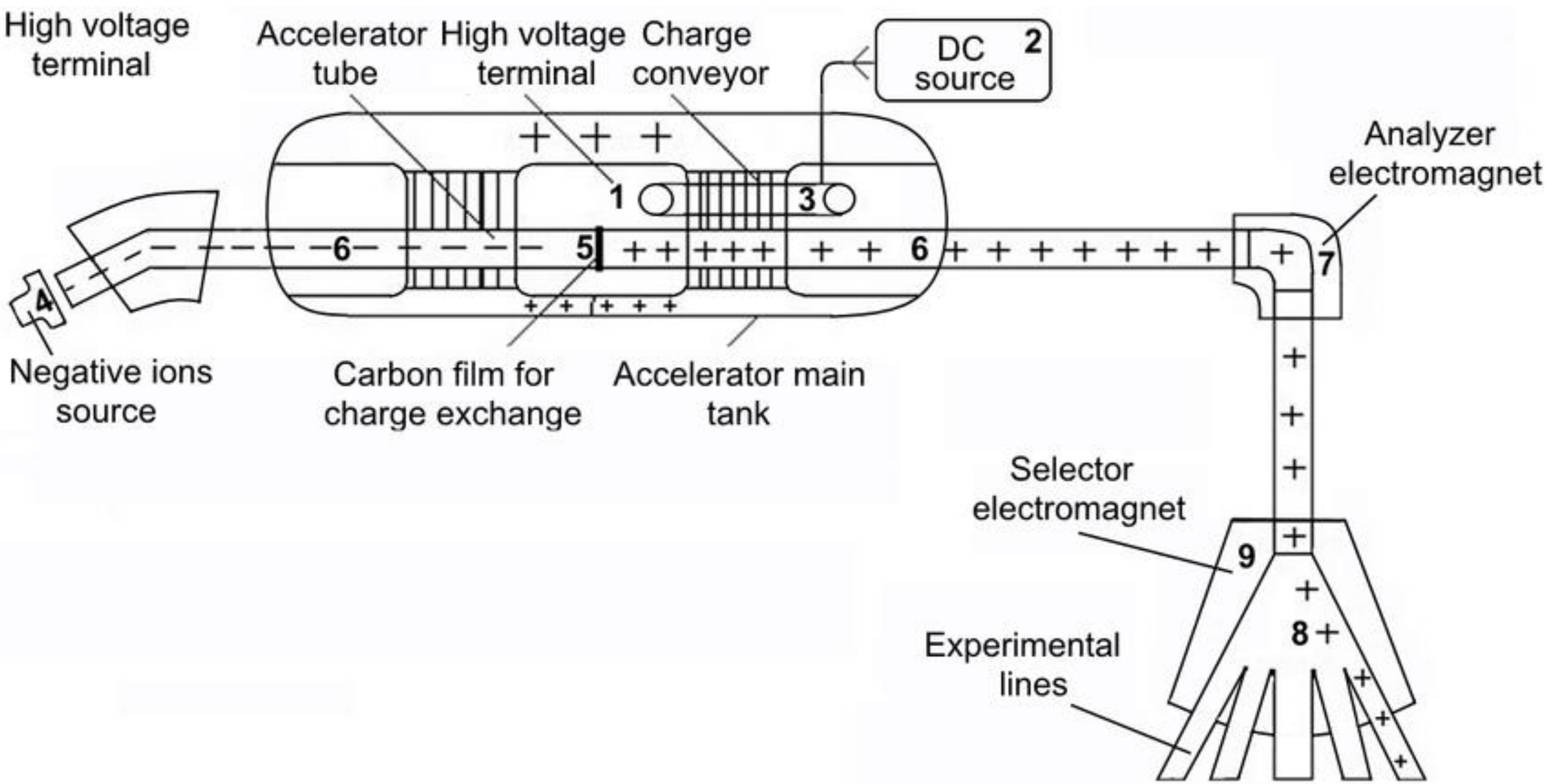
1933 Generator of 7 MV MIT



Concept of Van de Graaff Accelerator

Electrostatic accelerators :

Tandem Van de Graaff-> CMAM

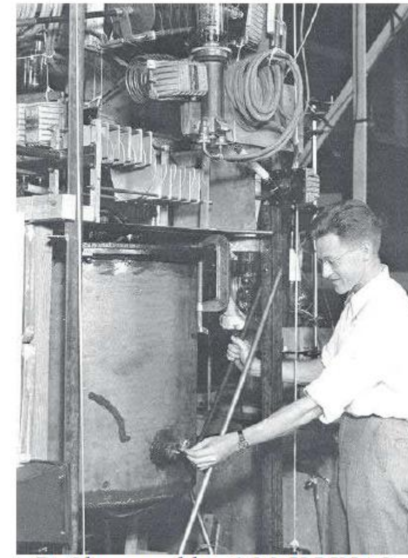


Aceleradores : los microscopios de la F.N.

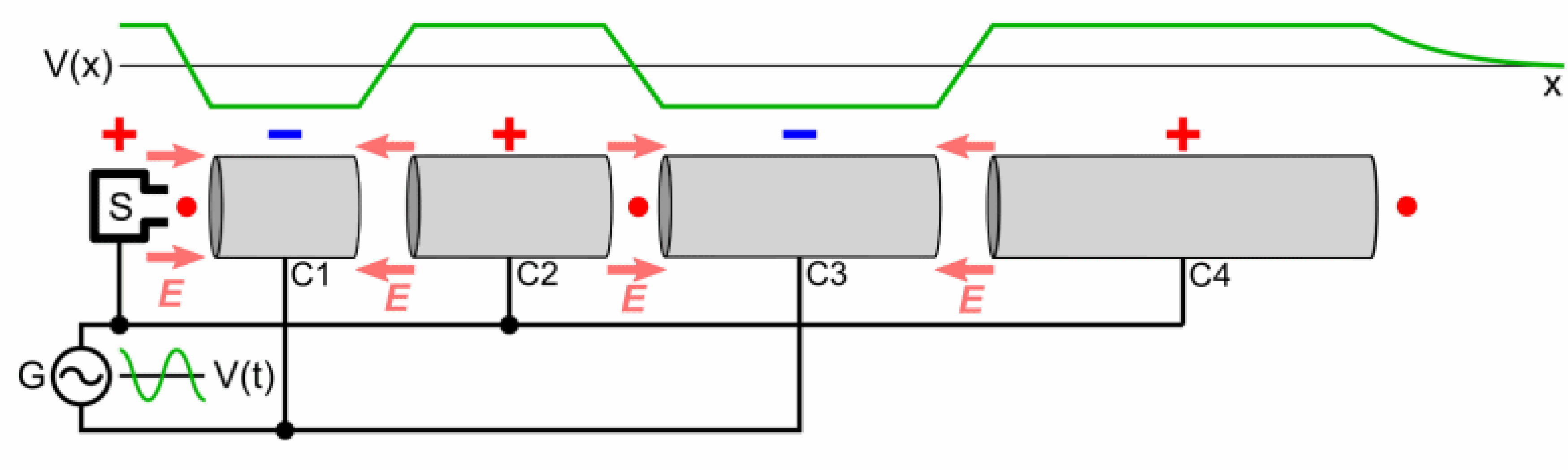
Aceleradores lineales

1929 **Wideroe** inventó el acelerador lineal (LINAC). Prototipo de 3-pasos

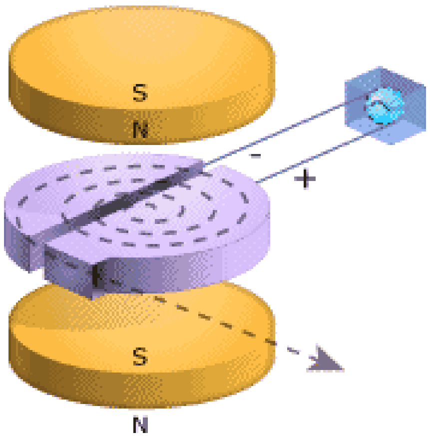
1931 **D. Sloan** (UC-Berkley, Lawrence group) construyó un LINAC que podía acelerar iones de Hg hasta 1 MeV.



D. Sloan and his 1 MeV LINAC



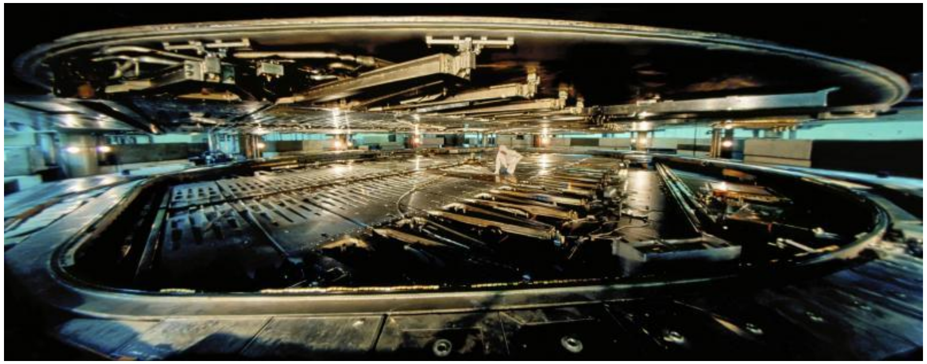
Aceleradores : los microscopios de la F.N.



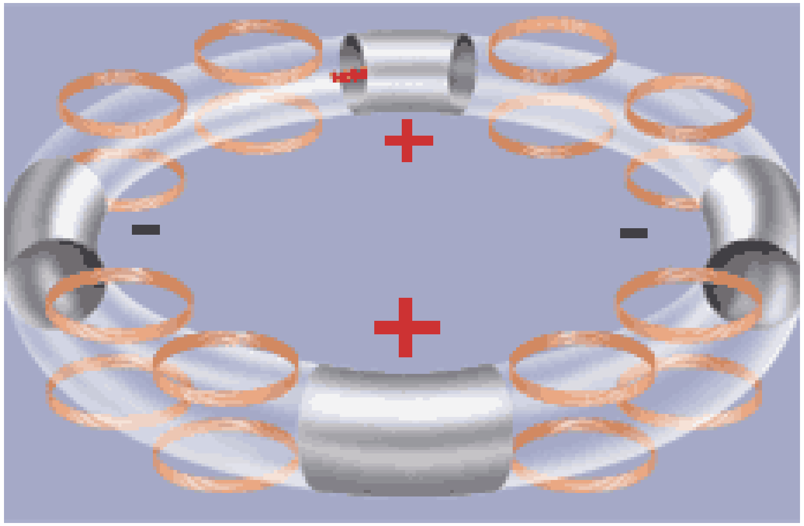
Ciclotrón

Lawrence (PN 1939)
propuso el ciclotrón

TRIUMF Cyclotron, Canada

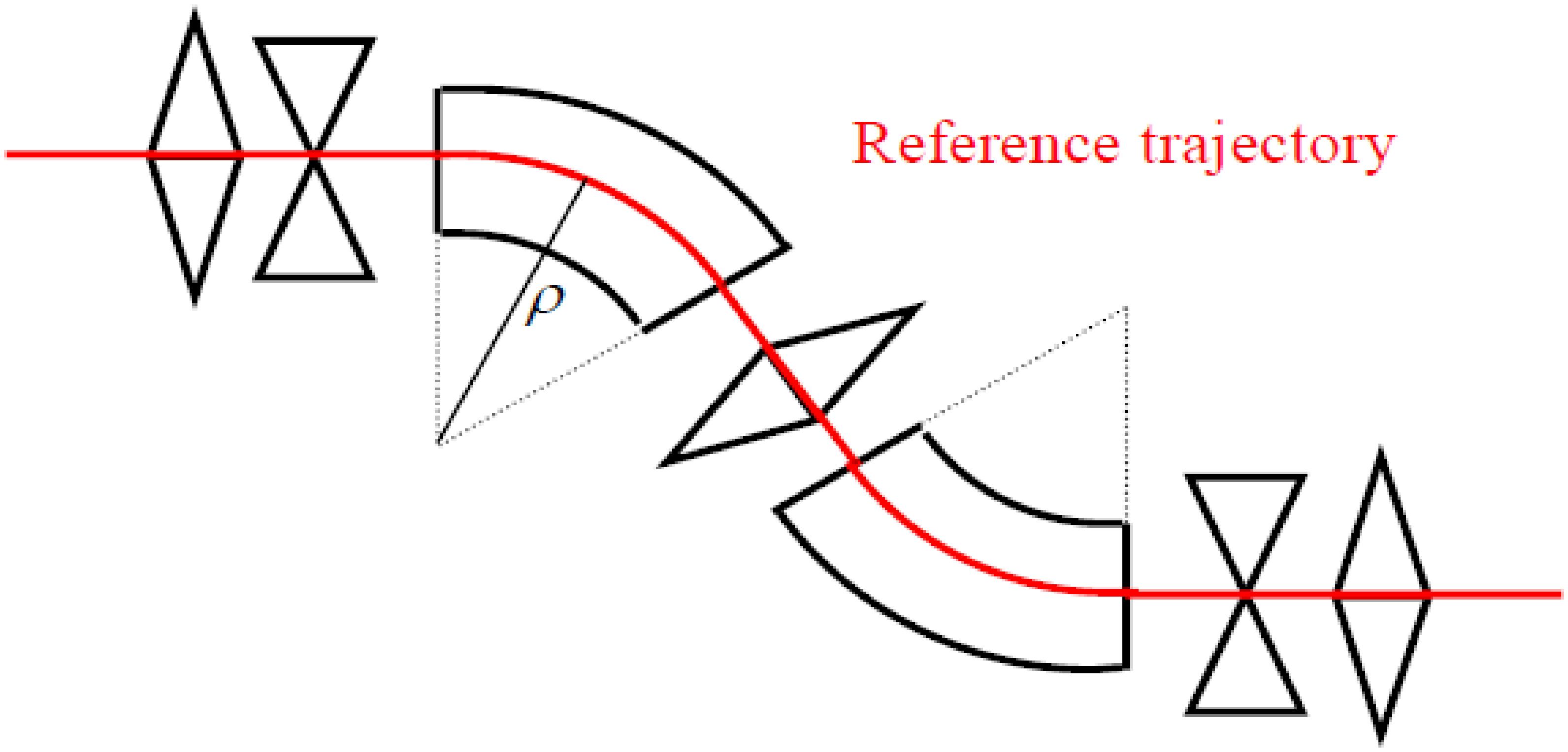


Complejo de aceleradores del CERN



Sincrotrón (1940)

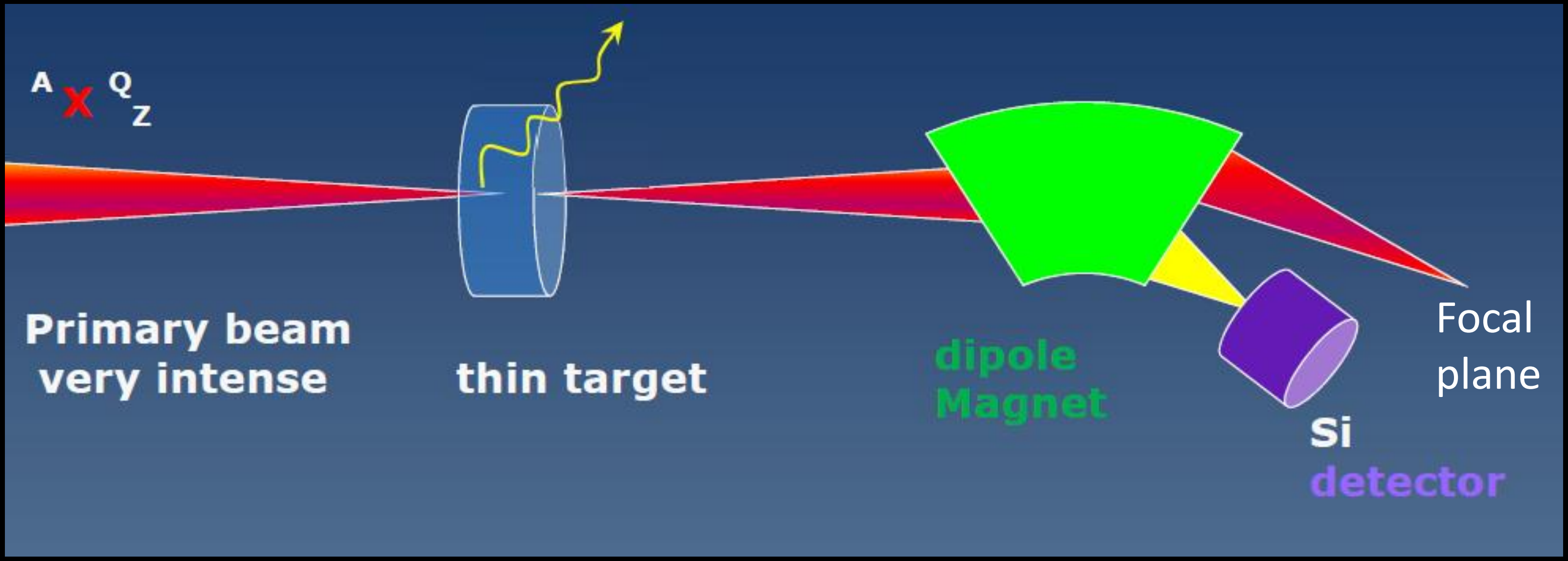
Ion optics



¿Qué es un espectrómetro?

En el sentido más amplio, un espectrómetro es cualquier instrumento que se utiliza para medir la variación de una característica física en un rango determinado.

Un dipolo magnético es el espectrómetro más simple para analizar la relación masa-carga(m/q)



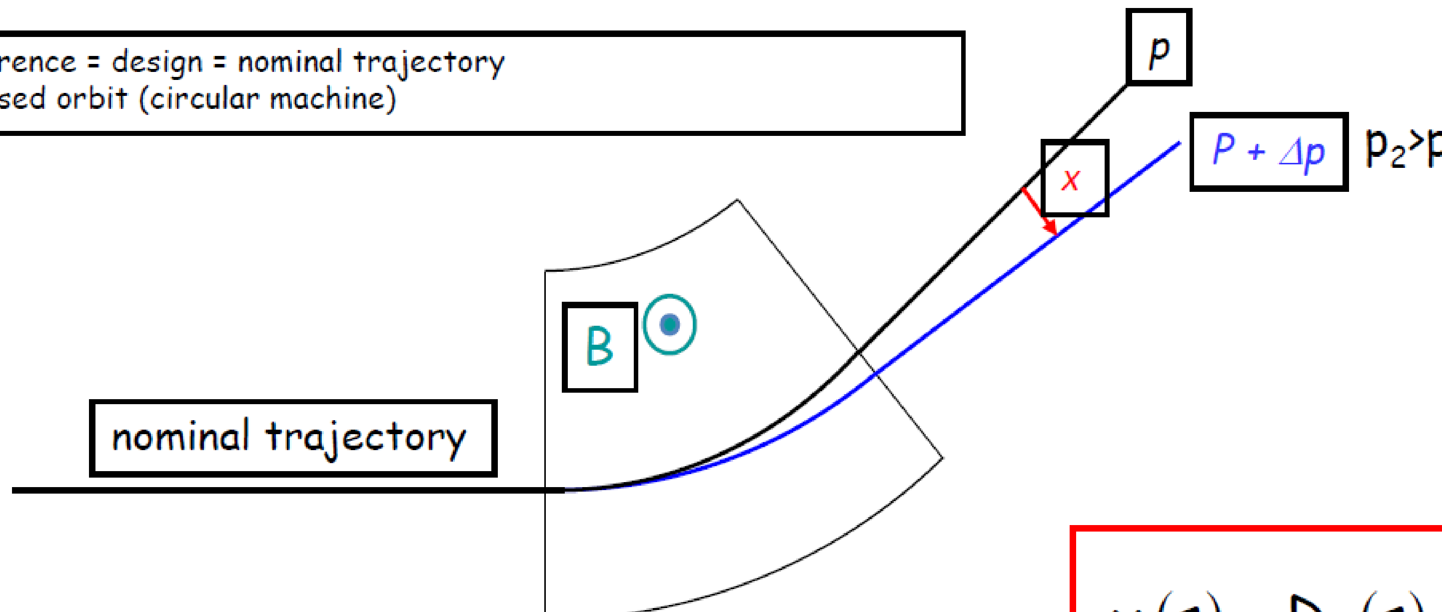
Dispersión

$$B\rho := p / q$$

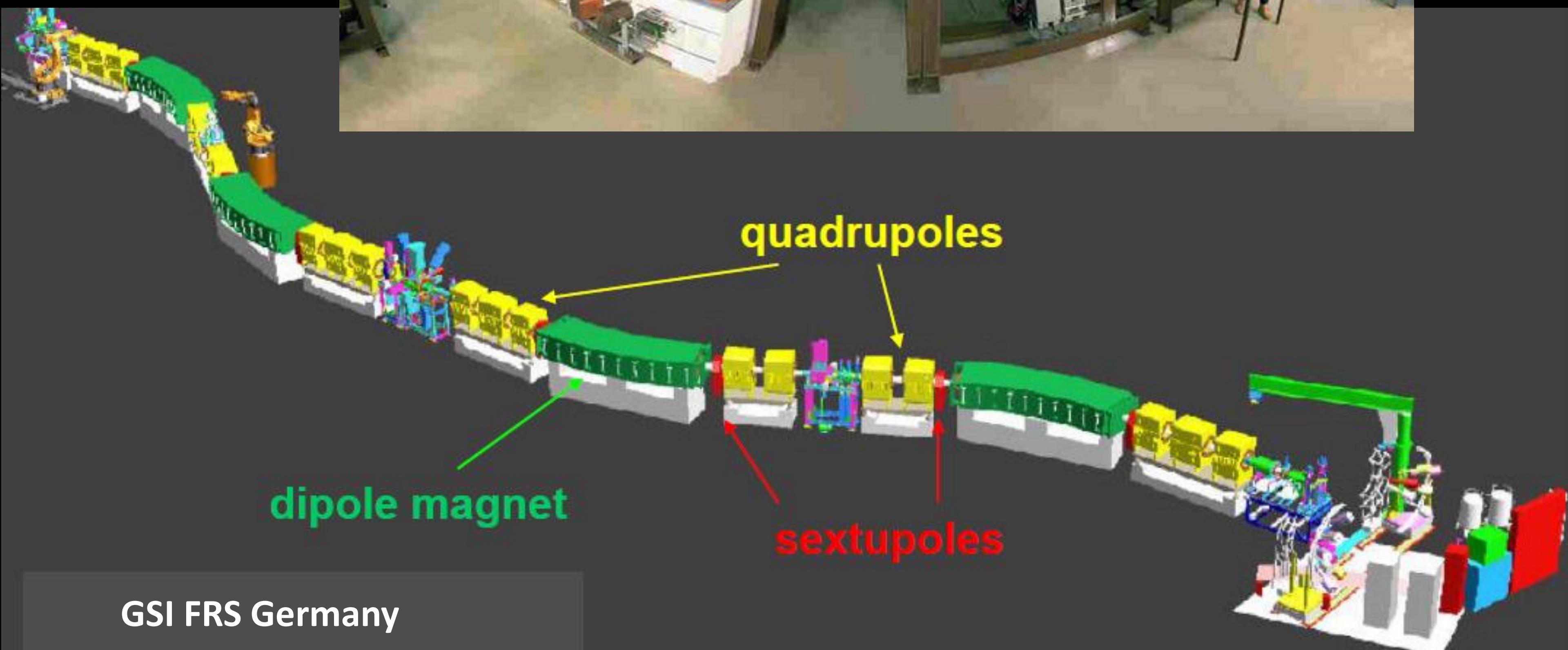
p = momentum
q = charge

In a homogenous field with flux density B perpendicular to the direction of motion, ions of magnetic rigidity $B\rho$ are bend on a radius ρ.

reference = design = nominal trajectory
= closed orbit (circular machine)



$$x(s) = D_x(s)\,\frac{\Delta p}{p}$$

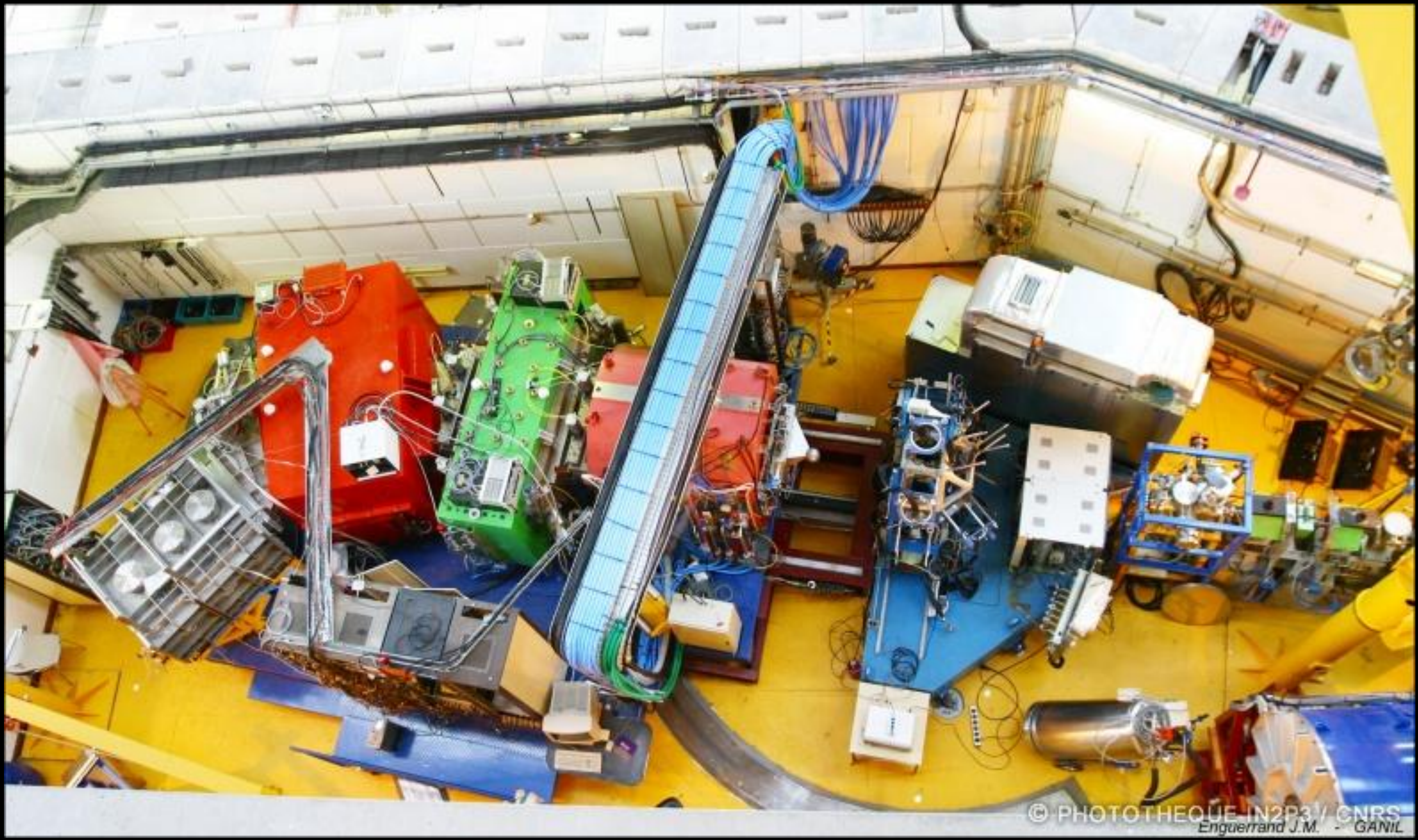


GSI FRS Germany

BIGRIPS RIKEN Japan

LISE (wien filter)
GANIL, France

VAMOS
GANIL, France



Why it works?

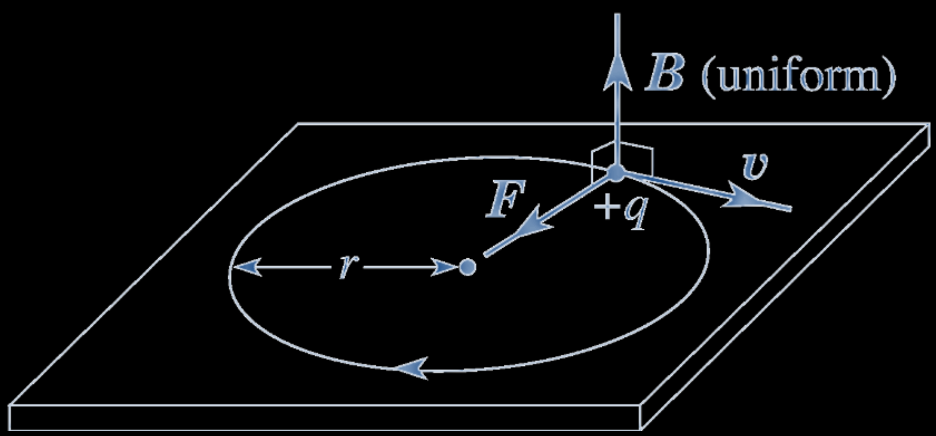
Thanks to the Lorentz force F and Newton's second law

1. Lorentz force: A charged particle moving in an electromagnetic field experiences a force.

$$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{F} = q(\underbrace{\boldsymbol{E}}_{\text{Electic Force}} + \underbrace{\boldsymbol{v} \times \boldsymbol{B}}_{\text{Magnetic Force}})$$

This force causes a centripetal acceleration and consequently a circular motion of the particle in the medium based on the equations described below.

2. **Newton's second law**

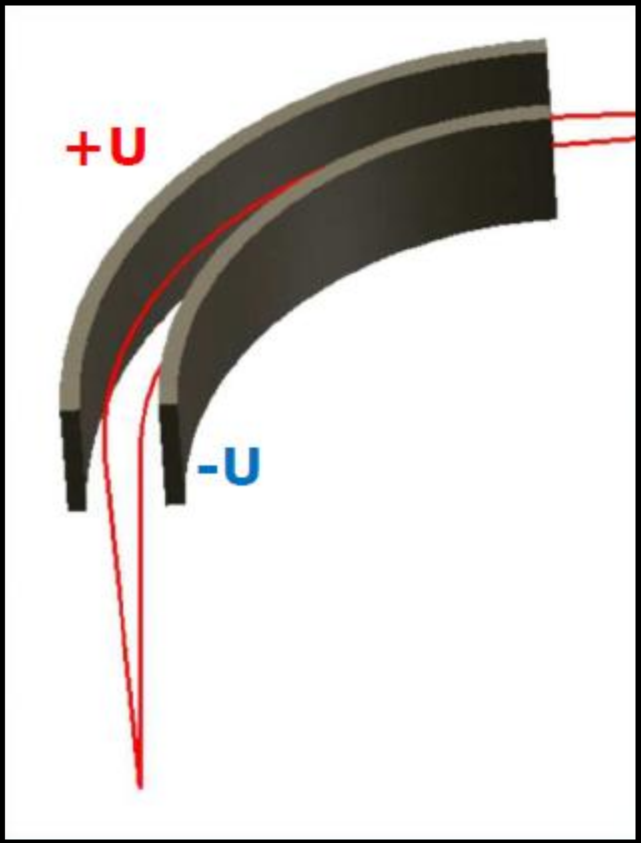


$$\boldsymbol{F} = m\,\boldsymbol{a}$$

$$F_{centripetal} = \frac{mv^2}{r}$$

Radius r → ρ

Electrostatic selection :



$$F = q\,E$$

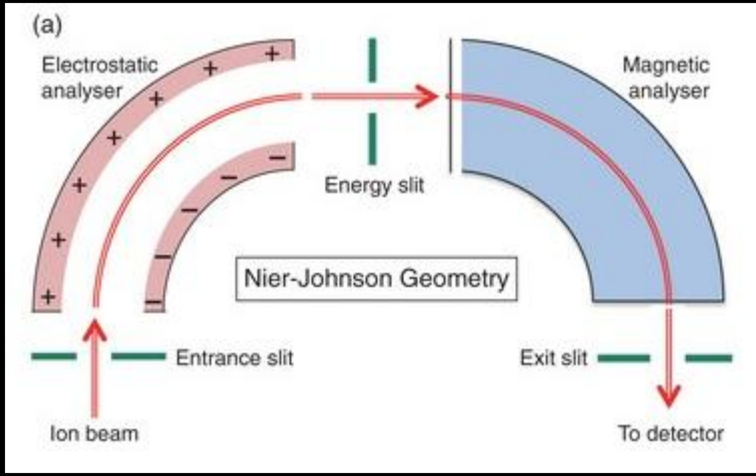
$$F_{Electic} = F_{centripetal}$$

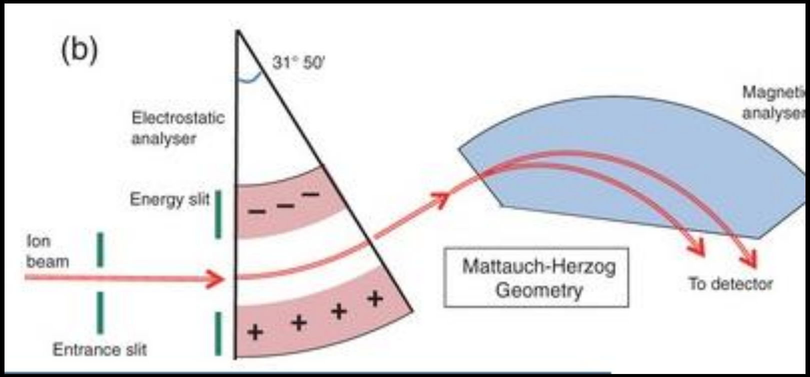
$$E\rho = \frac{mv^2}{q}$$

✓ **Difficult to bend energetic particles with raisonnable E field due to sparking**

➡ **Most used for low energy particles keV**

- Aston Nobel price (1919) : E+ B selection with a « mass spectrograph »
  - ✓ identification Stable isotopes : $^{20-22}Ne$; $^{35-37}Cl$ & mass measurement





Magnetic Separation:

$$\mathbf{F}_{\text{Magnetic}} = \mathbf{F}_{\text{centripetal}}$$

$$\mathbf{F}_{\text{magnetic}} = \text{q}\ \mathbf{v}\ \mathbf{B}$$

$$B\rho = \frac{mv}{q}, \ \rightarrow \text{Magnetic Rigidity}$$

Beam rigidity quantifies how difficult it is to bend the beam and is given by the total momentum divided by the total charge

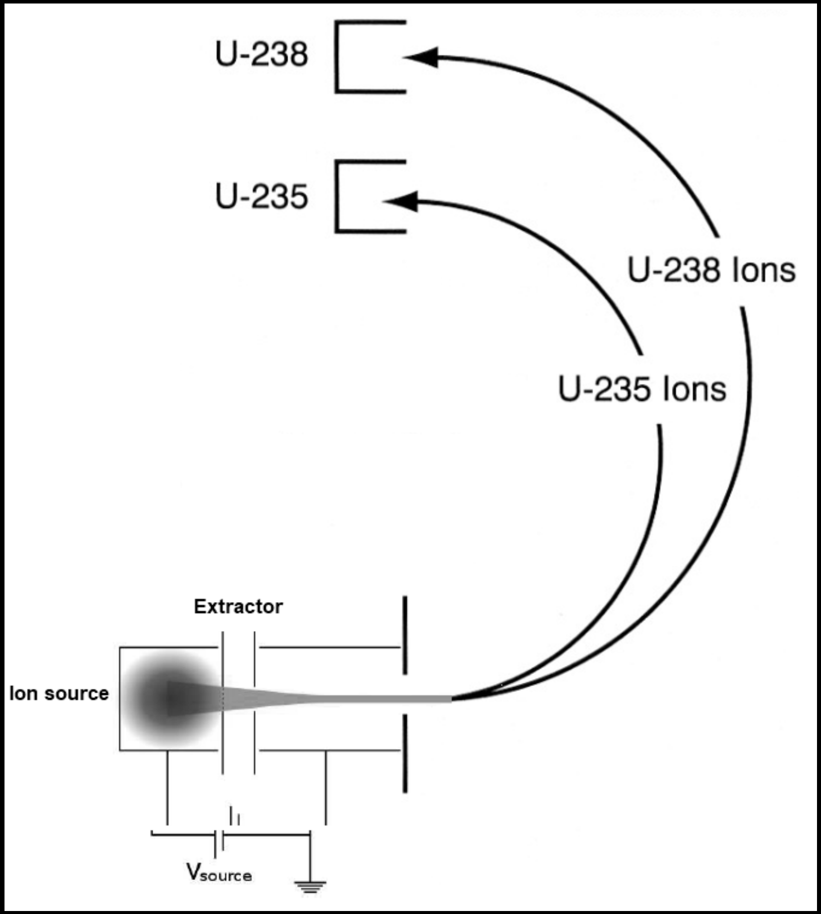
Wien Filter: $\mathbf{F}_{\text{electic}} = \mathbf{F}_{\text{magnetic}}$

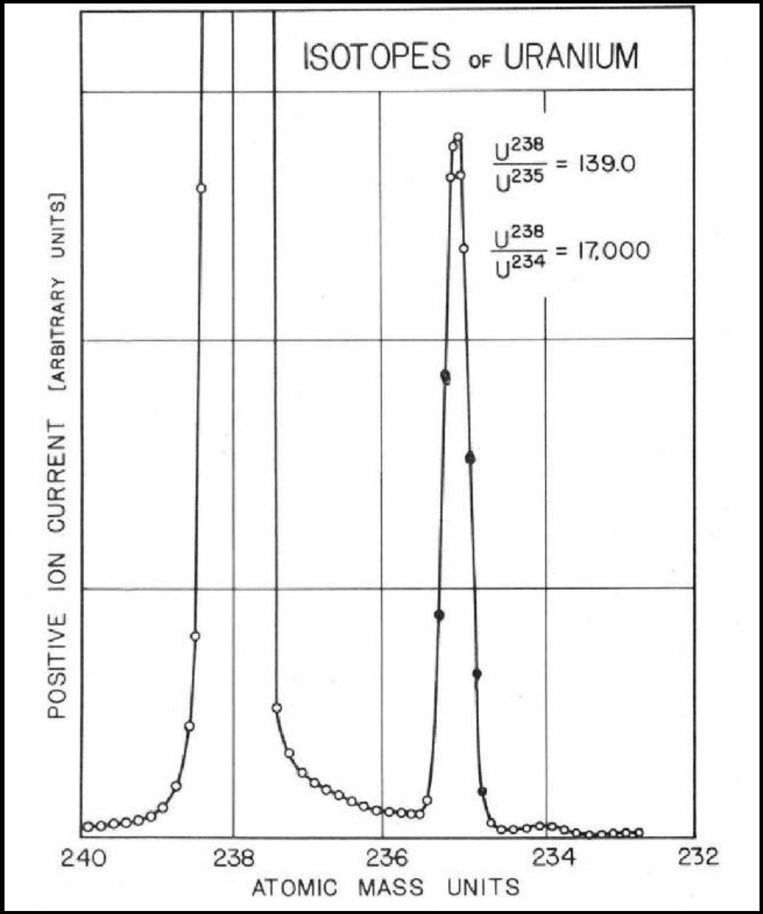
$$\text{v} = \text{E}/\text{B} \text{ with } \text{E} \perp \text{B}$$

$$\text{m/q} = \frac{2Ek}{qv^2}$$

The simplest m/q magnetic spectrometer : **1 dipole magnet**

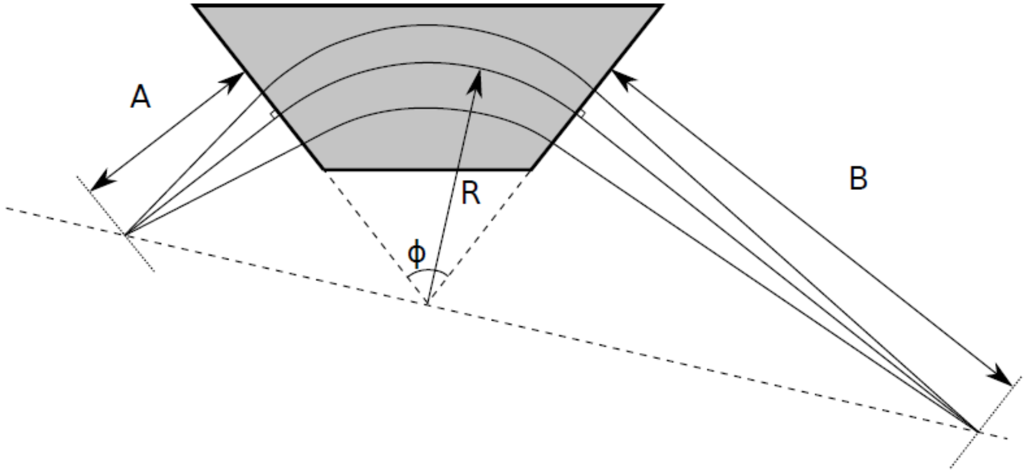
- 40's: Manhattan project U-235/U-238 enrichment (B selection)
- Dipole →mass dispersion



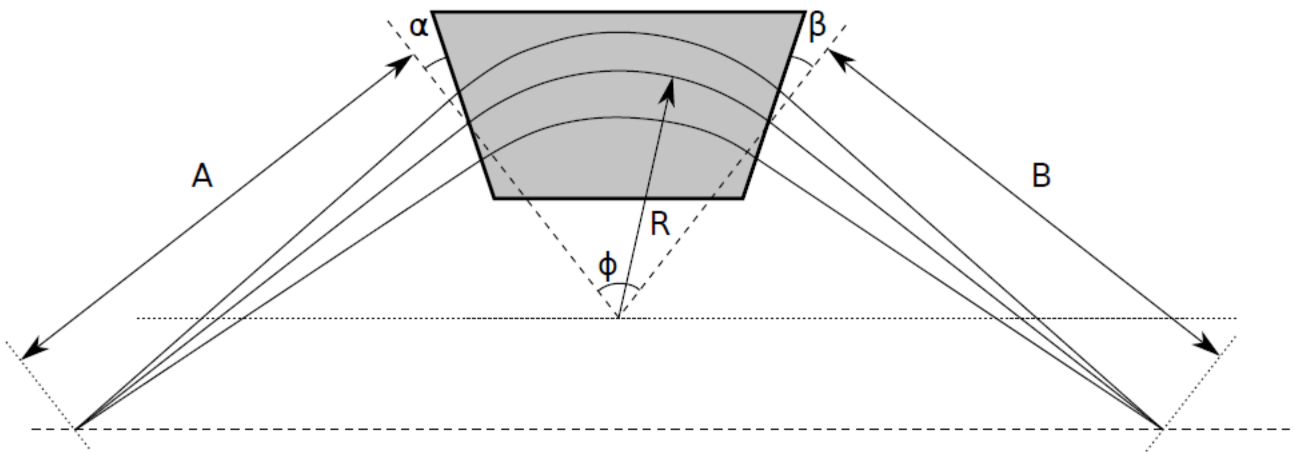


The dipole elements also have focusing/defocusing properties.

With edges perpendicular to the optical axis (edge angle 0◦) focuses the beam in the bending plane (x). There is no focusing action in the y direction.



If the magnet edge angles deviate from 90◦, the focusing power in the x direction can be adjusted. If the edge angle is made positive (as shown), there is weaker focusing in the x direction. If the angle is negative, there is stronger focusing in the x direction.

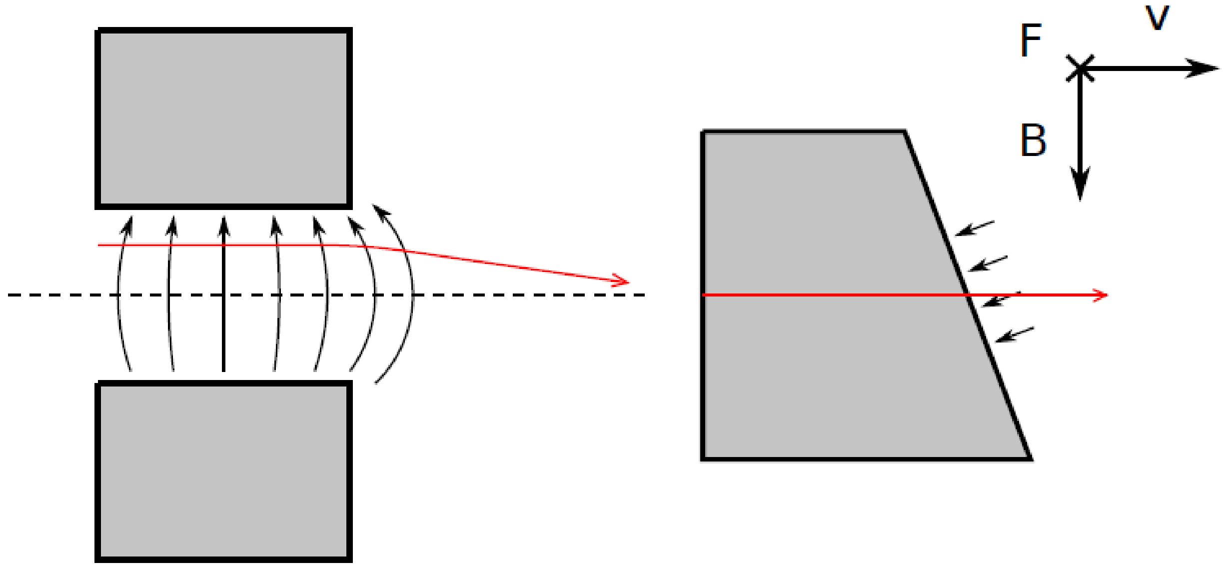


Changing the edge angle also has an important effect in the y direction:

if the angles are positive, the fringing field of the magnet will focus the beam in the y direction

Overall, this means that the focusing in the x direction can be traded for y focusing. The focal length from the edge focusing is given by.

$$f_Y = \frac{R}{\tan\alpha}$$



Dipôle : Traitement général

α	45	deg		K1	0,3
R	1000	mm		K2	4
gap	70	mm			
β entrée	0	deg		Indice	0
β sortie	0	deg			

Arête	414,213562	mm	Ro Theta	785,398163	mm
Equifocale	2414,21356	mm	Cd (dp/p)	2	mm/pm
Foc. Objet	1E+12	mm	Foc. Image	1000	mm
			Cd (dp/p)	1	mm/pm

ATTENTION : Erreur de calcul sur les distances focales pour aimants à indice

	Correction 1er niveau			Correction 2ème niveau	
	radian	degré		radian	degré
ψ entrée	0,021	1,20321137		0,021	1,20321137
ψ sortie	0,021	1,20321137		0,021	1,20321137

ar	0,78539816	a1	-0,40985932
te	0	a2	-3,04647909
ts	0	a3	0,63661977
ca	0,70710678	Q	-1,03415799
sa	0,70710678	R	-0,10765524
delta	1	D	-1,09442448
par1	7,0711E+11		
par2	707106780	par5	-707,106781
par3	0,00070711		4
par4	-1,41421356	-414,213562	2414,21356

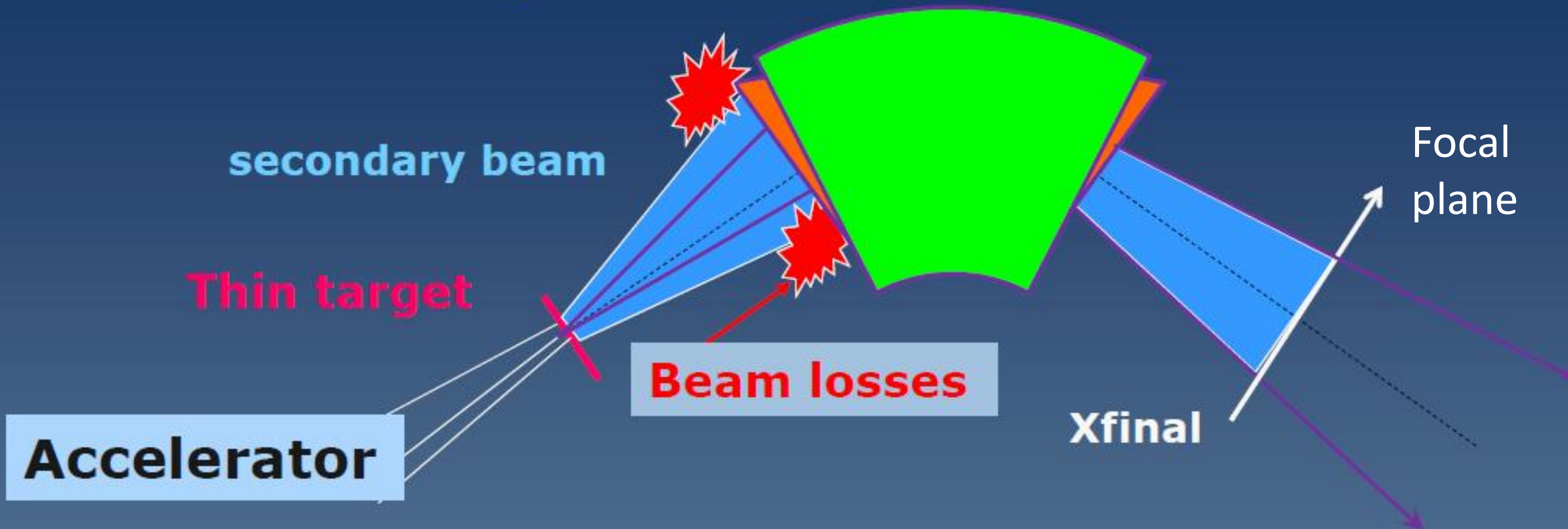
Dipôle à T11 = 0 (sans indice)		
α	45	deg
R	1000	mm
β (e/s)	0	deg
Focale	1000	mm

Dipôle à T11 = T33 = 0 (sans indice)		
α	45	deg
R	1000	mm
tan(ε)	0,20612762	*racine 1*
ε (e/s)	11,6471149	deg
Focale	2212,03854	mm
Cd (dp/p)	1,99058892	mm/pm
tan(ε)	-1,65848937	*racine 2*
ε (e/s)	-58,9117733	deg
Focale	-420,386128	mm
Cd (dp/p)	0,19984222	mm/pm
tan(ε)	1,86222106	*racine 3*
ε (e/s)	61,764505	deg
Focale	-462,221908	mm
Cd (dp/p)	-0,28605761	mm/pm

Dipôle secteur (sans indice)		
α	45	deg
R	1000	mm
ε (e/s)	0	deg
Focale	2414,21356	mm
Cd (dp/p)	2	mm/pm
ψ entrée (1)	0,021	1,20321137
ψ sortie (1)	0,021	1,20321137
ψ entrée (2)	0,021	1,20321137
ψ sortie (2)	0,021	1,20321137

Dipôle à double focalisation (sans indice)		
α	45	deg
R	1000	mm
ε (e/s)	11,7009195	deg
Focale	4828,42712	mm
Cd (dp/p)	4	mm/pm
ψ entrée (1)	0,02232769	1,27928238
ψ sortie (1)	0,02232769	1,27928238
ψ entrée (2)	0,02193926	1,25702674
ψ sortie (2)	0,02193926	1,25702674

2 problems with 1 dipole magnet :
Acceptance & identification

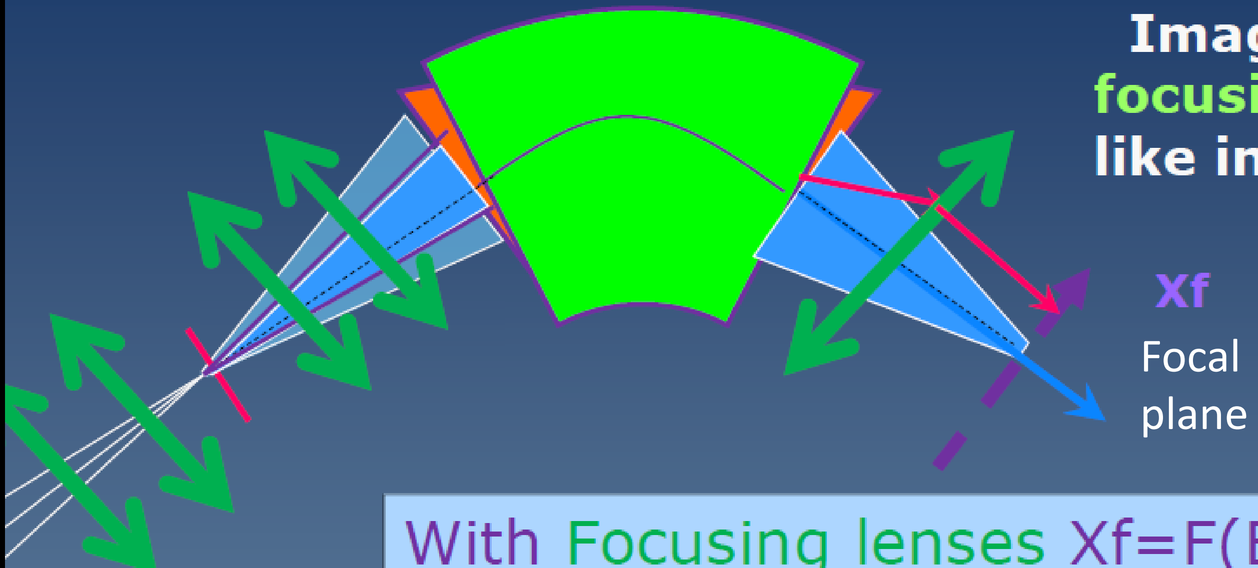


-1: Many particles are lost in the magnet (very bad)
-2: Trajectories are complex (bad)

Xfinal $=f(B\rho, \theta i, \phi i, X0, Y0)$

- Final position *xf* depend on the
  - $B\rho$ (good for identification or separation)
  - position & Angle after the reaction (bad)

Beam divergence after target
2 problems solved with focusing lenses



Imagine than focusing lenses exist like in light optics

With Focusing lenses $Xf=F(B\rho, \cancel{\theta i}, \cancel{\phi i}, \cancel{X0}, \cancel{Y0})$

less unknowns ! **Less beam losses**!!

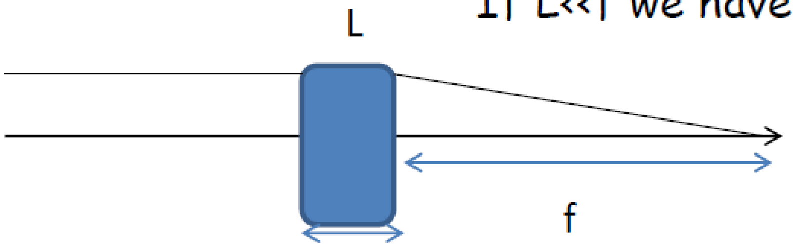
The trajectoires are independant of the angles $\theta i, \phi i$

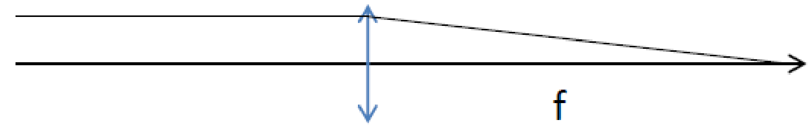
And the initial position is $x_0=0$, $y_0=0$

$$Xf=F(B\rho, \cancel{\theta i}, \cancel{\phi i}, \cancel{X0,Y0})$$

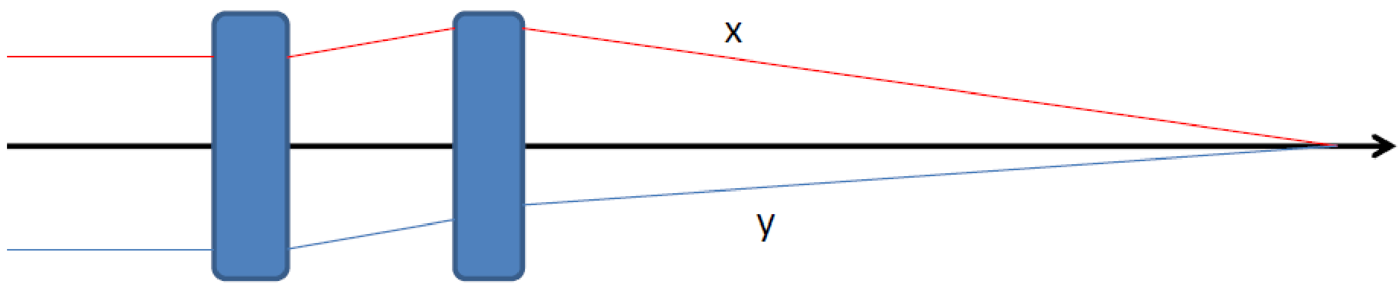
Focusing in both planes : doublets, triplets

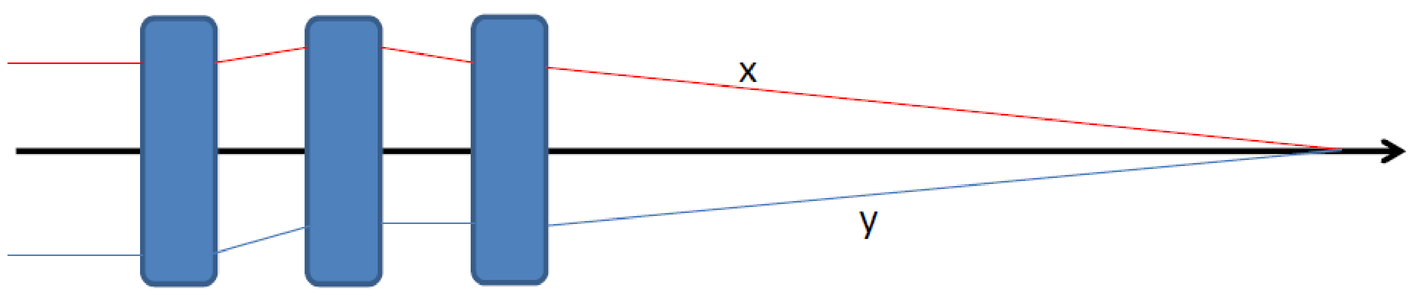
If L<<f we have the 'Thin Lens Model'





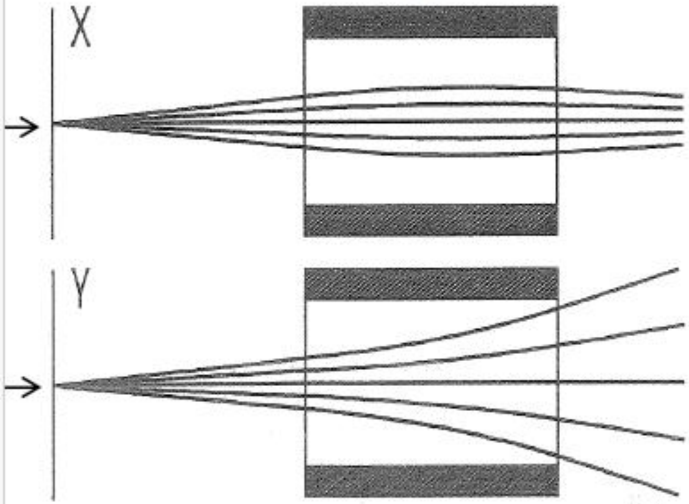
But how to have a Net Focusing effect in the two plans? : DOUBLETS/TRIPLETS



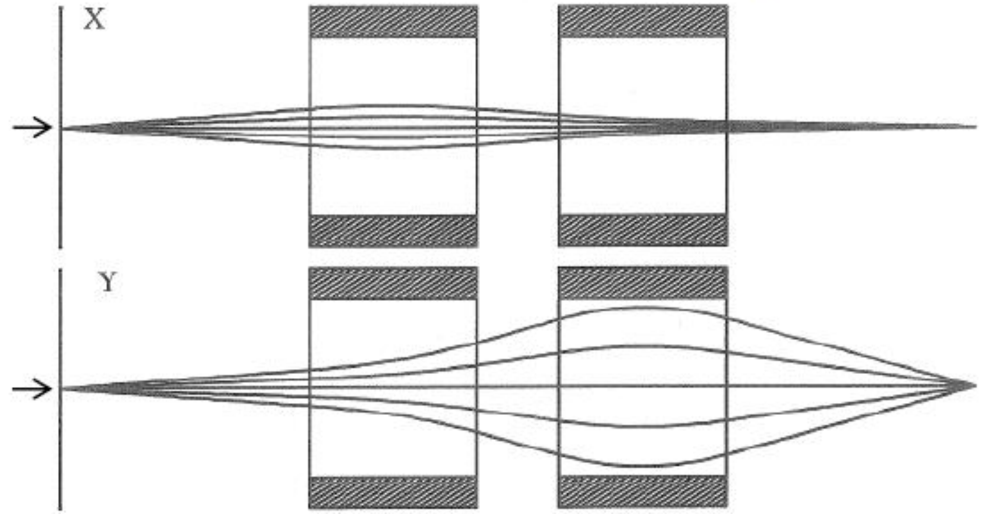


Focusing Elements

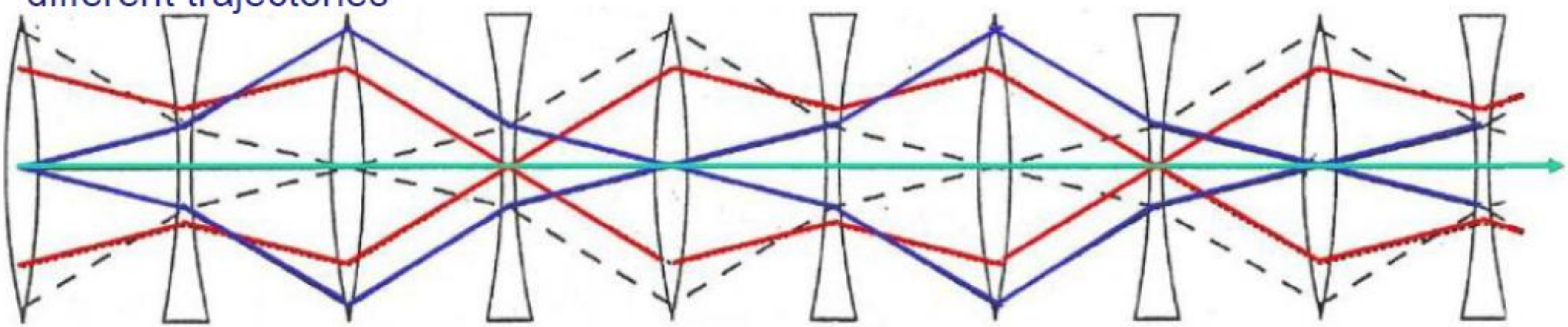
one quadrupole does not solve the problem



many quadrupole magnets combined can focus in x and y

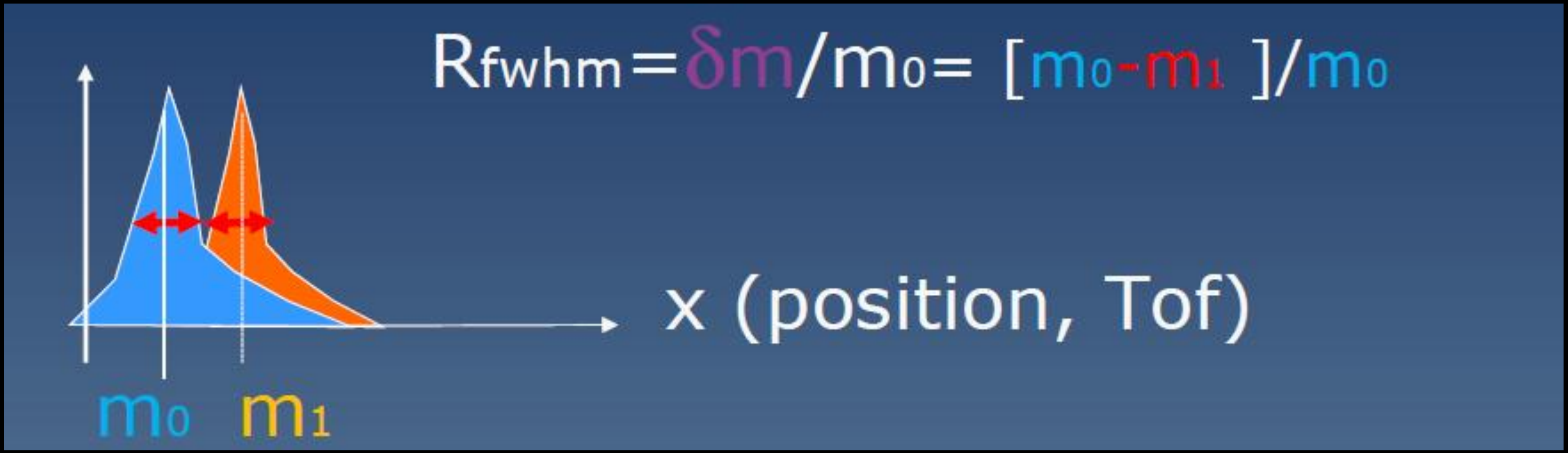


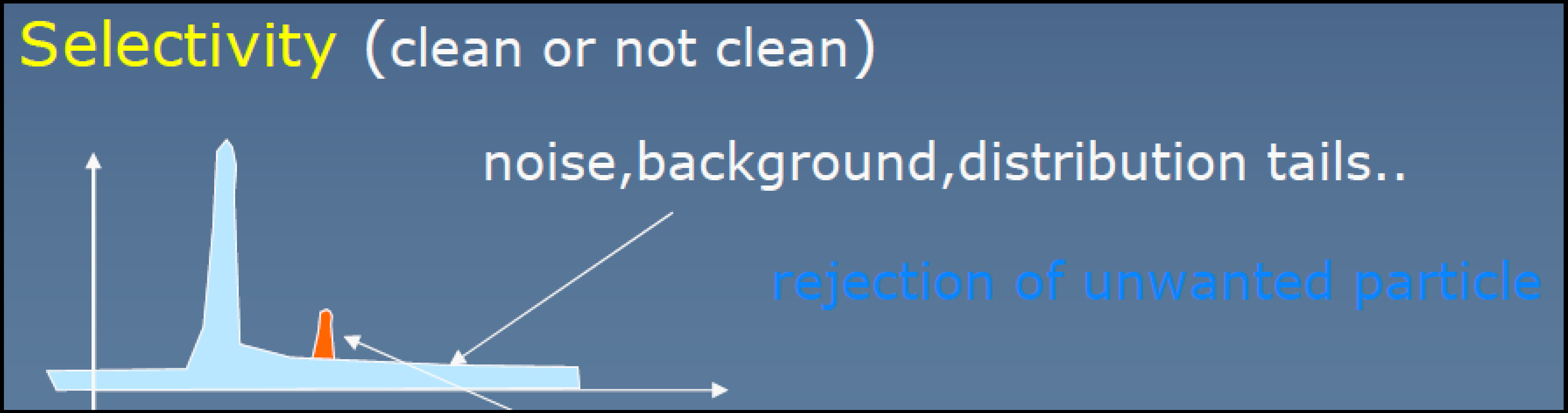
quadrupole channel with different trajectories



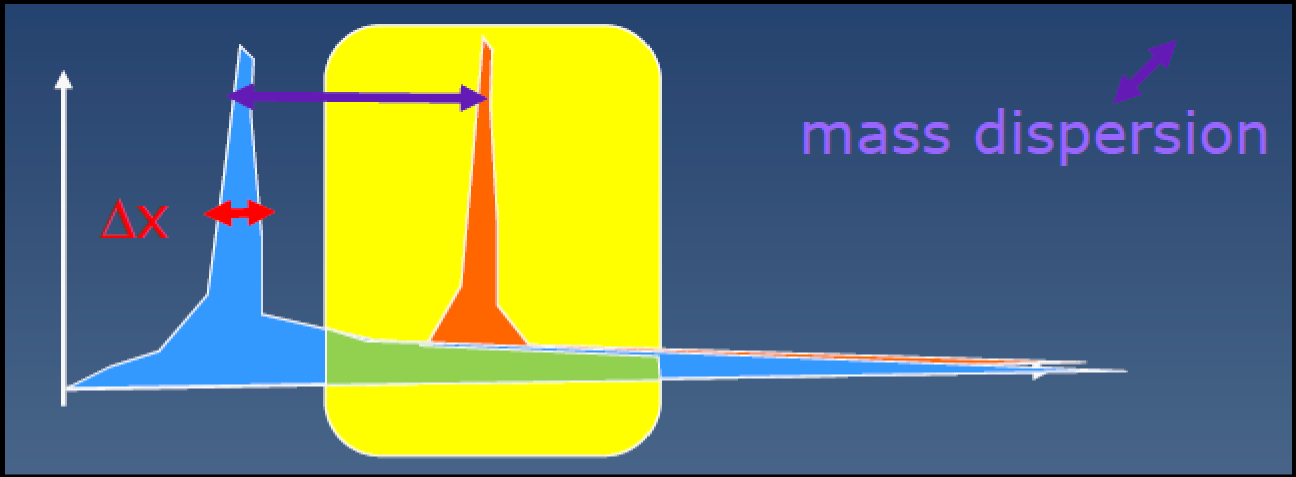
Resolving power

The term resolving power is the ability of a spectrometer to resolve adjacent peaks in a mass spectrum and is often used interchangeably with resolution. The separation of peaks for singly charged ions can be expressed as a mass difference *δm*





Resolution = Δx_{FWHM} / dx/dm, Resolving power = 1/Resolution

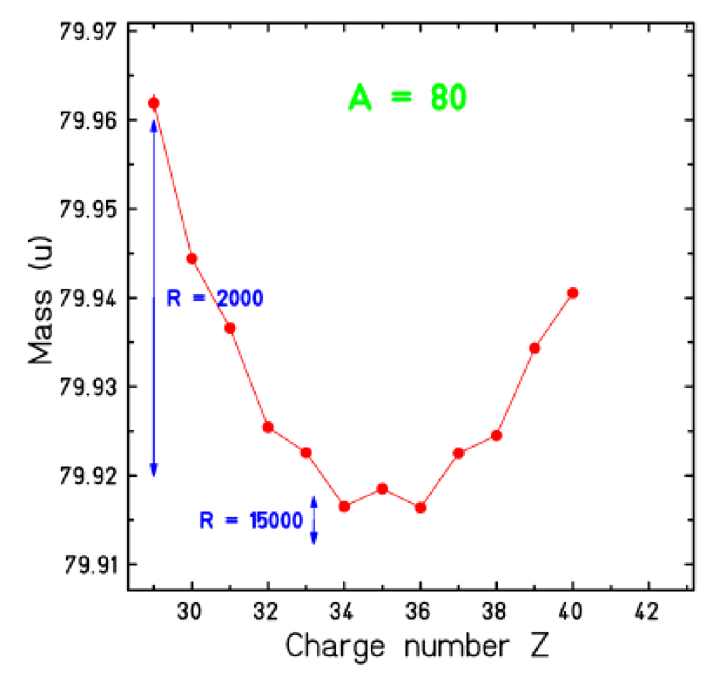


Mass dispersion usually expressed in meters (m) (SI):
cm/% (centimeters per 100%) ;
mm/‰

Notation :
- ✓ D_m
- ✓ dx/dm, physical meaning

Matricial notation (see later)
- ✓ (x|δ) Wollnik
- ✓ R16, T16, M16

- ✓ Resolving power R = $\frac{(x|\delta)}{\Delta x\ (FWHM)}$



Beam optics (basics)

Already seen:

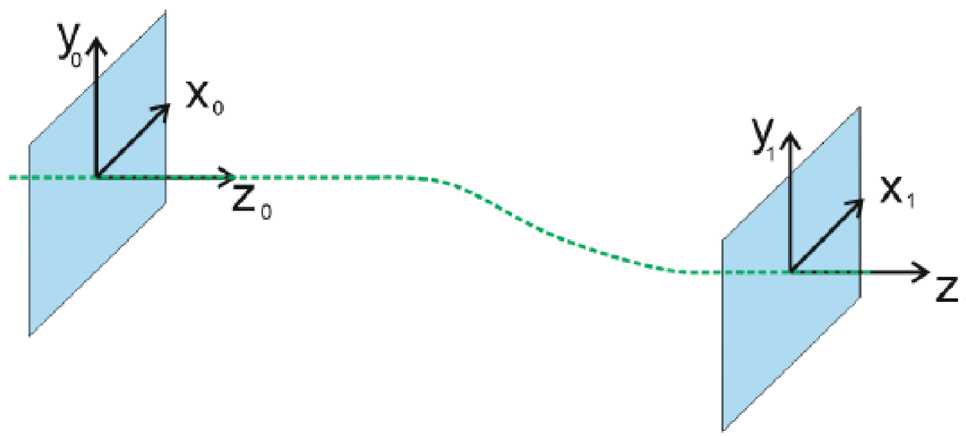
- ✓ Dispersion and focalisation with dipoles
- ✓ Focalisation with quadrupoles
- ✓ Resolution

Next concepts:

- **Particles coordinates**
- **Beam emittance**
- **Optical Matrices following Taylor expansion**
- **Angular Acceptance**
- **Bρ Acceptance**

Ion optical coordinates

We look at beamline, use coordinates relative to the nominal optical axis.



Transverse motion:

$x' = dx / dz$

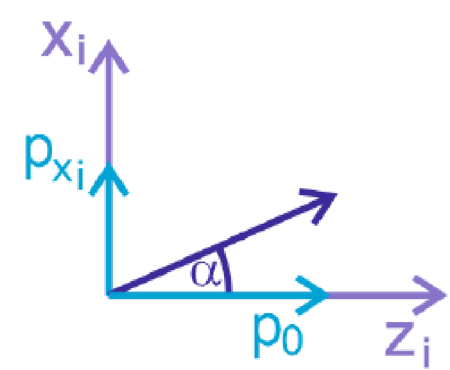
$y' = dy / dz$

Often defined as derivative in path with coordinates of single ions.

$a = p_x / p_0$

$b = p_y / p_0$

With common constant p_0 we can use a normal Hamiltonian.



for same forward momentum $x' = a$,
for small angles $x' = a = \tan(\alpha) \sim \alpha$

The Coordinates

Notations in the Literature is not consistent!

Wollnik GICOSY	Brown	TRANSPORT	COSY	Meaning
x	x	x	r1 = x	the horizontal displacement of the arbitrary ray with respect to the assumed central trajectory.
a	x’	θ	r2 = a = p_x/p_0	the angle this ray makes in the horizontal plane with respect to the assumed central trajectory.
y	y	y	r3 = y	the vertical displacement of the ray with respect to the assumed central trajectory
b	y’	ϕ	r4 = b = p_y/p_0	the vertical angle of the ray with respect to the assumed central trajectory
ℓ	ℓ		r5 = ℓ = $-(t - t_0)v_0\gamma/(1 + \gamma)$	the path length difference between the arbitrary ray and the central trajectory.
δ	δ	$dp/p=\frac{B\rho - B\rho_0}{B\rho_0}$		fractioned momentum deviation of the ray from the assumed central trajectory
δ_U			r6 = $\delta K = (K - K_0)/K_0$	energy difference ray with respect to the reference energy
δ_m			r7 = $\delta m = (m - m_0)/m_0$	mass difference ray with respect to the reference energy
δ_e			r8 = $\delta z = (z - z_0)/z_0$	charge difference ray with respect to the reference energy

Transfer Matrix Description

Transfer function on vector of coordinates

In practise use Taylor expansion of this function, $(x,a) = \dfrac{\partial x_f}{\partial a_i}$

1^{st} order transfer matrix T :

$$\begin{pmatrix} X \\ a \\ Y \\ b \\ \delta \end{pmatrix}_f = \begin{pmatrix} (X,X) & (X,a) & \boxed{=0} & & (X,\delta) \\ (a,X) & (a,a) & & & (a,\delta) \\ \boxed{=0} & & (Y,Y) & (Y,b) & \boxed{=0} \\ & & (b,Y) & (b,b) & \\ \boxed{=0} & & & & \boxed{=1} \end{pmatrix} \begin{pmatrix} X \\ a \\ Y \\ b \\ \delta \end{pmatrix}_i$$

Det (T) = 1
Liouville's theorem

with bending only in one plane
only forces in x or y direction
momentum conservation

Full system $\quad T_{tot} = T_n * \ldots * T_3 * T_2 * T_1$

Ion optics

Taylor expansion in x, a, y, b and δ

$$x_1 = (x|x)\ x_0 + (x|a)\ a_0 + (x|\delta)\delta + (x|x^2)x_0^2 + (x|xa)\ x_0 a_0 + (x|a^2)a_0^2$$

$$(x|x\delta)\ x_0 + (x|a\delta)\ a_0\delta + (x|\delta^2)\delta^2 + (x|y^2)y_0^2 + (x|yb)\ y_0 b_0 + (x|b^2)b_0^2 + \text{higher orders}$$

First order

$$(x|\ \dots) = \frac{\partial}{\partial x}$$

$$Higher\ orders : e.g.\ (x|a^2) = \frac{\partial x}{\partial a \partial a} = T_{122}$$

Transfer matrix formalism

Most crucial parameters :

$$T = \begin{pmatrix} (x|x) & (x|a) & (x|y) & (x|b) & (x|l) & (x|\delta) \\ (a|x) & (a|a) & (a|y) & (a|b) & (x|l) & (a|\delta) \\ (y|x) & (y|a) & (y|y) & (y|b) & (x|l) & (y|\delta) \\ (b|x) & (b|a) & (b|y) & (b|b) & (x|l) & (a|\delta) \\ (l|x) & (l|a) & (l|y) & (l|b) & (x|l) & (l|\delta) \\ (\delta|x) & (\delta|a) & (\delta|y) & (\delta|b) & (x|l) & (\delta|\delta) \end{pmatrix}$$

$$T_{11} = magnification\ in\ horizontal$$

$$T_{16} = dispersion\ in\ momentun = dispersion\ in\ B\rho$$

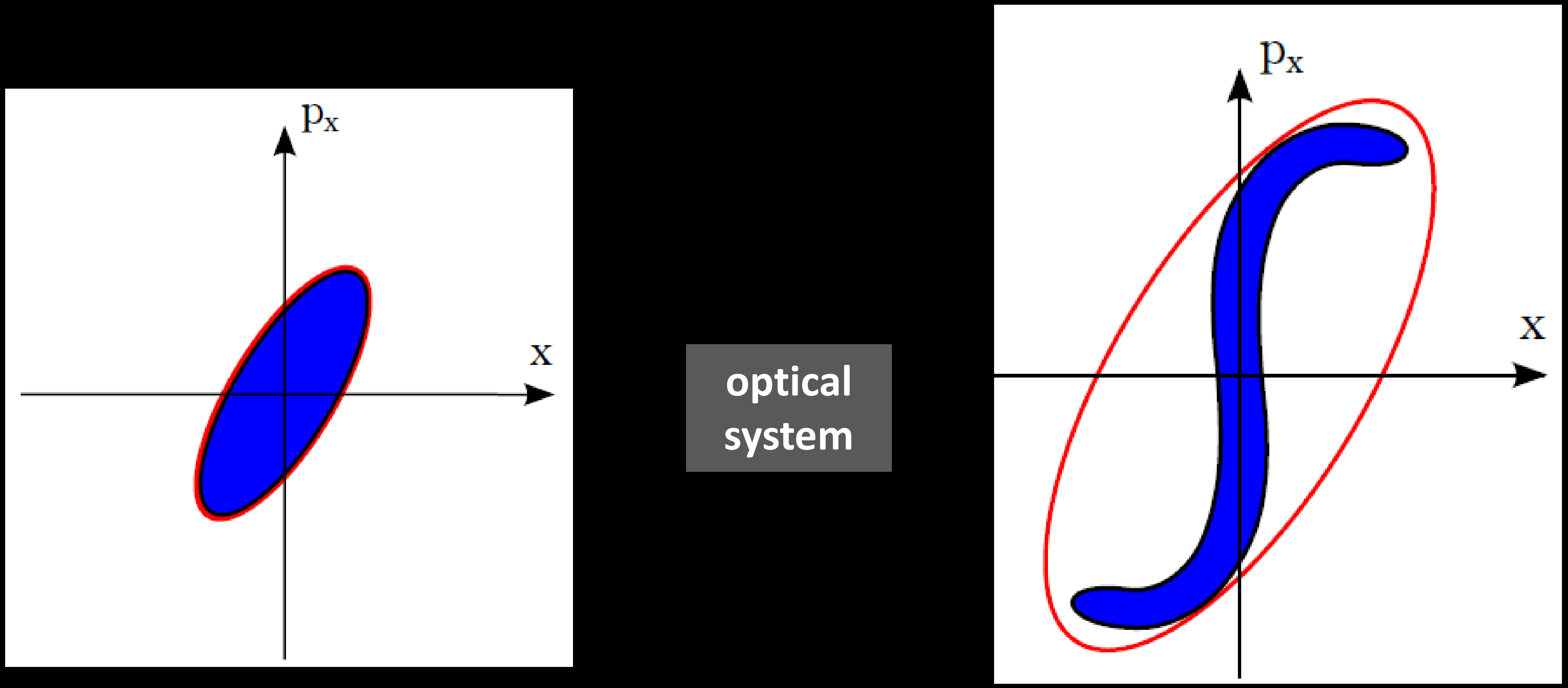
$$T_{33} = magnification\ in\ vertical$$

$$T_{12} = angular\ dependance\ in\ horizontal$$

$$T_{34} = angular\ dependance\ in\ vertical$$

Beam emittance

The emittance is defined as the six-dimensional volume limited by a contour of constant particle density in the (x, px, y, py, z, pz) phase space. This volume obeys the Liouville theorem and is constant in conservative fields

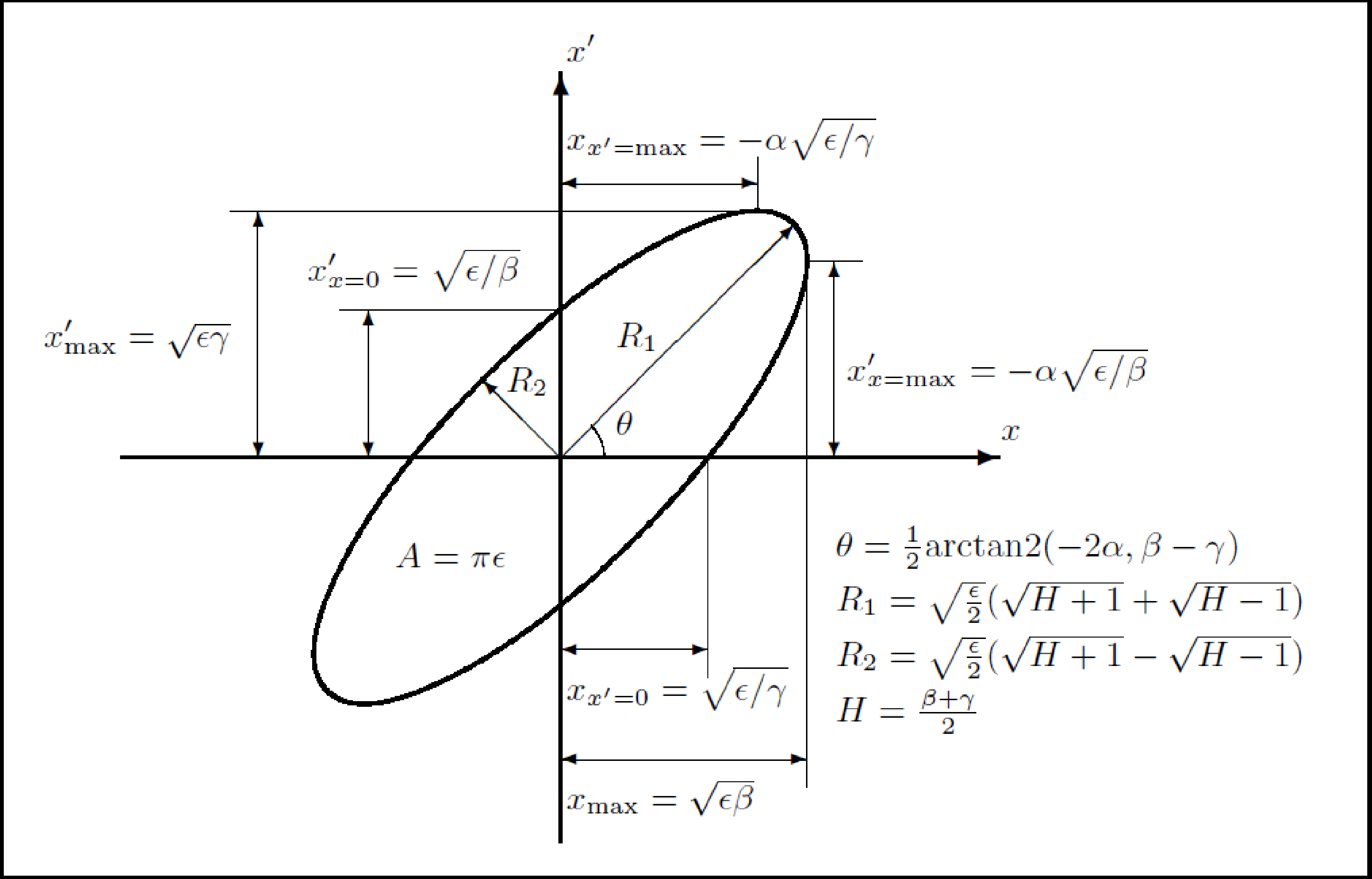


The area of the particle distribution is conserved but the area of the elliptical envelope increases.

Beam emittance

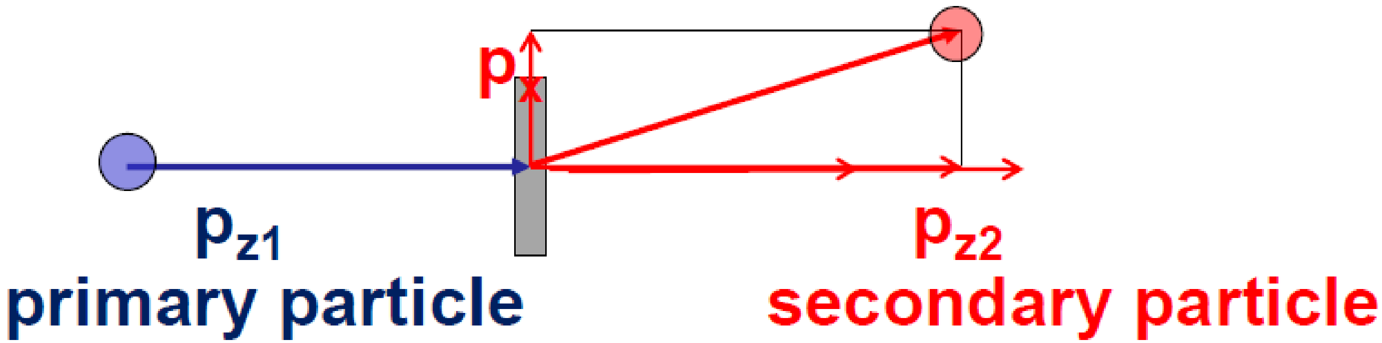
$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \varepsilon \qquad \beta\gamma - \alpha^2 = 1 \qquad A = \pi\varepsilon = \pi R_1 R_2$$

ε is the two-dimensional transverse emittance, and α, β and γ are known as the Twiss parameters

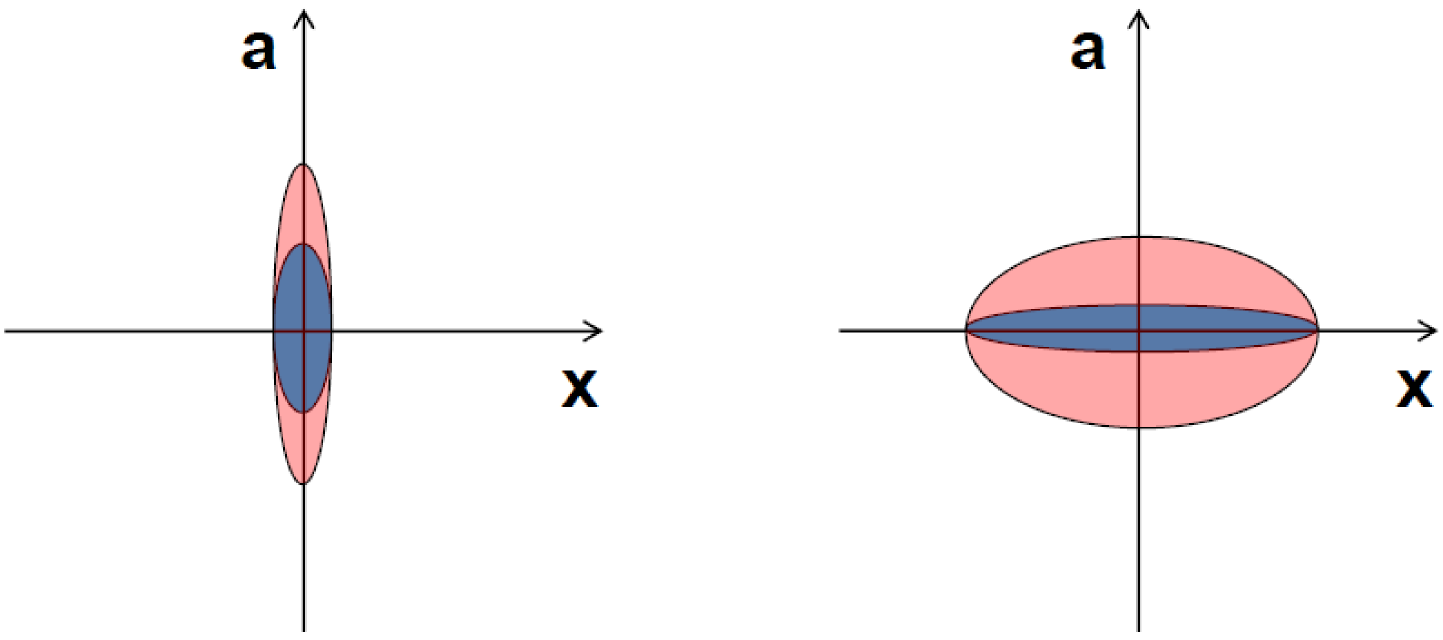


Emittance grow with targets

Momentum transfer by reaction in target increases transverse momentum spread, but in a thin target Δx does not change much.



Make small beam spot to avoid large emittance for secondary beam



The percentage of bivariate normally distributed data covered by an ellipse whose axes have a length of *numberOfSigmas* · σ can be obtained by integration of the probability distribution function over an elliptical area.

$$percentage = (1 - \exp(-numberOfSigmas^2/2)) \cdot$$

This results in the following equation,

$$(x/\sigma_x)^2 + (y/\sigma_y)^2 = numberOfSigmas^2.$$

where the *numberOfSigmas* is the radius of the "ellipse":

the *numberOfSigmas* =1 ellipse covers 39.3% of the data,
the *numberOfSigmas* =2 ellipse 86.5%,
and the *numberOfSigmas* =3 ellipse 98.9%.

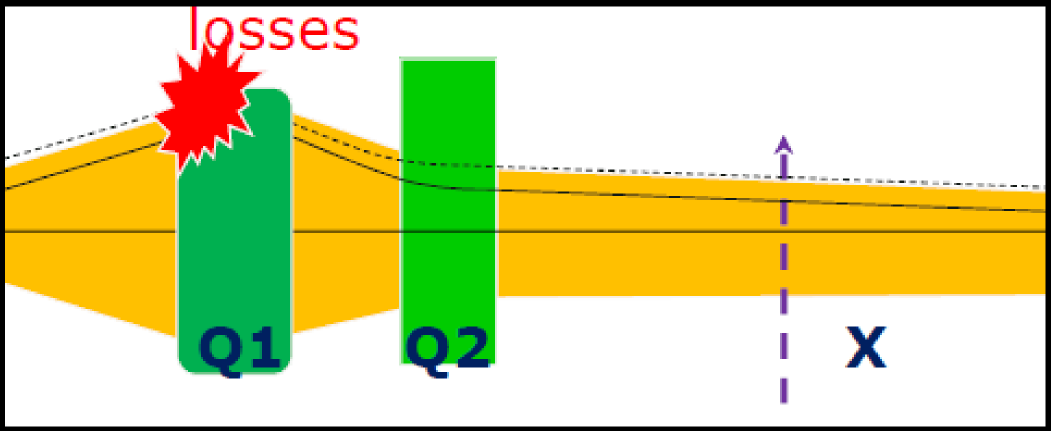
From the formula above we can show that if we want to cover *p* percent of the data, we have to chose *numberOfSigmas* as

$$numberOfSigmas = \sqrt{(-2 \ln(1-p/100))}.$$

For covering 95% of the data we calculate *numberOfSigmas* = **2.45.**

$$\textbf{Resolving power (95\%)} = \frac{(x|\delta)}{\Delta x\ (2.45\ \sigma)}$$

The beam size : important for the design



Ellipse Area = $\pi(\det \sigma)^{1/2}$

Emittance $\varepsilon = \det \sigma$ is constant for fixed energy & conservative forces (Liouville's Theorem)

Note: ε shrinks (increases) with acceleration (deceleration);
Dissipative forces: ε increases in gases; electron, stochastic, laser cooling

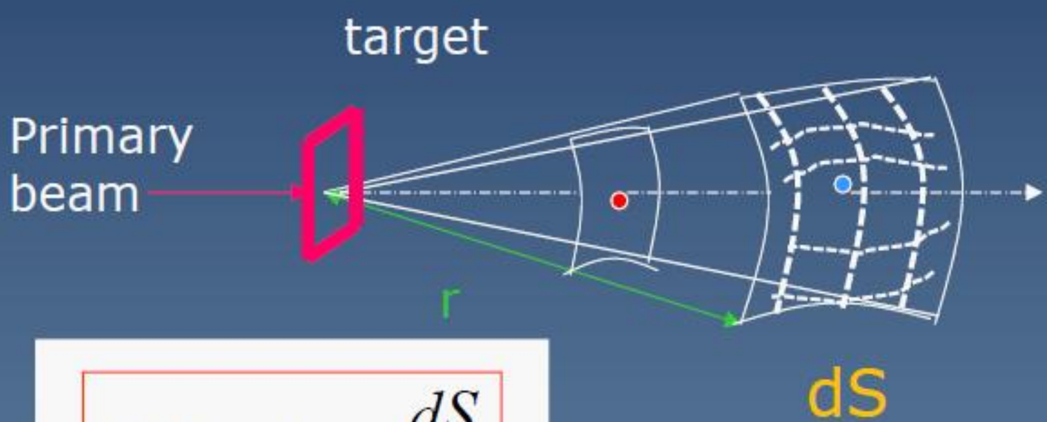
$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

Angular acceptance

The reaction products exit from the target with an

Angular dispersion

Vacuum chamber limitation induces beam losses = less transmission



The acceptance

is measured in steradian

$$d\Omega(strd) = \frac{dS}{r^2}$$

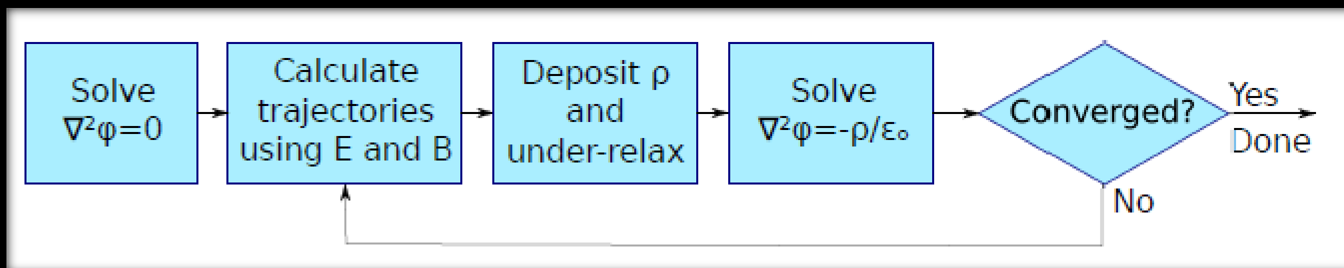
$B\rho$ Acceptance = $\pm$ Xmax / R_{16}

Modelling of ion optical transport lines

1. Trajectories : exact equations

integrate the particle equation of motion using mesh based maps for E and B fields [field map 3D]

$$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$



$$\frac{d}{ds}\left[m\gamma \dot{x}\right] = m\gamma \dot{s}(1+\frac{x}{\rho}) + q(t' Ex + y' B_s - \dot{s}(1+\frac{x}{\rho}) \cdot B_y)$$

$$\frac{d}{ds}\left[m\gamma \dot{y}\right] = q(t' E_y + (1+\frac{x}{\rho}) \cdot B_x - x' \cdot B_s)$$

$$\frac{d}{ds}\left[m\gamma \dot{s}(1+\frac{x}{\rho})\right] = -\frac{m\gamma \dot{x}}{\rho} + q(t' E_s + x' \cdot B_y - y' \cdot B_x)$$

Examples of codes : ZGOUBY

But generally we can do simpler : <u>Matrix approach</u>

Transfer matrix formalism

6x6 Matrix
representing first
optic element
(usually a Drift)

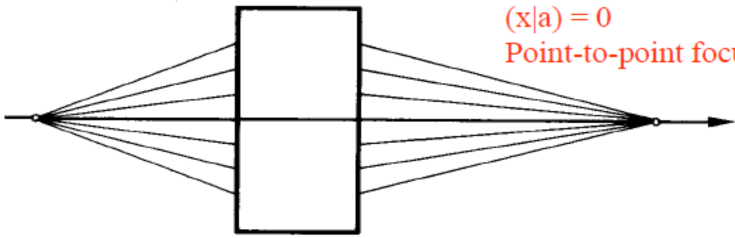
$$x_n = T_n T_{n-1} \ldots T_0 x_0$$

Ray at final
Location n

Ray at initial
Location 0
(e.g. a target
or slits)

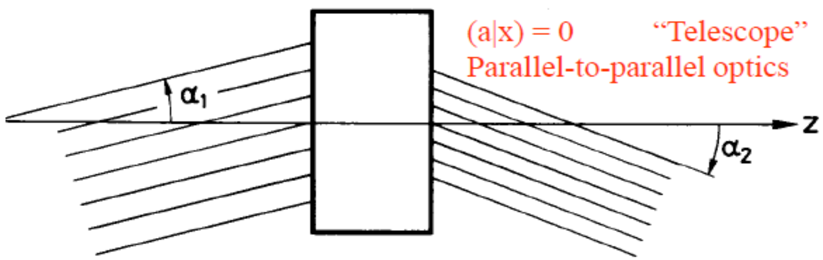
Complete system is represented in first order by one Matrix Rsystem = $T_n\ T_{n-1} \ldots T_0$

Focal points



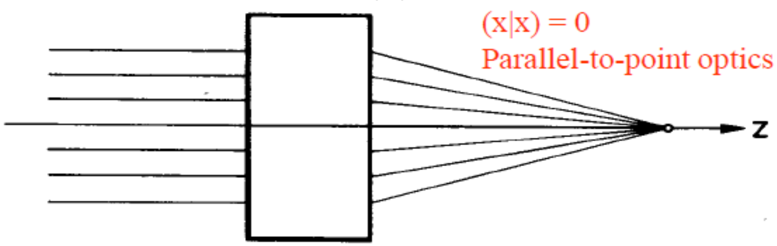
(a)

$(x|a) = 0$
Point-to-point focusing



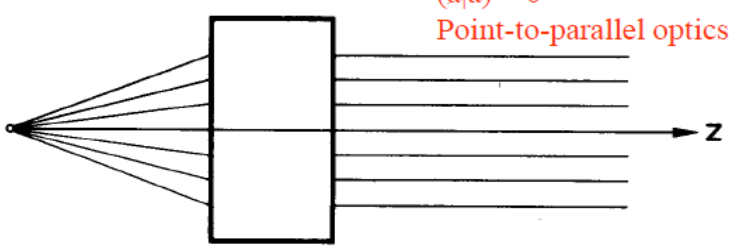
(b)

$(a|x) = 0$ "Telescope"
Parallel-to-parallel optics



(c)

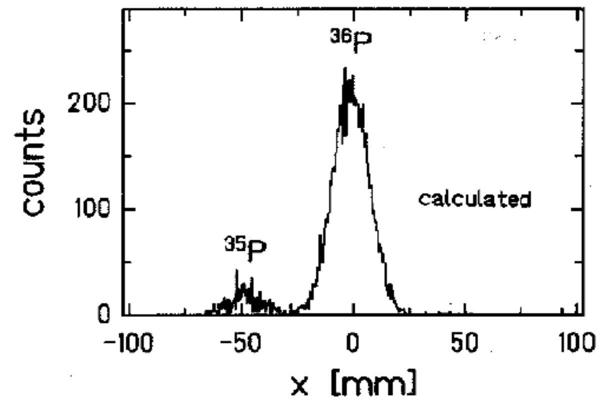
$(x|x) = 0$
Parallel-to-point optics

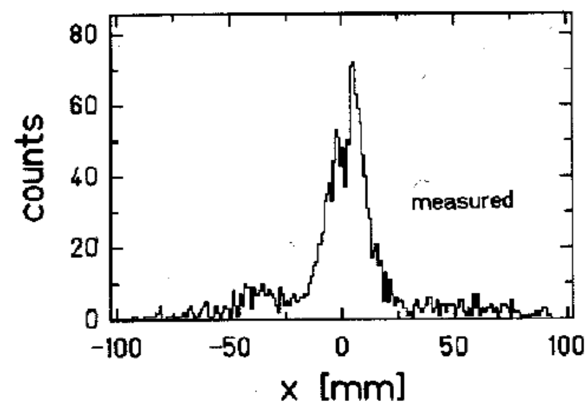


(d)

$(a|a) = 0$
Point-to-parallel optics

Achromatic system:
T16= T26= 0
$(x|\delta p) = (a|\delta p)$





Exercise 1:
Imagine a spectrometer with a dispersion of 30 cm/%
and beam width of 1 mm FWHM on the focal plan detector.

What is the resolving power R ?

a) **30**
b) **30000**
c) **1500**

Exercise 2:

For covering 95% of the beam ellipse data which value of sigma in ΔX we should use for calculating the resolving power?

a) **1 σ**
b) **2.35 σ (FWHM)**
c) **2.45 σ**

Supplemental slides

Transfer matrix formalism

Following Taylor expansion the trajectory component Xi after propagation through an ion optical element can be calculated from

$$X_i = \sum_j Y_j \left\{ (X_i \mid Y_j) + \sum_k \frac{Y_k}{2} \left\{ (X_i \mid Y_j Y_k) + \sum_l \frac{Y_l}{3} \{ (X_i \mid Y_j Y_k Y_l) + \cdots \} \right\} \right\},$$

where Yi are the components of the trajectory before the ion optical element, and (Xi | Yj), (Xi | YjYk), (Xi | YjYkYl), . . . are the first-order, second-order, third-order, . . . transfer coefficients

This can be described as matrix–vector multiplication with :

6×6 matrix in first order
6×6^2 matrix in second order,
6×6^3 matrix in third order, etc.

Transfer matrix formalism

$$\begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix} = \begin{pmatrix} T_{11} & T_{12} & T_{13} & T_{14} & T_{15} & T_{16} \\ T_{21} & T_{22} & T_{23} & T_{24} & T_{25} & T_{26} \\ T_{31} & T_{32} & T_{33} & T_{34} & T_{35} & T_{36} \\ T_{41} & T_{42} & T_{43} & T_{44} & T_{45} & T_{46} \\ T_{51} & T_{52} & T_{53} & T_{54} & T_{55} & T_{56} \\ T_{61} & T_{62} & T_{63} & T_{64} & T_{65} & T_{66} \end{pmatrix} \begin{bmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \\ l_0 \\ \delta_0 \end{bmatrix}$$

$$\mathrm{T} = \begin{pmatrix} (x|x) & (x|a) & (x|y) & (x|b) & (x|l) & (x|\delta) \\ (a|x) & (a|a) & (a|y) & (a|b) & (x|l) & (a|\delta) \\ (y|x) & (y|a) & (y|y) & (y|b) & (x|l) & (y|\delta) \\ (b|x) & (b|a) & (b|y) & (b|b) & (x|l) & (a|\delta) \\ (l|x) & (l|a) & (l|y) & (l|b) & (x|l) & (l|\delta) \\ (\delta|x) & (\delta|a) & (\delta|y) & (\delta|b) & (x|l) & (\delta|\delta) \end{pmatrix}$$