The Parton Branching method

collider predictions with Monte Carlo based on TMDs

Ola Lelek on behalf of the Parton Branching team

aleksandra.lelek@uantwerpen.be

Universiteit Antwerpen

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Introduction

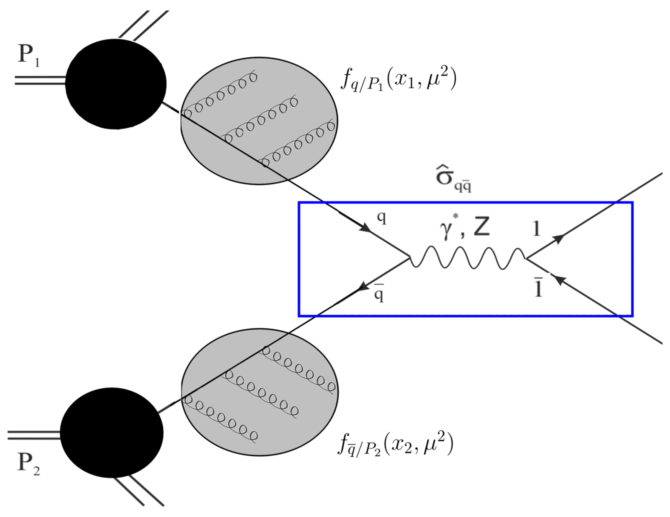
The structure of matter can be studied experimentally in accelerators like e.g. the LHC where new particles are born in the proton - proton collisions. Predictions from accelerators rely on Monte Carlo (MC) event generators. The current baseline MC generators are based on collinear factorization theorem which assumes that partons are collinear with the hadron: they carry the longitudinal momentum fractions x of the hadron and the transverse momentum is neglected. It works well for sufficiently inclusive, one scale observables. However, for observables with more scales involved, transverse degrees of freedom cannot be neglected [1]. In this work we present the Parton Branching (PB) method: MC approach based on Transverse Momentum Dependent (TMD) factorization and TMD parton distribution functions (PDFs), so called TMDs.

Factorization

The basic quantities measured at colliders are the cross sections σ- probabilities that a certain reaction takes place. The collinear factorization factorizes the cross section into the parton level process $\hat{\sigma}$ and the PDFs $f_a(x,\mu^2)$ which incorporate all the non-perturbative physics and perturbative evolution. The PDF gives the probability of finding inside the hadron a parton of a flavour a, with a momentum fraction x and at an evolution scale μ. PDFs cannot be calculated from the theory and have to be measured. However, if the PDF is known at a given scale, its change with the scale can be calculated from the evolution equations.
PDFs are universal for a given hadron - once measured they can be used in calculations for other processes.

Example: Collinear factorization for Drell-Yan (DY):

$$\sigma = \sum_{q\bar{q}} \int \mathrm{d}x_1 \mathrm{d}x_2 f_q(x_1,\mu^2) f_{\bar{q}}(x_2,\mu^2)\hat{\sigma}_{q\bar{q}}(x_1,x_2,\mu^2,Q^2)$$



Collinear factorization is a basis of many QCD calculations but for some observables also the transverse degrees of freedom have to be taken into account $\rightarrow$ Formalism to follow: TMD factorization theorems
low $q_\perp$ (Collins-Soper-Sterman CSS) or high energy ($k_\perp$-) factorization.

For practical applications MC approach needed $\rightarrow$ Parton Branching: MC based on TMDs
Idea: promote collinear formula to TMD form: PDF $\mathrm{f}(\mathrm{x},\mu) \rightarrow$ TMD $\mathrm{A}(\mathrm{x},\mathrm{k}_\perp,\mu)$

$$\sigma \sim \sum_{q\bar{q}} \int \mathrm{d}k_{\perp 1}^2 \mathrm{d}k_{\perp 2}^2 \int \mathrm{d}x_1 \mathrm{d}x_2 A_q(x_1, k_{\perp 1},\mu^2) A_{\bar{q}}(x_2, k_{\perp 2},\mu^2)\hat{\sigma}_{q\bar{q}}(x_1,x_2,k_{\perp 1},k_{\perp 2},\mu^2,Q^2)$$

PB TMD evolution equation

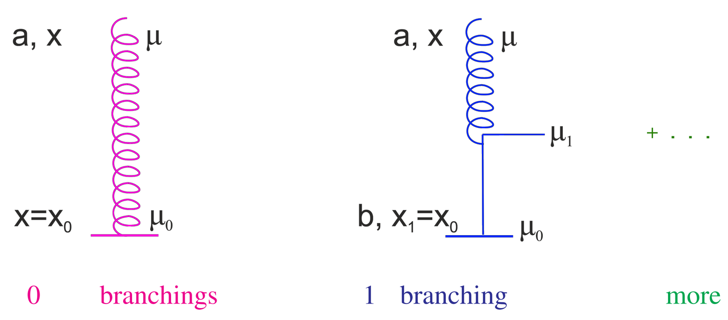
PB delivers TMDs for all flavors, in a wide kinematic range of x, μ and $k_\perp$ from the TMD evolution equation [2, 3, 4]:

$$\begin{aligned}\widetilde{A}_a\left(x,k_\perp,\mu^2\right) &= \Delta_a\left(\mu^2,\mu_0^2\right)\widetilde{A}_a\left(x,k_\perp,\mu_0^2\right) \\ &+ \sum_b \int_{\mu_0^2}^{\mu^2} \frac{\mathrm{d}\mu_1^2}{\mu_1^2}\int_0^{2\pi}\frac{d\phi}{2\pi}\Delta_a\left(\mu^2,\mu_1^2\right)\int_x^{z_M} \mathrm{d}z P_{ab}^R\left(z,\mu_1^2,\alpha_s((1-z)^2\mu_1^2)\right)\widetilde{A}_b\left(\frac{x}{z},k'_\perp,\mu_0^2\right)\Delta_b(\mu_1^2,\mu_0^2) \\ &+ \ldots\end{aligned}$$

where μ-evolution scale, $\widetilde{A}=xA$, $k=(k^0,k^1,k^2,k^3)=(E_k,\boldsymbol{k},k^3)$, $\boldsymbol{k}=(k^1,k^2)$, $k_\perp=|\boldsymbol{k}|$, $k'_\perp=|\boldsymbol{k}+(1-z)\boldsymbol{\mu}_1|$

PB equation is based on showering version of DGLAP equation and unitarity picture:
parton evolution is expressed in terms of resolvable, real emission DGLAP splitting functions P_{ab}^R and non-resolvable branching probabilities and the virtual contributions, included via Sudakov form factors

$$\Delta_a\left(\mu^2,\mu_0^2\right) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2}\frac{\mathrm{d}\mu'^2}{\mu'^2}\int_0^{z_M}\mathrm{d}z\, zP_{ba}^R\left(z,\mu'^2,\alpha_s\left((1-z)^2\mu'^2\right)\right)\right)$$



Soft gluon resolution scale z_M separates resolvable ($z<z_M$) and non-resolvable ($z>z_M$) branchings.
Transverse momentum is calculated at each branching. Transverse momentum $\mathbf{k}$ of the final parton is a sum of the intrinsic transverse momentum of the initial parton and all the transverse momenta emitted in the evolution process $\mathbf{k}=\mathbf{k}_0-\sum_i \mathbf{q}_i$.

Initial distribution at μ_0 consists of collinear part $\widetilde{f}_a$ and a Gaussian for intrinsic transverse momentum: $\widetilde{A}_a(x,k_\perp^2,\mu_0^2)=\widetilde{f}_{a,0}(x,\mu_0^2)\frac{1}{2\pi\sigma^2}\exp\left(\frac{-k_\perp^2}{2\sigma^2}\right)$.
Parameters of $\widetilde{f}_a(x,\mu_0^2)$ are fitted to experimental data using `xFitter` [5].
Officially released PB TMD sets [4] can be accessed via TMDlib - a library collecting different TMD approaches [6].

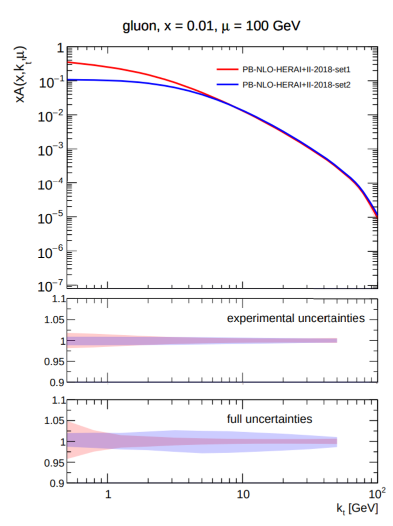


Fig.: Example of a gluon PB TMD as a function of $k_\perp$ from officially released PB sets [4].

iTMDs (= collinear PDFs) can be obtained from integration of PB TMD (and used in LHAPDF [7]):
$\widetilde{f}_a(x,\mu^2)=\int \mathrm{d}k_\perp^2 \widetilde{A}_a(x,k_\perp,\mu^2)$
$\rightarrow$ collinear physics applications.

Colour coherence in QCD causes the angles of the emitted partons to increase from the hadron side towards the hard scattering. This feature, known as angular ordering (AO), is taken into account in PB for a proper infrared resummation.

From PB TMD evolution to TMD Parton Shower

The PB TMD Parton Shower is implemented in the MC generator CASCADE [8].
The Sudakov form factor for the backward evolution between scales μ_{i-1}^2 and μ is given by

$$\Delta_{\mathrm{backward},a}(x,k_\perp,\mu,\mu_{i-1})=\exp\left(-\sum_b\int_{\mu_{i-1}^2}^{\mu^2}\frac{\mathrm{d}\mu'^2}{\mu'^2}\frac{\mathrm{d}\phi}{2\pi}\int_x^{z_M}\mathrm{d}zP_{ab}^R\frac{\widetilde{A}_b(x',k'_\perp,\mu')}{\widetilde{A}_a(x,k_\perp,\mu')}\right)$$

The kinematics in forward and backward equations corresponds perfectly to each other! The TMD obtained from PB TMD evolution equation guides the evolution in the shower.

Ingredients of the PB method

Determination of the TMD:

- TMD (forward) evolution equation (solved with MC methods),
- PDF fit procedure implemented within `xFitter`.

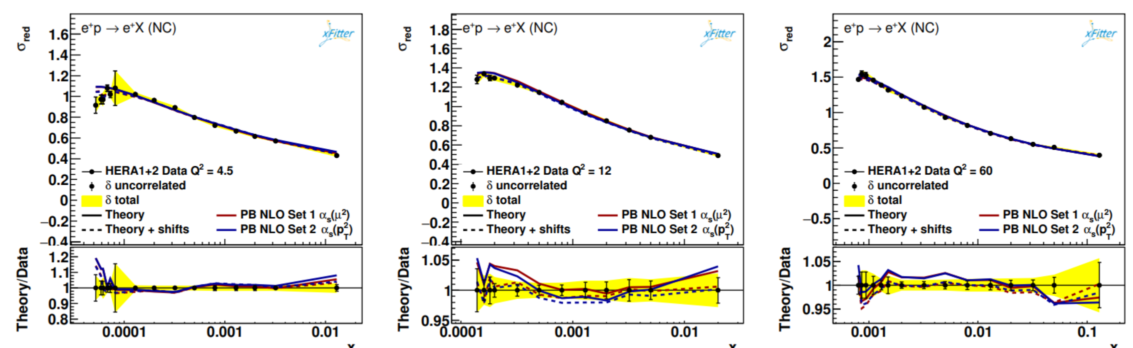
Application to measurements:

- recipe on how to use PB TMDs in the hard process generation (LO, NLO),
- backward PB Parton Shower implemented in Cascade MC generator,
- procedure to merge different jet multiplicities developed.

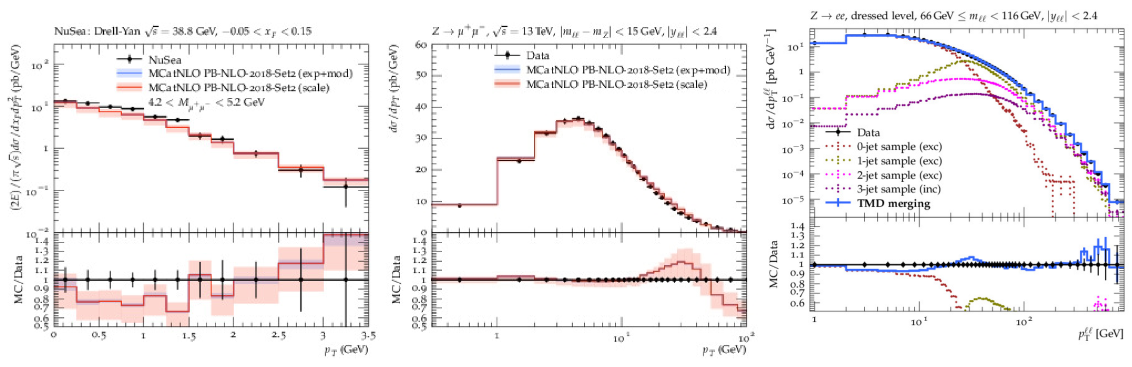
Predictions with PB TMDs

To obtain collider predictions with PB TMDs [4, 11, 12]:
• ME is generated by standard tools used in collinear physics (Pythia, Madgraph...) using the iTMD.
• ME is supplemented with transverse momentum $k_\perp$ by an algorithm in CASCADE. The TMD used in CASCADE corresponds to the iTMD from which the ME was initially generated.
• x of the incoming partons have to be adjusted to conserve energy-momentum and the mass of the DY system is kept unchanged.
For inclusive observables, like DY $p_\perp$, the whole kinematics is included by using PB TMDs. For exclusive observables CASCADE PS needs to be applied.

PB was successfully applied to multiple measurements (HERA DIS, inclusive Drell-Yan at different masses and center of mass energies, Drell-Yan + jets, ...).



Predictions for inclusive DIS reduced cross section obtained with officially released PB TMD sets compared to HERA data [4].



Predictions for DY $p_\perp$ spectra. Left and middle: obtained with Madgraph MCatNLO+PB TMD compared with NuSea (left) and CMS (middle) data [12]. Right: the fully TMD-merged calculation as well, as separate contributions from different jet samples, compared to 8 TeV ATLAS data [13].

For an overview of the applications: see poster of Sara Taheri Monfared.

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