Detector conception and momentum measurement optimization applied to Muon Scattering Tomography

Maxime Lagrange[1], A. Giammanco, G. Strong, T. Dorigo, P. Vischia, J. Kieseler, F. Nardi, H. Zaraket, M. Lamparth, F. Fanzago, O. Savchenko, N. Sharma, A. Bordignon

[1] Centre for Cosmology, Particle Physics and Phenomenology (CP3), Université Catholique de Louvain (UCL), Belgium

I - Muon Scattering Tomography (MST)

- **Cosmic muons** are produced from decay products of energetic protons striking molecules of the upper atmosphere (flux at sea level ≈ 1 muon/s/cm^2)
- Muons moving through material are **deflected** by the **Coulomb interaction** with atom's **nucleus**
- The total **deflection** depends on the **atomic number** Z and the **density** ρ of the material
- For monochromatic muons and given a material of radiation length $X^0(\rho, Z)$, the **deflection angle distribution** is Gaussian, with mean μ = 0 and RMS θ_{RMS} such as

$$\theta_{RMS} = \frac{13.6MeV}{\beta cp}\sqrt{\frac{x}{X_0(Z,\rho)}} \qquad (1)$$

- x muon's path in the material, X_0 radiation length, p muon's momentum

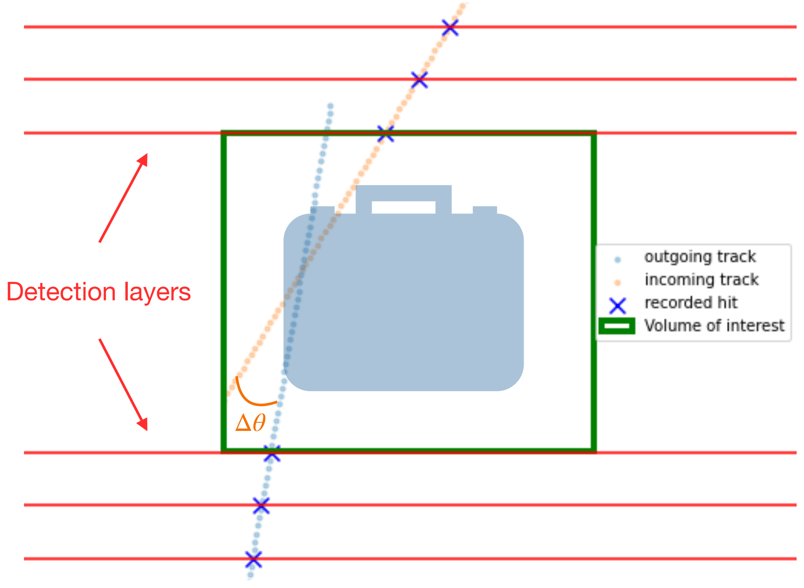


Figure 1: Illustration of multiple scattering tomography concept. Upper and lower detector planes respectively detect incoming and outgoing muon tracks, which are used to compute the scattering angle $\Delta\theta$

- By **measuring** the **scattering angle**, one can **infer** on material **radiation length**

II - Muon momentum knowledge and reconstruction

- **Scattering angle amplitude** depends both on muon **momentum** p and material **radiation length** X_0 (eq. 1)
- Low momentum muons can have large scattering angle and mimic the behavior of muons crossing dense materials (Figure 2)
- A **good knowledge** of p is **crucial** for an accurate material identification (*Figure 3*)

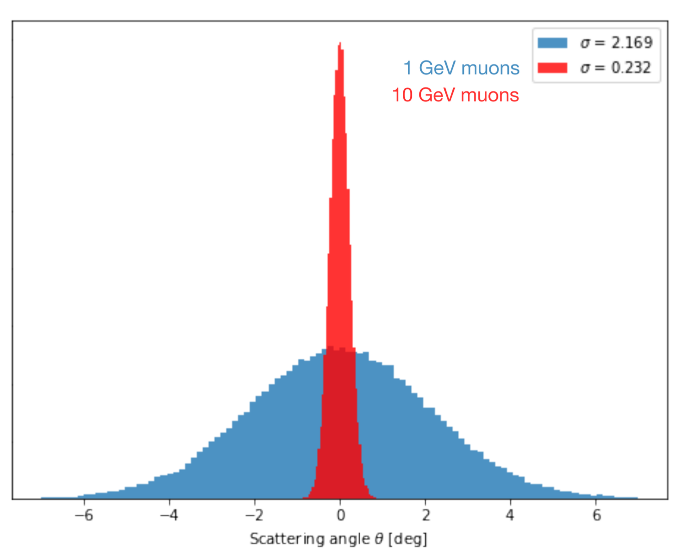


Figure 2: Illustration of scattering angle amplitude for 1 GeV and 10 GeV muons in a 4cm thick lead block

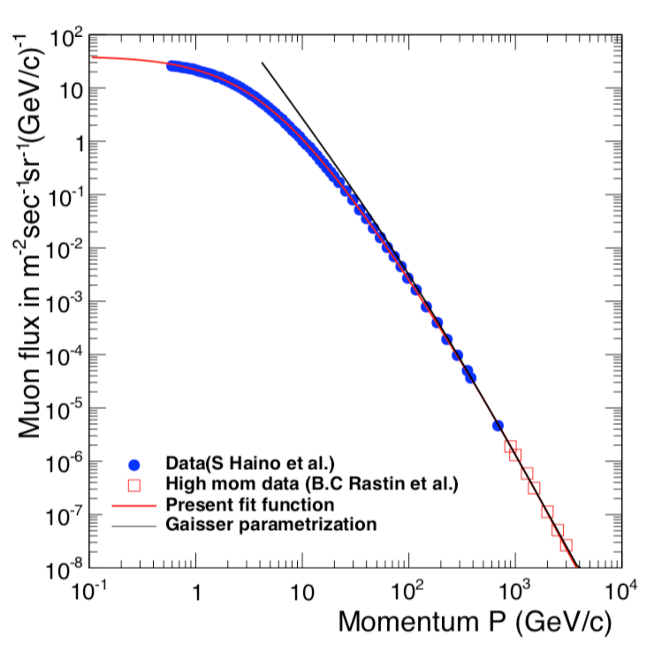


Figure 4: Cosmic muon energy spectrum at 0° zenith angle from [1]

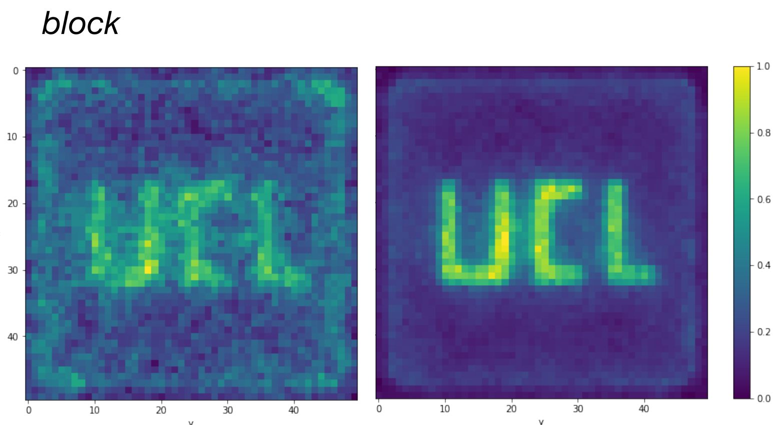


Figure 3: Illustration of MST image reconstruction for no momentum knowledge (left) and full momentum knowledge (right)

III - Muon momentum measurement

- Muon momentum can be **estimated** by **measuring scattering angle** $\Delta\theta$ in a **known material** as it was proposed in [2]. Inverting eq. (1):

$$p = \frac{13.6MeV}{\theta_{RMS}}\sqrt{\frac{x}{X_0}} \qquad (2)$$

θ_{RMS} the scattering angle RMS $\theta_{RMS} = \frac{1}{N}\sum_{i=0}^{N}\Delta\theta_i^2$

- Alternating **detection planes** and **scattering material** chosen for its scattering power (e.g lead), one can measure θ_{RMS}
- Mean momentum estimation **resolution** $\frac{\bar{\Delta p}}{p_{true}}$ quickly worsen with detector resolution

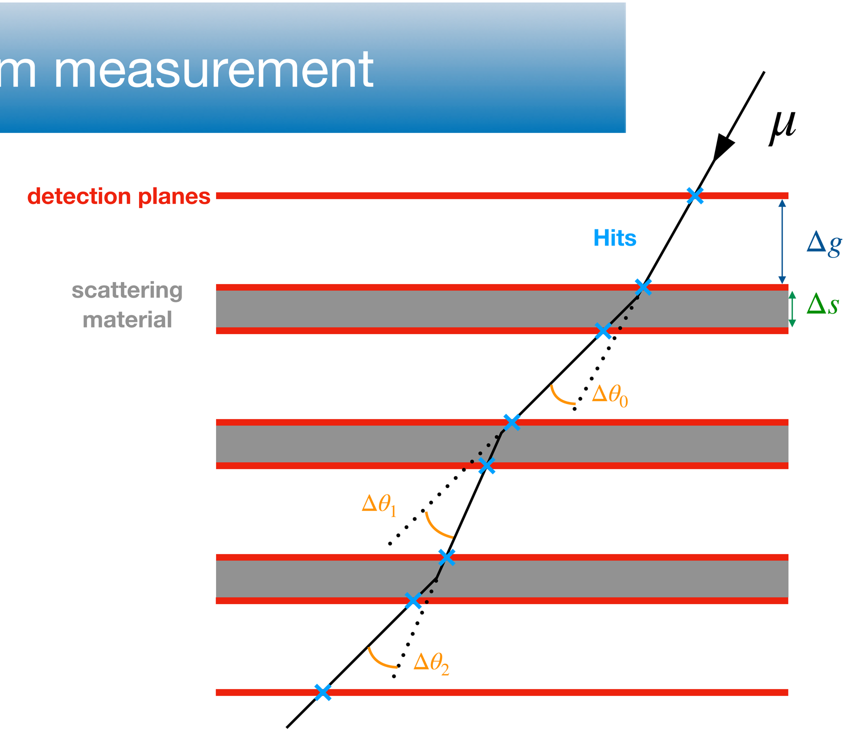


Figure 5: Illustration of momentum measurement module, to be placed at the bottom of a regular MST detection system.

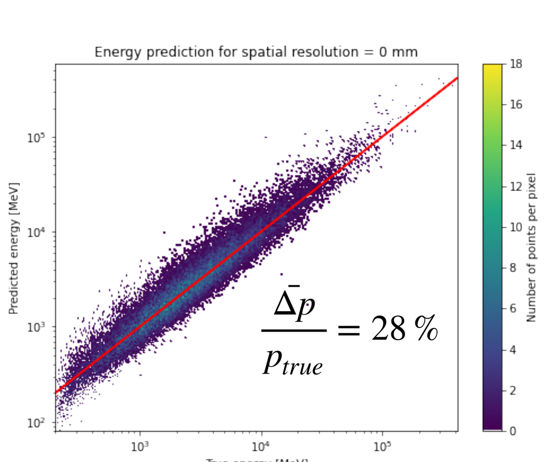
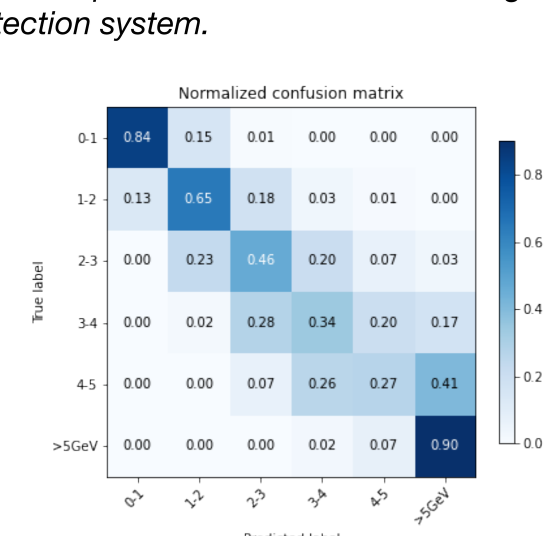


*Figure 6: Momentum prediction for detector design with Δs = 4cm, Δg = 10 cm, 3 lead blocks as scattering material and a **perfect spatial resolution***



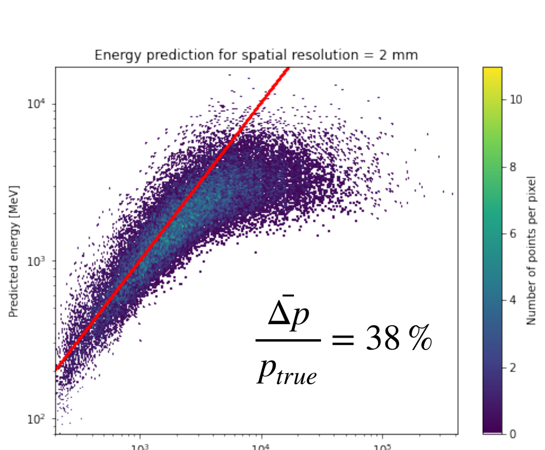
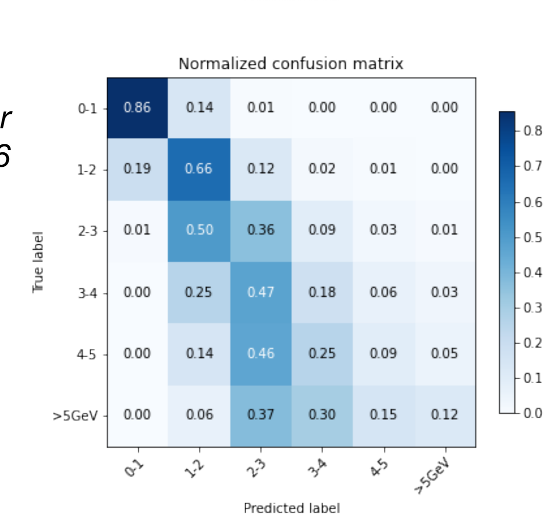


*Figure 7: Momentum prediction for same detector design as Figure 6 but with **2mm spatial resolution***



III- Detection system parameter space

One can **relate momentum measurement precision** to the **detector configuration.**

- **Momentum** is estimated from scattering angle RMS, then its **relative uncertainty** is $\frac{\Delta p}{p} = \frac{\Delta\theta_{RMS}}{\theta_{RMS}}$
- **Uncertainty** on scattering angle RMS is a **function** of **detector parameters**:

$$\frac{\Delta\theta_{RMS}}{\theta_{RMS}} = f(\delta\theta, S_p, N_{planes})$$

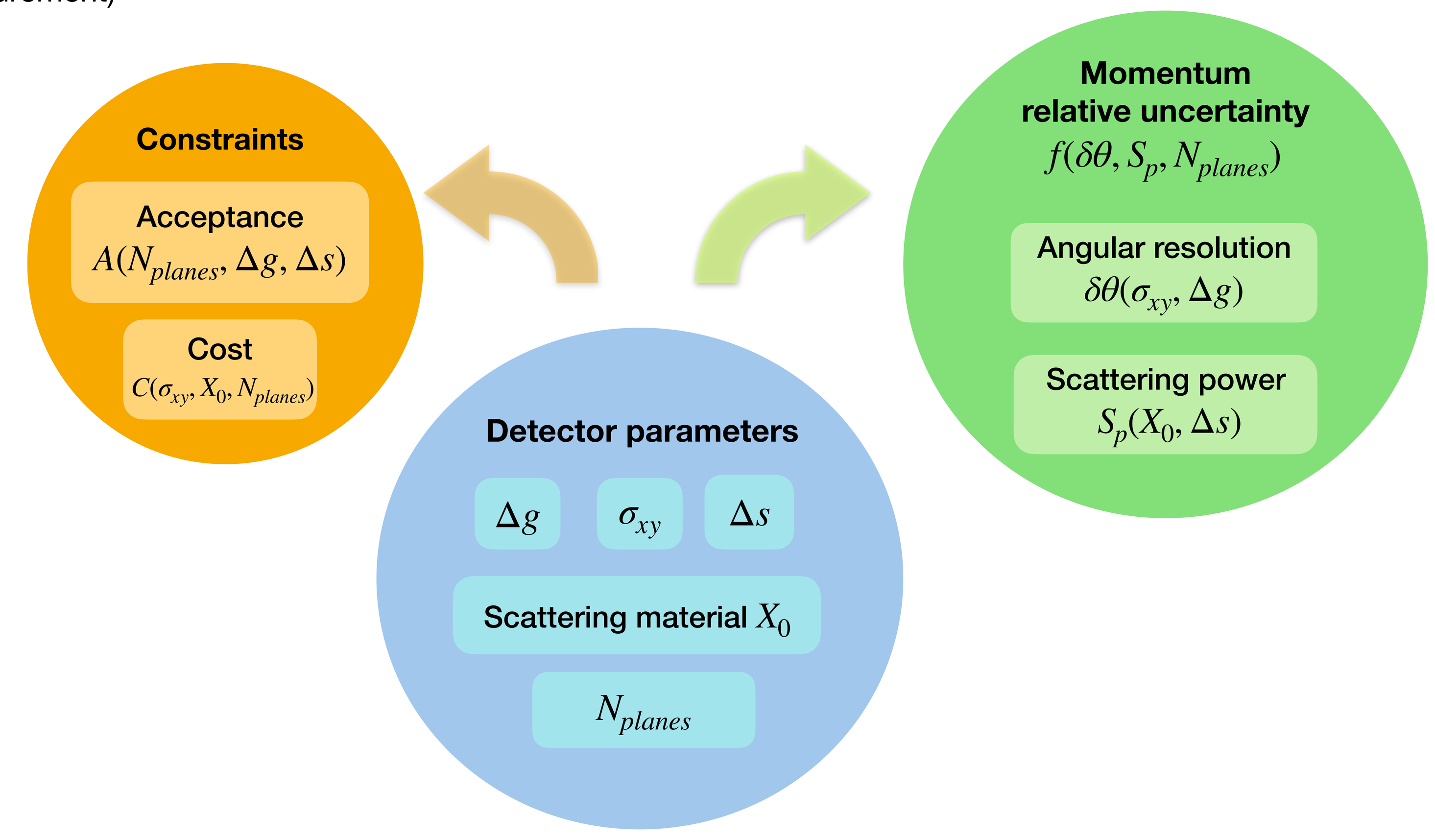
With $\delta\theta$ the **angular resolution** on track measurement, S_p **scattering power** of the material, N_{planes} the **number** of scattering **layers** (= number of $\Delta\theta$ measurement)

Angular resolution

- Angular resolution $\delta\theta = \delta\theta(\sigma_{xy}, \Delta g)$ depends on spatial resolution σ_{xy} and spacing between planes Δg

Scattering power

- Scattering power $S_p = S_p(X_0, \Delta s)$ is a function of radiation length X_0, and width of the material Δs. The more scattering power, the larger scattering angle muons undergo



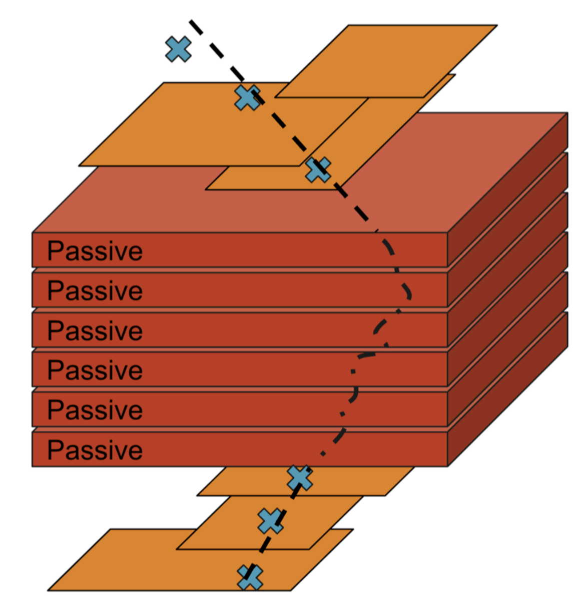
Momentum measurement module has an effect on the global MST detection system: adding detection layers **increases** the **cost** of the detector and **reduces** its **acceptance**. These **constraints** have to be taken into account when optimizing the momentum measurement system.

IV - Optimization: a mission for TomOpt

Multiple Scattering Tomography **detection system** aims to be as **performant** as possible in terms of material identification while being within a certain **budget** and **respecting constraints** such as **exposure time**. Optimization of such a system can be done using differentiable programming, and this is exactly what TomOpt proposes.

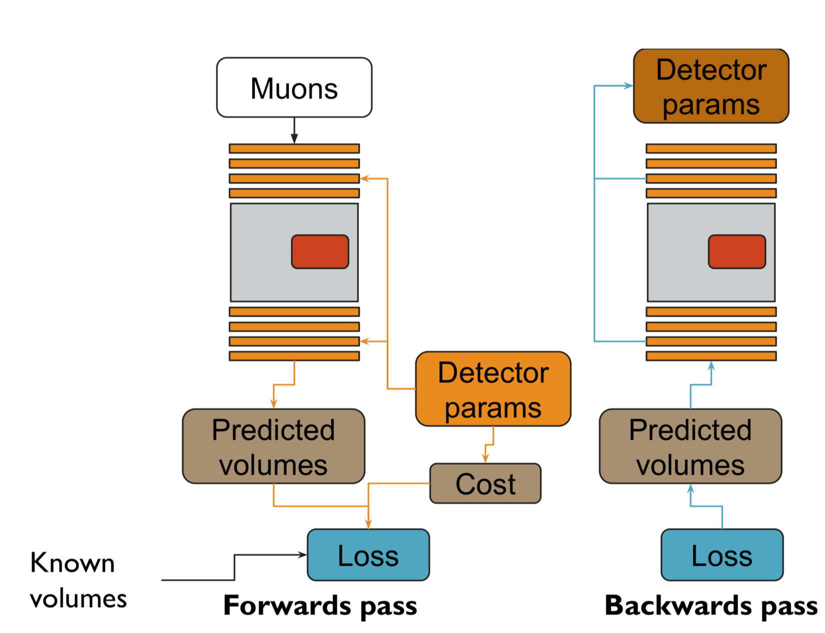
TomOpt: Differential Muon Tomography

- **TomOpt** is a first step within the **MODE collaboration** [3]
- TomOpt proposes a **full optimization pipeline** applied to muon scattering tomography. It provides differentiable functions of detector predictions and parameters, which allows **optimization** via **gradient descent**.



TomOpt: Package overview

1. Detector **configuration** (resolution, efficiency, orientation, etc.)
2. Muon **generation** sampling literature models
3. Muon **propagation** through passive volumes using GEANT4 parametric models
4. **Reconstruction** and material **identification**
5. **Loss function** computation based on **target performances** and cost
6. **Optimization** of detector system using **gradient-descent** optimizers



V - References

[1] - Energy and angular distribution of cosmic muons / P. Mitra, P. Shukla / Proceedings of the DAE-BRNS Symp. on Nucl. Phys. 60 (2015)

[2] - Muography of different structures using muon scattering and absorption algorithms / S. Vanini, P. Calvini, P. Checchia, A. Rigoni Garola, J. Klinger, G. Zumerle, G. Bonomi, A. Donzella, A. Zenoni / https://doi.org/10.1098/rsta.2018.0051

[3] - Toward the End-to-End Optimization of Particle Physics Instruments with Differentiable Programming: a White Paper / Tommaso Dorigo, Andrea Giammanco, Pietro Vischia (editors) MODE Collaboration / arXiv: 2203.13818v1

UCLouvain

MODE