MODEL-INDEPENDENT CALCULATIONS OF PROTON STRUCTURE EFFECTS IN MUONIC HYDROGEN

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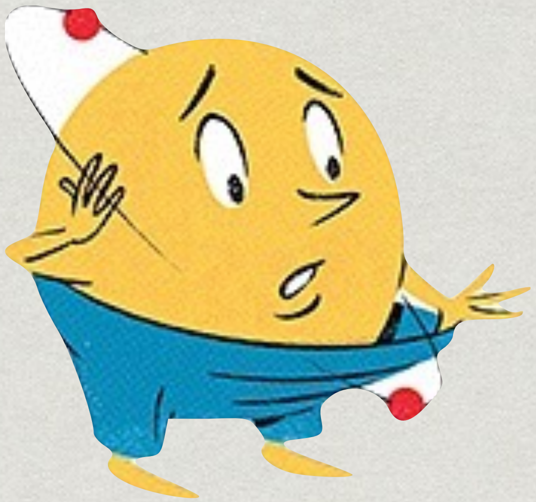
SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Cluster of Excellence
PRISMA
Precision Physics, Fundamental Interactions and Structure of Matter

JG|U

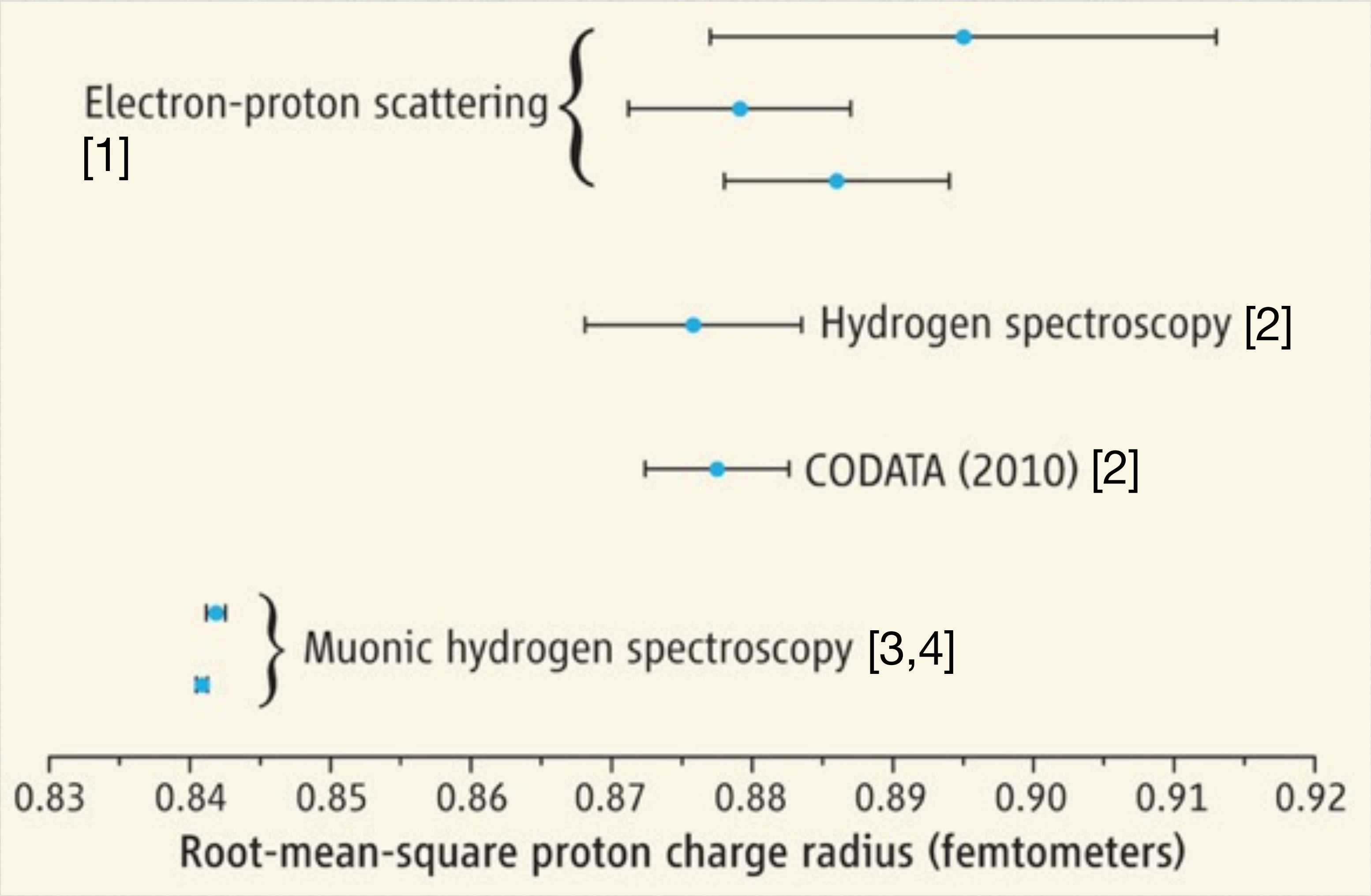
Outline



- *Finite-Size effects* by dispersive technique
  - Breakdown of the expansion in charge radii (de Rújula scenario)
- *Compton scattering* sum rules
- *Proton polarizability effects* in the hyperfine splitting of muonic hydrogen
  - pion-nucleon loops (LO ChPT)
  - Δ-exchange (NLO ChPT)
  - neutral-pion exchange (NLO ChPT)

☺ $(Z\alpha)^6 \ln(Z\alpha)$ polarizability contribution in light muonic atoms from *off-forward two-photon exchange*

Proton Radius Puzzle



Lamb shift discrepancy: 310 μeV

[1] J. C. Bernauer *et al.*, Phys. Rev. Lett. **105**, 242001 (2010).
[2] P. J. Mohr, et al., Rev. Mod. Phys. **84**, 1527 (2012).
[3] R. Pohl, A. Antognini *et al.*, Nature **466**, 213 (2010).
[4] A. Antognini *et al.*, Science **339**, 417 (2013).

seven standard-deviation discrepancy (7σ) !!!

$[R_E^{\mu\text{H}} = 0.84087(39)\,\text{fm}]$ ⟷ $[R_E^{\text{CODATA 2010}} = 0.8775(51)\,\text{fm}]$

Proton Radius Puzzle: Possible Explanations

Why do we observe different radii ?

Lamb shift difference of 310 ± 2 μeV

μH experiment wrong ?

eH theory wrong ?

BSM ?

μH theory wrong ?
- 2γ corrections
- missing QED or EW corrections
- soft hadronic corrections

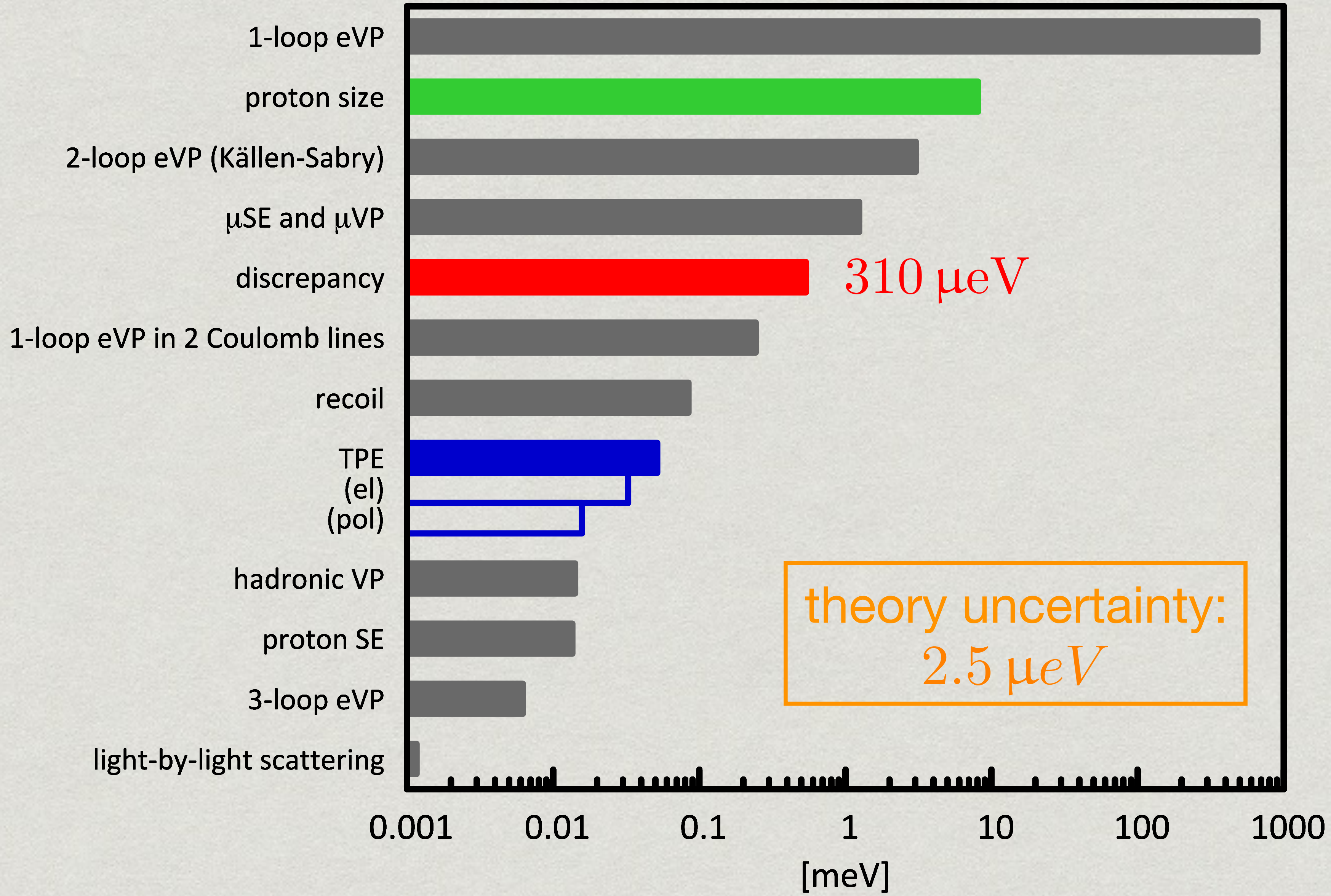
eH experiment
+ ep scattering wrong ?
- R_∞
- 2γ corrections
- low-Q^2 extrapolation

Theory of µH Lamb Shift

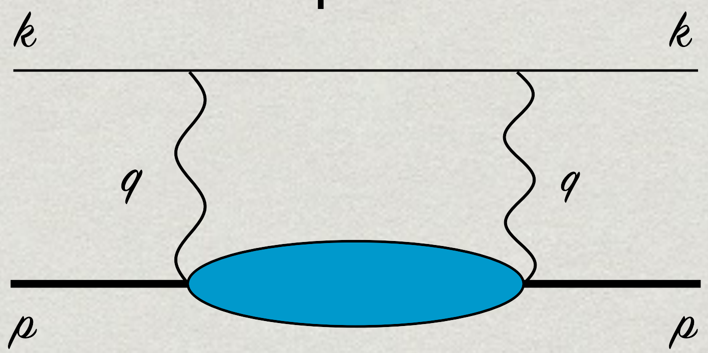
Assuming ChPT is working, it should be best applicable to atomic systems, where the energies are very small !

$$\Delta E_{\rm LS}^{\rm th} = 206.0668(25) - 5.2275(10)\,(R_E/{\rm fm})^2$$

numerical values reviewed in: A. Antognini et al., Annals Phys. **331** (2013) 127-145



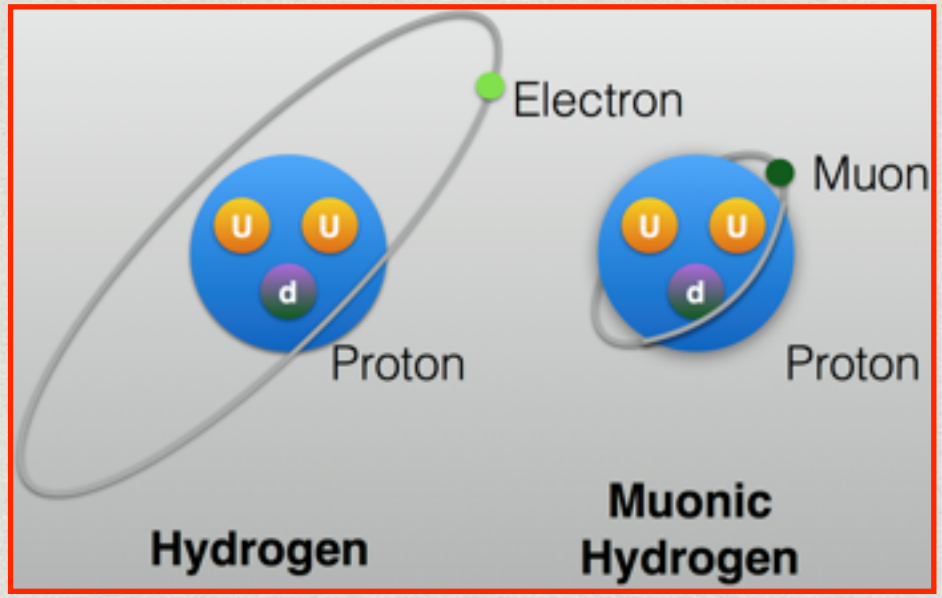
subleading effects of proton structure proposed to resolve the puzzle



$$\delta V^{(2\gamma)} = \delta V^{(2\gamma)}_{\rm elastic} + \delta V^{(2\gamma)}_{\rm polariz.}$$

Finite-Size Effects

Why muonic atoms ?



- HFS:

Fermi - Energy:
$$E_F(nS) = \frac{8}{3}\frac{Z\alpha}{a^3}\frac{1+\kappa}{mM}\frac{1}{n^3}$$
with Bohr radius $a = 1/(Z\alpha m_r)$

$$\Delta E_{nS}(\mathrm{LO}+\mathrm{NLO}) = E_F(nS)\,[1 - 2\,Z\alpha m_r\,R_Z]$$

NLO becomes appreciable in μH

- Lamb shift:

wave function at the origin

$$\Delta E_{nl}(\mathrm{LO{+}NLO}) = \delta_{l0}\,\frac{2\pi Z\alpha}{3}\,\frac{1}{\pi(an)^3}\,\left[R_E^2 - \frac{Z\alpha m_r}{2}R^3_{E(2)}\right]$$

Fourier transformation:
$$\rho_E(r) = \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3} G_E(\boldsymbol{q}^2)e^{-i\boldsymbol{q}\boldsymbol{r}}$$

$$R_E^2 = -6 \lim_{Q^2\to 0}\frac{\mathrm{d}}{\mathrm{d}Q^2}G_E(Q^2)$$
$$R^3_{E(2)} = \frac{48}{\pi}\int_0^\infty \frac{\mathrm{d}Q}{Q^4}\left\{G_E^2(Q^2) - 1 + \frac{1}{3}\langle r^2\rangle_E\,Q^2\right\}$$
J. L. Friar, Annals Phys. **122** (1979) 151

Phys. Rev. A **91** (2015) 040502
Phys. Rev. A **93** (2016) 026502

Lamb Shift: Expand or not ?

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **91**, 040502(R) (2015)

Breakdown of the expansion of finite-size corrections to the hydrogen Lamb shift in moments of charge distribution

Franziska Hagelstein and Vladimir Pascalutsa
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(Received 13 February 2015; published 20 April 2015)

PHYSICAL REVIEW A **93**, 026501 (2016)

Comment on "Breakdown of the expansion of finite-size corrections to the hydrogen Lamb shift in moments of charge distribution"

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(Received 15 December 2015; published 23 February 2016)

PHYSICAL REVIEW A **93**, 026502 (2016)

Reply to "Comment on 'Breakdown of the expansion of finite-size corrections to the hydrogen Lamb shift in moments of charge distribution' "

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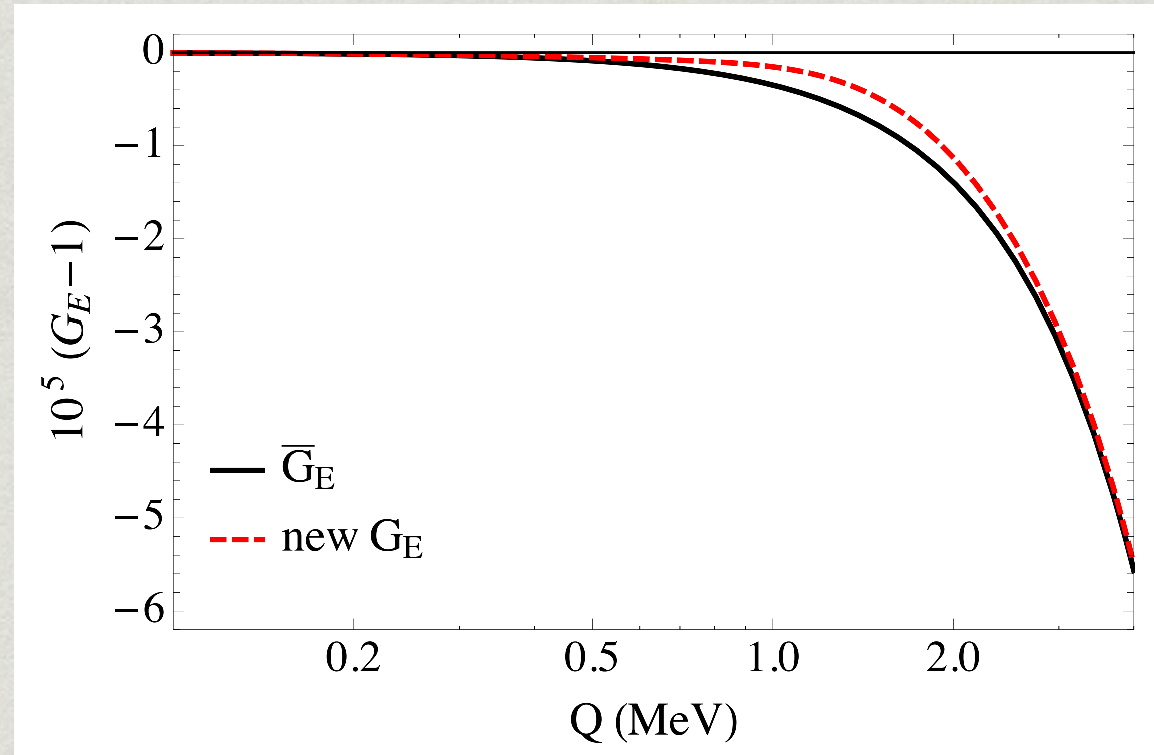
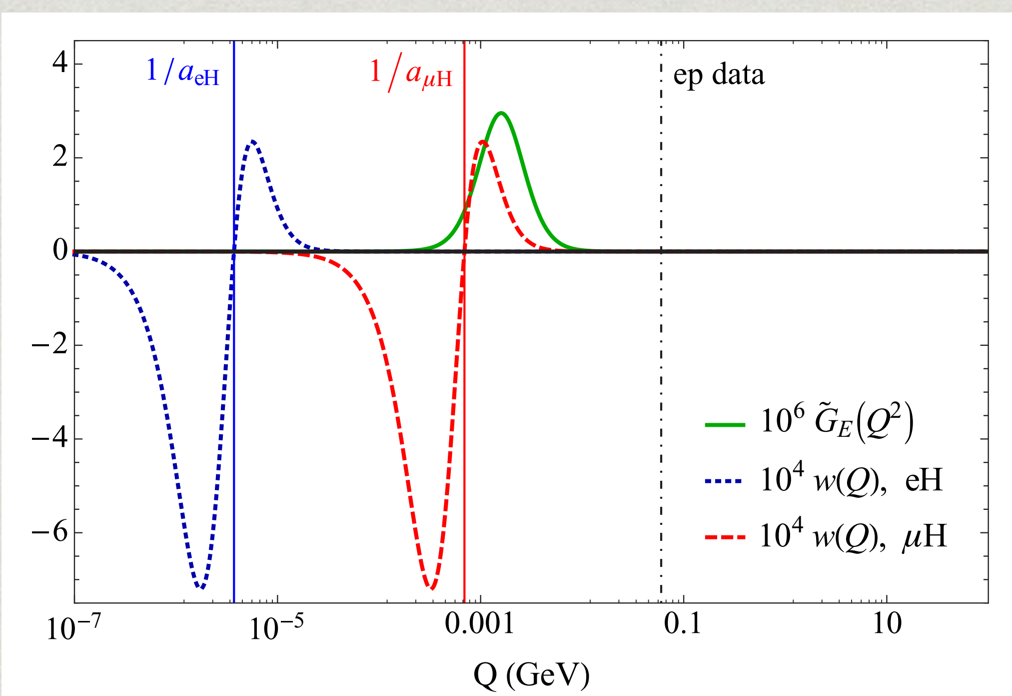
(Received 22 January 2016; published 23 February 2016)



$$\Delta E^{\mathrm{FF}(1)}_{2P-2S} = -\frac{(Z\alpha)^4 m_r^3}{2\pi}\int_{t_0}^{\infty}\mathrm{d}t\,\frac{\mathrm{Im}\,G_E(t)}{(\sqrt{t}+Z\alpha m_r)^4}$$

$$= -\frac{2Z\alpha}{\pi}\int_0^{\infty}\mathrm{d}Q\,w_E(Q)\,G_E(Q^2)$$

$$= -\frac{(Z\alpha)^4 m_r^3}{12}\left[\langle r^2\rangle_E - Z\alpha m_r \langle r^3\rangle_E\right] + O(\alpha^6)$$



convolution of momentum-space wave functions: $w_E(Q) = 2(Z\alpha m_r)^4\,Q^2\,\frac{(Z\alpha m_r)^2 - Q^2}{[(Z\alpha m_r)^2+Q^2]^4}$

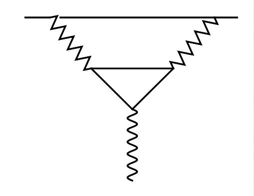
cf. Wichmann - Kroll contribution
S. G. Karshenboim, et al., JETP Lett. **92** (2010) 8

Lamb Shift: Expand or not ?

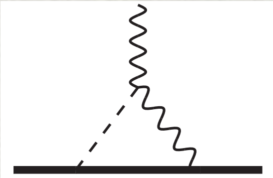
$$E^{\mathrm{FF}(1)}_{2P-2S} = \int_0^\infty \mathrm{d}Q\, w_E(Q)\, G_E(Q^2) \quad \text{with} \quad w_E(Q) = -\frac{4}{\pi}(Z\alpha)^5 m_r^4\, Q^2 \frac{(Z\alpha m_r)^2 - Q^2}{[(Z\alpha m_r)^2 + Q^2]^4}$$

- the finite-size effects are not always expandable in the moments of charge distribution
  - convergence radius of the Taylor expansion of $G_E(Q^2)$ has to be much larger than the inverse Bohr radius ($Z\alpha m_r$) of the given hydrogen-like system
- a tiny non-smoothness of the electric form factor at a MeV scale would be able to explain the puzzle
  - one needs to know all the "soft" (below several MeV) contributions to the proton electric FF to high accuracy

- missing standard model effects, e.g., muon-decay correction?



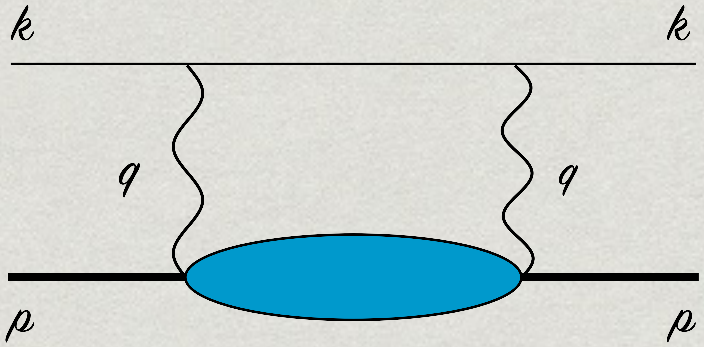
- light particles beyond the standard model



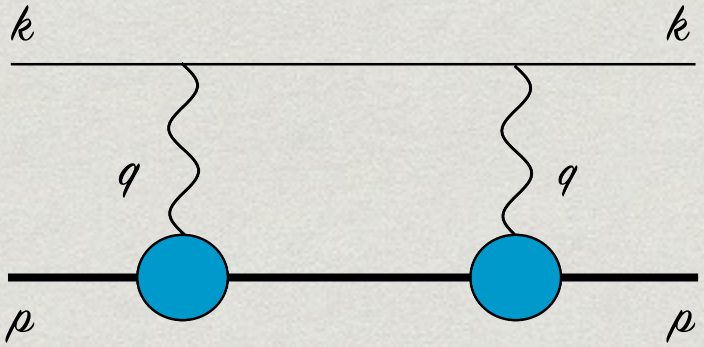
Structure Effects through 2γ

- proton structure effects at subleading orders arise through multi-photon processes

forward
two-photon exchange (2γ)



polarizability contribution



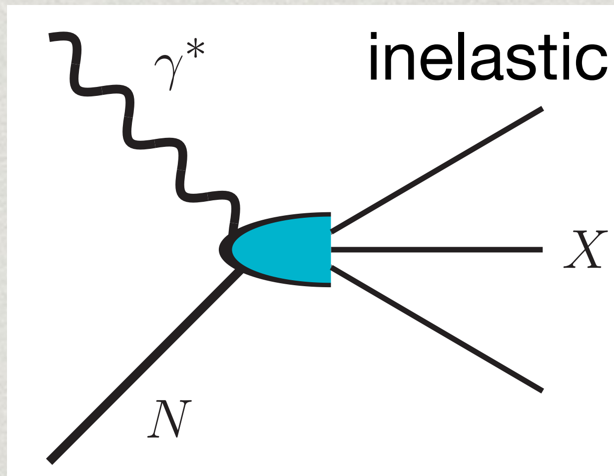
elastic contribution:
finite-size recoil,
3rd Zemach moment (Lamb shift),
Zemach radius (Hyperfine splitting)

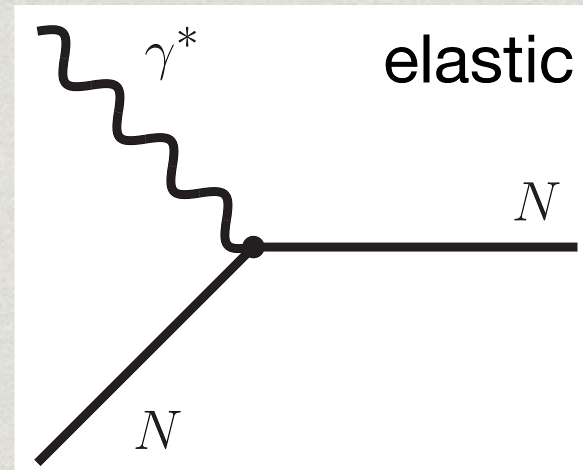
- “blob” corresponds to <u>doubly-virtual Compton scattering</u> (VVCS):

$$T^{\mu\nu}(q,p) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) \boxed{T_1(\nu, Q^2)} + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}\,q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}\,q^\nu\right) \boxed{T_2(\nu, Q^2)}$$
$$-\frac{1}{M}\gamma^{\mu\nu\alpha}q_\alpha \boxed{S_1(\nu, Q^2)} - \frac{1}{M^2}\left(\gamma^{\mu\nu}q^2 + q^\mu\gamma^{\nu\alpha}q_\alpha - q^\nu\gamma^{\mu\alpha}q_\alpha\right) \boxed{S_2(\nu, Q^2)}$$

Proton Structure in e^-p Scattering

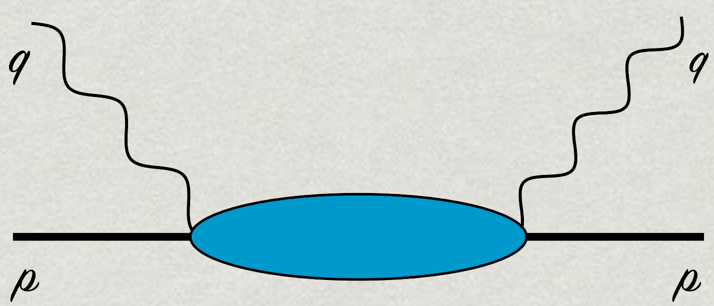
photoabsorption cross section:





elastic + inelastic
= Born + non-Born

Compton scattering (CS):



photon energy and virtuality: $\nu,\ Q^2$
Bjorken variable: $x = Q^2/2M\nu$
$\tau = Q^2/4M^2$

proton polarizabilities:

$$\alpha_{E1}(Q^2) + \beta_{M1}(Q^2) = \frac{8\alpha M}{Q^4} \int_0^{x_0} \mathrm{d}x\, x\, f_1(x, Q^2)$$

proton structure functions:

$$f_1(x, Q^2),\ f_2(x, Q^2),\ g_1(x, Q^2),\ g_2(x, Q^2)$$

Lamb shift (f_1, f_2) — Hyperfine splitting (HFS) (g_1, g_2)

elastic structure functions:

Sachs form factors: G_E, G_M
Dirac & Pauli form factors: F_1, F_2

$$f_1^{\mathrm{el}}(x, Q^2) = \frac{1}{2} G_M^2(Q^2)\,\delta(1-x)$$

$$f_2^{\mathrm{el}}(x, Q^2) = \frac{1}{1+\tau}\left[G_E^2(Q^2) + \tau G_M^2(Q^2)\right]\delta(1-x)$$

$$g_1^{\mathrm{el}}(x, Q^2) = \frac{1}{2} F_1(Q^2)\, G_M(Q^2)\,\delta(1-x)$$

$$g_2^{\mathrm{el}}(x, Q^2) = -\frac{\tau}{2} F_2(Q^2)\, G_M(Q^2)\,\delta(1-x)$$

CS Amplitudes & Structure Functions

<u>optical theorem:</u>
unitarity

$$\mathrm{Im}\, T_1(\nu,Q^2) = \frac{4\pi^2 Z^2\alpha}{M} f_1(x,Q^2)$$
$$\mathrm{Im}\, T_2(\nu,Q^2) = \frac{4\pi^2 Z^2\alpha}{\nu} f_2(x,Q^2)$$
$$\mathrm{Im}\, S_1(\nu,Q^2) = \frac{4\pi^2 Z^2\alpha}{\nu} g_1(x,Q^2)$$
$$\mathrm{Im}\, S_2(\nu,Q^2) = \frac{4\pi^2 Z^2\alpha M}{\nu^2} g_2(x,Q^2)$$

+

<u>dispersion relations:</u>
analyticity, crossing symmetries

$$T_i(\nu,Q^2) = \frac{2}{\pi}\int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu' \,\mathrm{Im}\, T_i(\nu',Q^2)}{\nu'^{\,2}-\nu^2-i0^+}$$
$$S_1(\nu,Q^2) = \frac{2}{\pi}\int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu' \,\mathrm{Im}\, S_1(\nu',Q^2)}{\nu'^{\,2}-\nu^2-i0^+}$$
$$\nu S_2(\nu,Q^2) = \frac{2}{\pi}\int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu'^{\,2} \,\mathrm{Im}\, S_2(\nu',Q^2)}{\nu'^{\,2}-\nu^2-i0^+}$$

→

$$T_1(\nu,Q^2) = \frac{2}{\pi}\int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu' \,\mathrm{Im}\, T_1(\nu',Q^2)}{\nu'^{\,2}-\nu^2-i0^+} = \frac{8\pi Z^2\alpha}{M}\int_0^1 \frac{\mathrm{d}x}{x}\, \frac{f_1(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}$$
$$T_2(\nu,Q^2) = \frac{2}{\pi}\int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu' \,\mathrm{Im}\, T_2(\nu',Q^2)}{\nu'^{\,2}-\nu^2-i0^+} = \frac{16\pi Z^2\alpha M}{Q^2}\int_0^1 \mathrm{d}x\, \frac{f_2(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}$$
$$S_1(\nu,Q^2) = \frac{2}{\pi}\int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu' \,\mathrm{Im}\, S_1(\nu',Q^2)}{\nu'^{\,2}-\nu^2-i0^+} = \frac{16\pi Z^2\alpha M}{Q^2}\int_0^1 \mathrm{d}x\, \frac{g_1(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}$$
$$\nu S_2(\nu,Q^2) = \frac{2}{\pi}\int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu'^2 \,\mathrm{Im}\, S_2(\nu',Q^2)}{\nu'^{\,2}-\nu^2-i0^+} = \frac{16\pi Z^2\alpha M^2}{Q^2}\int_0^1 \mathrm{d}x\, \frac{g_2(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}$$

with
$\nu_{\mathrm{el}} = Q^2/2M$

Compton Scattering Sum Rules

- *Compton scattering (CS) amplitudes* in terms of integrals of *total photoabsorption cross sections*
  - dispersion relations:

$$T_1(\nu,0) = -\frac{4\pi\alpha}{M} + \frac{2\nu^2}{\pi}\int_0^\infty \mathrm{d}\nu' \, \frac{\sigma_{\rm abs}(\nu')}{\nu'^{\,2}-\nu^2-i0^+}$$

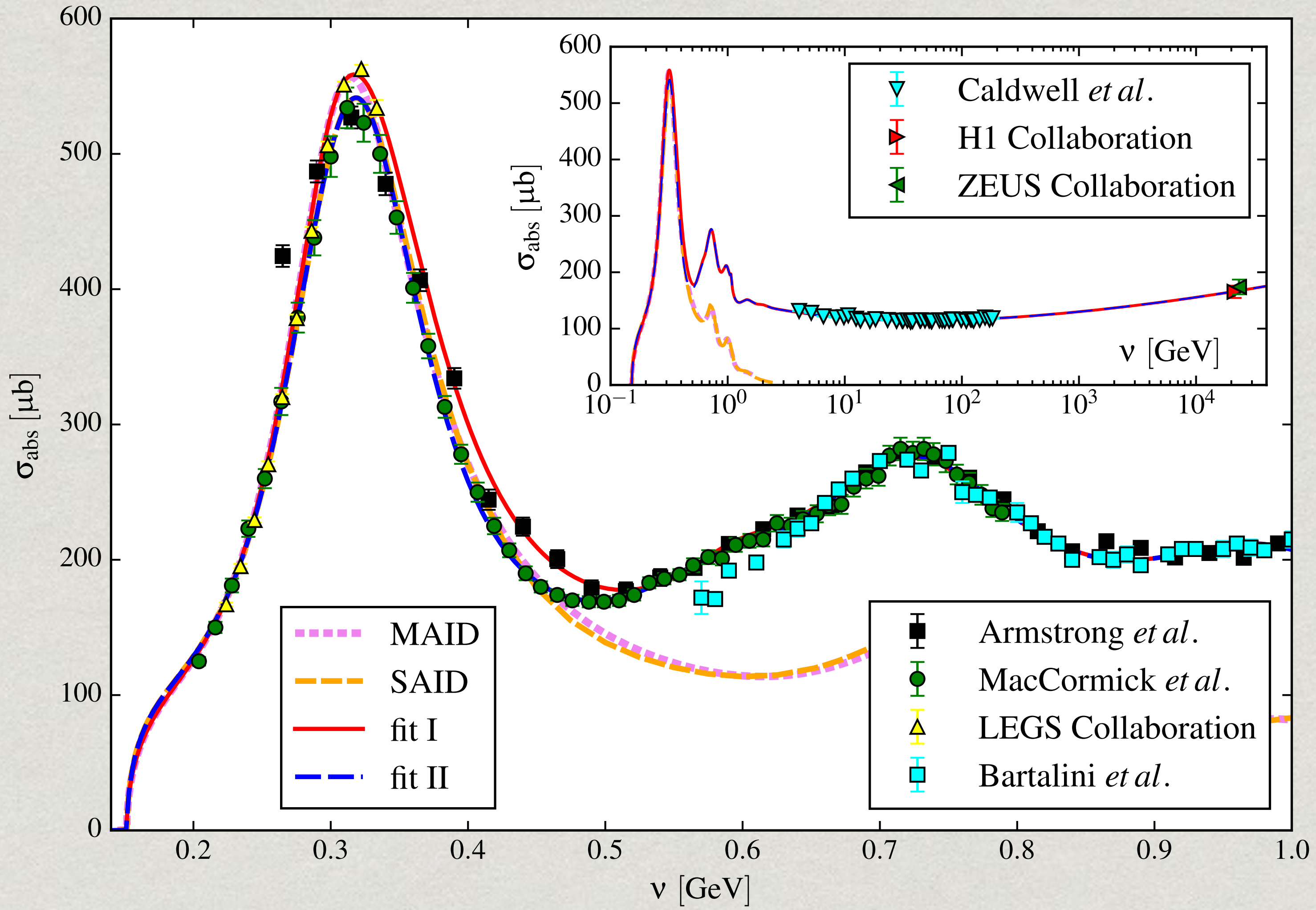
$$S_1(\nu,0) = \frac{M}{\pi}\int_0^\infty \mathrm{d}\nu' \, \frac{\nu'\,\Delta\sigma_{\rm abs}(\nu')}{\nu'^{\,2}-\nu^2-i0^+}$$

- *low-energy expansion* of CS amplitudes:

$$\frac{1}{4\pi}T_1(\nu,0) = -\frac{Z^2\alpha}{M} + (\alpha_{E1}+\beta_{M1})\nu^2 + \left[\alpha_{E1\nu}+\beta_{M1\nu}+{}^1\!/_{12}\,(\alpha_{E2}+\beta_{M2})\right]\nu^4 + O(\nu^6)$$

$$\frac{1}{4\pi}S_1(\nu,0) = -\frac{\alpha\varkappa^2}{2M} + M\gamma_0\nu^2 + M\bar{\gamma}_0\nu^4 + O(\nu^6)$$

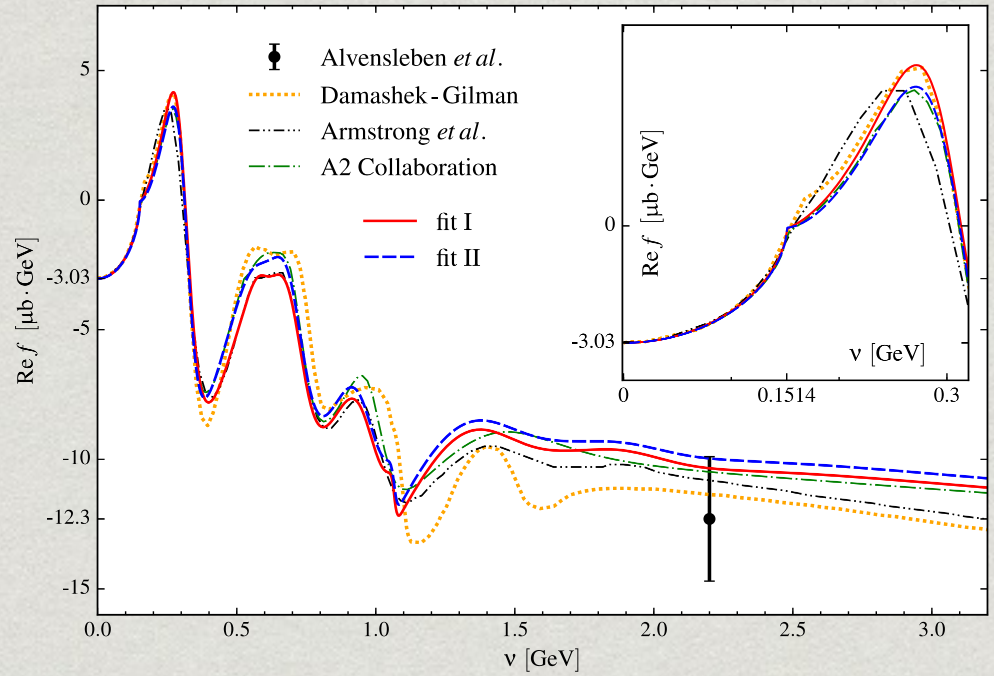
Spin-independent CS amplitude



Baldin sum rule:

$$\alpha_{E1} + \beta_{M1} = \frac{1}{2\pi^2} \int_{\nu_0}^{\infty} \mathrm{d}\nu \, \frac{\sigma_{\mathrm{abs}}(\nu)}{\nu^2}$$

$\alpha_{E1} + \beta_{M1} = 14.0(2) \times 10^{-4}\,\mathrm{fm}^3$



PDG 2014:

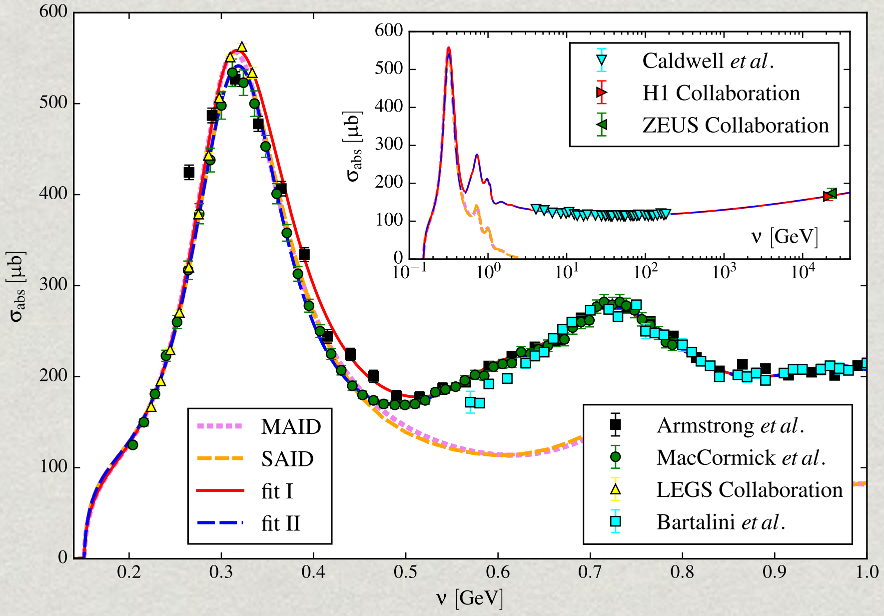
$\alpha_{E1} + \beta_{M1} = 13.7(6) \times 10^{-4}\,\mathrm{fm}^3$

BChPT:

$\alpha_{E1} + \beta_{M1} = 14.0(7) \times 10^{-4}\,\mathrm{fm}^3$

V. Lensky, et al., Phys. Rev. C 90 (2014) 055202

Spin-independent CS amplitude



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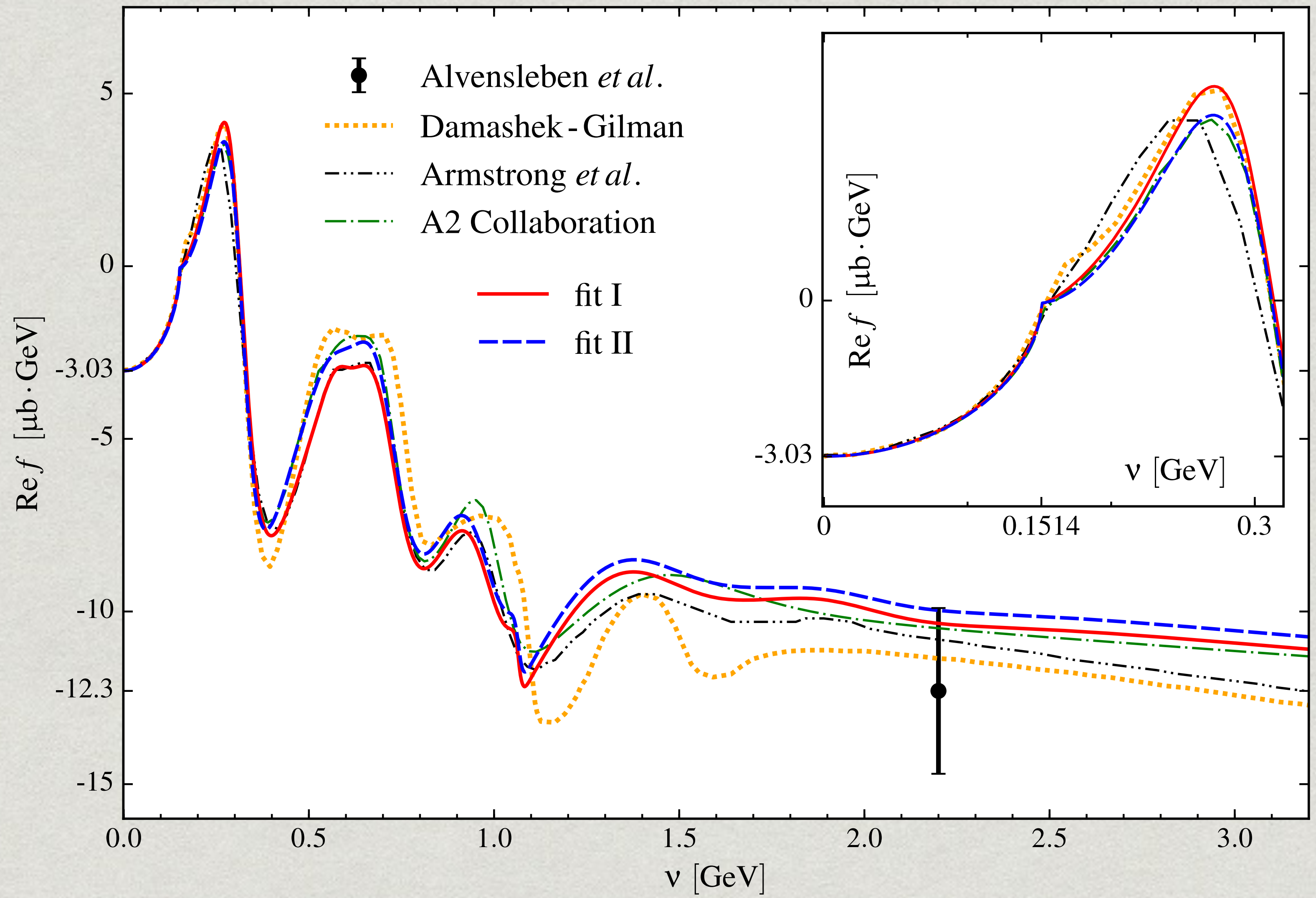

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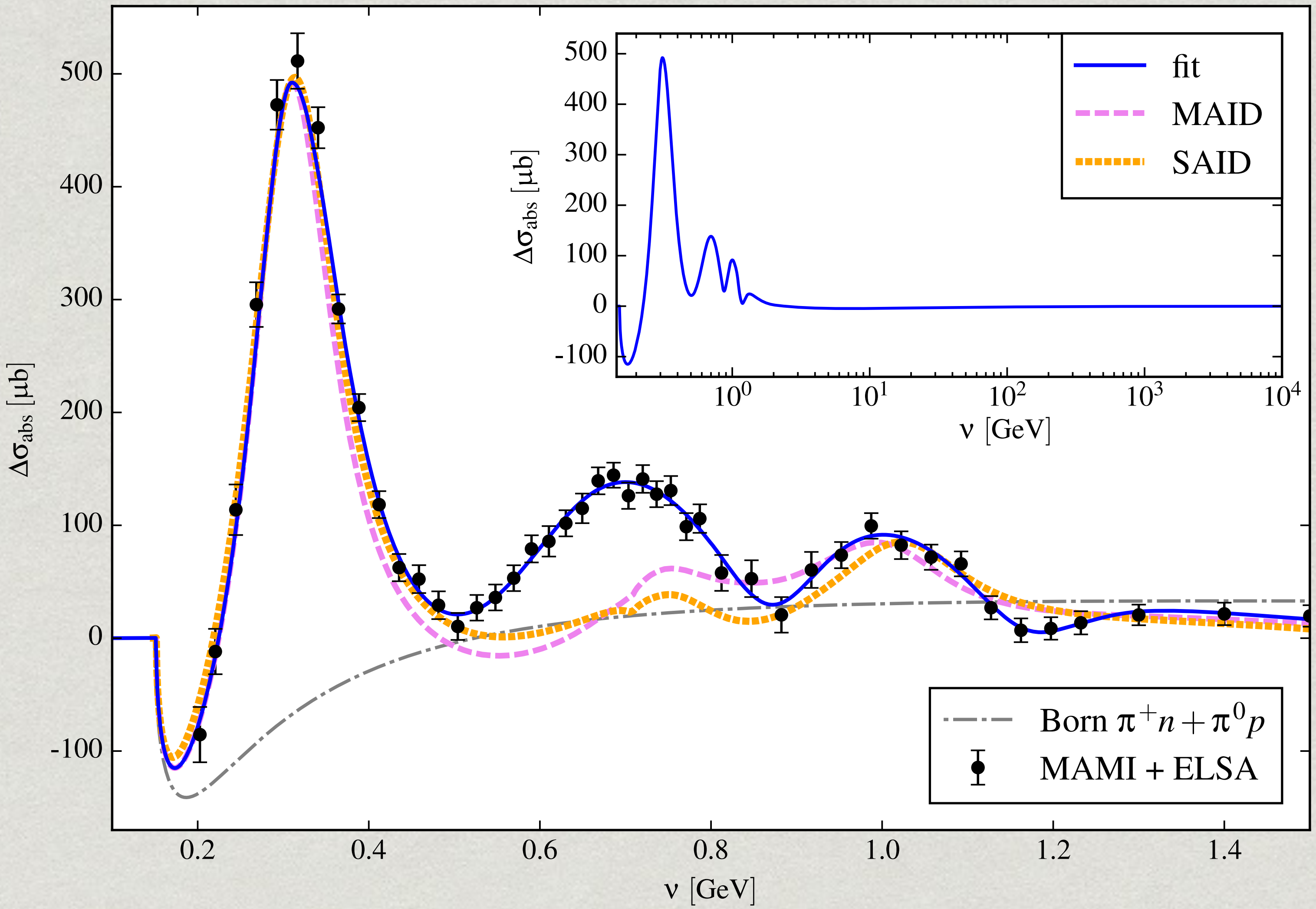
O. Gryniuk et al., Phys. Rev. D **92** (2015) 074031; ibid. **94** (2016) 034043

Spin-dependent CS amplitude



γ_0 = -92.9(10.5) × 10^{-6} fm^4

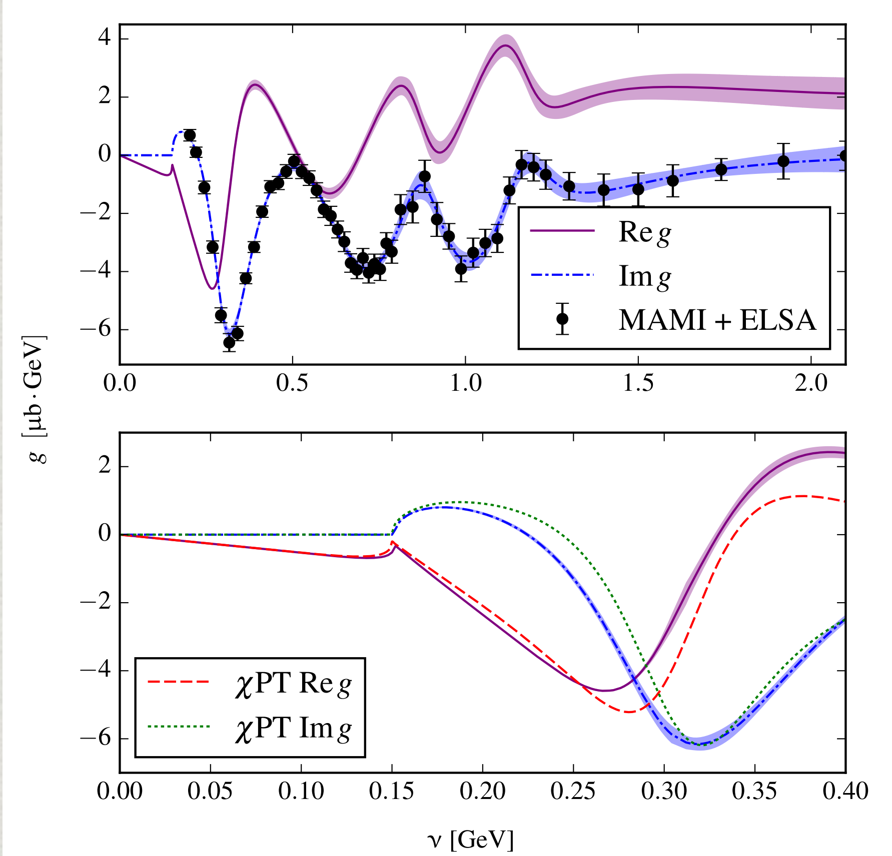
$\bar{\gamma}_0$ = 48.4(8.2) × 10^{-6} fm^6

BChPT:

γ_0 = -90(140) × 10^{-6} fm^4

$\bar{\gamma}_0$ = 110(50) × 10^{-6} fm^4

V. Lensky, et al., Eur. Phys. J. C 75 (2015) 604



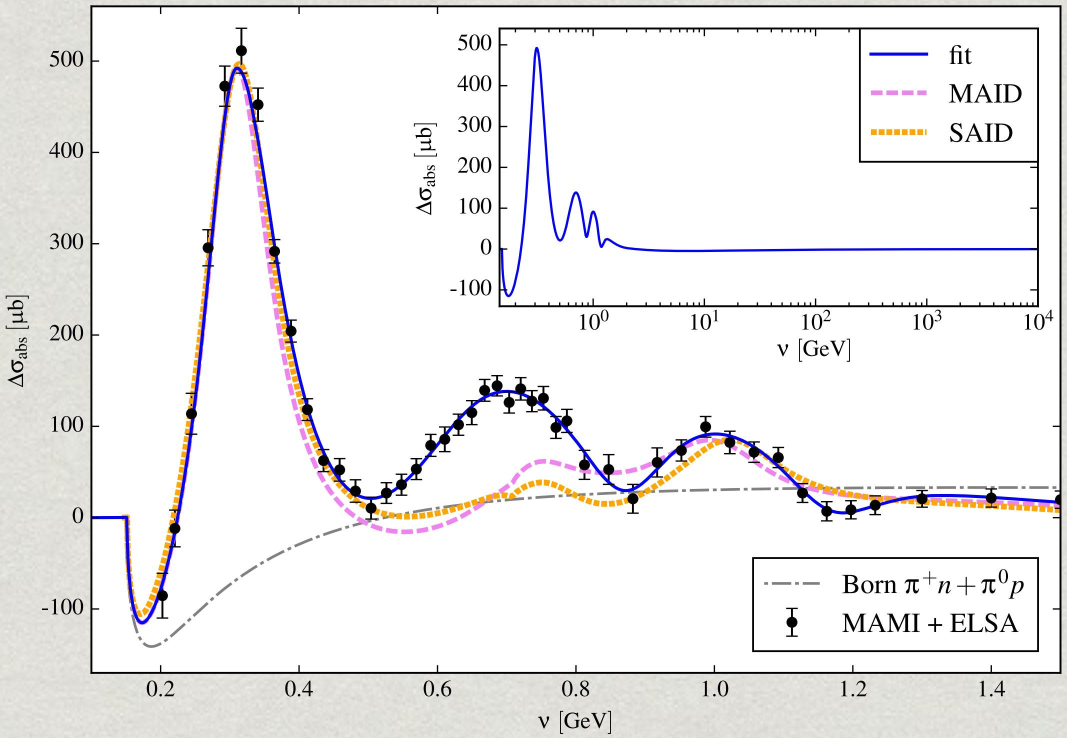
Forward spin polarizability sum rules:

$$\gamma_0 = -\frac{1}{4\pi^2}\int_{\nu_0}^{\infty} d\nu\, \frac{\Delta\sigma_{abs}(\nu)}{\nu^3}$$

$$\bar{\gamma}_0 = -\frac{1}{4\pi^2}\int_{\nu_0}^{\infty} d\nu\, \frac{\Delta\sigma_{abs}(\nu)}{\nu^5}$$

O. Gryniuk et al., Phys. Rev. D **92** (2015) 074031; ibid. **94** (2016) 034043

Spin-dependent CS amplitude



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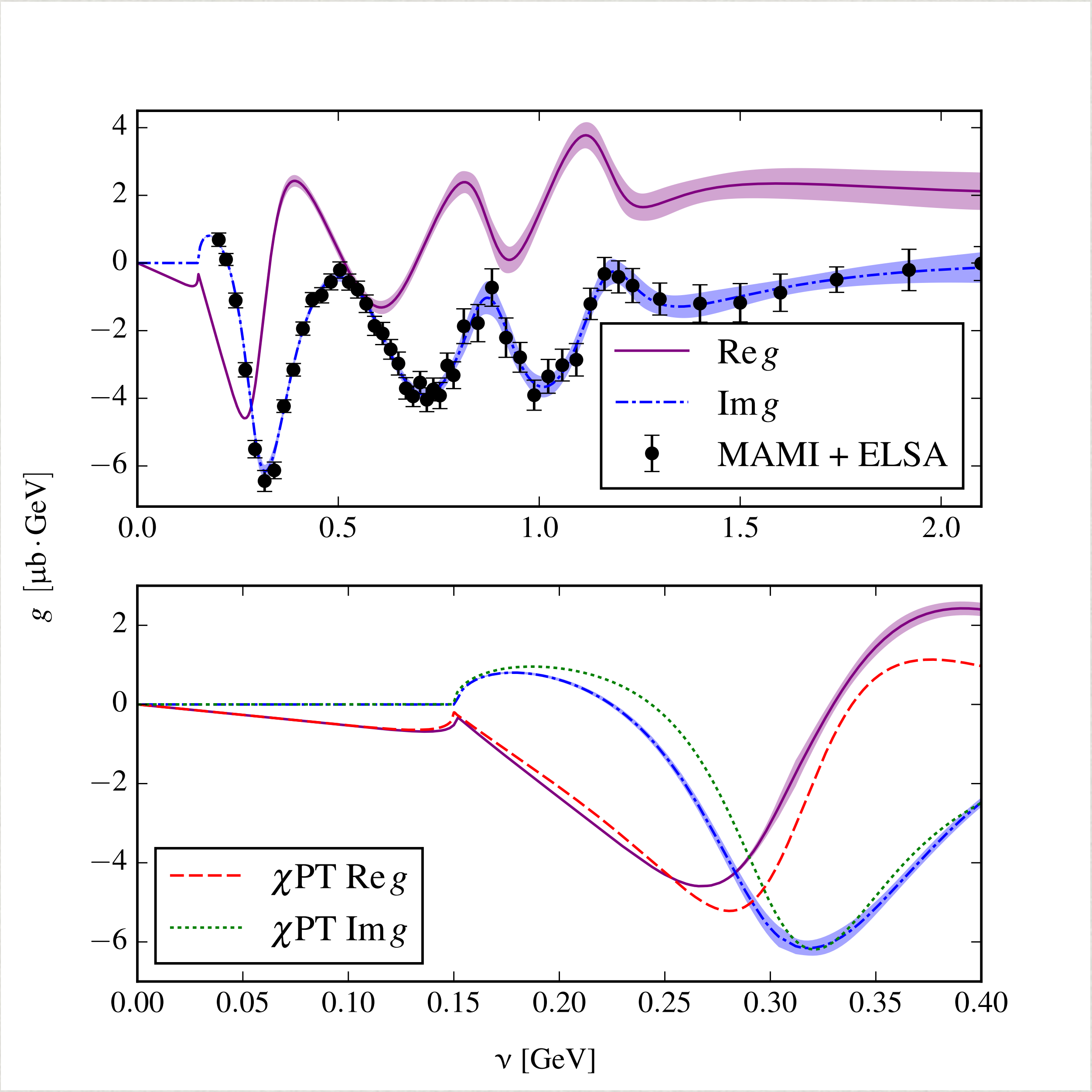

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$$\bar{\gamma}_0 = -\frac{1}{4\pi^2}\int_{\nu_0}^{\infty} d\nu \, \frac{\Delta\sigma_{abs}(\nu)}{\nu^5}$$

Dispersive Approach

[1] K. Pachucki, Phys. Rev. A **60** (1999) 3593–3598
[2] A. Martynenko, Phys. Atom. Nucl. **69** (2006) 1309–1316
[3] C. E. Carlson, M. Vanderhaeghen, hep-ph/1101.5965 (2011)
[4] M. C. Birse, J. A. McGovern, Eur. Phys. J. A **48** (2012) 120
[5] M. Gorchtein, et al., Phys. Rev. A **87** (2013) 052501

wave function at the origin

$$\Delta E(nS) = 8\pi\alpha m\,\phi_n^2\,\frac{1}{i}\int_{-\infty}^{\infty}\frac{\mathrm{d}\nu}{2\pi}\int\frac{\mathrm{d}\mathbf{q}}{(2\pi)^3}\,\frac{(Q^2-2\nu^2)\,T_1(\nu,Q^2)-(Q^2+\nu^2)\,T_2(\nu,Q^2)}{Q^4(Q^4-4m^2\nu^2)}$$

$$T_1(\nu,Q^2) = \boxed{T_1(0,Q^2)} + \frac{32\pi Z^2\alpha M\nu^2}{Q^4}\int_0^1 \mathrm{d}x\,\frac{x f_1(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}$$

$$T_2(\nu,Q^2) = \frac{16\pi Z^2\alpha M}{Q^2}\int_0^1 \mathrm{d}x\,\frac{f_2(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}$$

Caution:
in the dispersive approach the subtraction function is modelled!

low-energy expansion:

$$\lim_{Q^2\to 0}\overline{T}_1(0,Q^2)/Q^2 = 4\pi\beta_{M1}$$

modelled Q^2 behavior: [1,2]

$$\overline{T}_1(0,Q^2) = 4\pi\beta_{M1}\,Q^2/(1+Q^2/\Lambda^2)^4$$

	Pachucki	Martynenko	Carlson & Vanderhaeghen	Birse & McGovern	Gorchtein *et al.*[a]
β_{M1}	1.56(57)	1.9(5)	3.4(1.2)	3.1(5)	
$\Delta E_{2S}^{(\mathrm{subt})}$	1.9	2.3	5.3(1.9)	4.2(1.0)	−2.3(4.6)
$\Delta E_{2S}^{(\mathrm{inel})}$	−13.9	−16.1	−12.7(5)	−12.7(5)[b]	−13.0(6)
$\Delta E_{2S}^{(\mathrm{pol})}$	−12(2)	−13.8(2.9)	−7.4(2.0)	−8.5(1.1)	−15.3(4.6)
$\Delta E_{2S}^{(\mathrm{el})}$	−23.2(1.0)		{−27.8; **−29.5(1.3)**; −30.8}	−24.7(1.6)[c]	−24.5(1.2)
ΔE_{2S}	−35.2(2.2)		−36.9(2.4)	−33(2)	−39.8(4.8)

2γ in μH Lamb Shift:
ChPT vs. Dispersive Approach

ChPT talks by V. Lensky and A. Pineda in the next session!

	Nevado & Pineda HBχPT	Alarcón *et al.* BχPT	Alarcón *et al.* HBχPT	Peset & Pineda HBχPT[a]
$\Delta E_{2S}^{(\mathrm{subt})}$		-3.0	1.3	
$\Delta E_{2S}^{(\mathrm{inel})}$		-5.2	-19.1	
$\Delta E_{2S}^{(\mathrm{pol})}$	$-18.5(9.3)$	$-8.2(^{+1.2}_{-2.5})$	-17.85	$-26.2(10.0)$
$\Delta E_{2S}^{(\mathrm{el})}$	$-10.1(5.1)$			$-8.3(4.3)$
ΔE_{2S}	-28.6			$-34.4(12.5)$

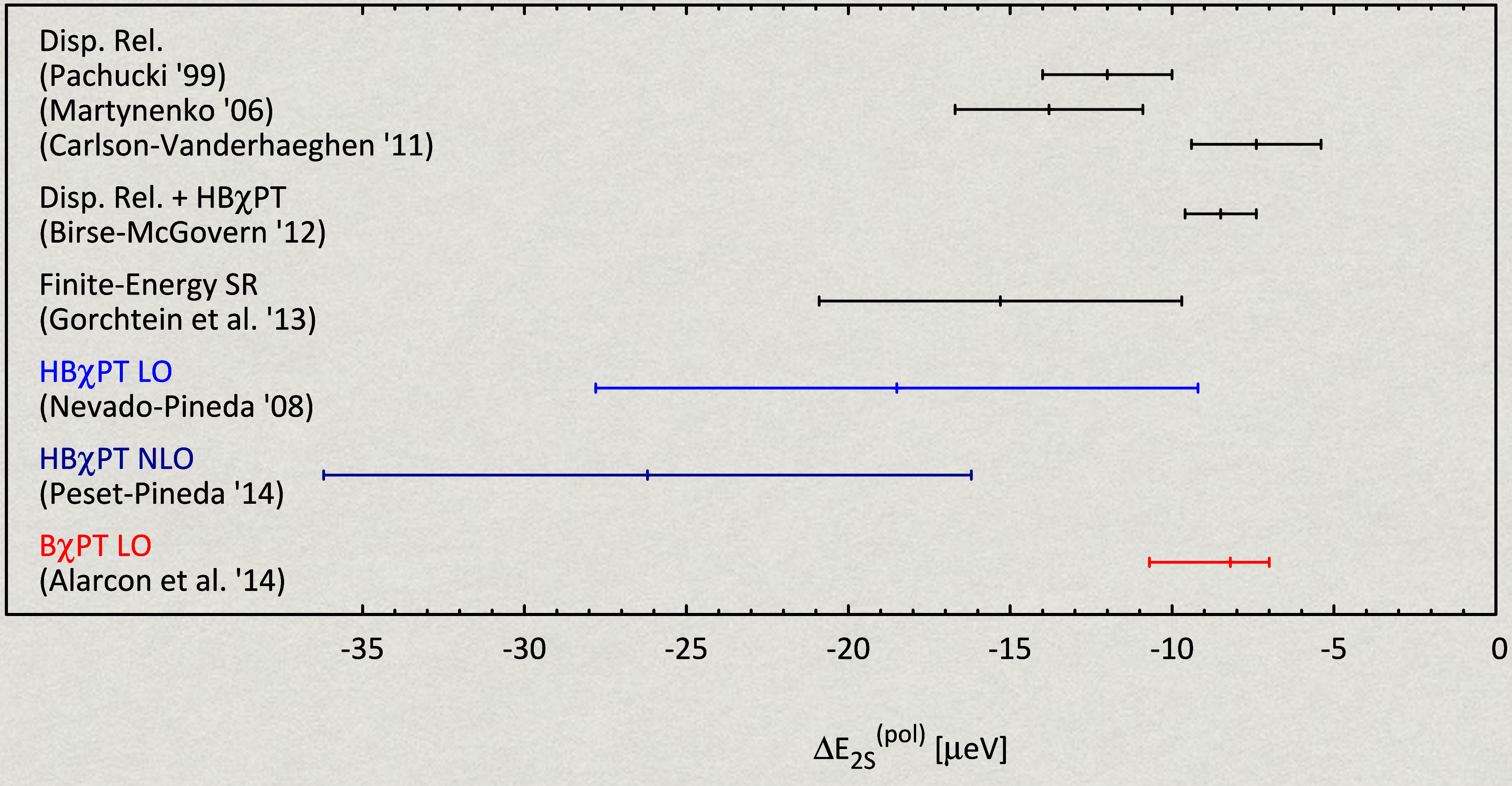
[a]prediction at LO and NLO (including pions and deltas)

D. Nevado, A. Pineda, Phys. Rev. C **77** (2008) 035202

A. Pineda, Phys. Rev. C **71** (2005) 065205

C. Peset, A. Pineda (2014)

J. M. Alarcon, V. Lensky, V. Pascalutsa, Eur. Phys. J. C **74** (2014) 2852



BChPT result is in good agreement with calculations based on dispersive sum rules!

HFS in Muonic Hydrogen

$$\Delta E_{\mathrm{HFS}}(nS) = [1 + \Delta_{\mathrm{QED}} + \Delta_{\mathrm{weak}} + \Delta_{\mathrm{FSE}}]\, E_F(nS)$$

Fermi - Energy:

$$E_F(nS) = \frac{8}{3}\frac{Z\alpha}{a^3}\frac{1+\kappa}{mM}\frac{1}{n^3}$$

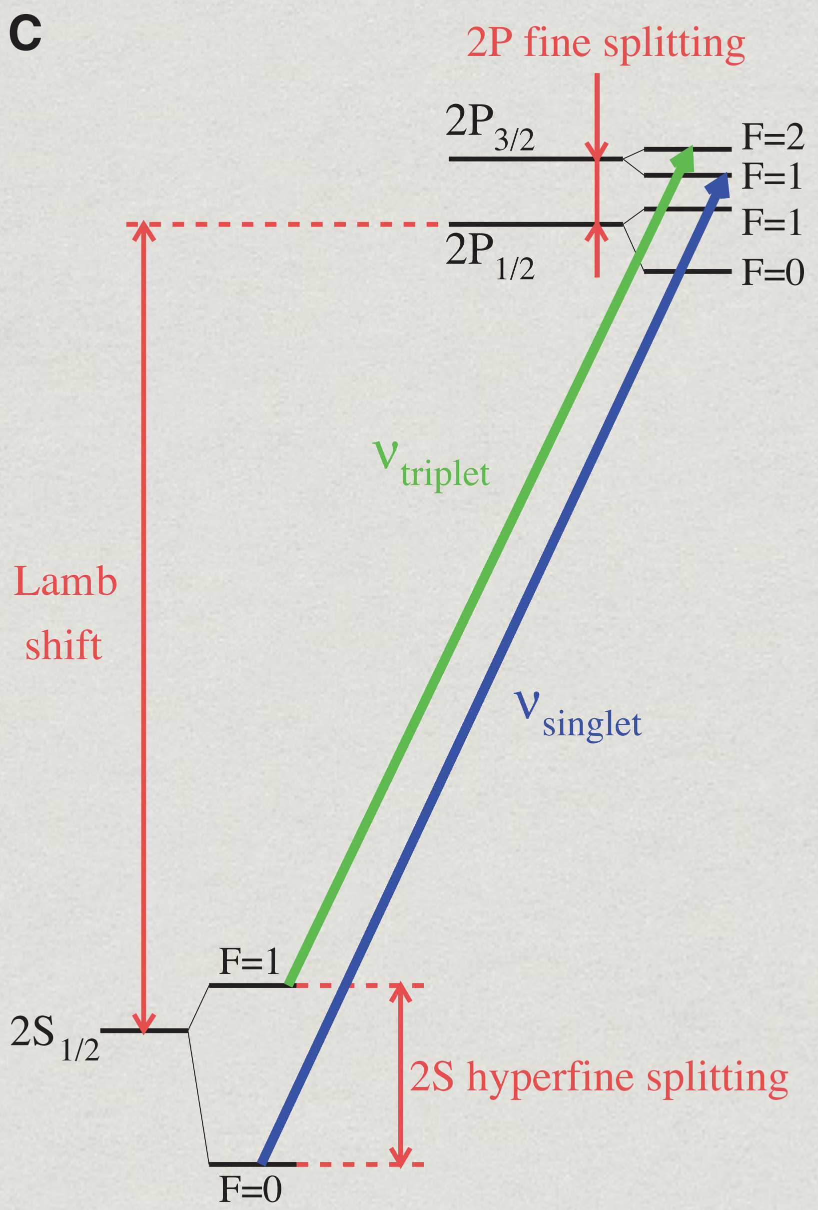
with $\Delta_{\mathrm{FSE}} = \Delta_Z + \Delta_{\mathrm{recoil}} + \Delta_{\mathrm{pol}}$

Zemach radius:

$$\Delta_Z = \frac{8Z\alpha m_r}{\pi}\int_0^\infty \frac{\mathrm{d}Q}{Q^2}\left[\frac{G_E(Q^2)G_M(Q^2)}{1+\kappa} - 1\right] \equiv -2Z\alpha m_r R_Z$$

experimental value: $R_Z = 1.082(37)\,\mathrm{fm}$

A. Antognini, et al., Science **339** (2013) 417–420



A. Antognini, et al., Annals Phys. **331** (2013) 127–145

2γ in μH HFS

$$\frac{E_{\mathrm{HFS}}(nS)}{E_F(nS)} = \frac{4m}{Z(1+\kappa)}\frac{1}{i}\int_{-\infty}^{\infty}\frac{\mathrm{d}\nu}{2\pi}\int\frac{\mathrm{d}\mathbf{q}}{(2\pi)^3}\frac{1}{Q^4-4m^2\nu^2}\left\{\frac{(2Q^2-\nu^2)}{Q^2}S_1(\nu,Q^2)+\frac{3\nu}{M}S_2(\nu,Q^2)\right\}$$

- *polarizability contribution* is given by the *non-Born* part of the *spin-dependent* amplitudes

$$S_1(\nu,Q^2) = S_1^{\mathrm{Born}}(\nu,Q^2) + \underbrace{\frac{2\pi Z^2\alpha}{M}F_2^2(Q^2) + \frac{16\pi Z^2\alpha M}{Q^2}\int_0^{x_0}\mathrm{d}x\,\frac{g_1(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}}_{\Delta_1}$$

$$\nu S_2(\nu,Q^2) = \nu S_2^{\mathrm{Born}}(\nu,Q^2) + \underbrace{\frac{64\pi Z^2\alpha M^4\nu^2}{Q^6}\int_0^{x_0}\mathrm{d}x\,\frac{x^2 g_2(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}}_{\Delta_2}$$

Δ_1 and Δ_2 → Δ_{Pol}

using dispersion relation & optical theorem

2γ in μH HFS

$$I_1(Q^2) = \frac{2M^2}{Q^2}\int_0^{x_0} \mathrm{d}x\, g_1(x,Q^2)$$

$$I_1^{\mathrm{non-pol}}(Q^2) = I_A^{\mathrm{non-pol}}(Q^2) = -\frac{1}{4}F_2^2(Q^2)$$

with $v = \sqrt{1+1/\tau}$

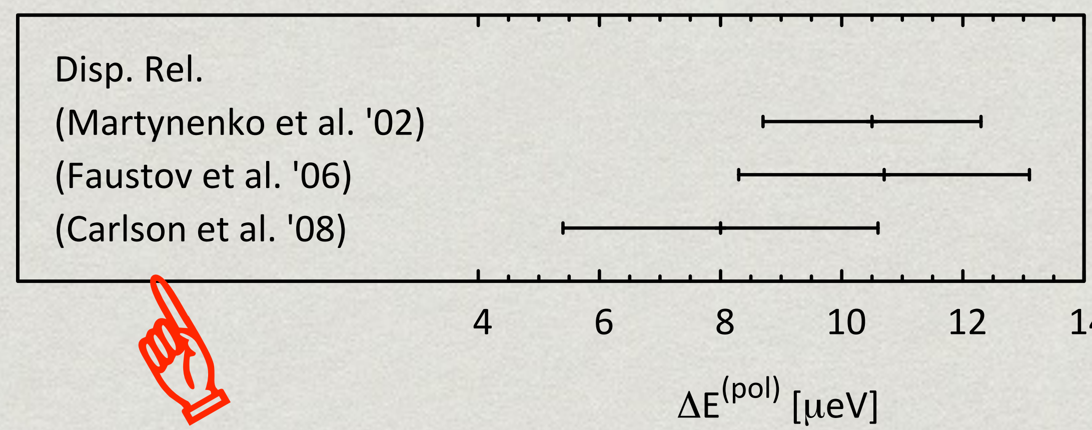
$$\Delta_{\mathrm{pol}} = \frac{Z\alpha m}{2\pi(1+\kappa)M}\left[\delta_1+\delta_2\right] = \Delta_1 + \Delta_2$$

$$\delta_1 = 2\int_0^\infty \frac{\mathrm{d}Q}{Q}\left(\frac{5+4v_l}{(v_l+1)^2}\left[4I_1(Q^2)+F_2^2(Q^2)\right] - \frac{32M^4}{Q^4}\int_0^{x_0}\mathrm{d}x\, x^2 g_1(x,Q^2)\right.$$
$$\left.\times\left\{\frac{1}{(v_l+\sqrt{1+x^2\tau^{-1}})(1+\sqrt{1+x^2\tau^{-1}})(1+v_l)}\left[4+\frac{1}{1+\sqrt{1+x^2\tau^{-1}}}+\frac{1}{v_l+1}\right]\right\}\right)$$

$$\delta_2 = 96M^2\int_0^\infty \frac{\mathrm{d}Q}{Q^3}\int_0^{x_0}\mathrm{d}x\, g_2(x,Q^2)\left\{\frac{1}{v_l+\sqrt{1+x^2\tau^{-1}}} - \frac{1}{v_l+1}\right\}$$

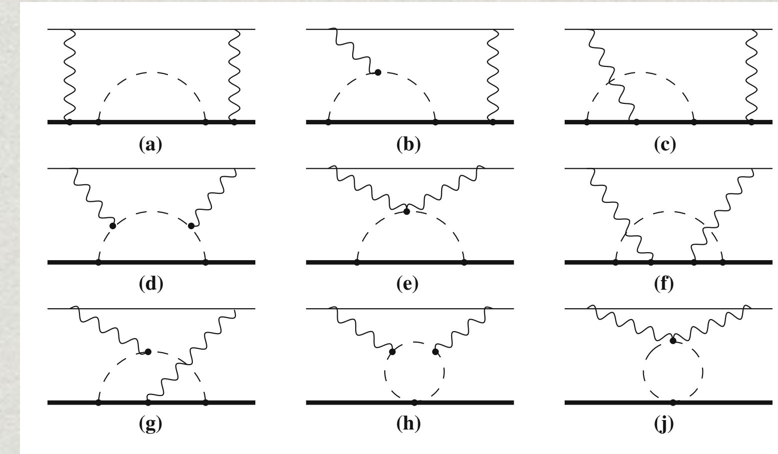
- 2γ effect on the HFS is completely *constrained by empirical information*
- a ChPT calculation of the HFS in μH will put the reliability of both ChPT and dispersive calculations to the test
  - leading chiral logarithms motivate the relative order of the Zemach and polarizability corrections
    A. Pineda, Phys. Rev. C **67** (2003) 025201

next talk



[1] C. E. Carlson, et al., Phys. Rev. A **83** (2011) 042509
[2] C. E. Carlson, et al., Phys. Rev. A **78** (2008) 022517
[3] R. Faustov, et al., Proc. SPIE Int. Soc. Opt. Eng. **6165** (2006) 0M
[4] A. Martynenko et al., Nucl. Phys. A **703** (2002) 365–377
[5] A. Martynenko, Phys. Rev. A **71** (2005) 022506

Chiral Dynamics (LO)



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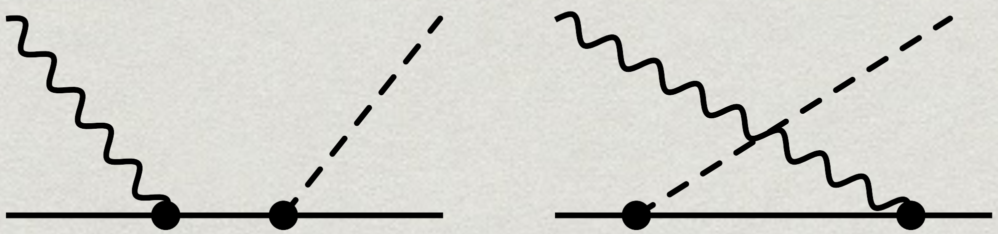
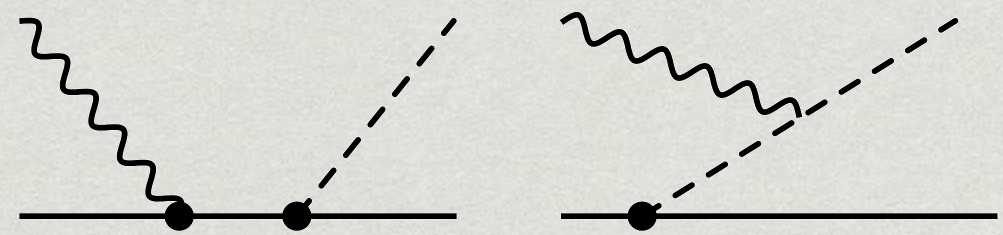
Lamb shift

<u>pion-production cross section:</u>

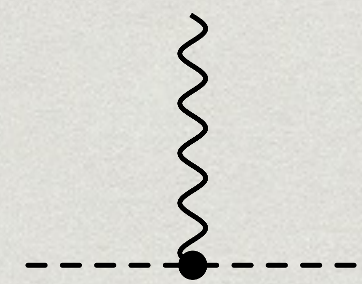


$\gamma^* + p \rightarrow \pi^0 + p$



$\gamma^* + p \rightarrow \pi^+ + n$

- improved cutoff behavior after adding the pion form factor $F_{\pi\pi\gamma}(Q^2)$



$$F_{\pi\pi\gamma} = \left(1 + \frac{Q^2}{\Lambda_\pi^2}\right)^{-1}$$

with $\Lambda_\pi^2 = 0.462\,\mathrm{GeV}^2$

$$E_{\mathrm{HFS}}^{(\pi N \text{ loops})}(2S) = -0.23^{+1.08}_{-0.23}\,\mu\mathrm{eV}$$

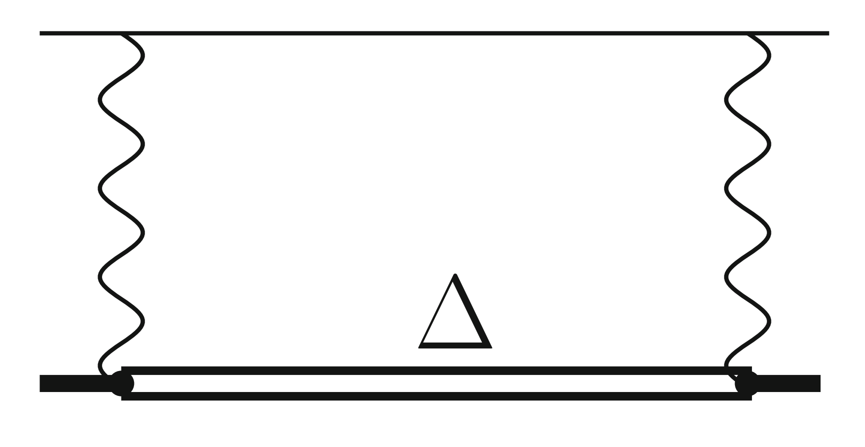
$$\Delta_1 = -18\,\mathrm{ppm}$$

$$\Delta_2 = 8\,\mathrm{ppm}$$

2γ with Δ-Excitation (NLO)

- the Δ-contribution to the Lamb shift is small compared to the leading order πN-loops

$$E_{\mathrm{LS}}^{(\Delta)} = 0.65 \pm 0.49\,\mu\mathrm{eV}$$



  - expected since β_{M1} *is suppressed*

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- multipole ratios are small, the result is dominated by $(G^*_M)^2$

$$E_{\mathrm{HFS}}^{(\Delta)}(2S) = -0.86 \pm 0.65\,\mu\mathrm{eV}$$

$$\Delta_1 = 34\,\mathrm{ppm}$$
$$\Delta_2 = -71\,\mathrm{ppm}$$

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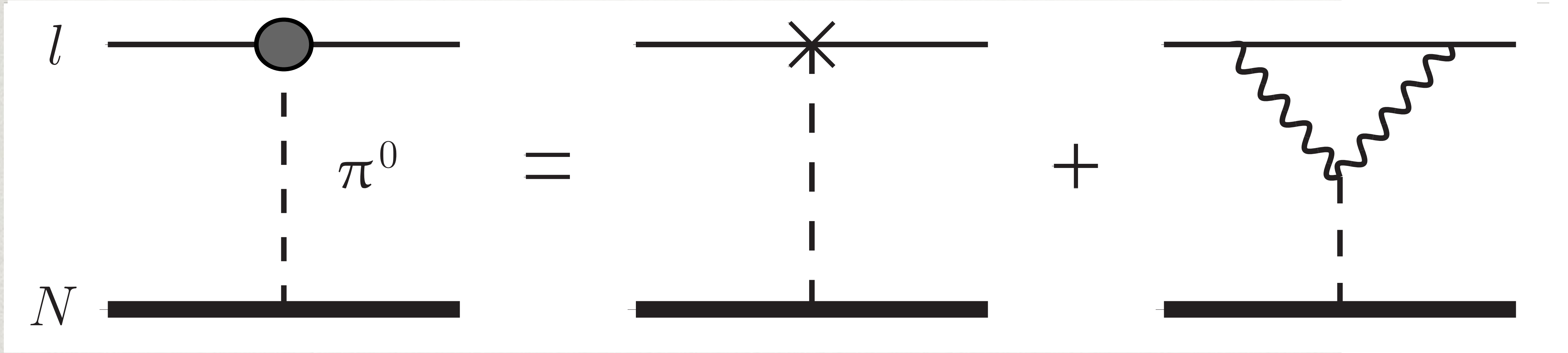
Neutral-Pion Exchange (NLO)



- $\mathcal{O}(\alpha^6)$ contribution from *off-forward* scattering
- result for muonic hydrogen:

$$E^{(\pi^0)}_{\text{HFS}}(2S) = 0.02 \pm 0.04\,\mu eV$$

$$E^{(\pi^0)}_{2S\,\text{HFS}} = -(0.09 \pm 0.06)\,\mu\text{eV}$$

N. T. Huong, E. Kou, B. Moussallam, Phys. Rev. D **93** (2016) 114005

Prog. Part. Nucl. Phys. **88** (2016) 29-97

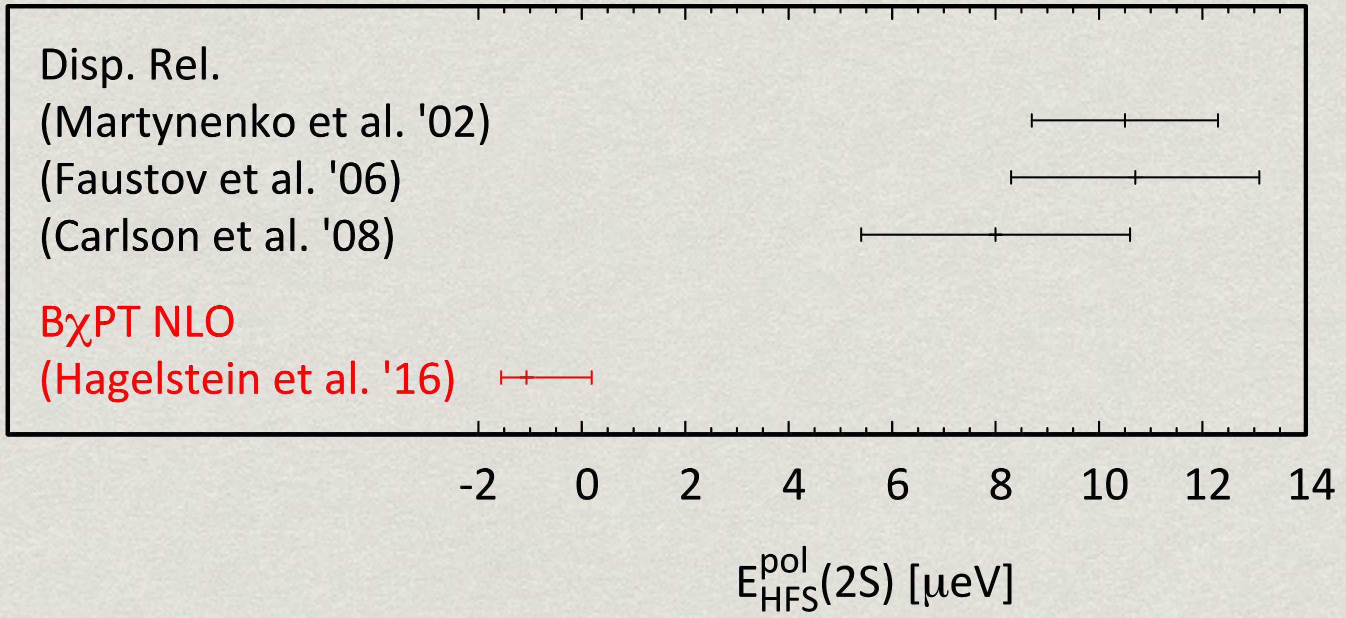
Summary: 2ɣ in HFS

$$E_{\mathrm{HFS}}^{(\pi N\ \mathrm{loops})}(2S) = -0.23^{+1.08}_{-0.23}\,\mu\mathrm{eV}$$

$$E_{\mathrm{HFS}}^{(\Delta)}(2S) = -0.86 \pm 0.65\,\mu\mathrm{eV}$$

$$E_{\mathrm{HFS}}^{(\pi^0)}(2S) = 0.02 \pm 0.04\,\mu eV$$

$$E_{\mathrm{HFS}}^{(\mathrm{pol})}(2S) = -1.07^{+1.26}_{-0.69}\,\mu\mathrm{eV}$$



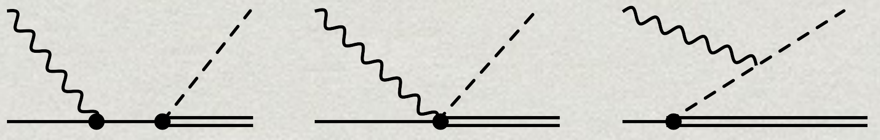
predictions of the polarizability contribution to the HFS based on BChPT disagree with the dispersive results

→ changes the Zemach radius into $R_Z = 1.025$ fm (smaller)

compared to $R_Z = 1.082$ fm (μH) and $R_Z = 1.022$ fm [1] or $R_Z = 1.065 - 1.108$ fm [2] (FF)

[1] R. Faustov, E. Cherednikova, A. Martynenko, Nuclear Phys. A 703 (2002) 365–377.
[2] C. E. Carlson, et al., Phys. Rev. A **78** (2008) 022517

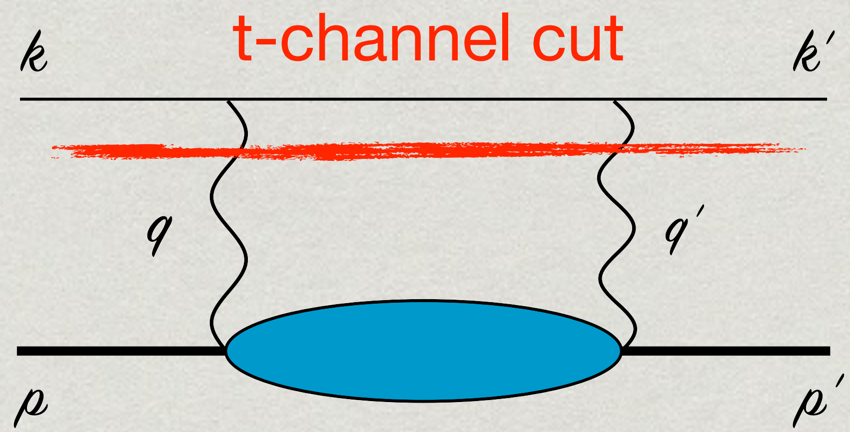
- empirical information on spin structure functions is limited (especially for g_2)
- problem in BChPT? next step: include πΔ-loops



- the low-Q region is very important

Nuclear Polarizability Effect at $(Z\alpha)^6 \ln(Z\alpha)$

- *off-forward 2γ*:



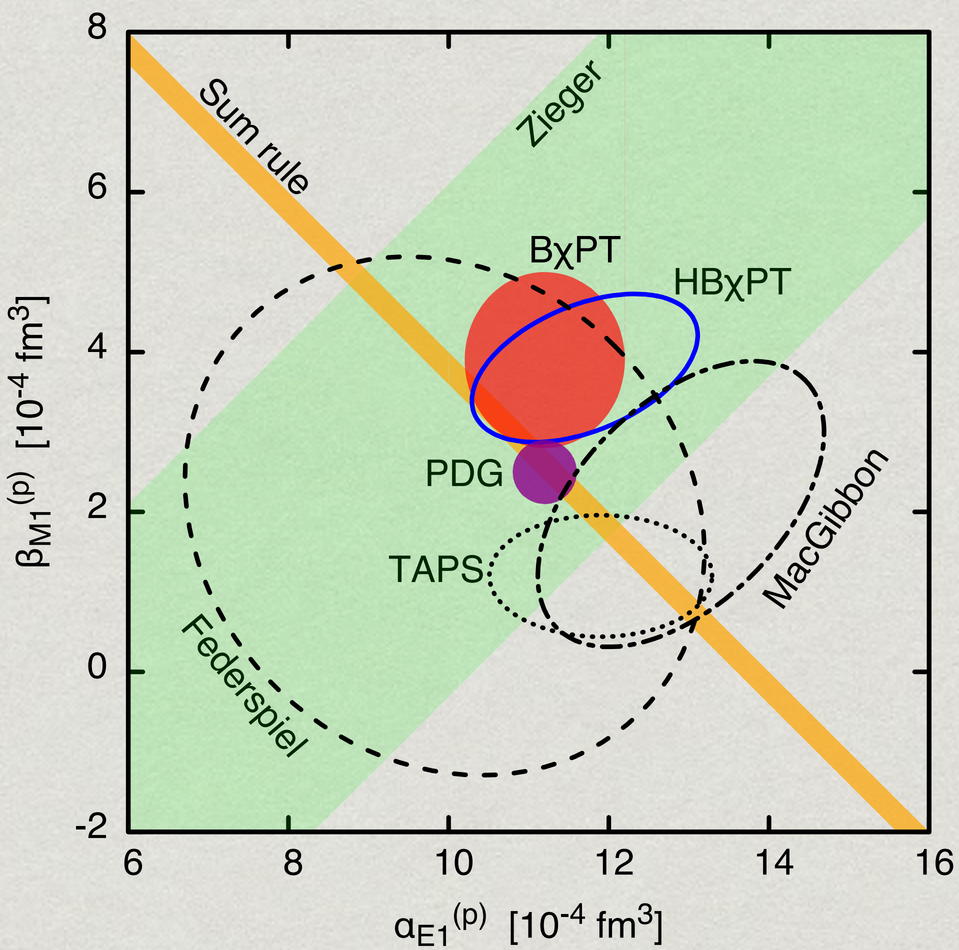
$$\mathrm{Im}\,\mathscr{M}(p_t^2) \approx -\frac{2\pi\alpha m}{(1-\tau)^{7/2}}\sqrt{\tau}\,\arccos\sqrt{\tau}\;\alpha_{E1} + \mathcal{O}(\tau)$$

- $(Z\boldsymbol{\alpha})^6 \ln(Z\boldsymbol{\alpha})$ effect in the Lamb shift is expressed entirely in terms of the static *electric dipole polarizability*

$$E_{nS} = -\frac{4(Z\alpha m_r)^4 \alpha\,\alpha_{E1}}{n^3}\ln\frac{Z\alpha m_r}{2nm}$$

  - no contribution from the *magnetic dipole polarizability* or the lowest order *spin polarizabilities*, i.e., not present in the HFS

$(Z\alpha)^6 \ln(Z\alpha)$ Polarizability Effect in μH



PDG 2014:

$$\alpha_{E1} = 11.2 \pm 0.4 \times 10^{-4}\,\mathrm{fm}^3$$

- off-forward 2γ:

$$E_{\mathrm{LS}}^{(\alpha^6 \ln \alpha)}(\mu\mathrm{H}) = -0.79 \pm 0.03\,\mu\mathrm{eV}$$

- forward 2γ:

$$E_{\mathrm{LS}}^{(\alpha^5)} = -8.2^{+1.2}_{-2.5}\,\mu\mathrm{eV}$$

talk by V. Lensky

J. M. Alarcon, V. Lensky, V. Pascalutsa, Eur. Phys. J. C **74** (2014) 2852

- total Lamb shift:

$$E_{\mathrm{LS}}^{\mathrm{th}}(\mu\mathrm{H}) = 206.0668(25) - 5.2275(10)\,(R_p/\mathrm{fm})^2$$

A. Antognini et al., Annals Phys. **331** (2013) 127-145

$(Z\alpha)^6 \ln(Z\alpha)$ Polarizability Effect in muonic atoms: μ^3H, μ^3He^+ and μ^4He^+

TABLE II: Summary of our numerical results for the polarizability contribution to the Lamb shift of light muonic atoms.

	μ^3H	μ^3He$^+$	μ^4He$^+$
M [GeV] α_{E1} [fm^3]	2.808 921 112(17) [1] 0.139(2) [2]	2.808 391 586(17) [1] 0.149(5) [2]	3.727 379 378(23) [1] 0.0683(8)(14) [2]
presently accounted for nucl. pol. effect [meV]	$\delta^{A}_{\text{pol}} = -0.476(10)(13)$ [3]	$\delta^{A}_{\text{pol}} = -4.16(06)(16)$ [3]	$\delta^{A}_{\text{pol}} = -2.47(15)$ [4]
this work: $E_{LS}^{(\alpha^6 \ln \alpha)}$ [meV]	$-0.128(2)$	$-1.950(65)$	$-0.925(22)$

- total Lamb shift in μ^4He^+ [in meV]: [5]

$$E_{\text{LS}}(\mu^4\text{He}^+) = 1572.186(205) - 106.358(7)(R_\alpha/\text{fm})^2$$

[1] P. J. Mohr, et al., Rev. Mod. Phys. **84** (2012) 1527; P. J. Mohr, et al., arXiv:1507.07956 (2015).

[2] I. Stetcu, et al., Phys. Rev. C **79** (2009) 064001.

[3] N. Nevo Dinur, et al., Phys. Lett. B **755** (2016) 380.

[4] C. Ji, et al., Phys. Rev. Lett. **111** (2013) 143402.

[5] M. Diepold, et al., arXiv:1606.05231 [physics.atom-ph].

Summary & Conclusions

- the finite-size effects are up to "soft" effects, $Q \sim \alpha m_r$, expandable in the moments of charge distribution

- (forward 2ɣ) polarizability contribution to the HFS:

$$E^{(\mathrm{pol})}_{\mathrm{HFS}}(2S) = -1.07^{+1.26}_{-0.69}\,\mu\mathrm{eV}$$

  - predictions of the polarizability contribution to the HFS based on BChPT disagree with the dispersive results
    → changes the Zemach radius into $R_Z = 1.025$ fm (smaller)

- $(Z\alpha)^6 \ln(Z\alpha)$ nuclear polarizability effect in the Lamb shift of muonic atoms is non-negligible

$$E_{nS} = -\frac{4(Z\alpha m_r)^4 \alpha\, \alpha_{E1}}{n^3} \ln \frac{Z\alpha m_r}{2nm}$$

Thank you for your attention !!!