Higher orders in ε'/ε

Sebastian Jaeger
University of Sussex

HC2NP
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Based on work in progress with
M Cerda Sevilla, M Gorbahn, A Kokulu

Outline

ε' as a precision observable

NNLO QCD penguin contribution

Full factorisation of scales

Dynamical charm

Summary

Direct CP violation in $K_L \to \pi\pi$

Precisely known experimentally for a decade

$$(\varepsilon'/\varepsilon)_{\rm exp} = (16.6 \pm 2.3) \times 10^{-4}$$

average of NA48 (CERN) and KTeV

$$\left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 \simeq 1 - 6\,\mathrm{Re}(\frac{\varepsilon'}{\varepsilon})$$

← **defines** Re(ε'/ε) experimentally
left-hand side is measured

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}, \qquad \eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}$$

(magnitudes directly measurable from decay rates)

Even more precise measurement possible in principle at NA62/CERN

Isospin limit

It is useful to formulate the problem in terms of isospin (as opposed to charge) final states.

Defining $\langle(\pi\pi)_I|\mathcal{H}_{\rm eff}|K\rangle \equiv A_I e^{i\delta_I}$ $\qquad I=0,2$

one has

$$\frac{\varepsilon'}{\varepsilon} = -\frac{\omega_+}{\sqrt{2}\,|\varepsilon_K|}\left[\frac{{\rm Im}A_0}{{\rm Re}A_0}(1-\hat{\Omega}_{\rm eff}) - \frac{1}{a}\frac{{\rm Im}A_2}{{\rm Re}A_2}\right]$$

Cirigliano, Pich, Ecker, Neufeld, Pich 2003
Buras, Gorbahn, SJ, Jamin arXiv:1507.06345

A small imaginary part on the l.h.s. has been neglected.

In the isospin limit, A_2 is pure electroweak penguin.

Moreover, the strong (rescattering) phases for a given isospin all coincide with the pi pi scattering phase shift (Watson's theorem).

Broken by QED and $m_u \neq m_d$: parameters $\Omega_{\rm eff}, a, \omega_+$

ε' master formula

Buras, Buchalla, Lautenbacher 1990; Buras, Jamin 1993;1996; Bosch et al 1999;
Buras, Gorbahn, SJ, Jamin arXiv:1507.06345

$$\omega_+ = a\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} = (4.53 \pm 0.02) \times 10^{-2}$$

from experiment
Cirigliano et al 2003

leading isospin breaking
Cirigliano et al 2003

$$\frac{\varepsilon'}{\varepsilon} = -\frac{\omega_+}{\sqrt{2}|\varepsilon_K|}\left[\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}(1-\Omega_{\mathrm{eff}}) - \frac{1}{a}\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right]$$

neglect small imaginary part (for simplicity; could easily be restored)

from experiment

$$A_I \equiv \langle(\pi\pi)_I|\mathcal{H}_{\mathrm{eff}}|K\rangle$$

QCD isospin amplitudes
factorise into Wilson coefficients (perturbative)
and matrix elements (nonperturbative)

$$A_I = \langle(\pi\pi)_I|H_{\mathrm{eff}}|K\rangle = \sum_{i=1}^{10} C_i\langle(\pi\pi)_I|Q_i|K\rangle$$

known to NLO Buras et al 1992,1993; Ciuchini et al 1993

NEW: partial NNLO Cerda Sevilla, Gorbahn, SJ, Kokulu 2016

(NNLO ADMs: Gorbahn, Haisch; Gorbahn, Brod
NNLO weak scale: Misiak et al; Gambino et al)

NEW: first-ever calculation with controlled errors by RBC-UKQCD (2015)

State of phenomenology (NLO)

$$(\varepsilon'/\varepsilon)_{\rm SM} = (1.9 \pm 4.5) \times 10^{-4}$$

Buras, Gorbahn, SJ, Jamin arXiv:1507.06345

$$(\varepsilon'/\varepsilon)_{\rm exp} = (16.6 \pm 2.3) \times 10^{-4}$$

2.9σ discrepancy

(see also Kitahara, Nierste, Tremper 1607.06727)

parameterise hadronic matrix elements values from RBC-UKQCD 2015

quantity	error on ε'/ε	quantity	error on ε'/ε
$B_6^{(1/2)}$	4.1	$m_d(m_c)$	0.2
NNLO	1.6	q	0.2
$\hat{\Omega}_{\rm eff}$	0.7	$B_8^{(1/2)}$	0.1
p_3	0.6	${\rm Im}\lambda_t$	0.1
$B_8^{(3/2)}$	0.5	p_{72}	0.1
p_5	0.4	p_{70}	0.1
$m_s(m_c)$	0.3	$\alpha_s(M_Z)$	0.1
$m_t(m_t)$	0.3		

all in units of 10^-4

(still) completely dominated by $\langle Q_6\rangle_0 \propto B_6^{1/2}$

next are NNLO and isospin breaking

What to make of the discrepancy

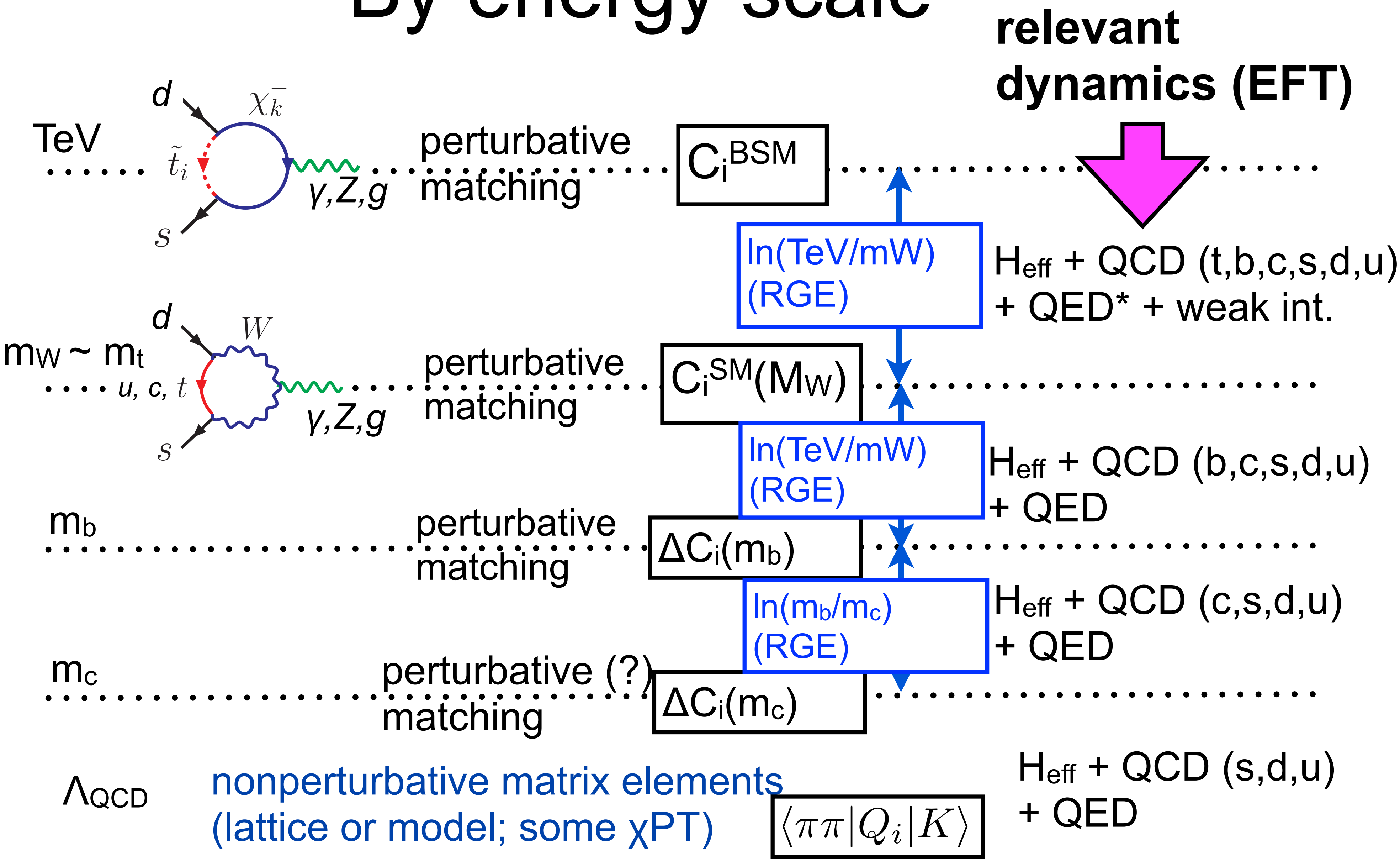
Possible explanations

- new physics
- missing SM electroweak corrections
- missing QED corrections
- missing perturbative QCD corrections
- hadronic matrix elements off

Likelihood of the SM explanations decreases from bottom to top (as per our error budget)

Prospects for improvement on hadronic matrix elements are good. Controlling other sources of uncertainties will become important soon.

By energy scale



Note - all this applies to any CP-violating or rare Kaon process !

* + Higgs force. Dynamics negligible in flavour physics, vacuum value of course fixes quark masses and mixings

Standard Model

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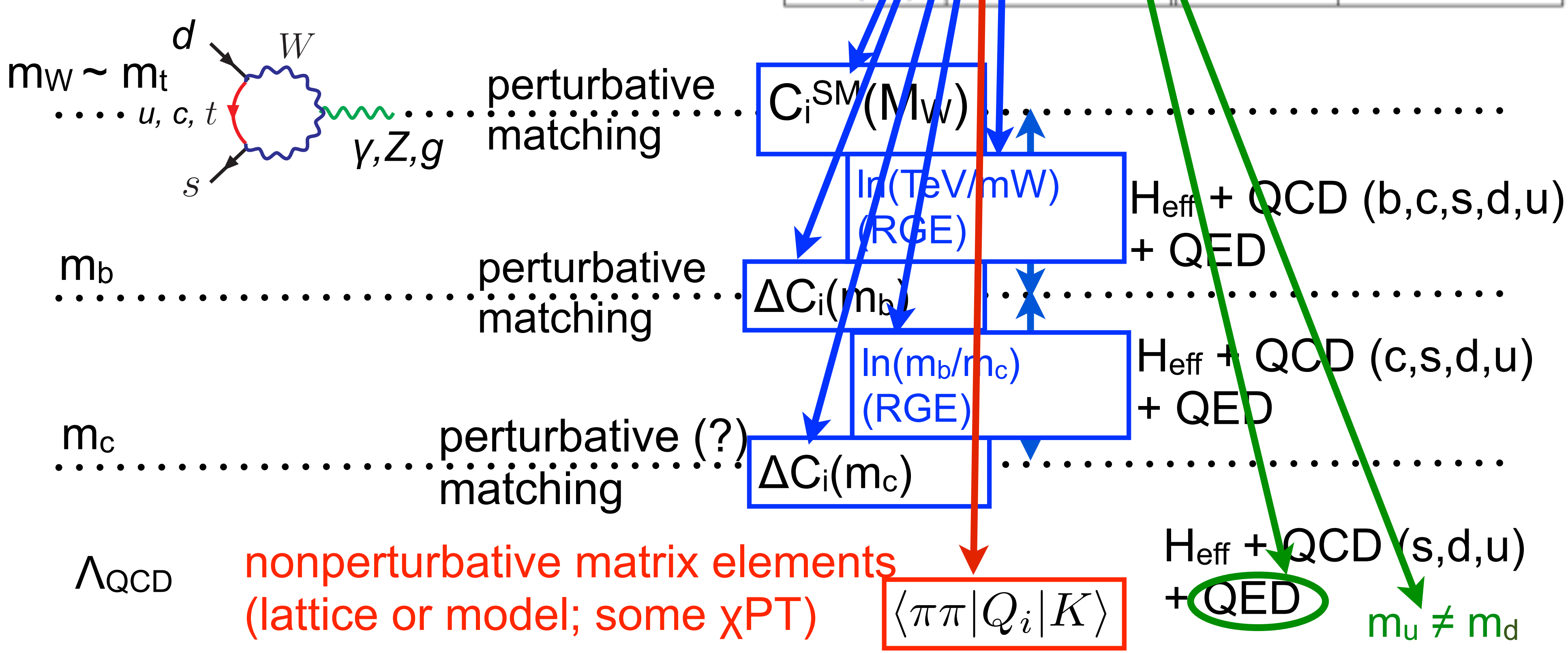

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Operators

Current–Current:

$$Q_1 = (\bar{s}_\alpha u_\beta)_{V-A} \, (\bar{u}_\beta d_\alpha)_{V-A} \qquad Q_2 = (\bar{s}u)_{V-A} \, (\bar{u}d)_{V-A}$$

Large coefficients, but CP-conserving (y=0). Account for K->pi pi decay rates.

QCD–Penguins:

$$Q_3 = (\bar{s}d)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}q)_{V-A} \qquad Q_4 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V-A}$$

$$Q_5 = (\bar{s}d)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}q)_{V+A} \qquad Q_6 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V+A}$$

$\mathcal{O}(\alpha_s)$ **but CP-violating (y=1). However, isospin-0 final state only**

Electroweak Penguins:

$$Q_7 = \frac{3}{2} (\bar{s}d)_{V-A} \sum_{q=u,d,s,c,b} e_q \, (\bar{q}q)_{V+A} \qquad Q_8 = \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s,c,b} e_q \, (\bar{q}_\beta q_\alpha)_{V+A}$$

$$Q_9 = \frac{3}{2} (\bar{s}d)_{V-A} \sum_{q=u,d,s,c,b} e_q \, (\bar{q}q)_{V-A} \qquad Q_{10} = \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s,c,b} e_q \, (\bar{q}_\beta q_\alpha)_{V-A}$$

$\mathcal{O}(\alpha_{\rm em})$ **but can create isospin-2 state**

Minimizing nonperturbative input

Why does a single matrix element dominate the error?

- Re A_0, Re A_2 dominate BR($\pi\pi$) $\Rightarrow$ known from CPC data

- EWP suppressed in I=0 (α/α_s) $\Rightarrow$ $C_{3..6}$ $Q_{3..6}$ dominate ImA_0

- QCDP cannot create I=2 $\Rightarrow$ Im A_2 due to $C_{7..10}$ $Q_{7..10}$

  [broken by QED, $m_u \neq m_d$ in matrix elements, estimated separately through Ω_{eff}]

- Operator identities (only 7 independent ones)

- Colour hierarchies between matrix elements, coefficients

- Better control over I=2 matrix element on lattice

Operator relations

Operator (Fierz) identities and isospin imply for the purely left-handed operators (in the 3-flavour effective theory):

$$\langle Q_9\rangle_2 = \langle Q_{10}\rangle_2 = \frac{3}{2}\langle Q_+\rangle_2 \quad \text{where} \quad Q_\pm = \frac{1}{2}(Q_2 \pm Q_1)$$

Hence (splitting $C_i = z_i - y_i \frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*} \equiv z_i + y_i\,\tau$) one has

$$\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right) = \mathrm{Im}\tau\,\frac{y_7\langle Q_7\rangle_2 + y_8\langle Q_8\rangle_2 + y_9\langle Q_9\rangle_2 + y_{10}\langle Q_{10}\rangle_2}{z_+\langle Q_+\rangle_2}$$

$$= \mathrm{Im}\tau\,\frac{y_9 + y_{10}}{z_+} - \frac{G_F}{\sqrt{2}}\,\mathrm{Im}\lambda_t\, y_8\,\frac{\langle Q_8\rangle_2}{\mathrm{Re}A_2}\left(1 + \frac{y_7}{y_8}\frac{\langle Q_7\rangle_2}{\langle Q_8\rangle_2}\right)$$

perturbatively calculable **without** nonperturbative input
do **not** use data (would spoil cancellation of matrix element!)

from CPC (BR) data

remaining hadronic input

small

small (colour) p_{72} in error budget

Operator relations (I=0)

Analogously,

$$\left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)_{V-A} = \mathrm{Im}\tau\,\frac{[4y_4 - \quad (3y_9 - y_{10})]}{2(1+q)z_-} + \mathrm{Im}\tau\,\frac{3q(y_9 + y_{10})}{2(1+q)z_+}$$

where $q \equiv \frac{z_+(\mu)\langle Q_+(\mu)\rangle_0}{z_-(\mu)\langle Q_-(\mu)\rangle_0}$ is the only hadronic input (numerically,

<~ 0.1 (RBC-UKQCD), ~0.1 (Buras-Bardeen-Gerard approach) - negligible impact on error budget. No input from data here.

The remainder

dominant
hadronic input

$$\left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)_{V+A} = -\frac{G_F}{\sqrt{2}}\,\mathrm{Im}\lambda_t \left\{ y_6 \frac{\langle Q_6\rangle_0}{\mathrm{Re}A_0}\left(1 + \frac{y_5}{y_6}\frac{\langle Q_5\rangle_0}{\langle Q_6\rangle_0}\right) + y_8 \frac{\langle Q_8\rangle_0}{\mathrm{Re}A_0}\left(1 + \frac{y_7}{y_8}\frac{\langle Q_7\rangle_0}{\langle Q_8\rangle_0}\right)\right\}$$

from CPC
(BR) data

small (colour)
p_5 in error budget

small (EWP)

is again dominated by one matrix element.

Matrix element summary

From a phenomenological perspective, in the isospin limit by the most important goal is reducing the error on

$$\langle Q_6(\mu)\rangle_0 = -\,4h\left[\frac{m_{\mathrm{K}}^2}{m_s(\mu)+m_d(\mu)}\right]^2 (F_K - F_\pi)\, B_6^{(1/2)}$$

None of the other matrix elements contributes above 1/4 or below of the current **experimental** error, if phenomenology is done appropriately.

Apart from this, calculation of isospin breaking on the lattice, and interfacing with perturbation theory, will be important.

Will now discuss three aspects

1) NNLO computation (partial) of Wilson coefficients
2) Combining perturbative and nonperturbative input
3) Formula with dynamical charm (n_f=4)

NNLO corrections

After the matrix element of Q_6, missing NNLO corrections are the next most important item on the error budget

- 2-loop matching at weak scale known

Misiak et al
Buras, Gambino, Haisch

- missing ingredients: 2-loop matching at bottom and charm threshold, and 3-loop mixing into electroweak penguins

needed for Im A_2

sufficient for Im A_0/Re A_0 to NNLO, neglecting tiny NNLO EWP effects

Setup

Operator basis:

$$\mathcal{O}_1^{q'} = (\bar{s}_L\gamma_\mu T^a q'_L)(\bar{q'}_L\gamma^\mu T^a d_L), \quad \mathcal{O}_2^{q'} = (\bar{s}_L\gamma_\mu q'_L)(\bar{q'}_L\gamma^\mu d_L),$$

$$\mathcal{O}_3 = (\bar{s}_L\gamma_\mu d_L)\sum_q(\bar{q}\gamma^\mu q), \quad \mathcal{O}_4 = (\bar{s}_L\gamma_\mu T^a d_L)\sum_q(\bar{q}\gamma^\mu T^a q),$$

$$\mathcal{O}_5 = (\bar{s}_L\gamma_{\mu\nu\rho} d_L)\sum_q(\bar{q}\gamma^{\mu\nu\rho} q), \quad \mathcal{O}_6 = (\bar{s}_L\gamma_{\mu\nu\rho} T^a d_L)\sum_q(\bar{q}\gamma^{\mu\nu\rho} T^a q)$$

Chetyrkin, Misiak, Münz 1998

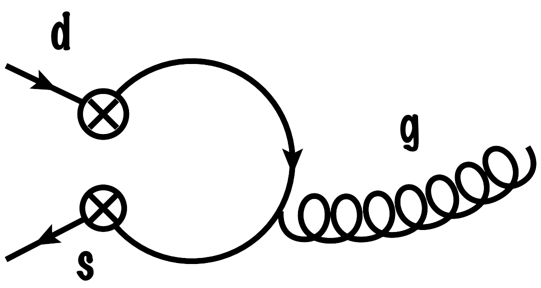
- avoids traces over γ_5

- differs from “traditional” basis (Buras-Jamin-Lautenbacher-Weisz),

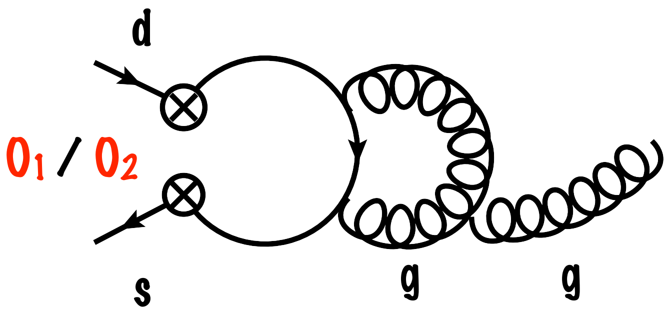
- relation to traditional basis nontrivial in d dimensions; also the renormalisation of the traditional basis has never been defined at NNLO.

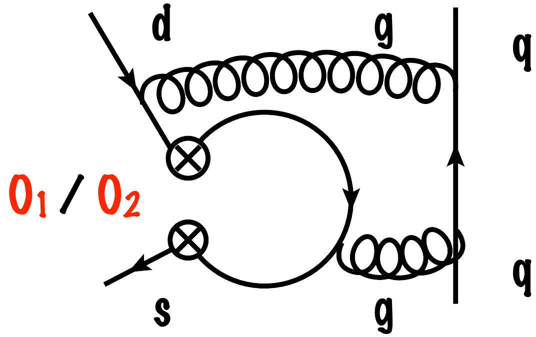
Calculation

NLO (1-loop)



NNLO (2-loop)





in the theories with n_f = 3,4,5 flavours [or rather, differences]

matching for penguin operators known Brod, Gorbahn 2010

Result (schematic)

Two matching matrices at the bottom and charm scales:

$$\langle Q_i \rangle^{(n_f=5)} = \sum_j M_{ji}^{(b)} \langle Q_j \rangle^{(n_f=4)}$$

$$\langle Q_i \rangle^{(n_f=4)} = \sum_j M_{ji}^{(c)} \langle Q_j \rangle^{(n_f=3)}$$

$$M_{ji}^{b} = M_{ji}^{b(0} + \frac{\alpha_s}{4\pi} M_{ji}^{b(1)}(m_b/\mu_b) + \left(\frac{\alpha_s}{4\pi}\right)^2 M_{ji}^{b(2)}(m_b/\mu_b) + \ldots$$

matching scale (unphysical)

(Similarly for the charm threshold.)

Anomalous dimensions for n_f=3,4 reconstructible from known n_f=5 results.

Factorisation

The perturbative corrections have the factorised structure

$$C_i(\mu, n_f = 3) = U^{(3)}_{ij}(\mu, \mu_c) M^{(34)}_{jk}(\mu_c) U^{(45)}_{kl}(\mu_c, \mu_b) M^{(45)}_{lm}(\mu_b) U^{(5)}_{mn}(\mu_b, \mu_W)) C_n(\mu_W)$$

Gorbahn, Haisch 2005

NNLO (QCD) RGE
Gorbahn, Haisch
Gorbahn, Brod

Misiak et al
Buras, Gambino, Haisch

NNLO threshold matching (QCD penguins)
Cerda Sevilla, Gorbahn, SJ, Kokulu 2016

NNLO for the isospin-0 amplitudes now complete.

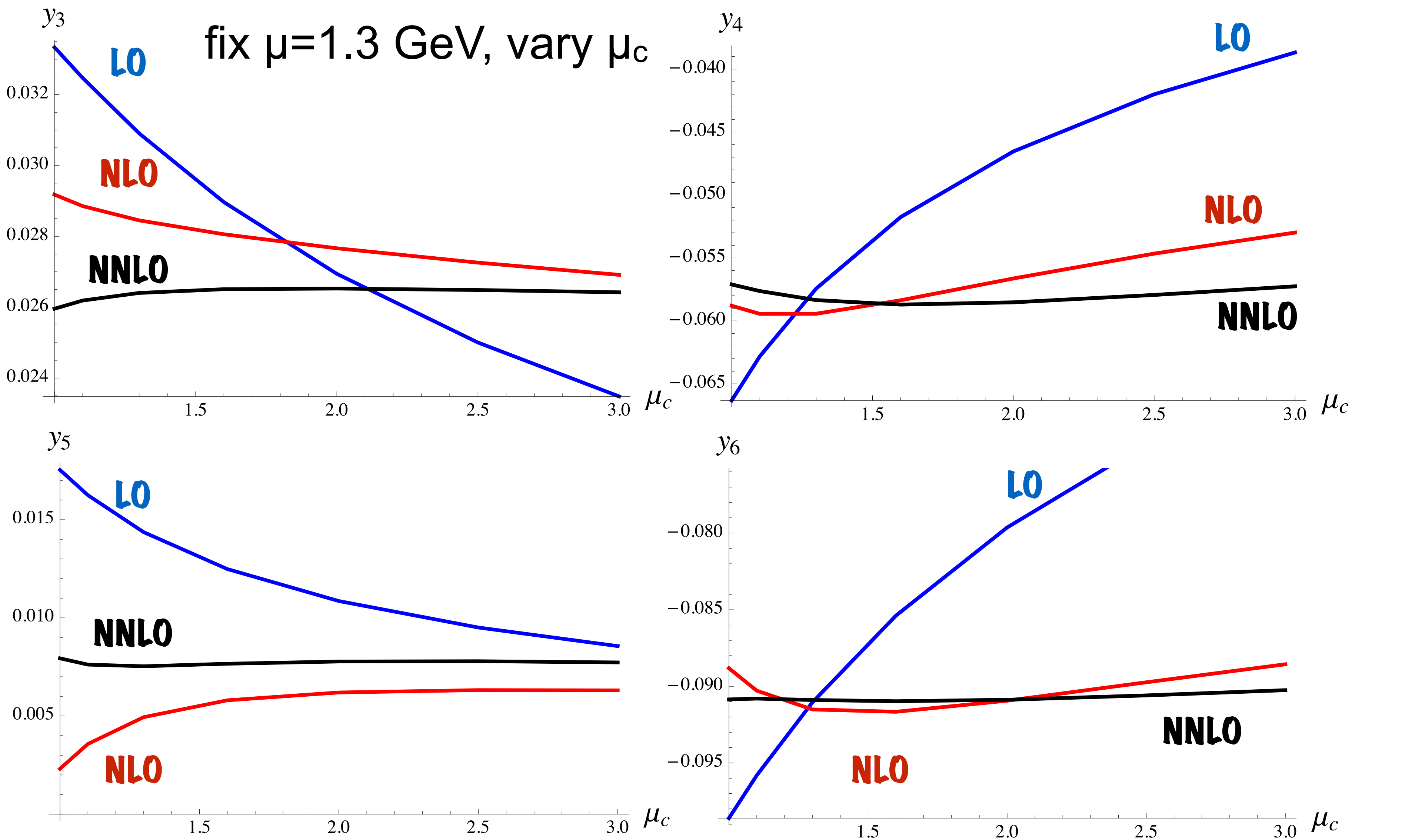
Still μ-dependent and scheme-dependent - not observables!

Will (only) cancel in the sum $\sum_i C_i \langle Q_i(\mu) \rangle$

However, may fix μ and study dependence on matching scales μ_b, μ_c as probe of importance of N^3LO corrections

Wilson coefficients

Cerda Sevilla, Gorbahn, SJ, Kokulu 2016 (preliminary)

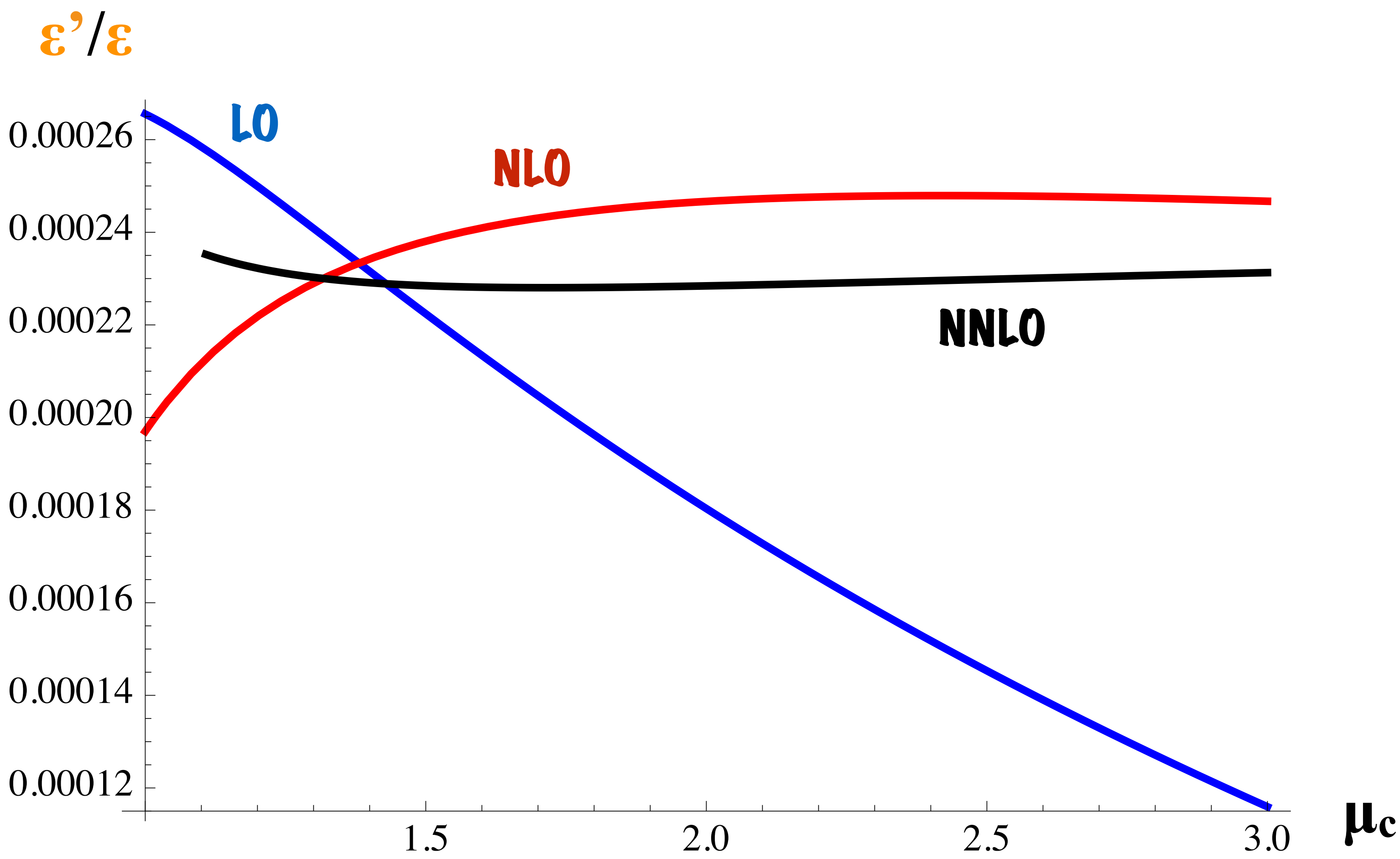


Very small residual scale dependence

NNLO accuracy of ~1% for the most important coefficient y_6

ε'/ε

Cerda Sevilla, Gorbahn, SJ, Kokulu 2016 (preliminary)



Preliminary! EWP fixed to NLO central value; some parametric dependence (m_t, ...); do not pay too much attention to central value

Tiny scale variation suggests negligible N^3LO QCD penguin effects

Schemes

Wilson coefficients and matrix elements μ-dependent and scheme-dependent - not observables!

Will (only) cancel in the sum $\sum_i C_i \langle Q_i(\mu) \rangle$

This means $\langle Q_i(\mu) \rangle$ are needed in the same scheme and for the same scale (or ideally as a function of μ)

Perturbation theory is easiest and most transparent in dimensional regularisation with minimal subtraction. Not defined beyond perturbation theory.

One possibility (employed by RBC-UKQCD): 2 steps

1) renormalise in a momentum-space subtraction scheme (RBC-UKQCD: RI/SMOM)

2) perform perturbative conversion to MSbar

Step 2) involves unknown master Feynman integrals from two loops. More complicated than perturbative Wilson coefficients.

Separate calculation needed for different lattice schemes.

RG-invariant factorisation

Instead of factoring traditionally as ...

$$\langle Q_i(\mu)\rangle C_i(\mu, n_f = 3) = \langle Q_i(\mu)\rangle \Big(u^{(3)}_{ij}(\mu)(u^{(3)})^{-1}_{jk}(\mu_c)\Big) M^{(34)}_{kl}(\mu_c)\Big(u^{(4)}_{kl}(\mu_c)$$

$$\times (u^{(4)})^{-1}_{lm}(\mu_b)\Big) M^{(45)}_{mn}(\mu_b)\Big(u^{(5)}_{nr}(\mu_b)(u^{(5)})^{-1}_{rs}(\mu_W)\Big) C_s(\mu_W)$$

This relies on the fact that $U(\mu_1, \mu_2) = u(\mu_1)u(\mu_2)^{-1}$
which can be shown to all orders in perturbation theory.

All hatted objects are scale- and scheme-independent.
They satisfy “naive” (d=4) Fierz relations.
$\hat{M}^{(34)}$, $\hat{M}^{(45)}$, $\hat{C}^{(5)}$ contain physics from precisely one scale each.
Can estimate uncertainties individually from residual scale dep.

RG-invariant factorisation

... we can also factorize as:

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$$= \langle \hat{Q}_j\rangle\, \hat{M}^{(34)}_{jl}\, \hat{M}^{(45)}_{lr}\, \hat{C}^{(5)}_r$$

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(compare $\hat{B}_K$)

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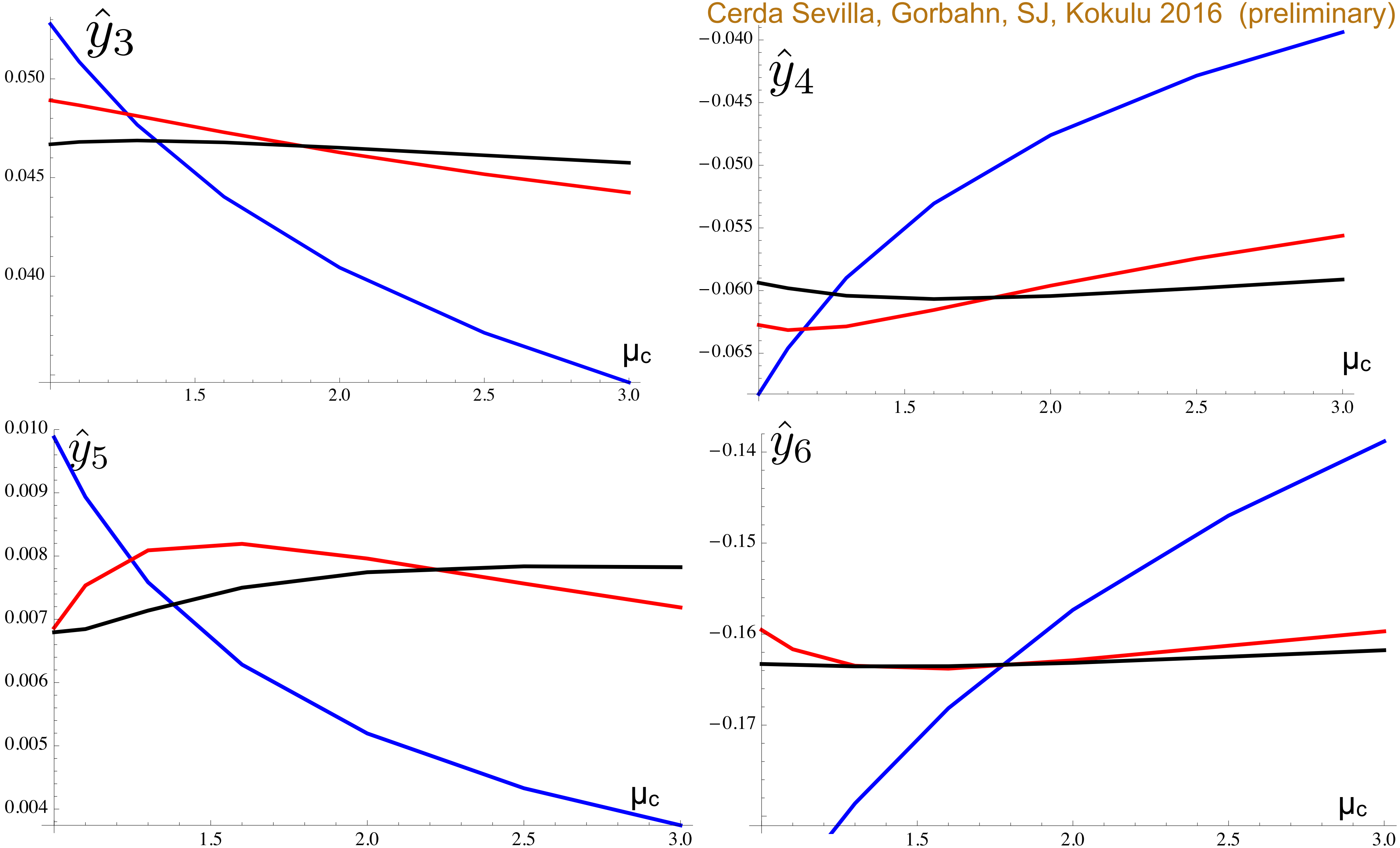
RGI Wilson coefficients

Cerda Sevilla, Gorbahn, SJ, Kokulu 2016 (preliminary)



NNLO accuracy of ~1% for the most important coefficient $\hat{y}_6$

RG-invariant matrix elements

Cerda Sevilla, Gorbahn, SJ, Kokulu, wip

$$\langle \hat{Q}_i \rangle = u^{-T}(\mu)\langle Q_i(\mu)\rangle$$

encapsulate the nonperturbative part in the RGI formalism. Can, for example, be computed from RI/SMOM: One needs the u-factor for this scheme (difficult computation).

However, a direct computation on the lattice would be preferable (with step scaling?). Because

$$u(\mu) = H(\mu)u^{(0)}(\mu) = \left(I + H^{(1)}\frac{\alpha_s}{4\pi} + \ldots\right)u^{(0)}(\mu)$$

we have

$$\langle \hat{Q}_i \rangle = \lim_{\mu\to\infty} u^{-T}(\mu)\langle Q_i(\mu)\rangle = \boxed{\lim_{\mu\to\infty} u^{(0)}(\mu)^{-T}\langle Q_i(\mu)\rangle}$$

where we have used asymptotic freedom and where

$$u^{(0)}(\mu) = \left(\frac{\alpha_s}{4\pi}\right)^{-\gamma_0^T/(2\beta_0)}$$

is the leading-order evolution. Similar to RGI mass or $\hat{B}_K$

The phenomenological formula is unchanged, apart form putting a hats over all symbols, such as

$$\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right) = \mathrm{Im}\tau\,\frac{\hat{y}_9 + \hat{y}_{10}}{\hat{z}_+} - \frac{G_F}{\sqrt{2}}\,\mathrm{Im}\lambda_{\mathrm{t}}\,\hat{y}_8\,\frac{\langle \hat{Q}_8\rangle_2}{\mathrm{Re}A_2}\left(1 + \frac{\hat{y}_7}{\hat{y}_8}\,\frac{\langle \hat{Q}_7\rangle_2}{\langle \hat{Q}_8\rangle_2}\right)$$

obtaining an expression entirely in terms of scheme-and scale-independent quantities.

Dynamical charm

No evidence for a failure of perturbation theory at the charm scale (the contrary is true). Very different from Kaon mixing.

Still one may ask about nonperturbative virtual-charm effects.

Lattice simulations with dynamical charm are becoming feasible.

Translation between the theories:

$$\langle \hat{Q}_i^{(3)} \rangle \, \hat{C}_i^{(3)} = \langle \hat{Q}_i \rangle \, \hat{M}_{ij}^{(4)} \hat{C}_j^{(4)} = \langle \hat{Q}_j^{(4)} \rangle \, \hat{C}_j^{(4)}$$

n_f=4 matrix elements

available at NNLO (CC,QCDP)
NLO (EWP)

The phenomenological formula needs modification, as it is specialised to n_f=3 operator matrix elements and operator relations

Cirigliano, Pich, Ecker, Neufeld

$n_f=4$ phenomenological formula

Cerda Sevilla, Gorbahn, SJ, Kokulu, wip

There are two new operators Q_1^c and Q_2^c, and the penguin operators contain charm quark.

The I=2 amplitude ratio is unchanged in form.
The I=0 ratio depends explicitly on the new operators:

$$\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} = \mathrm{Im}\tau\left[\frac{(2\,y_4-\frac{1}{2}[3y_9-y_{10}])(1+2\,q_-^c)}{z_-(1+\tilde{q})} - \frac{q_-^c}{1+\tilde{q}} + \frac{\left(\frac{3}{2}[y_9+y_{10}](1+q_+^c)\right)\tilde{q}}{z_+(1+\tilde{q})} - \frac{q_+^c\tilde{q}}{1+\tilde{q}} + \frac{\left(y_3+y_4-\frac{1}{2}[y_9+y_{10}]\right)\tilde{p}_3}{z_-(1+\tilde{q})} + \frac{G_F V_{ud}V_{us}^*}{\sqrt{2}\,\mathrm{Re}A_0}\left(\langle Q_6\rangle_0\left(y_6+p_5y_5+p_{8g}y_{8g}\right)+\langle Q_8\rangle_0\left(y_8+p_{70}y_7+p_{70\gamma}y_{7\gamma}\right)\right)\right]$$

$$\tilde{q} = \frac{z_+\langle Q_+ - Q_+^c\rangle_0}{z_-\langle Q_- - Q_-^c\rangle_0},\quad q_-^c = \frac{\langle Q_-^c\rangle_0}{\langle Q_- - Q_-^c\rangle_0},\quad q_+^c = \frac{\langle Q_+^c\rangle_0}{\langle Q_+ - Q_+^c\rangle_0}$$

$$\tilde{p}_3 = \frac{\langle Q_3\rangle_0}{\langle Q_- - Q_-^c\rangle_0},\quad p_5 = \frac{\langle Q_5\rangle_0}{\langle Q_6\rangle_0},\quad p_{8g} = \frac{\langle Q_{8g}\rangle_0}{\langle Q_6\rangle_0},\quad p_{70} = \frac{\langle Q_7\rangle_0}{\langle Q_8\rangle_0},\quad p_{70\gamma} = \frac{\langle Q_{7\gamma}\rangle_0}{\langle Q_8\rangle_0}$$

new parameters would be $O(\alpha_s)$ for perturbative charm

redefinition of $n_f=3$ parameters

Isospin breaking

complicated, particularly QED effects (IR subtractions, real emission, lattice matching, …)
- don't respect the two-amplitude structure
- violate Watson's theorem on strong phases

Now in principle understood on the lattice in QED perturbation theory.

talk by G Martinelli @ Kaon 2016

In practice need to
- carefully define&express observable at O(α)
- obtain appropriate perturbative ingredients
- match properly with lattice calculations of O(α) terms

For the time being, use chiral perturbation theory results (n_f=3 only)

Cirigliano, Pich, Ecker, Neufeld

Summary

ε'/ε at NLO perturbation theory with RBC-UKQCD matrix elements shows a tension with the data.

New NNLO calculation of the non-EW-penguin part of the weak Hamiltonian does not move the central value (while shrinking the perturbative error).

ε'/ε (and other observables) can be expressed in terms of RGI objects, to achiever a fuller factorization between perturbative and non-perturbative pieces.

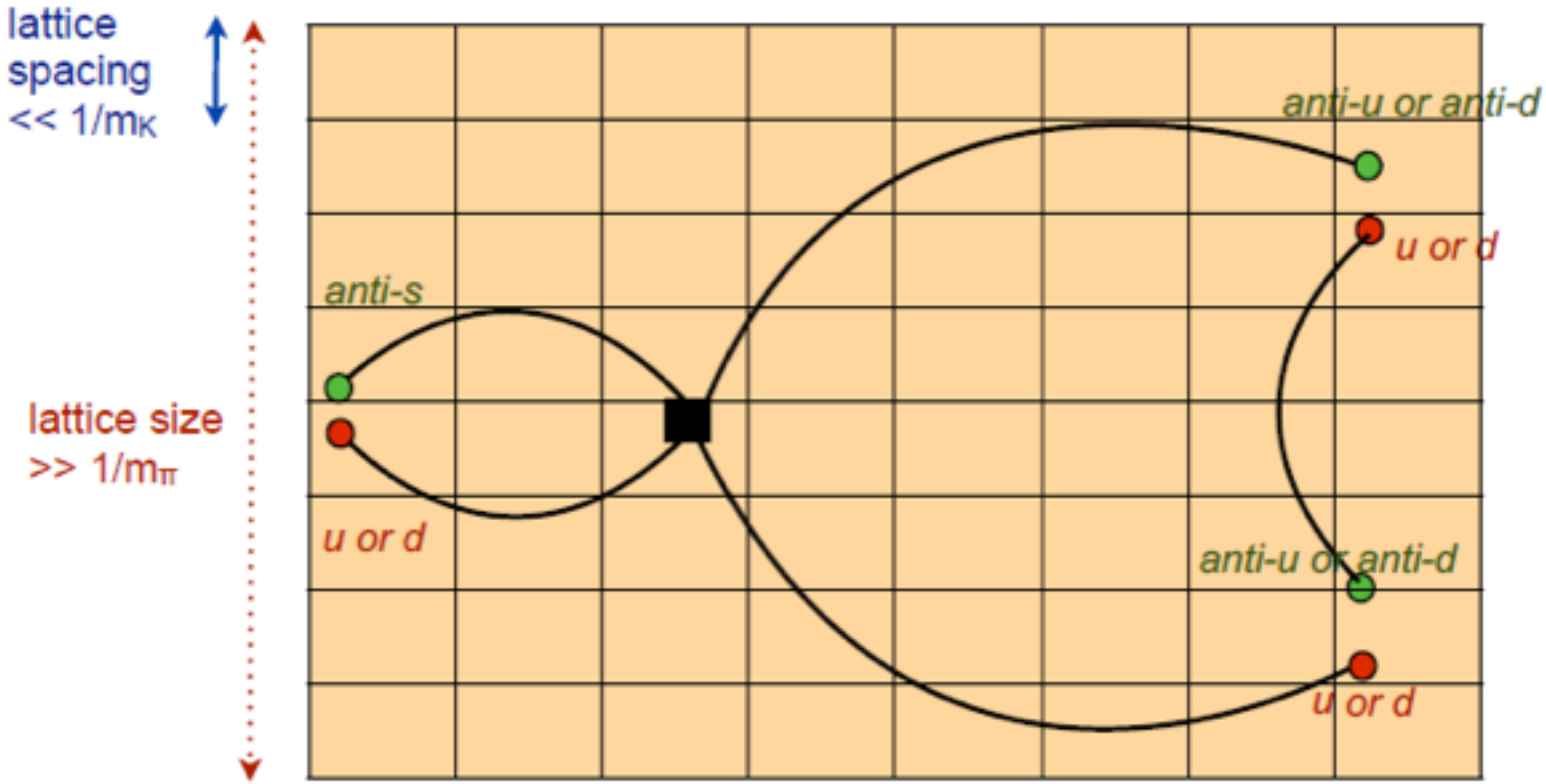
ε'/ε phenomenology benefits from systematic use of operator identities as long as matrix elements dominate the error budget

Formalism can be extended to n_f=4 dynamical quarks

EW NNLO including systematic treatment of O(α) (as well as m_d-m_u) about the isospin limit are next steps on perturbative side

BACKUP

Lattice QCD computation



Need sufficiently large and fine lattice.
Need to confront several no-go theorems

Chiral quarks

Nielsen-Ninomya:
Cannot have chiral symmetry in d=4 lattice

one way to circumvent: 5d domain-wall fermions

gives exact 4d chiral symmetry, zero modes localised at boundary

employed by RBC-UKQCD

Strong phases

Maiani-Testa

no access to rescattering phases on Euclidean lattice

Luescher

pi pi phase shifts can be determined from the volume dependence of the pi pi energy spectrum

(lattice calculation anyway done for several finite volumes)

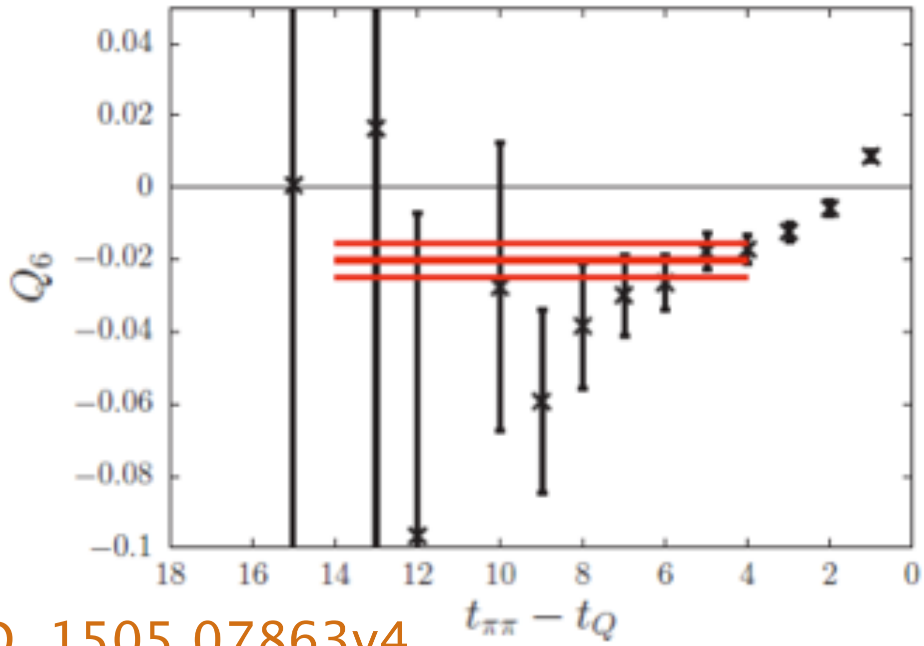
Then used to restore the phases of the matrix elements (cf Watson's theorem)

Vacuum subtraction

Lattice calculations are done in position space (and real time), fitting Green's functions to sums of exponentials.

The isospin-2 state, by a clever choice of Green's functions and boundary conditions, can be made the state of smallest energy, so $\langle Q_i \rangle_2$ can be extracted.

For isospin-0, the vacuum is always lowest. Use a variety of tricks to suppress the vacuum contributions; sizable statistical uncertainty remains



RBC-UKQCD, 1505.07863v4

Inputs

	value range
$B_6^{(1/2)}$	0.57 ± 0.19
$B_8^{(3/2)}$	0.76 ± 0.05
q	0.05 ± 0.05
$B_8^{(1/2)}$	1.0 ± 0.2
p_{72}	0.222 ± 0.033
p_3	0 ± 0.5
p_5	0 ± 0.5
p_{70}	$0 \pm 1/3$
$\mathrm{Im}\lambda_t$	$(1.4 \pm 0.1) \times 10^{-4}$
$m_t(m_t)$	(163 ± 3) GeV
$m_s(m_c)$	(109.1 ± 2.8) GeV
$m_d(m_c)$	(5.4 ± 1.9) GeV
$\alpha_s(M_Z)$	0.1185 ± 0.0006
s_W^2	0.23126
$\hat{\Omega}_{\mathrm{eff}}$	$(14.8 \pm 8.0) \times 10^{-2}$

parameterisation of hadronic matrix elements

CKM input

isospin breaking