Nucleon Compton scattering and muonic hydrogen in χPT

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Outline

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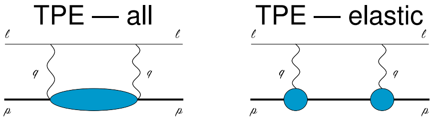
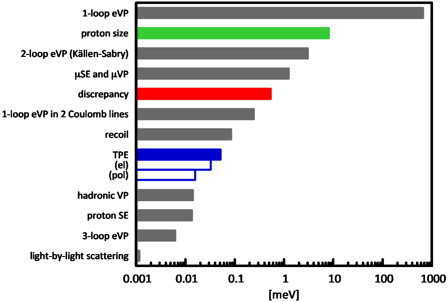
Two-photon corrections



polarisability = all − elastic

elastic is calculated from formfactors (empirical data)



review by Hagelstein, Miskimen, Pascalutsa (2016)

- muonic hydrogen Lamb shift in theory [energies in meV, R_p in fm]:

$$\Delta E_{\rm LS} = 206.0336(15) - 5.2275(10) R_p^2 + E^{\rm TPE} \quad \text{Antognini et al (2013)}$$

$$E^{\rm TPE} = 0.0332(20); \quad E^{\rm (pol)} = 0.0085(11) \quad \text{Birse, McGovern (2012)}$$

- $E^{\rm TPE}$ is an important source of th. uncertainty see R. Pohl's talk
- polarisability corrections depend on the VV Compton scattering amplitude

Lamb shift and Compton scattering

- Compton scattering amplitude (forward VVCS)

$$T^{\mu\nu}(q,p) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu, Q^2) + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right) T_2(\nu, Q^2)$$
$$-\frac{1}{M}\gamma^{\mu\nu\alpha}q_\alpha\, S_1(\nu, Q^2) - \frac{1}{M^2}\left(\gamma^{\mu\nu}q^2 + q^\mu\gamma^{\nu\alpha}q_\alpha - q^\nu\gamma^{\mu\alpha}q_\alpha\right) S_2(\nu, Q^2)$$

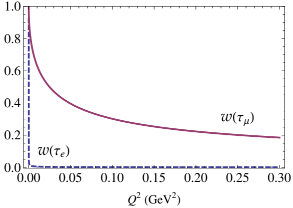
- spin-dependent terms contribute to hyperfine splitting see talk by F. Hagelstein
- n^{th} S-level shift is given by

$$\Delta E^{(\text{pol})}_{nS} = \frac{\alpha_{em}}{\pi}\,\phi_n^2 \int_0^\infty \frac{dQ}{Q^2}\, w(\tau_\ell)\left[T_1^{(\text{NB})}(0,Q^2) - T_2^{(\text{NB})}(0,Q^2)\right]$$

$w(\tau_\ell)$: the lepton weighting function

$$w(\tau_\ell) = \sqrt{1+\tau_\ell} - \sqrt{\tau_\ell}, \qquad \tau_\ell = \frac{Q^2}{4m_\ell^2}$$

weighted at low virtualities



Compton amplitudes

- $T_1(\nu, Q^2)$ and $T_2(\nu, Q^2)$ can be related, via dispersive integrals, with nucleon structure functions ($\tilde{\nu} = 2M\nu/Q^2$):

$$T_1(\nu, Q^2) = \frac{8\pi\alpha}{M} \int_0^1 \frac{dx}{x} \frac{f_1(x, Q^2)}{1 - x^2\tilde{\nu}^2 - i0}, \quad T_2(\nu, Q^2) = \frac{16\pi\alpha M}{Q^2} \int_0^1 dx \frac{f_2(x, Q^2)}{1 - x^2\tilde{\nu}^2 - i0}$$

- the integral for T_1 needs a subtraction: unknown function $T_1(0, Q^2)$
- high-Q^2 behaviour of $T_1(0, Q^2)$ needs to be modelled
  formfactors Pachucki (1999), Martynenko (2006);

  ChPT-inspired formfactors Carlson and Vanderhaeghen (2011), Birse and McGovern (2012)
- ChPT gives a prediction at $\mathcal{O}(p^3)$
  Nevado, Pineda (2008), Peset, Pineda (2014) HB;

  Alarcon, VL, Pascalutsa (2014) covariant
- something we know about $T_1(0, Q^2)$: low-energy theorem

$$T_1^{NB}(0, Q^2) = 4\pi\beta_{M1} Q^2 + \cdots, \quad T_2^{NB}(0, Q^2) = 4\pi(\alpha_{E1} + \beta_{M1})Q^2 + \cdots$$

$\Longrightarrow$ need to know the nucleon polarisabilities

Polarisabilities

- point particle (or low energies) $\Longrightarrow$ charge, mass, a.m.m.
- higher energies: response of the nucleon to external e.m. field

$\Longrightarrow$ static polarisabilities: low-energy constants of effective γN interaction

$$\mathcal{H}^{(2)}_{\rm eff} = -\frac{1}{2}\,4\pi(\alpha_{E1}\vec{E}^2 + \beta_{M1}\vec{H}^2),$$

$$\mathcal{H}^{(3)}_{\rm eff} = -\frac{1}{2}\,4\pi\Big(\gamma_{E1E1}\vec{\sigma}\cdot\vec{E}\times\dot{\vec{E}} + \gamma_{M1M1}\vec{\sigma}\cdot\vec{H}\times\dot{\vec{H}} - 2\gamma_{M1E2}E_{ij}\sigma_i H_j + 2\gamma_{E1M2}H_{ij}\sigma_i E_j\Big),$$

$$\mathcal{H}^{(4)}_{\rm eff} = -\frac{1}{2}\,4\pi(\alpha_{E1\nu}\dot{\vec{E}}^2 + \beta_{M1\nu}\dot{\vec{H}}^2) - \frac{1}{12}\,4\pi(\alpha_{E2}E_{ij}^2 + \beta_{M2}H_{ij}^2),\ldots$$

$$A_{ij} = \frac{1}{2}(\nabla_i A_j + \nabla_j A_i), \quad A = \vec{E}, \vec{H}$$

- this EFT breaks down around the pion production threshold
- we can calculate the polarisabilities from our more high-energy theory — ChPT
- ... or find from fits to data (with some help of ChPT)

Outline

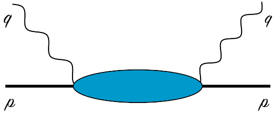
ChPT for nucleon Compton scattering

- we can calculate muonic hydrogen Lamb shift in ChPT
  - prediction at $\mathcal{O}(p^3)$
- we want to know nucleon polarisabilities
  - calculate RCS in ChPT
  - fit to data in order to pin down the polarisabilities
- we can also see what our theory gives for VVCS and for VCS
  - test our ChPT predictions
  - complementary information about properties of the nucleon (sum rules etc.)

$\Longrightarrow$ RCS, VCS, VVCS and Lamb shift calculated in a single ChPT framework

- our choice is to use the covariant formulation of ChPT

ChPT framework

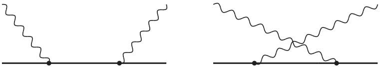


- inlcude nucleons, photons, pions
  - count powers of small momenta $p \sim m_\pi$
  - numerically $e \sim m_\pi/M_N$ — count as p
- also include the Delta isobar Pascalutsa, Phillips (2002)
  - $\Delta = M_\Delta - M_N$ is a new energy scale $\Longrightarrow$ δ-counting: numerically $\delta = \Delta/M_N \sim \sqrt{m_\pi/M_N}$, count $\Delta \sim p^{1/2}$ if $p \sim m_\pi$
  - complications due to the spin-3/2 field (consistent couplings etc.)
- two energy regimes:
  - $\omega \sim m_\pi$:
$$n = 4L - 2N_\pi - N_N - \frac{1}{2}N_\Delta + \sum kV_k$$
  - $\omega \sim \Delta$:
$$n = 4L - 2N_\pi - N_N - N_\Delta - 2N_{1\Delta R} + \sum kV_k$$

see earlier talks (H. Leutwyler, J. Alarcon, D. Siemens, ...) for more ChPT

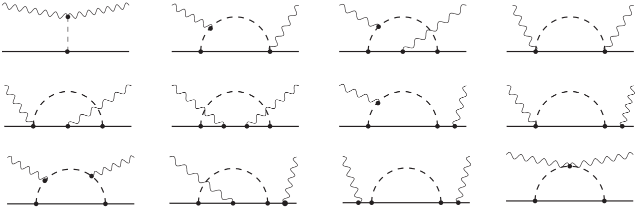
πN Loops

- Born graphs



  - responsible for low energy (Thomson) limit; point-like nucleon
  - $\mathcal{O}(p^2)$ and $\mathcal{O}(p^3)$ (a.m.m. coupling)

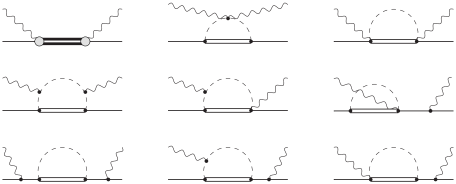
- π^0 anomaly and πN loops



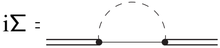
  - leading-order contribution to *polarisabilities*: $\mathcal{O}(p^3)$

Delta pole and $\pi\Delta$ loops

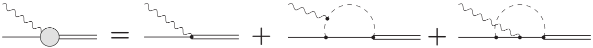
- Delta pole and $\pi\Delta$ loops



  - different counting in different energy regimes
    - Delta pole: $\mathcal{O}(p^4/\Delta) = \mathcal{O}(p^{7/2})$ at $\omega \sim m_\pi$; $\mathcal{O}(p)$ at $\omega \sim \Delta$
    - $\pi\Delta$ loops: $\mathcal{O}(p^4/\Delta) = \mathcal{O}(p^{7/2})$ at $\omega \sim m_\pi$; $\mathcal{O}(p^3)$ at $\omega \sim \Delta$
  - at $\omega \sim \Delta$ one needs to dress the 1ΔR propagator



  - at $\omega \sim \Delta$ corrections to $\gamma N\Delta$ vertex are $\mathcal{O}(p^2)$



Outline

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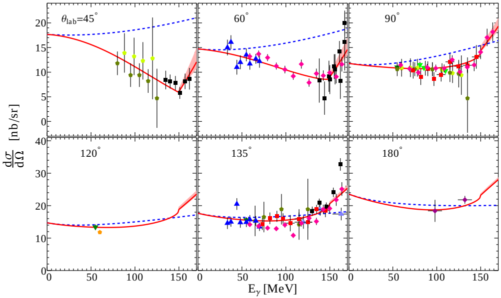
Results: RCS observables

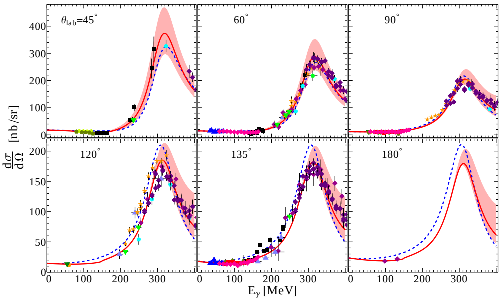
- covariant baryon calculation

  VL, McGovern, Pascalutsa (2015)

  VL, Pascalutsa (2009)
- NNLO ($\mathcal{O}(p^{7/2})$) at $\omega \sim m_\pi$
- NLO ($\mathcal{O}(p^2)$) at $\omega \sim \Delta$
- prediction of ChPT



- polarisabilities are seen starting at $\sim$ 50 MeV
- pion loops are important at low energies and around pion production threshold
- Delta pole dominates in the resonance region



One more step: fit at $\mathcal{O}(p^4)$ (partial)

- add a dipole polarisabilities contact term; fit $\delta\alpha_{E1}$ and $\delta\beta_{M1}$ to data

$$\mathcal{L}^{(4)}_{\pi N} = \pi e^2 \overline{N}\left(\delta\beta_{M1} F^{\mu\rho} F_{\mu\rho} + \frac{2}{M^2}(\delta\alpha_{E1} + \delta\beta_{M1})\partial_\mu F^{\mu\rho} F^{\nu}{}_{\rho}\partial_\nu\right) N$$

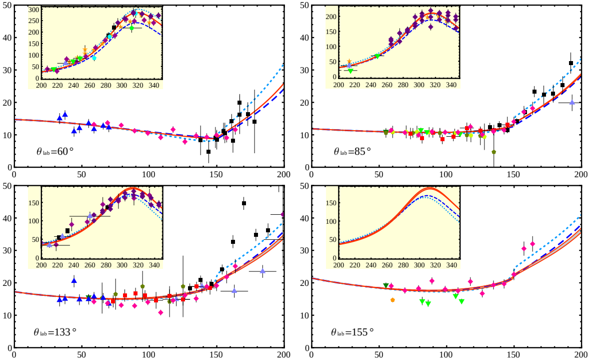
- results:

VL, McGovern (2014)

$$\alpha_{E1} = 10.6 \pm 0.5$$

$$\beta_{M1} = 3.2 \pm 0.5$$

Baldin constrained
$\chi^2/\text{d.o.f.} = 112.5/136$



Results: proton polarisabilities

- dipole polarisabilities (10^{-4} [fm^3])

Source	α_{E1}	β_{M1}
$\mathcal{O}(p^3)$ πN loops	6.9	-1.8
$\mathcal{O}(p^{7/2})$ $\pi\Delta$ loops	4.4	-1.4
Δ pole	-0.1	7.1
Total	11.2 ± 0.7	3.9 ± 0.7
Fixed-*t* DR [1,2]	12.1^*	1.6^*
Fixed-*t* DR [3,4]	...	...
HBχPT fit [5]	10.65 ± 0.50	3.15 ± 0.50
BχPT fit [6]	10.6 ± 0.5	3.2 ± 0.5
PDG	11.2 ± 0.4	2.5 ± 0.4

[1] Babusci et al. (1998)

[2] de Leon et al. (2001)

[3] Holstein et al. (2000)

[4] Hildebrandt et al. (2004)

[5] McGovern et al. (2012)

[6] VL, McGovern (2014)

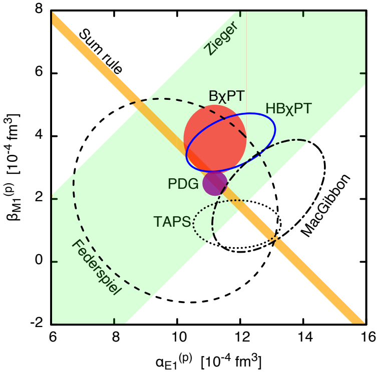
[7] Pasquini et al. (2007)

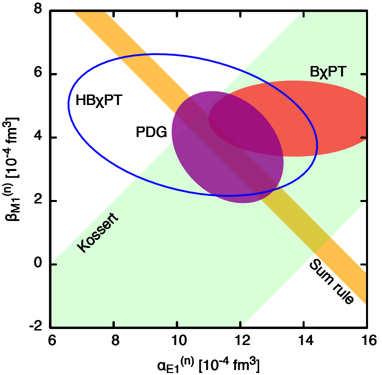
[8] McGovern et al. (2015)

- spin polarisabilities (10^{-4} fm^4)

Source	γ_{E1E1}	γ_{M1M1}	γ_{E1M2}	γ_{M1E2}
$\mathcal{O}(p^3)$ πN loops	-3.4	-0.1	0.5	0.9
$\mathcal{O}(p^{7/2})$ $\pi\Delta$ loops	0.4	-0.2	0.1	-0.2
Δ pole	-0.4	3.3	-0.4	0.4
Total	-3.3 ± 0.8	2.9 ± 1.5	0.2 ± 0.2	1.1 ± 0.3
Fixed-*t* DR [1]	-3.4	2.7	0.3	1.9
Fixed-*t* DR [3,4,7]	-4.3	2.9	-0.02	2.2
HBχPT [5,8]	-1.1 ± 1.9	2.2 ± 0.8	-0.4 ± 0.6	1.9 ± 0.5
MAMI 2015	-3.5 ± 1.2	3.16 ± 0.85	-0.7 ± 1.2	1.99 ± 0.29

Dipole polarisabilities: status

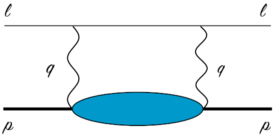




- for the proton, χPT calculations somewhat differ from other extractions (in particular, PDG value)
  - there are hints that the issue might be due to exp. data
    Krupina, VL, Pascalutsa, *in preparation*
- neutron polarisabilities are less well constrained — there is no free neutron target

Outline

VVCS



- forward VVCS amplitude

$$T(\nu, Q^2) = f_L(\nu, Q^2) + (\vec{\epsilon}'^* \cdot \vec{\epsilon})\, f_T(\nu, Q^2) \\ + i\vec{\sigma} \cdot (\vec{\epsilon}'^* \times \vec{\epsilon})\, g_{TT}(\nu, Q^2) - i\vec{\sigma} \cdot [(\vec{\epsilon}'^* - \vec{\epsilon}) \times \hat{q}]\, g_{LT}(\nu, Q^2)$$

- low-energy expansion of the amplitude is

$$\begin{aligned} f_T(\nu, Q^2) &= f_T^B(\nu, Q^2) + 4\pi \left[Q^2 \beta_{M1} + (\alpha_{E1} + \beta_{M1})\nu^2\right] + \ldots \\ f_L(\nu, Q^2) &= f_L^B(\nu, Q^2) + 4\pi(\alpha_{E1} + \alpha_L \nu^2) Q^2 + \ldots \\ g_{TT}(\nu, Q^2) &= g_{TT}^B(\nu, Q^2) + 4\pi \gamma_0 \nu^3 + \ldots \\ g_{LT}(\nu, Q^2) &= g_{LT}^B(\nu, Q^2) + 4\pi \delta_{LT} \nu^2 Q + \ldots \end{aligned}$$

- ν-dependent terms can be treated as functions of Q^2 and related to moments of nucleon structure functions

VVCS: results

NLO/LO [$\mathcal{O}(p^4/\Delta)/\mathcal{O}(p^3)$]

VL, Alarcón, Pascalutsa (2014)

HB χPT $\mathcal{O}(p^4)$

Kao, Spitzenberg, Vanderhaeghen (2003)

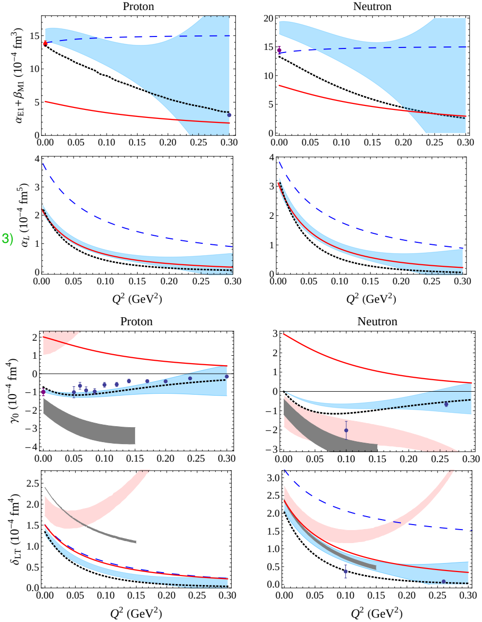
IR $\mathcal{O}(p^4)$

Bernard, Hemmert, Meissner (2003)

covariant χPT $\mathcal{O}(\epsilon^3)$

Bernard, Epelbaum, Krebs, Meissner (2013)

MAID

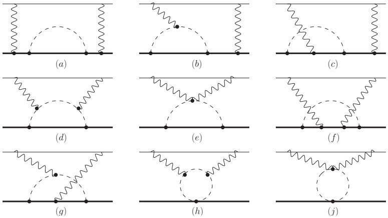


- HB and IR do not provide adequate description
- covariant χEFT works much better, especially in γ_0 (HB is off the scale there)
- δ_{LT} puzzle: difference between the two covariant calculations (the one of Bernard et al. contains $\pi\Delta$ loops subleading in our counting)
- a calculation of those $\pi\Delta$ loops is in progress in order to try to solve the puzzle

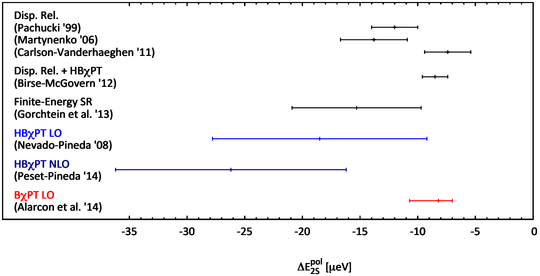
Outline

Lamb shift

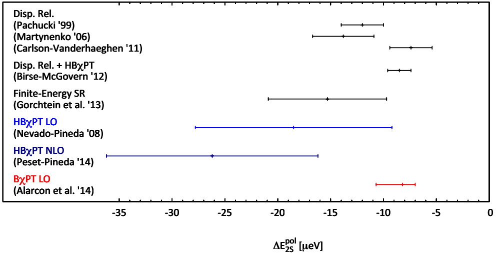
- πN loops give the leading contribution
- Delta pole turns out to be strongly suppressed
- Delta loops give small contributions to Q^2-dependent polarisabilities



- the $\mathcal{O}(p^3)$ result is
  $$\Delta E_{2S}^{(\mathrm{pol})} = -8.2(^{+1.2}_{-2.5})\ \mu\mathrm{eV}$$
  Alarcon, VL, Pascalutsa (2014)
- consistent with other calculations

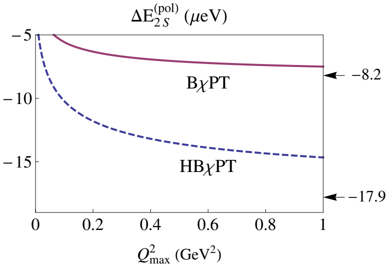


Lamb shift: HB vs. covariant



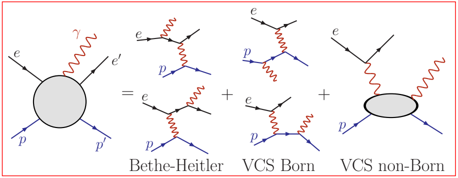
$$\Delta E_{nS}^{(\mathrm{pol})} = \frac{\alpha_{em}}{\pi}\,\phi_n^2 \int_0^{Q_{\max}} \frac{dQ}{Q^2}\, w(\tau_\ell) \left[T_1^{(\mathrm{NB})}(0,Q^2) - T_2^{(\mathrm{NB})}(0,Q^2) \right]$$

- one can expect larger error in HB since the integral converges more slowly there
- neither HB nor covariant (nor other results), however, can explain the missing $\sim 300\ \mu$eV



Outline

VCS: response functions



- low-energy expansion (small ω'):

  Guichon, Liu, Thomas (1995)

$$d^5\sigma^{\text{VCS}} = d^5\sigma^{\text{BH+Born}} + \omega' \Phi\, \Psi_0(Q^2, \epsilon, \theta, \phi) + \mathcal{O}(\omega'^2);$$

$$\Psi_0(Q^2, \epsilon, \theta, \phi) = V_1 \left[P_{LL}(Q^2) - \frac{P_{TT}}{\epsilon} \right] + V_2 \sqrt{\epsilon(1+\epsilon)} P_{LT}(Q^2)$$

- at $Q^2 = 0$:

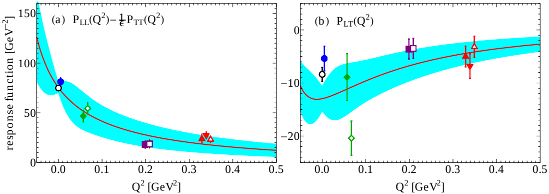
$$P_{LL}(0) = \frac{6M}{\alpha_{\text{em}}} \alpha_{E1}$$

$$P_{LT}(0) = -\frac{3M}{4\alpha_{\text{em}}} \beta_{M1}$$

- response functions of VCS in covariant χET

  VL, Pascalutsa, Vanderhaeghen

  in preparation



- new data from MAMI expected soon
- prospects for low-Q^2 expts. at MESA

Sum rules

- another tool to test χEFT
- sum rules connecting VCS, RCS, and VVCS:
  Pascalutsa, Vanderhaeghen (2015)

$$\delta_{LT} = -\gamma_{E1E1} + 3M\alpha_{\rm em}\left[P'^{\,(M1M1)1}(0) - P'^{\,(L1L1)1}(0)\right],$$

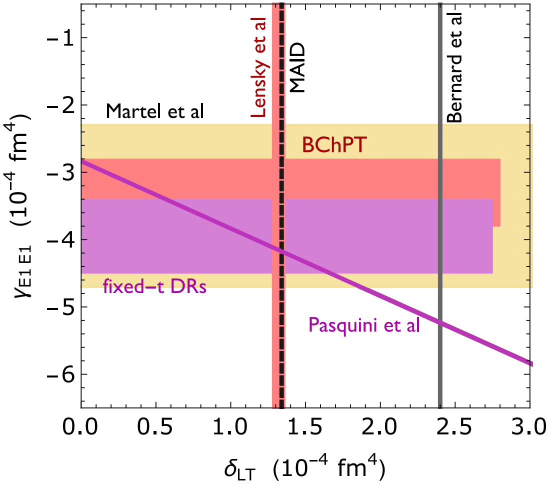
$$I_1'(0) = \frac{\kappa_N^2}{12}\langle r_2^2\rangle + \frac{M^2}{2}\left\{\frac{1}{\alpha_{\rm em}}\gamma_{E1M2} - 3M\left[P'^{\,(M1M1)1}(0) + P'^{\,(L1L1)1}(0)\right]\right\}$$

- verified in covariant and HB χEFT VL, Pascalutsa, Vanderhaeghen *in preparation*
- connect experimentally accessible quantities
- allow to obtain complementary information on, e.g., static spin polarisabilities

Sum rules and δ_{LT} puzzle

$$\delta_{LT} = -\gamma_{E1E1} + 3M\alpha_{\text{em}}\left[P'^{(M1M1)1}(0) - P'^{(L1L1)1}(0)\right]$$

- δ_{LT} puzzle shows here
- the result of Bernard et al. for δ_{LT} seems to be in contradiction with MAID
- new JLab data for the proton δ_{LT} are expected to shed light on this puzzle



Outline

Summary and outlook

- covariant baryon χEFT works reasonably well in nucleon CS
- RCS:
  - good description of exp. data, unpolarised and polarised
  - need more good-quality data at lower energies
  - $\mathcal{O}(p^4)$ calculation would decrease the theoretical error on the polarisabilities
- VVCS:
  - covariant vs. HB vs. IR: covariant shows a better description of the data
  - δ_{LT} puzzle: needs to be solved (calculations in progress)
  - more data are needed [updates are expected from JLab]
- VCS and sum rules:
  - VCS generalised polarisabilities and response functions show a good description of the data (to appear soon)
  - sum rules connecting RCS, VCS, VVCS are verified (to appear soon); can shed some light on the δ_{LT} puzzle
  - new data, especially at low Q^2, are needed
- polarisability contribution to μH Lamb shift:
  - prediction of ChPT at $\mathcal{O}(p^3)$ is consistent with other calculations
  - further work on nucleon CS will help to decrease the th. uncertainty in Lamb shift $\Delta E^{(\text{pol})}$