Nucleon σ-terms, charge radius and electric dipole moment

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Hadronic Contributions to New Physics Searches
15-30 September 2016, Puerto de la Cruz, Tenerife

Outline

Quantum ChromoDynamics (QCD)

QCD-Gauge theory of the strong interaction

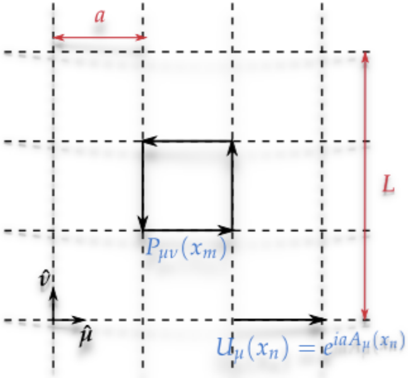
Lagrangian: formulated in terms of quarks and gluons

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t}\bar{\psi}_f\left(i\gamma^\mu D_\mu - m_f\right)\psi_f$$

$$D_\mu = \partial_\mu - ig\frac{\lambda^a}{2}A^a_\mu$$

H. Fritzsch, M. Gell-Mann, H. Leutwyler, Phys.Lett. B47 (1973) 365

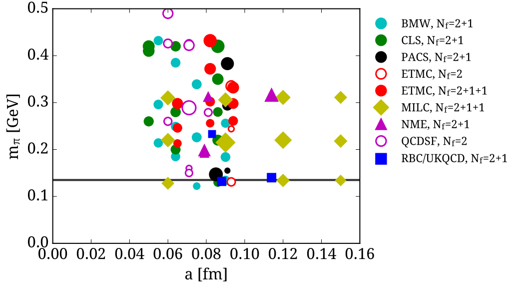
Choice of fermion discretisation scheme e.g. Clover, Twisted Mass, Staggered, Overlap, Domain Wall
Each has its advantages and disadvantages

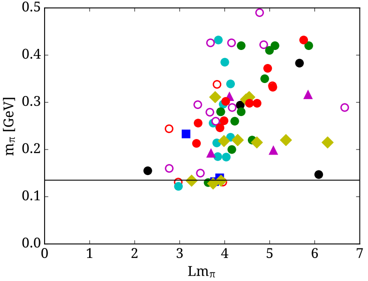


Eventually,

- all discretization schemes must agree in the continuum limit $a \to 0$
- observables extrapolated to the infinite volume limit $L \to \infty$

Status of simulations





Size of the symbols according to the value of $m_\pi L$: smallest value $m_\pi L \sim 3$ and largest $m_\pi L \sim 6.7$.

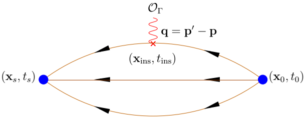
In this talk I will discuss three topics:

- Nucleon σ-terms - using simulations with pion mass close to its physical value
- Nucleon form factors and radii - using simulations with pion mass close to its physical value
- Neutron electric dipole moment - using simulations with heavier than physical pion mass

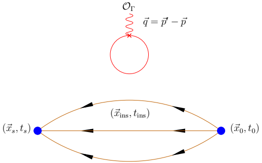
Evaluation of matrix elements

Three-point functions:

$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\rm ins}) = \sum_{\vec{x}_s, \vec{x}_{\rm ins}} e^{i\vec{x}_{\rm ins}\cdot\vec{q}}\, \Gamma_{\beta\alpha} \langle J_\alpha(\vec{x}_s, t_s) \mathcal{O}_\Gamma^{\mu\nu}(\vec{x}_{\rm ins}, t_{\rm ins}) \bar{J}_\beta(\vec{x}_0, t_0)\rangle$$



connected contribution



disconnected contribution

- Plateau method:

$$R(t_s, t_{\rm ins}, t_0) \xrightarrow[(t_s - t_{\rm ins})\Delta \gg 1]{(t_{\rm ins} - t_0)\Delta \gg 1} \mathcal{M}[1 + \ldots e^{-\Delta(\mathbf{p})(t_{\rm ins} - t_0)} + \ldots e^{-\Delta(\mathbf{p}')(t_s - t_{\rm ins})}]$$

- Summation method: Summing over $t_{\rm ins}$:

$$\sum_{t_{\rm ins}=t_0}^{t_s} R(t_s, t_{\rm ins}, t_0) = \text{Const.} + \mathcal{M}[(t_s - t_0) + \mathcal{O}(e^{-\Delta(\mathbf{p})(t_s - t_0)}) + \mathcal{O}(e^{-\Delta(\mathbf{p}')(t_s - t_0)})].$$

  Excited state contributions are suppressed by exponentials decaying with $t_s - t_0$, rather than $t_s - t_{\rm ins}$ and/or $t_{\rm ins} - t_0$
  However, one needs to fit the slope rather than to a constant or take differences and then fit to a constant
  L. Maiani, G. Martinelli, M. L. Paciello, and B. Taglienti, Nucl. Phys. B293, 420 (1987); S. Capitani *et al.*, arXiv:1205.0180

- Fit keeping the first excited state, T. Bhattacharya *et al.*, arXiv:1306.5435

All should yield the same answer in the end of the day!

The quark content of the nucleon

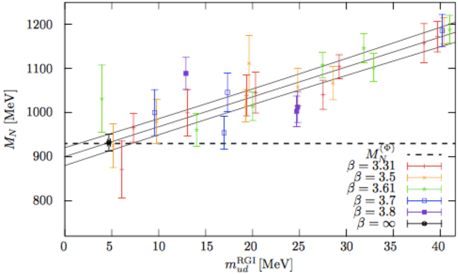
- $\sigma_f \equiv m_f \langle N|\bar{q}_f q_f|N\rangle$: measures the explicit breaking of chiral symmetry
  Largest uncertainty in interpreting experiments for direct dark matter searches - Higgs-nucleon coupling depends on σ,
  e.g. spin-independent cross-section can vary an order of magnitude if $\sigma_{\pi N}$ changes from 35 MeV to 60 MeV, J. Ellis, K. Olive, C. Savage, arXiv:0801.3656
- In lattice QCD:
  - Feynman-Hellmann theorem: $\sigma_l = m_l \frac{\partial m_N}{\partial m_l}$
    Similarly $\sigma_s = m_s \frac{\partial m_N}{\partial m_s}$

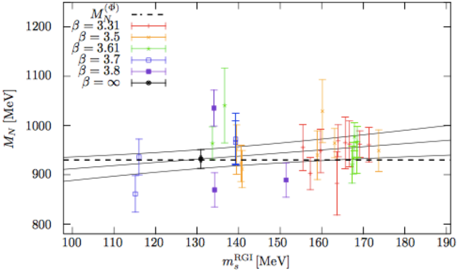
    S. Dürr *et al.* (BMW$_c$) Phys.Rev.Lett. 116 (2016) 172001
  - Direct computation of the scalar matrix element

    G. Bali, *et al.* (RQCD) Phys.Rev. D93 (2016) 094504, arXiv:1603.00827; Yi-Bo Yang *et al.* (χQCD) Phys.Rev. D94 (2016) no.5, 054503; A. Abdel-Rehim *et al.* arXiv:1601.3656, PRL116 (2016) 252001;

The quark content of the nucleon via Feynman-Hellmann

BMW Collaboration: 47 lattice ensembles with $N_f = 2 + 1$ clover fermions, 5 lattice spacings down to 0.054 fm, lattice sizes up to 6 fm and pion masses down to 120 MeV.





$\sigma_{\pi N} = 38(3)(3)$ MeV

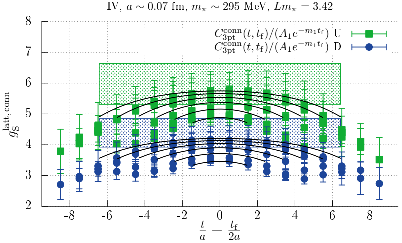
$\sigma_s = 105(41)(37)$ MeV

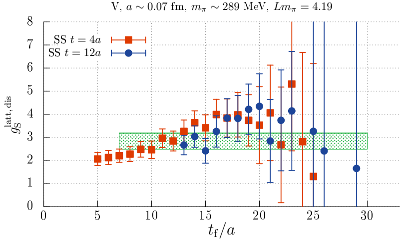
The quark content of the nucleon via direct determination

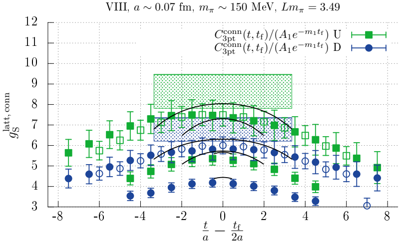
- RQCD: $N_f = 2$ clover fermions with a range of pion masses down to $m_\pi = 150$ MeV and $a = 0.06 - 0.08$ fm G. Bali, *et al.*, Phys.Rev. D93 (2016) 094504, arXiv:1603.00827
- χQCD: Valence overlap fermions on $N_f = 2 + 1$ flavor domain-wall fermion (DWF) configurations, 3 ensembles of $m_\pi = 330$ MeV, $m_\pi = 300$ MeV and $m_\pi = 139$ MeV Yi-Bo Yang *et al.*, Phys.Rev. D94 (2016) no.5, 054503; M/ Gong *et al.*, Phys. Rev. D 88 (2013) 014503 arXiv:1304.1194
- ETM Collaboration: $N_f = 2$ twisted mass plus clover, $48^3 \times 96$, a = 0.093(1) fm, m_π = 131 MeV, A. Abdel-Rehim *et al.*, arXiv:1601.3656, PRL116 (2016) 252001

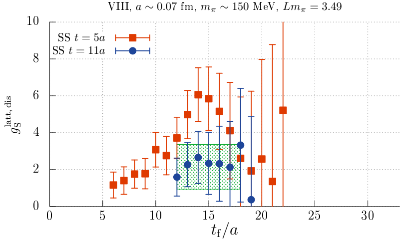
The quark content of the nucleon from RQCD

RQCD: $N_f = 2$ dynamical clover fermions with a range of pion masses down to $m_\pi = 150$ MeV and $a = 0.06 - 0.08$ fm G. Bali, *et al.* (RQCD) Phys.Rev. D93 (2016) 094504, arXiv:1603.00827







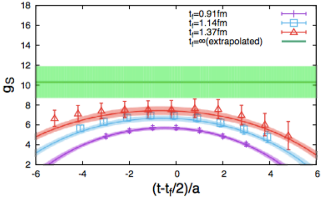


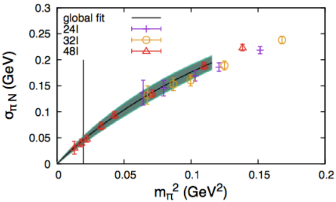
Perform chiral extrapolation, estimate finite lattice spacing and volume effects

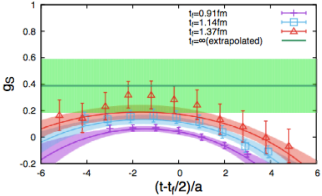
$\rightarrow$ $\sigma_{\pi N} = 35.0(6.1)$ MeV $\sigma_s = 34.7(12.2)$ MeV

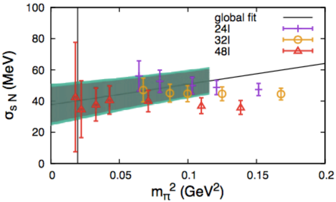
The quark content of the nucleon from χQCD

χQCD: Valence overlap fermions on $N_f = 2 + 1$ flavor domain-wall fermion (DWF) configurations, 3 ensembles of $m_\pi = 330$ MeV, $m_\pi = 300$ MeV and $m_\pi = 139$ MeV . Yi-Bo Yang *et al.* (χQCD) Phys.Rev. D94 (2016) 054503









Upper: Light (connected and disconnected);
Lower: strange, for $m_\pi = 148$ MeV
Perform chiral extrapolation

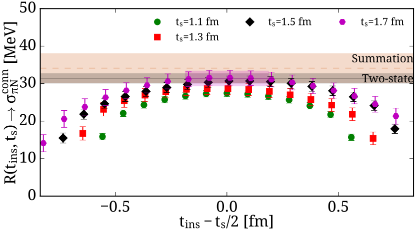
$\sigma_{\pi N} = 45.9(7.4)(2.8)$ MeV $\sigma_s = 40.2(11.7)(3.5)$ MeV

The quark content of the nucleon from ETMC

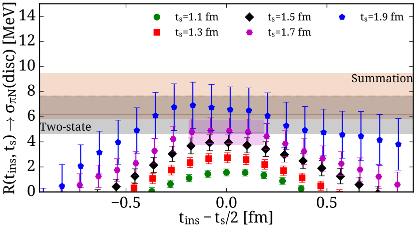
$N_f = 2$ twisted mass plus clover, $48^3 \times 96$, $a = 0.093(1)$ fm, $m_\pi = 131$ MeV

- Connected: $t/a = 10, 12, 14$ 9264 statistics, $t/a = 16$ $\sim$47,600 statistics and $t/a = 18$ $\sim$70,000 statistics
- Disconnected: $\sim$213,700 statistics

A. Abdel-Rehim *et al.* arXiv:1601.3656, PRL116 (2016) 252001



Connected



Disconnected

Our results are: $\sigma_{\pi N} = 36(2)$ MeV

The quark content of the nucleon from ETMC

$N_f = 2$ twisted mass plus clover, $48^3 \times 96$, $a = 0.093(1)$ fm, $m_\pi = 131$ MeV

- Connected: $t/a = 10, 12, 14$ 9264 statistics, $t/a = 16$ ~47,600 statistics and $t/a = 18$ ~70,000 statistics
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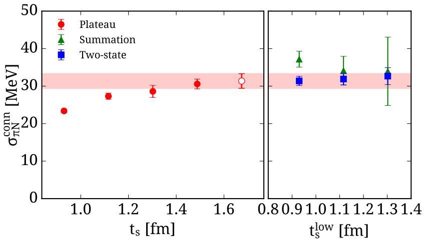


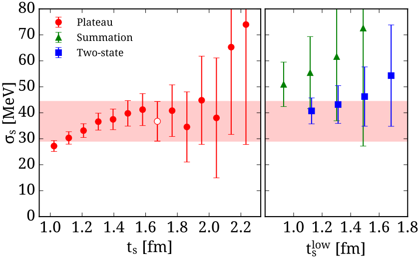
Connected

Our results are: $\sigma_{\pi N} = 36(2)$ MeV

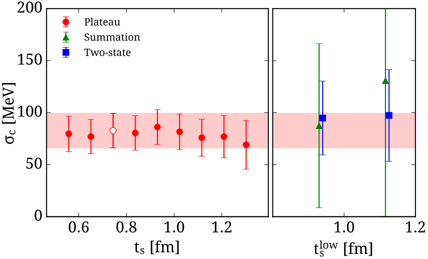
The quark content of the nucleon from ETMC

$N_f = 2$ twisted mass plus clover, $48^3 \times 96$, a = 0.093(1) fm, m_π = 131 MeV

A. Abdel-Rehim *et al.* arXiv:1601.3656, PRL116 (2016) 252001



Strange, ∼213,700 statistics

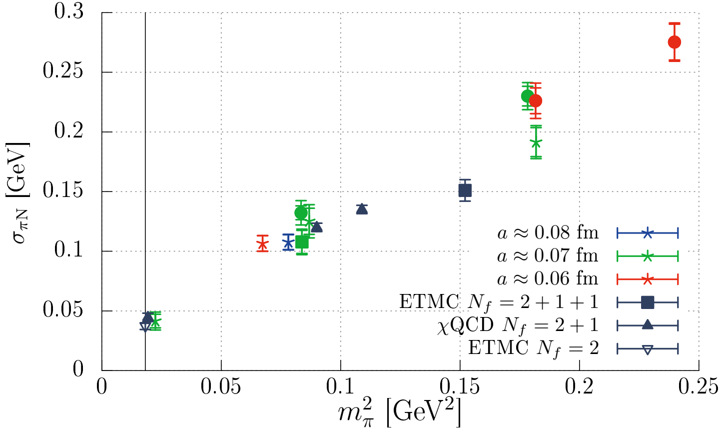


Charm, ∼213,700 statistics

Our results are: $\sigma_{\pi N} = 36(2)$ MeV $\sigma_s = 37(8)$ MeV $\sigma_c = 83(17)$ MeV

The quark content of the nucleon

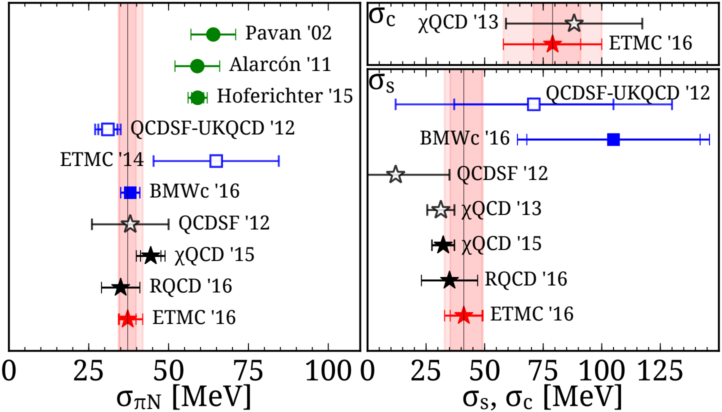
Comparison of results



G. Bali, *et al.*, Phys.Rev. D93 (2016) 094504, arXiv:1603.00827

The quark content of the nucleon

Comparison of results



Recent results from lattice QCD at the physical point and from phenomenology. Filled symbols for lattice QCD results include simulations with pion mass close to its physical value, A. Abdel-Rehim *et al.* arXiv:1601.3656, PRL116 (2016) 252001

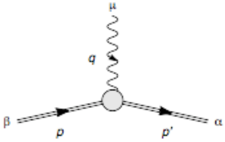
Electromagnetic form factors

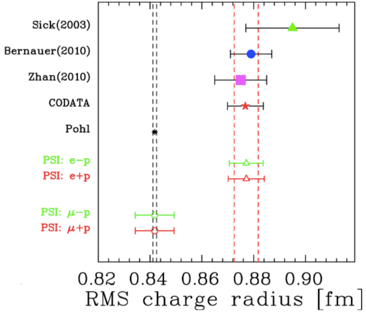
Electromagnetic form factors

$$\langle N(p',s')|j^\mu(0)|N(p,s)\rangle = \bar{u}_N(p',s')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u_N(p,s)$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_N^2}F_2(q^2)$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$



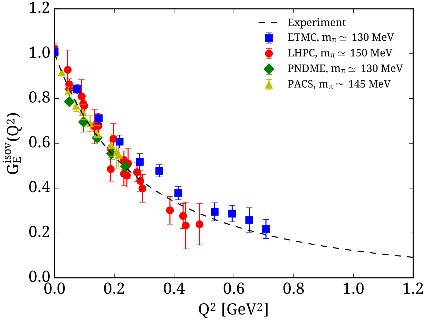


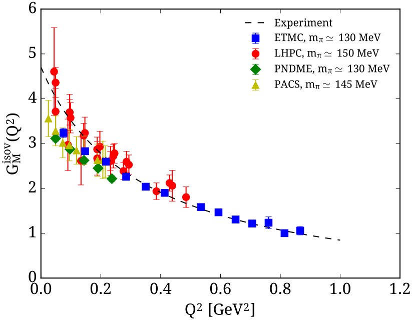
E. J. Downie, EPJ Conf. 113 (2016) 05021

- Proton radius extracted from muonic hydrogen is 7.9 σ different from the one extracted from electron scattering, R. Pohl *et al.*, Nature 466 (2010) 213
- Muonic measurement is ten times more accurate and a reanalysis of electron scattering data may give agreement with muonic measurement
- The Mainz A1 collaboration at MAMI has measured at low Q^2 and find $r_p = 0.879(5)_{\rm stat}(4)_{\rm syst}(2)_{\rm model}(4)_{\rm group}$ fm in agreement with the CODATA06 value of 0.8768(69) fm J. C. Bernauer et al., Phys. Rev. C 90, 015206 (2014)
  Other analyses of electron scattering data that include the Mainz data yield consistency with muonic results e.g. K. Griffioen, C. Carlson, S. Maddox, PRC 93 (2016) 015204; I. T. Lorenz, H.-W. Hammer, and Ulf-G. Meissner, EPJ A (2012), arXiv:1205.6628; D. W. Higinbotham *et al.*, 1510.01293

Recent results on the electric and magnetic form factors

Isovector form factors

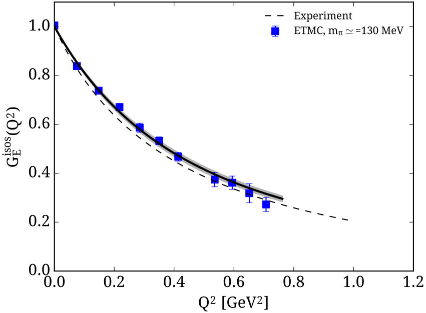


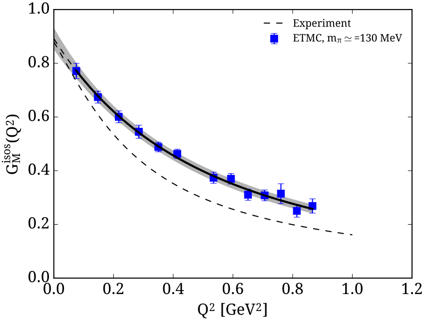


- ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$ G_E with $t_s = 1.7$ fm and 66,000 statistics, G_M with $t_s = 1.3$ fm and 9,300 statistics
- LHPC using $N_f = 2 + 1$ clover fermions, $a = 0.116$ fm, 48^4, summation method with 3 values of t_s from 0.9 fm to 1.4 fm and $\sim 7,800$ statistics, 1404.4029
- PNDME mixed action HISQ $N_f = 2 + 1 + 1$ and clover valence, $a = 0.087$ fm, $64^3 \times 96$, summation method with 3 values of t_s from 0.9 fm to 1.4 fm and $\sim 7,00$ HP and $\sim 85,000$ NP, Yong-Chull Jang, Lattice 2016
- PACS using $N_f = 2 + 1$ clover fermions, $a = 0.085$ fm, $96^3 \times 192$, $t_s = 1.3$ fm, 9,300 statistics, Y. Kuramashi, Lattice 2016

Recent results on the electric and magnetic form factors

Isoscalar form factors - connected contributions

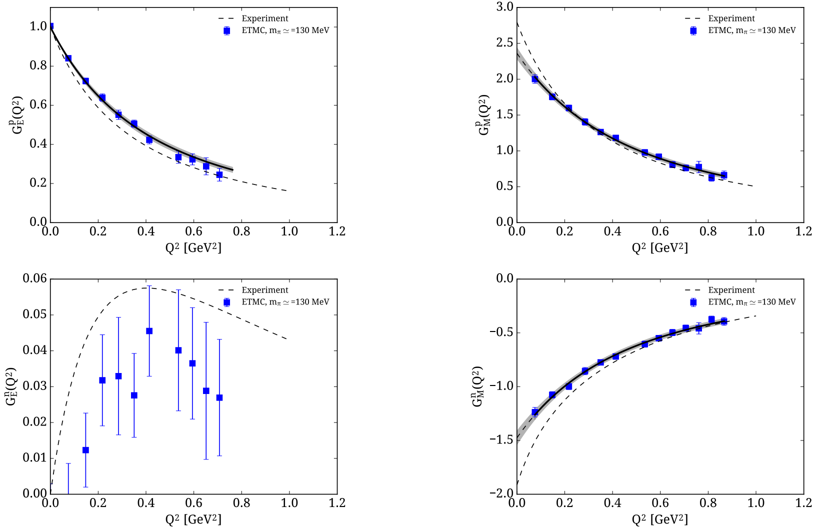




- ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$ G_E with $t_s = 1.7$ fm and 66,000 statistics, G_M with $t_s = 1.3$ fm and 9,300 statistics

Recent results on the electric and magnetic form factors

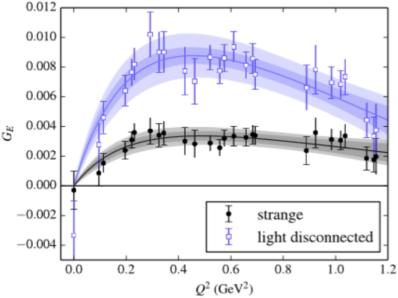
Isoscalar form factors - connected contributions

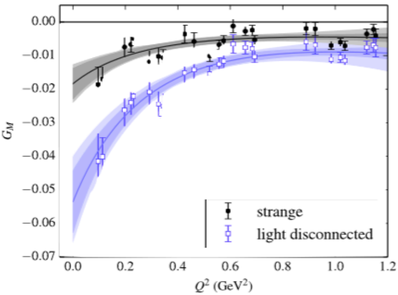


- ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$ G_E with $t_s = 1.7$ fm and 66,000 statistics, G_M with $t_s = 1.3$ fm and 9,300 statistics

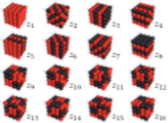
Strange Electromagnetic form factors

Experimental determination: Parity violating $e - N$ scattering
HAPPEX experiment finds $G_M^s(0.62) = -0.070(67)$
New methods for disconnected fermion loops: hierarchical probing, A. Stathopoulos, J. Laeuchli, K. Orginos, arXiv:1302.4018





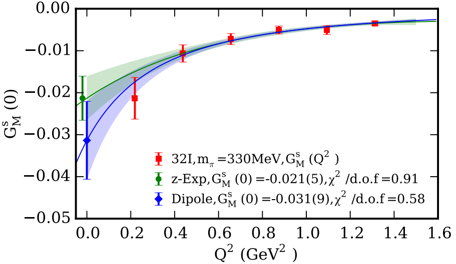
$N_f = 2 + 1$ clover fermions, $m_\pi \sim 320$ MeV, J. Green et al., Phys.Rev. D92 (2015) 3, 031501, arXiv: 1505.01803

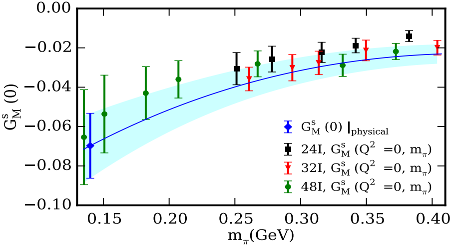


Sampling of the fermion propagator using site colouring schemes

Strange Electromagnetic form factors

Experimental determination: Parity violating $e - N$ scattering
HAPPEX experiment finds $G_M^s(0.62) = -0.070(67)$
R. S. Sufian *et al.* (χQCD Collaboration) 1606.07075





Overlap valence on $N_f = 2 + 1$ domain wall fermions, $24^3 \times 64$, $a = 0.11$ fm, $m_\pi = 330$ MeV; $32^3 \times 64$, $a = 0.083$ fm, $m_\pi = 300$ MeV and 48^3, a=0.11 fm, $m_\pi = 139$ MeV

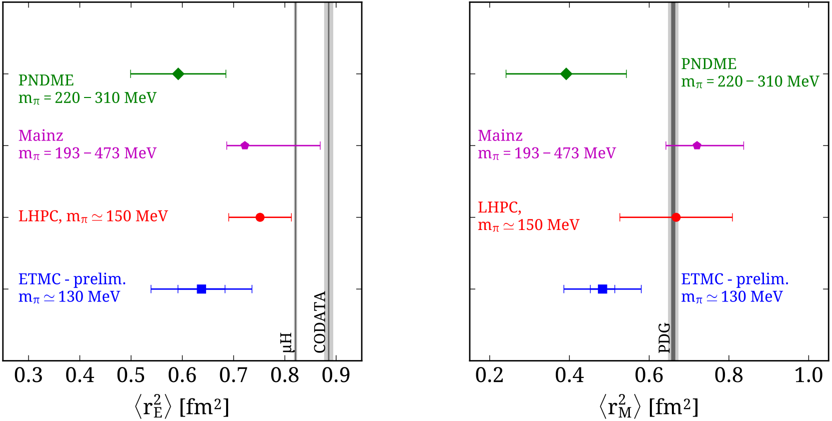
Electromagnetic radii

Slope at $Q^2 \to 0$ yields the radius : $\langle r^2_{EM} \rangle = -\frac{6}{G_{EM}(0)} \frac{dG_{EM}(Q^2)}{dQ^2}|_{Q^2=0}$

We need:

- An Ansatz for the Q^2-dependence of the form factors $\to$ dipole fit: $\frac{G_0}{(1+Q^2/M^2)^2}$, $z-$expansion
- Low values of Q^2, lowest momentum $2\pi/L \to$ large spatial length L

Only connected



Need further study and better accuracy

Position methods

- Avoid model dependence-fits
- Application to Sachs form factors $\rightarrow$ nucleon isovector magnetic moment $G_M^{\rm isov}(0)$

$$\lim_{t\rightarrow\infty}\lim_{t_s-t\rightarrow\infty}\frac{C^{3pt\,\mu}(t_s,t,\vec{q},\Gamma_\nu)}{C^{2pt}s}=\Pi^\mu\,(\vec{q},\Gamma_\nu)\;,$$

G_M is extracted from:

$$\Pi_i\,(\vec{q},\Gamma_k)=-C\frac{1}{4m_N}\epsilon_{ijk}q_jG_M\left(Q^2\right)$$

$\Rightarrow$ **Due to the factor q_j the magnetic moment $G_M(0)$ cannot be extracted directly**

Use instead

$$\lim_{q^2\rightarrow 0}\frac{\partial}{\partial q_j}\Pi_i(t,\vec{q},\Gamma_k)=\frac{1}{2m_N}\,\epsilon_{ijk}G_M(0)\,.$$

- Check with G_E:

$$\Pi_i\,(\vec{q},\Gamma_0)=-C\frac{i}{2m_N}q_iG_E\left(Q^2\right)$$

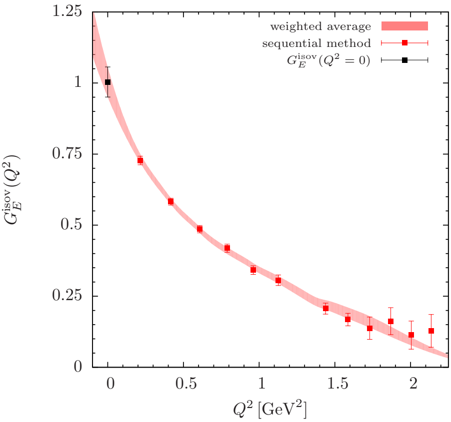
- Then apply to Isovector rms charge radius of the nucleon and the Neutron electric dipole moment

As a first step we calculated $G_M(0)$ (equivalently $F_2(0)$) at $m_\pi=373$ MeV.

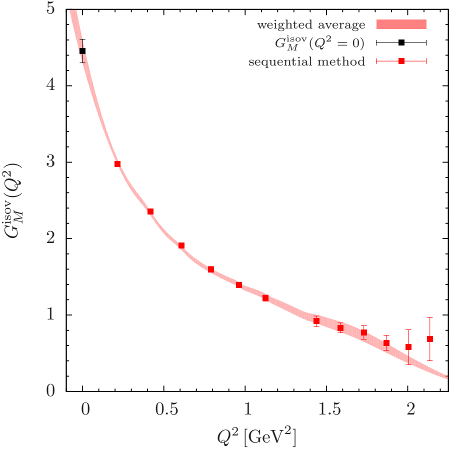
C.A., G. Koutsou, K. Ottnad, M. Petschlies, PoS(Lattice2014), 144

Magnetic moment $G_M^{\rm isov}(0)$

- Value for $G_M^{\rm isov} = 4.45(15)_{\rm stat}$ larger than result from dipole fit $3.99(9)_{\rm stat}$, Closer to exp. value (4.71)



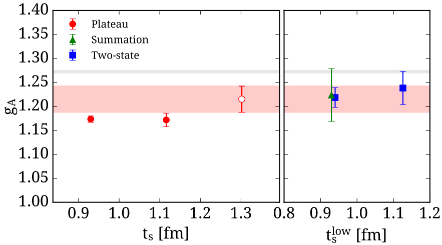
$G_E(0) = 1$ confirmed



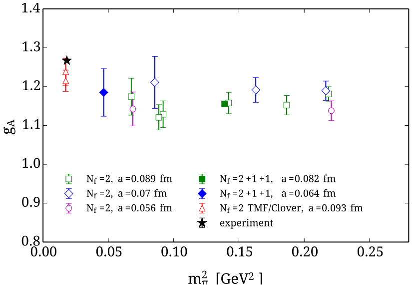
$G_M^{\rm isov}(0)$ from $\mathcal{O}(4700)$ gauge confs of B55; $t_s/a = 14$

Nucleon charges: g_A

- $N_f = 2$ twisted mass plus clover, $48^3 \times 96$, $a = 0.093(1)$ fm, $m_\pi = 131$ MeV
- 9264 statistics
- 3 sink-source time separations ranging from 0.9 fm to 1.3 fm



Isovector axial charge (t_s is the sink-source time separation and t_s^{low} is the lowest value of t_s used in the fits)



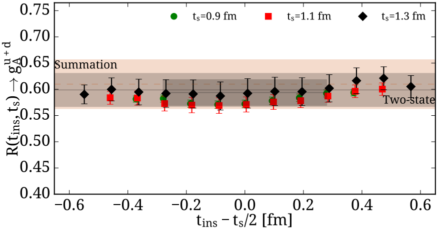
Results using twisted mass fermions

At the physical point we find from the plateau method: $g_A = 1.22(3)(2)$, where the first error is statistical and the second systematic determined by the difference between the values from the plateau and two-state fits.

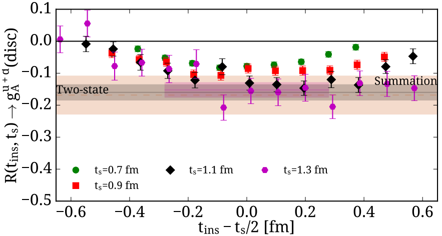
A. Abdel-Rehim *et al.* (ETMC):1507.04936, 1507.05068, 1411.6842, 1311.4522

Disconnected contributions to g_A^q

Updated results using $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm



Connected isoscalar axial charge (9264 statistics)



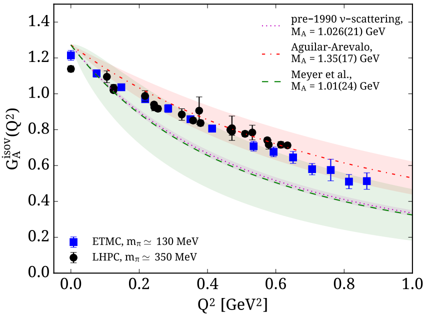
Disconnected isoscalar axial charge (854,400 statistics)

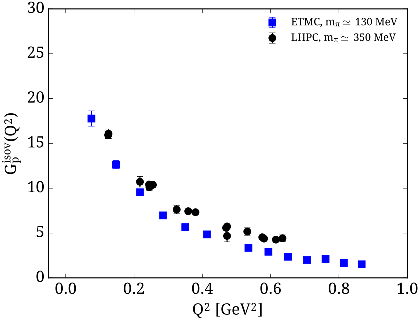
We find from the plateau method:

- $g_A^{u+d} = -0.15(2)$ with 854,400 statistics
- Combining with the isovector we find: $g_A^u = 0.828(21)$, $g_A^d = -0.387(21)$
- $g_A^s = -0.042(10)$ with 861,200 statistics

Recent results on nucleon axial form factors

Isovector form factors

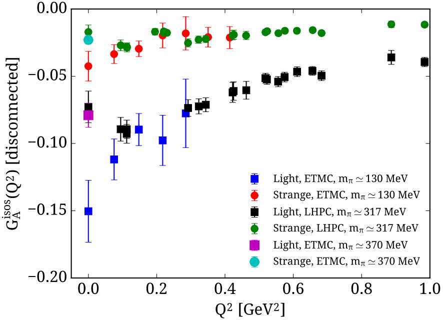




- ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$ G_E with $t_s = 1.7$ fm and 66,000 statistics, G_M with $t_s = 1.3$ fm and 9,300 statistics
- LHPC using $N_f = 2+1$ clover fermions, $a = 0.116$ fm, 48^4, summation method with 3 values of t_s from 0.9 fm to 1.4 fm and $\sim 7,800$ statistics, 1404.4029

Nucleon axial form factors

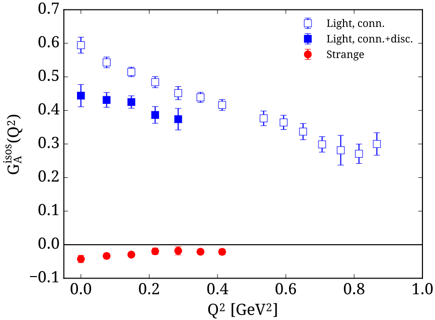
Disconnected contributions

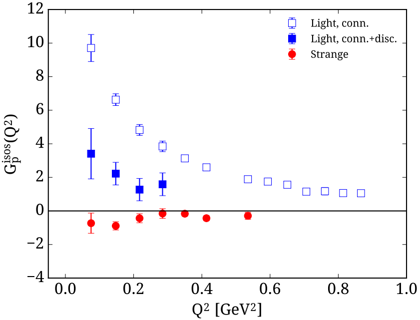


- ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$, $a = 0.093$ fm, $m_\pi = 131$ MeV, 855,000 statistics for disconnected
- LHPC using $N_f = 2+1$ clover fermions, $32^3 \times 96$, $a = 0.114$ fm, $m_\pi = 317$ MeV, 98,700 statistics for disconnected

Nucleon axial form factors

Isoscalar form factors





Large disconnected contributions

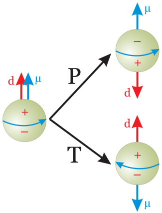
- ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$, $a = 0.093$ fm, $m_\pi = 131$ MeV, 855,000 statistics for disconnected

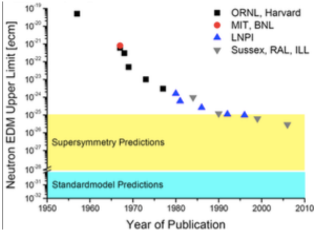
Neutron Electric Dipole Moment (nEDM)

Neutron Electric dipole moment

Possible experimental observation of nEDM

$\rightarrow$ Flags violation of P and T



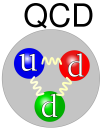


Best experimental bound:

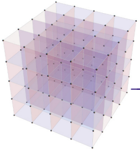
$$|\vec{d}_N| < 3.0 \times 10^{-13} e \cdot \text{fm}$$

C. A. Baker *et al*, Phys. Rev. Lett. **97**, 131801 (2006) [arXiv:hep-ex/0602020] and new analysis (2015)

$\rightarrow$ Use *ab initio* Lattice QCD



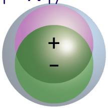
$\longrightarrow$



$\longrightarrow$

$\longrightarrow$

$|\vec{d}_N|/\theta$



General Considerations

- QCD Lagrangian density:

$$\mathcal{L}_{\rm QCD}\left(x\right) = \frac{1}{2g^2}\mathrm{Tr}\left[G_{\mu\nu}\left(x\right)G^{\mu\nu}\left(x\right)\right] + \sum_f \overline{\psi}_f\left(x\right)\left(i\gamma_\mu D^\mu + m_f\right)\psi_f\left(x\right)\ ,$$

is invariant under *C*, *P* and *T* transformations.

$\rightarrow$ cannot induce a non-vanishing nEDM.

- Insert the *CP*-violating Cherns-Simons (CS) term:

$$\mathcal{L}_{\rm CS}\left(x\right) \equiv -i\theta\frac{1}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left[G_{\mu\nu}\left(x\right)G_{\rho\sigma}\left(x\right)\right] \equiv -i\theta q(x)\,.$$

- Consider QFT with Lagrangian density:

$$\mathcal{L}\left(x\right) = \mathcal{L}_{\rm QCD}\left(x\right) + \mathcal{L}_{\rm CS}\left(x\right)\ .$$

- Model dependent studies as well as ChPT predictions:

$$|d_N| \sim \theta\cdot\mathcal{O}\left(10^{-2}\sim 10^{-3}\right)e\cdot\mathrm{fm}\,.$$

$\rightarrow \theta \lesssim \mathcal{O}\left(10^{-10}\sim 10^{-11}\right)$

General Considerations

- Need to compute expectations values with $\mathcal{L}(x)$ (in Euclidean time):

$$\langle \mathcal{O}(x_1, \ldots, x_n) \rangle_\theta = \frac{1}{Z_\theta} \int d[U] d[\psi_f] d[\bar{\psi}_f] \, \mathcal{O}(x_1, \ldots, x_n) \, e^{-S_{\rm QCD} + i\theta \int q(x) d^4x} \,.$$

- Sign problem: θ-term makes the action complex
  - Measure the neutron energy in an external electric field
  - Simulate with imaginary θ as done by e.g. QCDSF Guo *et al.* 2015
  - Assume θ is small and expand to first order

$$\langle \mathcal{O}(x_1, \ldots, x_n) \rangle_\theta = \langle \mathcal{O}(x_1, \ldots, x_n) \rangle_{\theta=0} + \left\langle \mathcal{O}(x_1, \ldots, x_n) \left(i\theta \int d^4x q(x) \right) \right\rangle_{\theta=0} + O(\theta^2) \,.$$

- Measure the neutron CP-violating electromagnetic form factor $F_3(Q^2)$ and extract $F_3(0) \rightarrow$

$$|d_n| = \lim_{q^2 \to 0} \frac{F_3(Q^2)}{2m_N}$$

- But $F_3(0)$ cannot be determined directly $\rightarrow$ use:
  - Fit the q^2-dependence
  - Use new methods referred to as position space methods to extract it directly at $Q^2 = 0$

Nucleon matrix element for nEDM

Form factor decomposition for the nucleon electromagnetic form factor reads

$$\langle N^\theta(\vec{p}_f, s)|J_\mu^{\rm EM}|N^\theta(\vec{p}_i, s')\rangle \sim \bar{u}_N^\theta(\vec{p}_f, s)\Lambda_\mu^\theta(q)u_N^\theta(\vec{p}_i, s')$$

where $\Lambda_\mu^\theta(q) = \Lambda_\mu^{\rm even}(q) + i\theta\, \Lambda_\mu^{\rm odd}(q) + \mathcal{O}(\theta^2)$ contains a (standard) *CP*–even and a *CP*–odd part

$$\begin{aligned} \Lambda_\mu^{\rm even}(q) &= \gamma_\mu F_1(q^2) + \frac{F_2(q^2)}{2m_N} q_\nu \sigma_{\mu\nu}\,, \\ \Lambda_\mu^{\rm odd}(q) &= \frac{F_3(q^2)}{2m_N} q_\nu \sigma_{\mu\nu}\gamma_5 \end{aligned} \tag{1}$$

- Extract the matrix element form the 3pt function

$$\begin{aligned} C_{3pt}^{\theta,\mu}(t_s t, \vec{q}, \Gamma_\nu) &= \langle N(\vec{p}_f, t_s) J_\mu^{\rm EM}(\vec{q}, t)\bar{N}(\vec{p}_i, 0)e^{i\theta\mathcal{Q}}\rangle \\ &= \langle N(\vec{p}_f, t_s) J_\mu^{\rm EM}(\vec{q}, t)\bar{N}(\vec{p}_i, 0)\rangle + i\theta\langle N(\vec{p}_f, t_s) J_\mu^{\rm EM}(\vec{q}, t)\bar{N}(\vec{p}_i, 0)\mathcal{Q}\rangle + \mathcal{O}(\theta^2) \end{aligned}$$

- What is new here is the correlation of the topological charge $\mathcal{Q}$ with the nucleon 2pt and 3pt functions

Main steps

- From 3pt-function we extract

$$\Pi_0^{\theta}\left(\vec{q},\Gamma_k\right) = \theta C \frac{i}{4m_N}\left[\alpha^1 q_k F_1(Q^2) + \frac{q_k(E+3m_N)\alpha^1 F_2(Q^2)}{2m_N} + \frac{q_k(E+m_N)F_3(Q^2)}{2m_N}\right].$$

- Need α^1 as input

- Build linear combination of CP–even ($\Pi_i\left(\vec{q},\Gamma_0\right)$, $\Pi_i\left(\vec{q},\Gamma_k\right)$) and CP–odd ratio $\Pi_0^{\theta}\left(\vec{q},\Gamma_k\right)$ to isolate $F_3(Q^2)$

  $\Rightarrow$ Can apply "derivative" to remove q_k for F_3

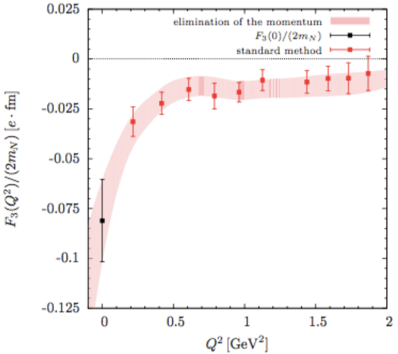
  ... or use standard method, i.e. fit Ansatz to extract $F_3(0)/(2m_N)$

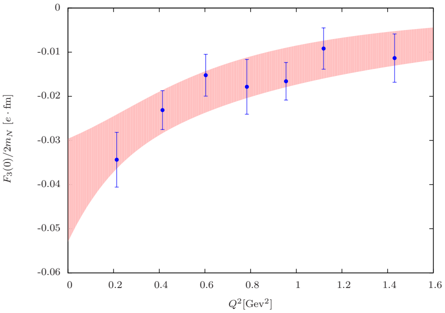
- α^1 can be determined from ratios suitably projected of 2pt functions at large t

$$\frac{C^{\theta}_{2\text{pt}}(t,\gamma_5)}{C_{2\text{pt}}(t,1+\gamma_0)} \to 2i\alpha^1\theta$$

Results for nEDM

- **We find a non-zero signal for the nEDM**
- All definitions of $\mathcal{Q}$ give signal
- Momentum elimination method and dipole fit yield results that are caompatible within the errors

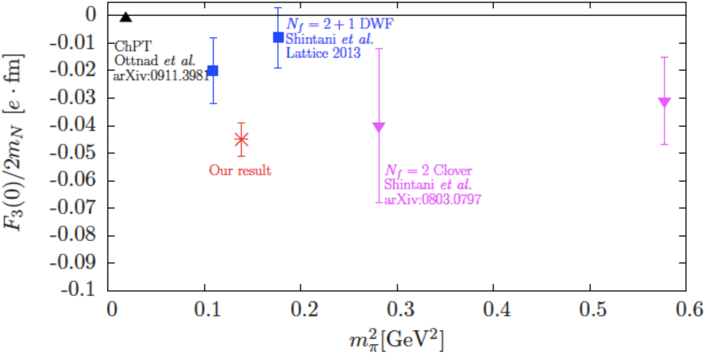




$\mathcal{O}(4700)$ gauge confs of $B55.32$ using improved gluonic $Q_{\rm top}$ and gradient flow to define $Q_{\rm top}$

Results on nEDM

Comparison of results



Shintani *et al.* '08 ($N_f = 2$ clover, external electric field); Shintani '13 *et al.* ($N_f = 2+1$, DWF, $F_3(Q^2)$); C.A. *et al.* '15 ($N_f = 2$ TMF)
Computation at the physical point under study

Conclusions

- Results for g_A, electromagnetic and axial form factors at the physical point are emerging from a number of collaborations
- New position space methods have been tested $\rightarrow$ applicable for extracting the proton radius
- Computation of gluonic observables have been advanced
- Methods are being developed for excited states and resonance properties, scattering lengths, $\cdots$
- Noise-reduction techniques will be crucial for precision baryon physics

European Twisted Mass Collaboration

Cyprus (Univ. of Cyprus, Cyprus Inst.), France (Orsay, Grenoble), Germany (Berlin/Zeuthen, Bonn, Frankfurt, Hamburg, Münster), Italy (Rome I, II, III, Trento), Netherlands (Groningen), Poland (Poznan), Spain (Valencia), Switzerland (Bern), UK (Liverpool)

Collaborators:
A. Abdel-Rehim, a. Athenodorou, S. Bacchio, K. Cichy, M. Constantinou, J. Finkenrath, K. Hadjiyiannakou, K.Jansen, Ch. Kallidonis, G. Koutsou, K. Ottnad, M. Petschlies, A. Vaquero

Thank you for your attention