Hadronic contribution from light by light processes in (g-2) of the muon in a nonlocal quark model.

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- Motivation
- Nonlocal quark model
- $(g-2)_\mu$ and density function.
- Contribution and comparison
- Conclusion

Light by light is last puzzle in picture of $(g-2)_\mu$ that cannot a direct connection with experiments.

Future experiments requires to increase accuracy of theoretical calculation.

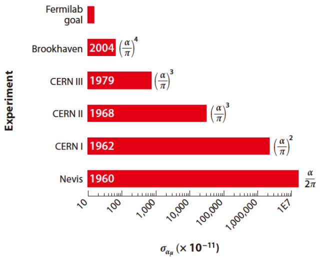


Figure: SeungCheon Kim talk on PhiPsi2015

Plan of FermiLab:
Statistical uncertainty on a_μ: 0.14 ppm

talk of M.Lancaster

J-Park experiment also have a goood plans:
Statistical uncertainty on a_μ: 0.37ppm
Statistical uncertainty on dμ: 1.3E21 e·cm

it will be good test of E821 experiment

BNL E821 experiment had a paremeters:
Statistical uncertainty on a_μ: 0.46ppm
Statistical uncertainty on dμ: 9E20 e·cm

Table 1. Summary of a_μ measurement and Standard Model prediction. Two values are quoted because of the two recent evaluations of the lowest order hadronic vacuum polarization.

	VALUE ($\times 10^{-11}$) UNITS
QED [6]	$116\ 584\ 718.951 \pm 0.009 \pm 0.019 \pm 0.007 \pm 0.077$
HVP(lo) [7]	$6\ 923 \pm 42$
HVP(lo) [8]	$6\ 949 \pm 43$
HVP(ho) [8]	-98.4 ± 0.7
HLbL [10]	105 ± 26
EW [11]	153.6 ± 1.0
Total SM [7]	$116\ 951\ 802 \pm 42 \pm 26 \pm 2 (\pm 49_{\text{tot}})$
Total SM [8]	$116\ 951\ 828 \pm 43 \pm 26 \pm 2 (\pm 50_{\text{tot}})$
Exp [5]	$116\ 592\ 089 \pm 63$
Δa_μ(Exp - SM)	287 ± 80 [7]
	261 ± 80 [8]

Also should to increase accuracy

problem

Lagrangian of nonlocal quark model have a form

$$\mathcal{L} = \bar{q}(x)(i\hat{\partial} - m_c)q(x) + \frac{G}{2}[J^a_S(x)J^a_S(x) + J^a_P(x)J^a_P(x)]$$
$$- \frac{H}{4}T_{abc}[J^a_S(x)J^b_S(x)J^c_S(x) - 3J^a_S(x)J^b_P(x)J^c_P(x)] \quad (1)$$

$$T_{abc} = \frac{1}{6}\epsilon_{ijk}\epsilon_{mnl}(\lambda_a)_{im}(\lambda_b)_{jn}(\lambda_c)_{kl},$$

Nonlocal current:

$$J^a_M(x) = \int d^4x_1 d^4x_2\, f(x_1)f(x_2)\, \bar{q}(x - x_1)\, \Gamma^a_M q(x + x_2), \quad (2)$$

where $\Gamma_S = \lambda^a$, $\Gamma_{PS} = \gamma_5\lambda^a$,

$$f(p^2) = \exp(-\frac{p^2}{2\Lambda^2}) \qquad f(p^2) = (1 + \frac{p^2}{\Lambda^2})^{-1}$$

$$S(p) = (\hat{p} - M(p))^{-1}; \qquad M_i(p) = m^i_c + m^i_d\, f^2(p); \quad (3)$$

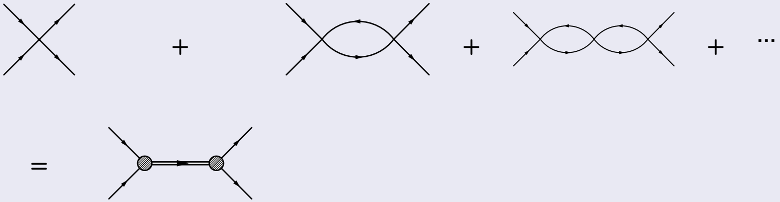
Parameters of model have a connection across of **Gap equations**

$$m_{d,u} + GS_u + \frac{H}{2} S_u S_s = 0,$$
$$m_{d,s} + GS_s + \frac{H}{2} S_u^2 = 0,$$
$$S_i = -8N_c \int \frac{d_E^4 K}{(2\pi)^4} \frac{f^2(K^2) m_i(K^2)}{D_i(K^2)},$$

where $m_i(K^2) = m_{c,i} + m_{d,i} f^2(K^2)$, $D_i(K^2) = K^2 + m_i^2(K^2)$

In results after fitting paramets on mass of pion, on weak coupling constant of pion decay we received one free parameter - m_d dynamical mass which can be varies on physical band of dynamical mass.

Propagator of meson is:



$$\hat{T}(q) = \Gamma^k_{ch} \left(\frac{1}{-\mathbf{G}^{-1}_{ch} + \Pi_{ch}(q^2)} \right)_{kl} \Gamma^l_{ch}. \quad (4)$$

$$\Pi_{ij}(P^2) = 8N_c \int \frac{d^4_E K}{(2\pi)^4} \frac{f^2(K^2_+) f^2(K^2_-)}{D_i(K^2_+) D_j(K^2_-)} \left[(K_+ \cdot K_-) \mp m_i(K^2_+) m_j(K^2_-) \right], \quad (5)$$

where $K_\pm = K \pm P/2$, $D = p^2 + m^2(p^2)$.

The gauge invariant interaction quarks field with external photon field can be introduced by Shwinger phase factor:

$$q(x) \to Q(x,y) = \mathrm{P}\exp\left(ie\int_y^x du_\mu V_\mu(u)\right) q(x). \qquad (6)$$

apart from kinetic term the additional terms in nonlocal interations are generated

$$J_I(x) = \int d^4x_1 d^4x_2\, f(x_1) f(x_2)\, \bar{Q}(x - x_1, x)\, \Gamma_I\, Q(x, x + x_2)$$

The following equations are used for obtaining of nonlocal vertices

$$\frac{\partial}{\partial y^\mu}\int_x^y dz^\nu\ F_\nu(z) = F_\mu(y), \qquad \delta^{(4)}(x-y)\int_x^y dz^\nu\ F_\nu(z) = 0.$$

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Nonlocal quark model - Vertex

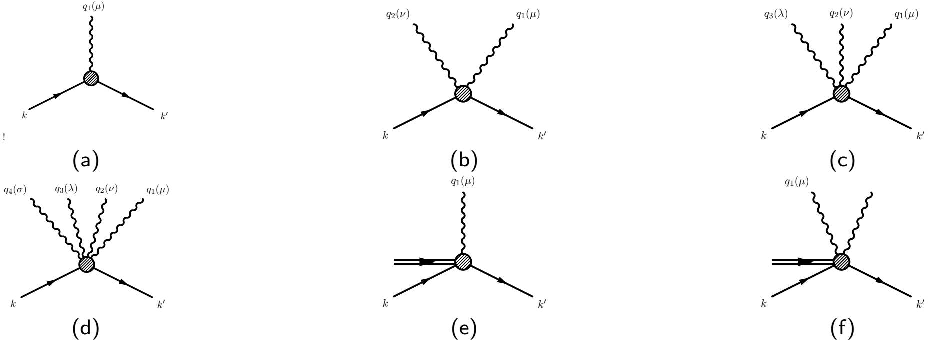


Figure: Diagrams wich discribe of interaction quarks with meson and external gauge filds. These diagrams take part in diagrams of 1/Nc contribution in LbL.

Nonlocal quark model

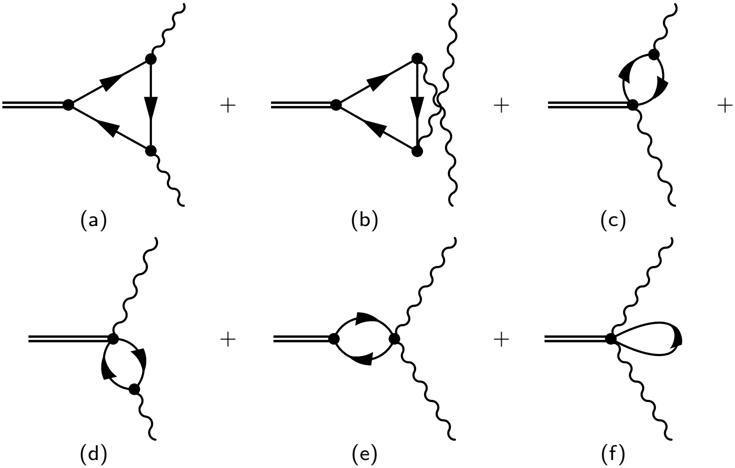
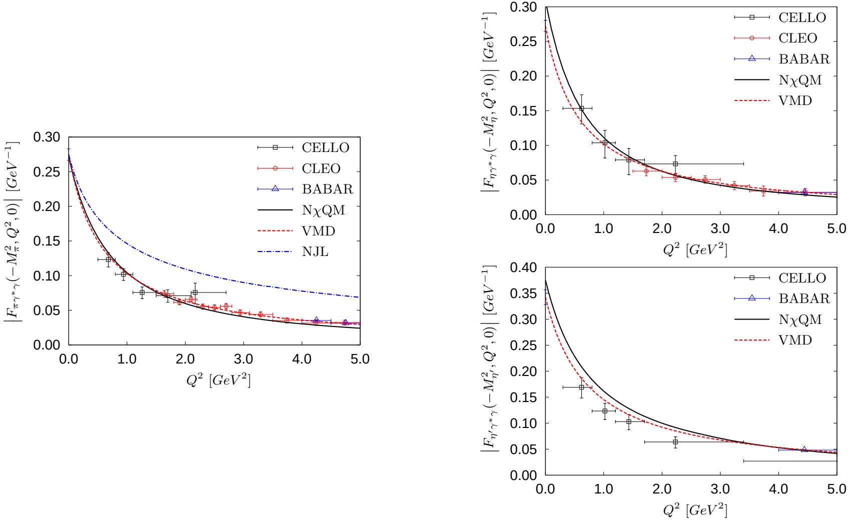


Figure: Diagrams of decay meson into two photons.


CELLO
CLEO
BABAR
NχQM
VMD
NJL
$|F_{\pi\gamma^*\gamma}(-M_\pi^2, Q^2, 0)|$ $[GeV^{-1}]$
Q^2 $[GeV^2]$
$|F_{\eta\gamma^*\gamma}(-M_\eta^2, Q^2, 0)|$ $[GeV^{-1}]$
$|F_{\eta'\gamma^*\gamma}(-M_{\eta'}^2, Q^2, 0)|$ $[GeV^{-1}]$

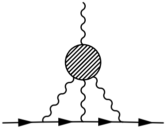
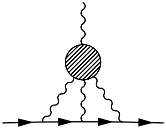


Figure: Diagrams of light by light processes.

$$a_\mu^{\rm LbL} = \frac{e^6}{48 m_\mu}\int\frac{d^4q_1}{(2\pi)^4}\int\frac{d^4q_2}{(2\pi)^4}\times$$
$$\times\,\frac{\Pi_{\rho\mu\nu\lambda\sigma}(q_2,-q_3,q_1)\mathrm{T}^{\rho\mu\nu\lambda\sigma}(q_1,q_2,p)}{q_1^2q_2^2q_3^2((p+q_1)^2-m_\mu^2)((p-q_2)^2-m_\mu^2)}, \quad (7)$$

where the tensor $\mathrm{T}^{\rho\mu\nu\lambda\sigma}$ is the Dirac trace

$$\mathrm{T}^{\rho\mu\nu\lambda\sigma}(q_1,q_2,p) = \mathrm{Tr}\Big((\hat{p}+m_\mu)[\gamma^\rho,\gamma^\sigma](\hat{p}+m_\mu)\times$$
$$\times\,\gamma^\mu(\hat{p}-\hat{q}_2+m_\mu)\gamma^\nu(\hat{p}+\hat{q}_1+m_\mu)\gamma^\lambda\Big).$$



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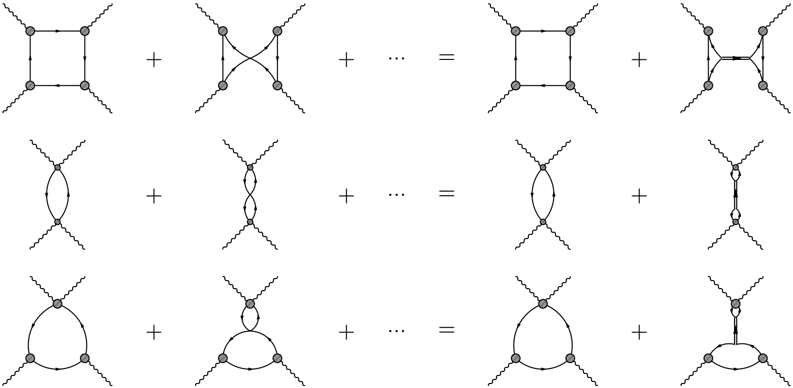


Figure: Sum of loops with 4-quarks interaction are create meson channel which will be associated with intermediate meson.

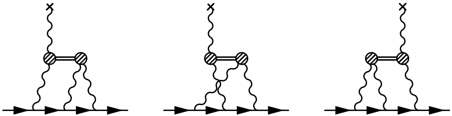
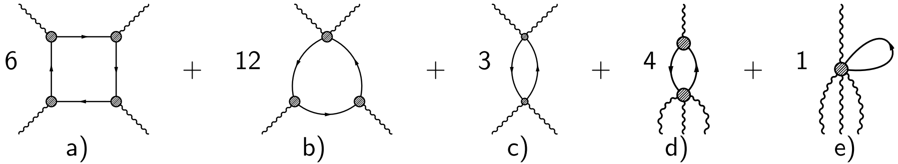
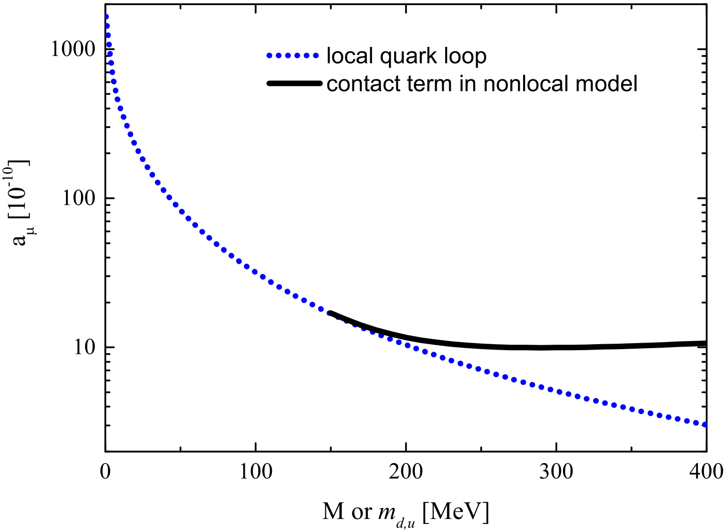


Figure: Contribution in process of light by light scaterring with intermediate mesons state.

Contribution of contact term is sum of set diagrams when polarization operator is:





local quark loop
contact term in nonlocal model
1000
100
10
a_μ [10^{-10}]
0
100
200
300
400
M or $m_{d,u}$ [MeV]

It is instructive to investigate "density" which is defined by

$$a_\mu^{\rm LbL} = \int\limits_0^\infty dQ_1 \int\limits_0^\infty dQ_2 \quad \rho^{\rm LbL}(Q_1, Q_2) \tag{8}$$

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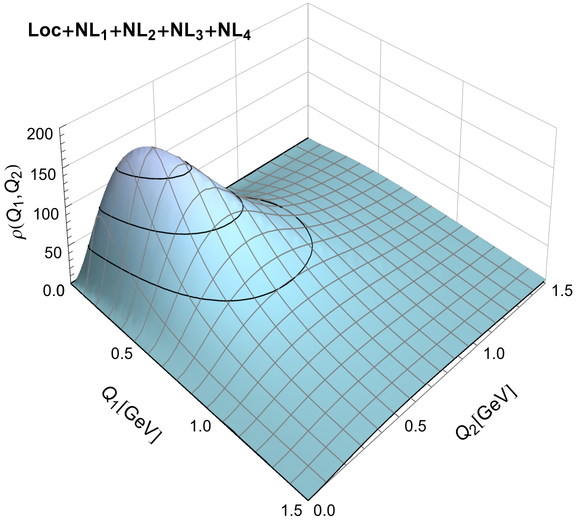
with the density $\rho^{\rm LbL}(Q_1, Q_2)$ being defined as

$$\rho^{\rm LbL}(Q_1, Q_2) = \frac{Q_1 Q_2}{2\pi^2} \sum_{a=1}^{6} \int\limits_{-1}^{1} dt \, \frac{\sqrt{1-t^2}}{Q_3^2} \langle A_a \rangle \tilde{\Pi}_a. \tag{9}$$

$$\frac{{\rm T}^{\rho\mu\nu\lambda\sigma} \Pi_{\rho\mu\nu\lambda\sigma}}{D_1 D_2} = \sum_{a=1}^{6} \langle A_a \rangle \tilde{\Pi}_a, \tag{10}$$

when $\langle A_a \rangle$ are function after avereging to muon momentum [PhysRevD.65.073034 (Knecht, Nyffeler)]



Loc+NL1+NL2+NL3+NL4
ρ(Q1,Q2)
200
150
100
50
0.0
0.5
1.0
1.5
Q1[GeV]
0.0
0.5
1.0
1.5
Q2[GeV]

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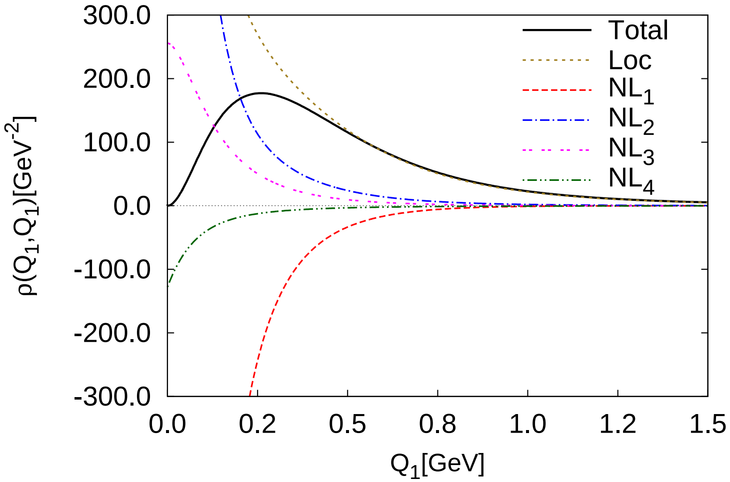
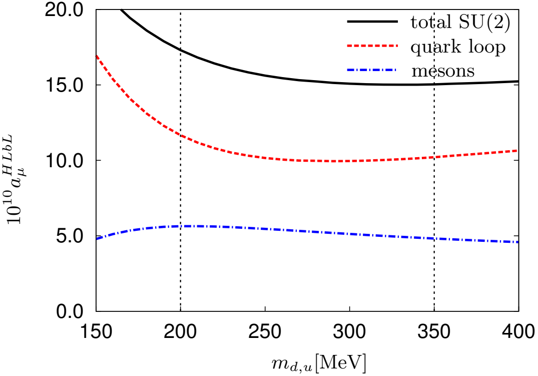


300.0
200.0
100.0
0.0
-100.0
-200.0
-300.0
0.0
0.2
0.5
0.8
1.0
1.2
1.5
ρ(Q1,Q1)[GeV-2]
Q1[GeV]
Total
Loc
NL1
NL2
NL3
NL4

result



$$a_\mu^{LbL} = 16.8 \pm (1.2) \cdot 10^{-10}; \qquad (11)$$

Model error for prediction of contribution is estimate as band of value of contribution in range of dynamical mass of quark 200 - 350 MeV.

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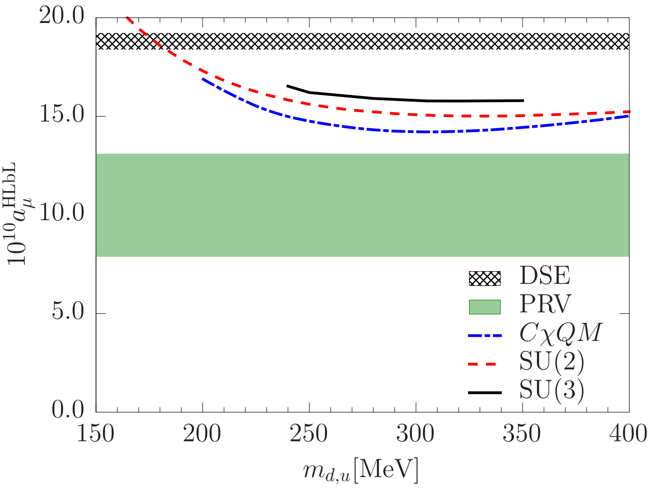
20.0
15.0
10.0
5.0
0.0
150
200
250
300
350
400
$10^{10}a_\mu^{\mathrm{HLbL}}$
$m_{d,u}$[MeV]
DSE
PRV
$C\chi QM$
SU(2)
SU(3)

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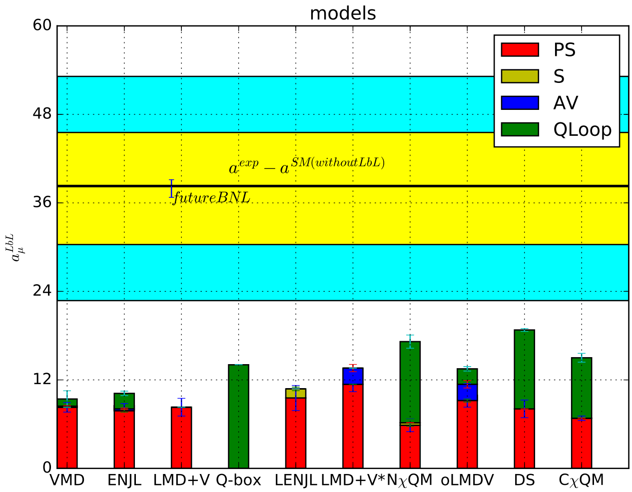

Figure: Comparison of contribution from approach, a_μ^{LbL} in 10^{-10}

- Main contributions in NχQM are due to contact term and term with intermediate pseudoscalar and scalar channels. The total contribution is estimated as $a_\mu^{\mathrm{HLbL}} = 16.8(1.25) \cdot 10^{-10}$.
- Next step: 1/Nc correction that have a connection with experimental data in hadron physycs.
- Model-independent evaluations needed and more precision data on different meson decays. (VEPP-2000, BESIII and etc.)