Dyson-Schwinger approach to the muon g-2 and the structure of the LbL amplitude

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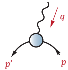
Hadronic Contributions to New Physics Searches (HC2NP)
September 29
Puerto de la Cruz, Tenerife, Spain

Introduction

- **Muon anomalous magnetic moment:**
  total SM prediction deviates from exp. by ~3σ



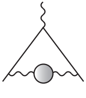
$$= ie\,\bar{u}(p')\left[F_1(q^2)\,\gamma^\mu - F_2(q^2)\,\frac{\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

$$F_2(0) = \underbrace{\frac{\alpha_{\rm QED}}{2\pi}} + \mathcal{O}(\alpha_{\rm QED}^2) \approx 1\,‰$$

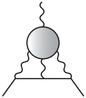
Schwinger 1948

- **QED corrections:** overwhelming part, **electroweak** and **QCD corrections** very small: 10^{-12} for electron, 10^{-8} for muon

- Theory uncertainty dominated by **QCD:** Is QCD contribution under control?



Hadronic vacuum polarization



Hadronic light-by-light scattering

$a_\mu\ [10^{-10}]$ — Jegerlehner, Nyffeler, Phys. Rept. 477 (2009)

Exp:	11 659 208.9	(6.3)	
QED:	11 658 471.9	(0.0)	
EW:	15.3	(0.2)	
Hadronic:			
• VP (LO+HO)	685.1	(4.3)	
• LBL	**10.5**	**(2.6)**	**?**
SM:	11 659 182.8	(4.9)	
Diff:	26.1	(8.0)	

Introduction

Dyson-Schwinger / Bethe-Salpeter approach:

- ab-initio, but (systematically improvable) truncations
- symmetries are exact: Poincaré invariance, chiral symmetry, electromagnetic gauge invariance
- successful applications in other systems: QCD's n-point functions, meson & baryon spectra, elastic & transition FFs, tetraquarks, QCD phase diagram, . . .

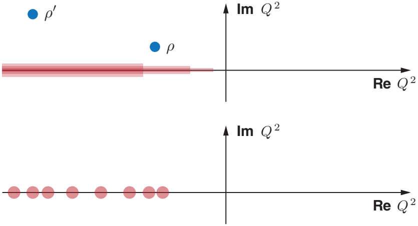
Outline:

- **Hadronic vacuum polarization:**
  basic ideas & results from DSEs & BSEs
  Review: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, 1606.09602, PPNP 91 (2016)
- **LbL scattering:**
  microscopic decomposition, quark loop, gauge invariance
  Mini-review: GE, Fischer, Heupel, Williams, 1411.7876, AIP Conf. Proc. 1701 (2016)
- **Structure of the LbL amplitude:**
  permutation group S4, kinematic phase space, tensor decomposition
  GE, Fischer, Heupel, 1505.06336, PRD 92 (2015)

Hadronic vacuum polarization

Vector current correlator from **lattice QCD:**

$$\Pi^{\mu\nu}(x-y) = \langle 0 \,|\, T\, \underbrace{[\bar\psi\,\gamma^\mu\psi](x)}_{j^\mu(x)}\, \underbrace{[\bar\psi\,\gamma^\nu\psi](y)}_{j^\nu(y)} \,|\, 0\,\rangle = \int \mathcal{D}[\psi,\bar\psi,A]\, e^{-S}\, j^\mu(x)\, j^\nu(y)$$



- Spectral decomposition:

$$\sum_\lambda |\,\lambda\,\rangle\langle\,\lambda\,| \quad \rightarrow \quad \sum_\lambda \frac{\cdots}{P^2+m_i^2}$$

- Pole in momentum space $\Longrightarrow$ exp. decay in Euclidean time

$$\Pi(x-y) \;\rightarrow\; e^{-m\tau}$$

Hadronic vacuum polarization

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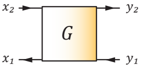
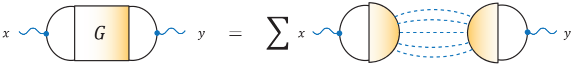
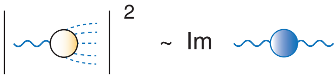
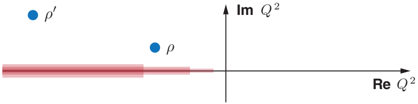
$$= \lim_{\substack{x_i\to x\\ y_i\to y}} \gamma^{\mu}_{\alpha\beta}\,\gamma^{\nu}_{\rho\sigma}\,\langle 0\,|\,T\,\bar\psi_\alpha(x_1)\,\psi_\beta(x_2)\,\bar\psi_\rho(y_1)\,\psi_\sigma(y_2)\,|\,0\,\rangle$$





Timelike side determined by $e^+e^- \to$ hadrons ⇒
spacelike correlator from **dispersion relations:**



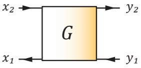


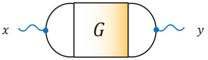
Hadronic vacuum polarization

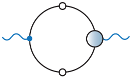
Microscopic decomposition:

$$\Pi^{\mu\nu}(x-y) = \langle 0 \,|\, T\, \underbrace{[\bar\psi\,\gamma^\mu\psi](x)}_{j^\mu(x)}\, \underbrace{[\bar\psi\,\gamma^\nu\psi](y)}_{j^\nu(y)} \,|\, 0 \rangle = \int \mathcal{D}[\psi,\bar\psi,A]\, e^{-S}\, j^\mu(x)\, j^\nu(y)$$

$$= \lim_{\substack{x_i \to x \\ y_i \to y}} \gamma^\mu_{\alpha\beta}\, \gamma^\nu_{\rho\sigma} \; \boxed{\langle 0 \,|\, T\, \bar\psi_\alpha(x_1)\, \psi_\beta(x_2)\; \bar\psi_\rho(y_1)\, \psi_\sigma(y_2) \,|\, 0 \rangle}$$

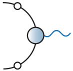
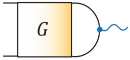


$=$ 

$=$ 

exact!

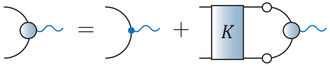
Need to know dressed **quark propagator** and **quark-photon vertex:**

 $=$ 

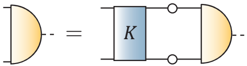
$= \langle 0 \,|\, T\, \bar\psi_\alpha(x_1)\, \psi_\beta(x_2)\; j^\nu(y) \,|\, 0 \rangle$

Bethe-Salpeter

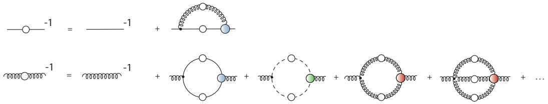
- **Bethe-Salpeter equation** for quark-photon vertex:



- Analogous for **bound states:**



- Depends on QCD's n-point functions as input, satisfy **DSEs = quantum equations of motion**

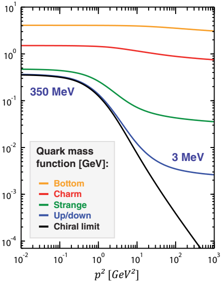


infinitely many coupled equations,
in practice truncations:
model / neglect higher
n-point functions to obtain
closed system

QCD's n-point functions

- **Quark propagator**

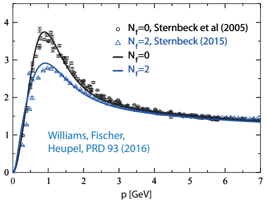
$A(p^2)\,(i\not p + M(p^2))^{-1}$



Dynamical chiral symmetry breaking generates 'constituent-quark masses'

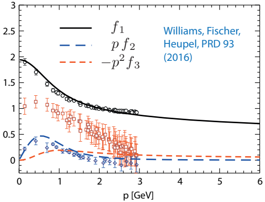
- **Gluon propagator**

$\frac{D(p^2)}{p^2}\left(\delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)$



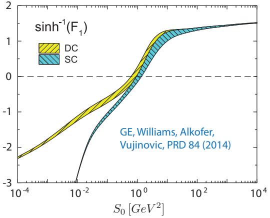
- **Quark-gluon vertex**

$f_1\,\gamma^\mu + f_2\, i p^\mu + f_3\, p^\mu \not p + \dots$



- **Three-gluon vertex**

$F_1\,[\,\delta^{\mu\nu}(p_1-p_2)^\rho + \delta^{\nu\rho}(p_2-p_3)^\mu + \delta^{\rho\mu}(p_3-p_1)^\nu] + \dots$

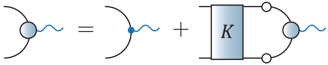


Agreement between lattice, DSE & FRG within reach

(→ see e.g. **Confinement 2016** talks: Sternbeck, Williams, Huber, Blum, Mitter, Cyrol, Campagnari, . . .)

Bethe-Salpeter

- **Bethe-Salpeter equation** for quark-photon vertex:



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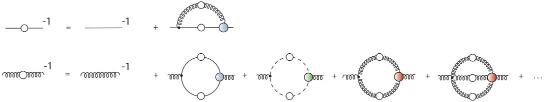
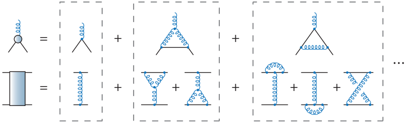


- Kernel can be derived systematically (nonperturbative!):



Review: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog. Part. Nucl. Phys. 91 (2016)

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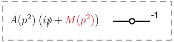

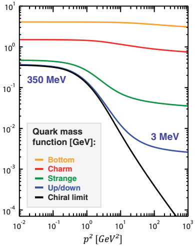

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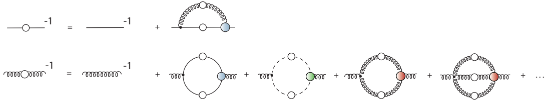

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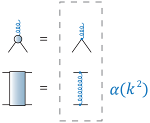
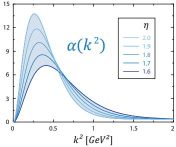




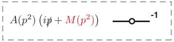
Rainbow-ladder:
effective gluon exchange

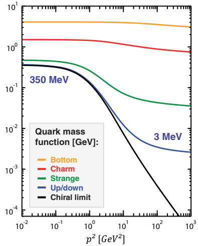
$$\alpha(k^2) = \alpha_{\mathrm{IR}}\left(k^2/\Lambda^2, \eta\right) + \alpha_{\mathrm{UV}}(k^2)$$

adjust scale Λ to observable,
keep width η as parameter

Maris, Tandy, PRC 60 (1999)

- **Quark propagator**





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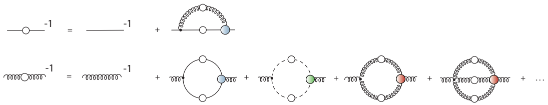

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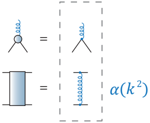

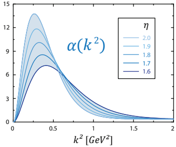

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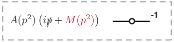
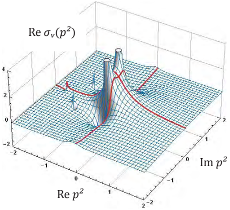
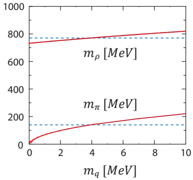




Calculated in **complex plane:** singularities pose restrictions (no physical threshold!)

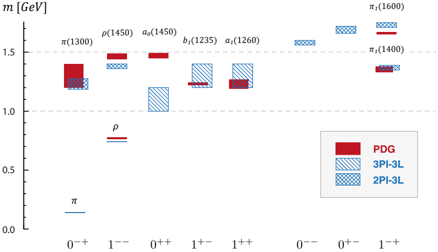
Spectroscopy

Review: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog. Part. Nucl. Phys. 91 (2016)

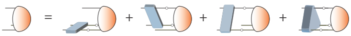


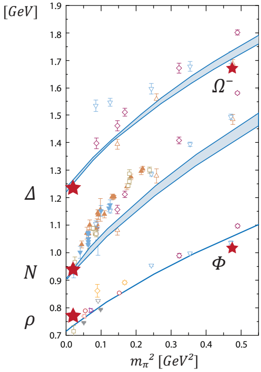
- Pion is **Goldstone boson:** $m_\pi^2 \sim m_q$

- **Light meson spectrum** beyond rainbow-ladder:
  Williams, Fischer, Heupel, PRD 93 (2016)



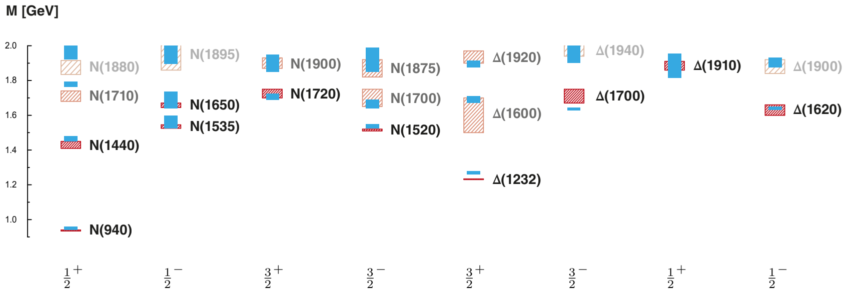
- **Baryons** from three-body BSE:
  GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010), GE, PRD 84 (2011), Sanchis-Alepuz, Fischer, PRD 90 (2014), . . .



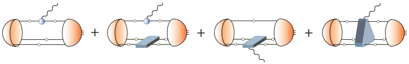


Spectroscopy

- **Baryon excitation spectrum:** quark-diquark structure GE, Fischer, Sanchis-Alepuz, 1607.05748

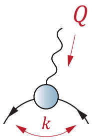


- **Electromagnetic, axial, transition form factors**
  GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog. Part. Nucl. Phys. 91 (2016)



- Light scalar mesons as **tetraquarks**
  GE, Fischer, Heupel, PLB 753 (2016)

Quark-photon vertex



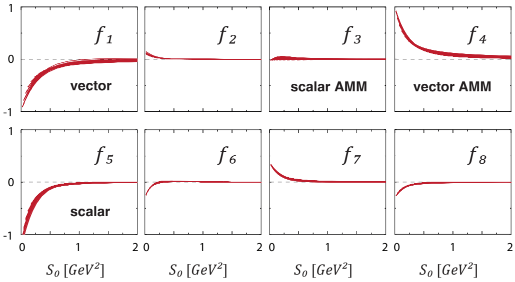
$$\Gamma^\mu(k,Q) = \left[i\gamma^\mu \Sigma_A + 2k^\mu (i\not{k}\,\Delta_A + \Delta_B) \right] + \left[i \sum_{j=1}^{8} f_j\, \tau_j^\mu(k,Q) \right]$$

Ball-Chiu vertex,
determined by WTI,
depends only on
quark propagator

Ball, Chiu, PRD 22 (1980)

Transverse part,
contains dynamics:
VM poles & cuts

Kizilersu et al, PRD 92 (1995),
GE, Fischer, PRD 87 (2013)



$\tau_1^\mu = t_{QQ}^{\mu\nu}\,\gamma^\nu$	**vector**
$\tau_2^\mu = t_{QQ}^{\mu\nu}\,k\cdot Q\,\frac{i}{2}[\gamma^\nu, \not{k}]$	
$\tau_3^\mu = \frac{i}{2}\,[\gamma^\mu, \not{Q}]$	**scalar AMM**
$\tau_4^\mu = \frac{1}{6}\,[\gamma^\mu, \not{k}, \not{Q}]$	**vector AMM**
$\tau_5^\mu = t_{QQ}^{\mu\nu}\,ik^\nu$	**scalar**
$\tau_6^\mu = t_{QQ}^{\mu\nu}\,k^\nu\not{k}$	
$\tau_7^\mu = t_{Qk}^{\mu\nu}\,k\cdot Q\,\gamma^\nu$	
$\tau_8^\mu = t_{Qk}^{\mu\nu}\,\frac{i}{2}\,[\gamma^\nu, \not{k}]$	

$$t_{ab}^{\mu\nu} := a\cdot b\,\delta^{\mu\nu} - b^\mu a^\nu$$

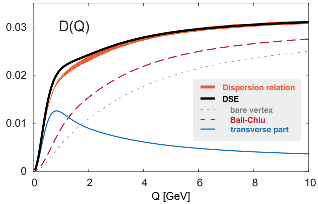
GE, Acta Phys. Polon. Supp. 7 (2014)

Hadronic vacuum polarization

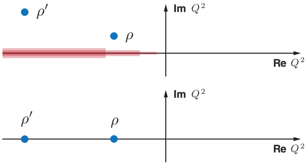
Adler function:

DSE: Goecke, Fischer, Williams, PLB 704 (2011)
DR: Eidelman, Jegerlehner, Kataev, Veretin, PLB 454 (1999)

$$D(Q^2) = -Q^2\, d\Pi(Q^2)/dQ^2$$



- **Dispersion relations** and **DSEs** (almost) identical on spacelike side, although timelike structure different: in rainbow-ladder, **bound states** without widths

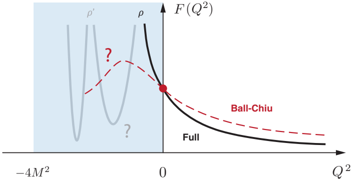


but we only need **spacelike** region for g-2!

- $a_\mu^{\text{HVP}} = 676 \times 10^{-10}$ (1.3 %)

- Similar in **hadronic form factors:** spacelike properties + hadronic poles reproduced, but missing meson-baryon interactions

- Separation into Ball-Chiu + transverse part in **any electromagnetic process!**



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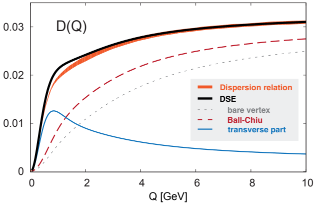
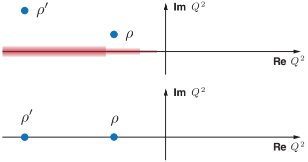
DR: Eidelman, Jegerlehner, Kataev, Veretin, PLB 454 (1999)

$$D(Q^2) = -Q^2\, d\Pi(Q^2)/dQ^2$$



- **Dispersion relations** and **DSEs** (almost) identical on spacelike side, although timelike structure different: in rainbow-ladder, **bound states** without widths

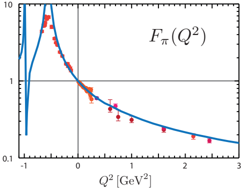


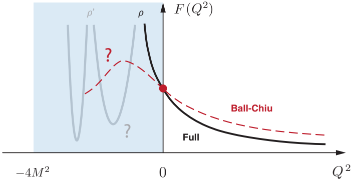
but we only need **spacelike** region for g-2!

- $a_\mu^{\text{HVP}} = 676 \times 10^{-10}$ (1.3 %)

- e.g. **Pion em. form factor:**
  Maris & Tandy, PRC 61 (2000),
  Krassnigg, Schladming 2010

- $\pi\to\gamma\gamma$ **transition:**
  Maris & Tandy, PRC 65 (2002)

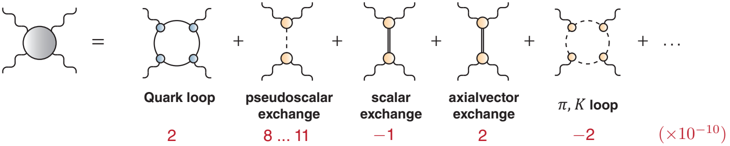




Light-by-light scattering

LbL amplitude: model results

Bijnens 1995, Hakayawa 1995, Knecht 2002, Melnikov 2004, Prades 2009, Jegerlehner 2009, Dorokhov 2011, Pascalutsa 2012, Pauk 2014, Colangelo 2015, ...



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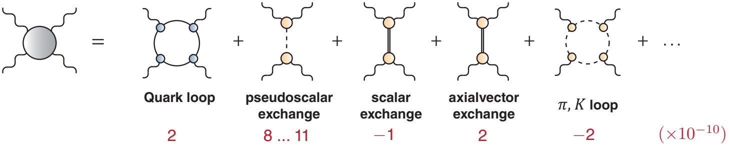
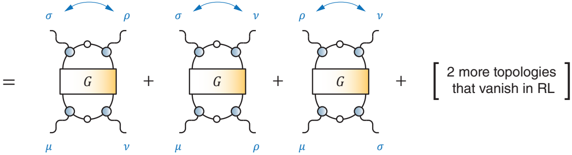


Exact expression:

GE, Fischer, PRD 85 (2012), Goecke, Fischer, Williams, PRD 87 (2013)



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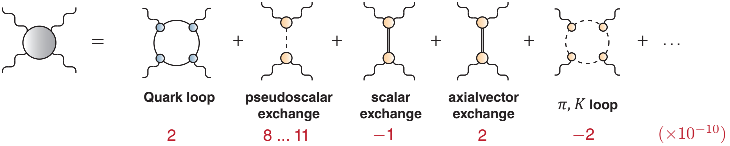

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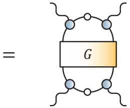

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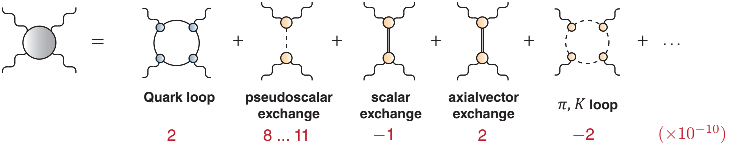

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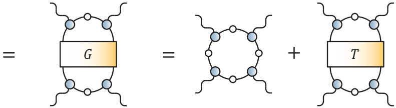

Light-by-light scattering

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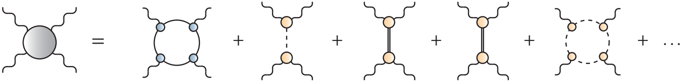
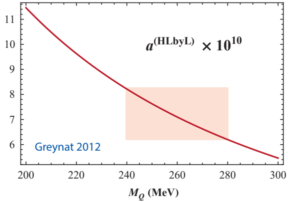


Quark loop	pseudoscalar exchange	scalar exchange	axialvector exchange	π, K loop	
2	8 ... 11	−1	2	−2	$(\times 10^{-10})$

How important is the **quark loop?**

- **Constituent quark loop** known analytically: 6 ... 8

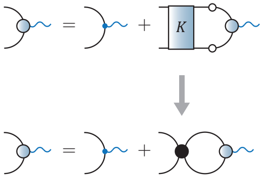


- **ENJL:** VM poles by summing up quark bubbles

Bijnens 1995

$$\gamma^\mu - \frac{1}{Q^2+m_V^2}\, t^{\mu\nu}_{QQ}\, \gamma^\nu$$

Large reduction: 2



Light-by-light scattering

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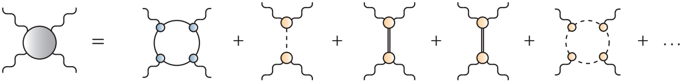
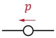


Quark loop	pseudoscalar exchange	scalar exchange	axialvector exchange	π, K loop	
2	8 ... 11	-1	2	-2	$(\times 10^{-10})$

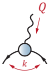
How important is the **quark loop?**

- Quark mass is not a constant:



$$S_0(p) = \frac{-i\not{p}+m}{p^2+m^2} \;\longrightarrow\; S(p) = \frac{1}{A(p^2)}\,\frac{-i\not{p}+M(p^2)}{p^2+M^2(p^2)}$$

- Quark-photon vertex is not bare:



$$\Gamma^\mu(k,Q) = \Big[i\gamma^\mu\,\Sigma_A + 2k^\mu(i\not{k}\,\Delta_A + \Delta_B)\Big] + \Big[i\sum_{j=1}^{8} f_j\,\tau_j^\mu(k,Q)\Big]$$

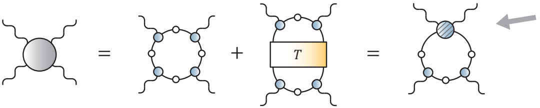
- **DSE result** for quark loop: $a_\mu = 10.7 \times 10^{-10}$
  but full Ball-Chiu vertex problematic

$A(p^2)$	$M(p^2)$	γ^μ	Γ_T^μ	$a_\mu\ [10^{-10}]$
1	0.2 GeV	1	0	10
1	$M(p^2)$	1	0	10
$A(p^2)$	$M(p^2)$	1	0	5
$A(p^2)$	$M(p^2)$	Σ_A	0	10
$A(p^2)$	$M(p^2)$	Σ_A	$k=0$	4
$A(p^2)$	$M(p^2)$	Σ_A	Full	10

Goecke, Fischer, Williams, PRD 87 (2013)

Light-by-light scattering

Goal: calculate **LbL amplitude** directly



Quark Compton vertex, enters in **Compton scattering**

GE, Fischer, PRD 87 (2013)

Two strategies:

- Calculate **quark loop,** approximate T-matrix by **meson exchanges**
  → calculate **two-photon currents:** $a_\mu^{\text{PS}} = 8.1\,(1.2)\,\times 10^{-10}$ Goecke, Fischer, Williams, PRD 83 (2011)

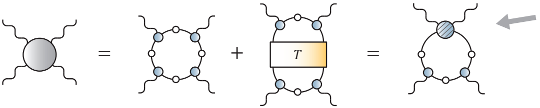
  Problem: only sum (without approximations!) is gauge invariant; how to deal with gauge artifacts?

- Calculate **quark loop + T-matrix** explicitly:
  gauge invariant, but more difficult

Either way, we first need to understand **structure of LbL amplitude!**

Light-by-light scattering

Goal: calculate **LbL amplitude** directly



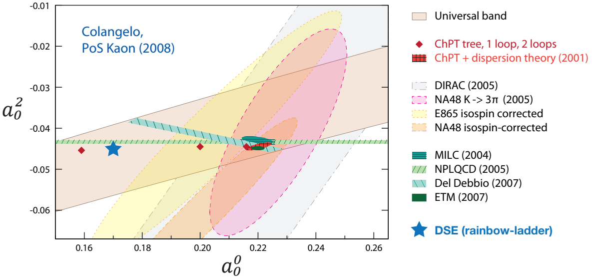
Quark Compton vertex, enters in **Compton scattering**

GE, Fischer, PRD 87 (2013)

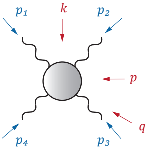
Similar:
$\pi\pi$ **scattering**

Bicudo et al., PRD 65 (2002),

Cotanch, Maris, PRD 66 (2002)



LbL amplitude



3 independent momenta: $p = p_2 + p_3$, $q = p_3 + p_1$, $k = p_1 + p_2$

6 Lorentz invariants:
$p^2, \quad q^2, \quad k^2, \quad p \cdot q, \quad p \cdot k, \quad q \cdot k$

⇒ Calculating LbL amplitude means determining **136 FFs** which depend on **6 variables** . . .

$$\Gamma^{\mu\nu\rho\sigma}(p,q,k) = \sum_{i=1}^{136} f_i(\dots)\ \tau_i^{\mu\nu\rho\sigma}(p,q,k)$$

Any constraints?

- Amplitude is **Bose-symmetric.** With symmetric tensor basis:
  ⇒ FFs only depend on symmetric combinations of variables
- Amplitude is **gauge invariant** ⇒ transverse to $p_1{}^\mu$, $p_2{}^\nu$, $p_3{}^\rho$ and $p_4{}^\sigma$.

  ⇒ should be separated into "gauge part" and transverse part:

  $$\Gamma = \Gamma_\perp + \Gamma_G$$

  $\Gamma_\perp$: physical, transverse part (41 tensors)

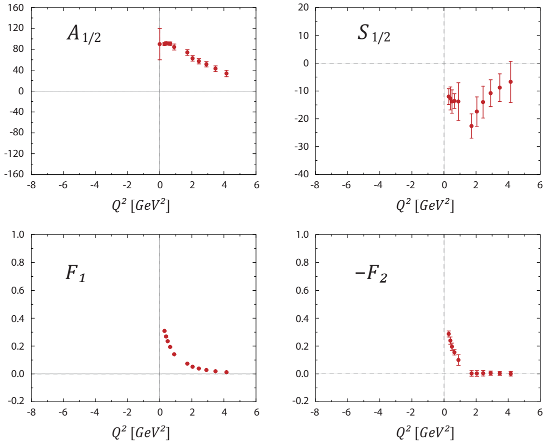
  Γ_G: vanishes by gauge invariance

  ⇒ With **'minimal' tensor basis** free of kinematic singularities:
  FFs free of kinematic singularities and zeros,
  only singularities are **physical poles and cuts** ⇒ 'simple'
- But this is not automatic ⇒ **choice of basis matters!**

Only physical poles and cuts?

Example: $\gamma N \to N^*(1535)$ **helicity amplitudes:**

CLAS data: Aznauryan et al., PRC 80 (2009)



Helicity amplitudes

in $[10^{-3} GeV^{-1/2}]$

Form factors:

no kinematic constraints

Only physical poles and cuts?

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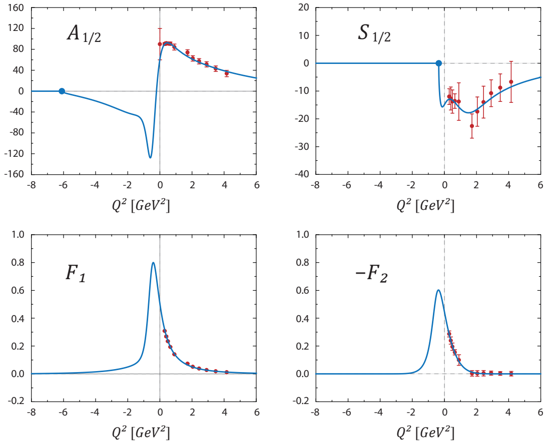


Helicity amplitudes

in $[10^{-3} GeV^{-1/2}]$

kinematic zeros at

$Q^2 = -(m_R \pm m)^2$

Form factors:

no kinematic constraints

Toy parametrization with "ρ bump"

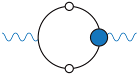
GE, 1602.03462

Ramalho & Tsushima, PRD 84 (2011)

Gauge invariance

Simplest example: **hadronic vacuum polarization**

$$\Pi^{\mu\nu}(Q) = \int d^4x\; e^{iQ\cdot x}\,\langle 0|\,\mathsf{T} j^\mu(x) j^\nu(0)\,|0\rangle \;=\; a(Q^2)\,\delta^{\mu\nu} + b(Q^2)\,Q^\mu Q^\nu$$



- **Analyticity** ⇒ a, b cannot have poles at $Q^2 = 0$ (intermediate massless particle, but $\Pi^{\mu\nu}$ = 1PI)
- **Transversality** ⇒ Ward identity: $Q^\mu \Pi^{\mu\nu}(Q) = 0 \quad \Rightarrow \quad a = -b\,Q^2$ (not $b = -a/Q^2$!!!)

$$\Rightarrow \quad \Pi^{\mu\nu}(Q) = \underbrace{\Pi(Q^2)\, t^{\mu\nu}_{QQ}}_{\text{transverse part}} \qquad\qquad t^{\mu\nu}_{QQ} = Q^2\,\delta^{\mu\nu} - Q^\mu Q^\nu$$

Gauge invariance

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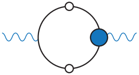

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What if calculation breaks gauge invariance?

- 1-loop in dim. reg: $\widetilde{\Pi}(Q^2) = 0$
- 1-loop with cutoff: $\widetilde{\Pi}(Q^2) \sim \Lambda^2 \neq 0$
  quadratic divergence, but only in gauge part!

Gauge invariance

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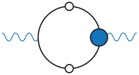

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- 1-loop with cutoff: $\widetilde{\Pi}(Q^2) \sim \Lambda^2 \neq 0$
  quadratic divergence, but only in gauge part!

- Different basis?

$$= \left[\Pi(Q^2) + \frac{\widetilde{\Pi}(Q^2)}{Q^2} \right] t^{\mu\nu}_{QQ} \; + \; \frac{\widetilde{\Pi}(Q^2)}{Q^2}\, Q^\mu Q^\nu$$

⇒ bad: kinematic singularities

- Must project onto full **transverse + gauge basis**, subtract gauge part.
  Also necessary if gauge invariance is violated by **more** than cutoff (e.g., incomplete calculation)!

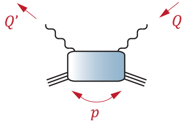
Compton scattering

Tensor decomposition for CS amplitude:
GE & Fischer, PRD 87 (2013), GE & Ramalho, in preparation

$$\bar{u}(p_f)\left(\underbrace{\frac{c_1}{m^4}\,t^{\mu\alpha}_{Q'p}\,t^{\alpha\nu}_{pQ}+\frac{c_2}{m^2}\,t^{\mu\nu}_{Q'Q}+\dots}_{\text{transverse part, 18 tensors}}\;+\underbrace{g_1\,\delta^{\mu\nu}+\dots}_{\text{gauge part, 14 tensors}}\right)u(p_i)$$

transverse part, 18 tensors

gauge part, 14 tensors



Tarrach's construction: Tarrach, Nuovo Cim. A28 (1975)

write down all possible tensors (# = 32),
apply transversality constraints,
divide and subtract poles ⟹ 18 transverse tensors

	#Q		#Q		#Q		#Q
T_1	2	T_7	3	T_{13}	7	T_{19}	4
T_2	4	T_8	3	T_{14}	5	T_{20}	5
T_3	2	T_9	5	T_{15}	7	T_{21}	3
T_4	4	T_{10}	3	T_{16}	5		
T_5	6	T_{11}	5	T_{17}	3		
T_6	5	T_{12}	5	T_{18}	3		

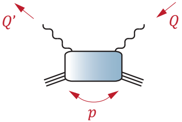
⟹ **minimal basis:**

- transverse
- no kinematic singularities
- **permutation-group singlets**
- **minimal powers in photon momenta**

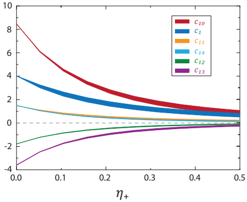
Compton scattering

Tensor decomposition for CS amplitude:
GE & Fischer, PRD 87 (2013), GE & Ramalho, in preparation

$$\bar{u}(p_f)\left(\underbrace{\frac{c_1}{m^4}\,t^{\mu\alpha}_{Q'p}\,t^{\alpha\nu}_{pQ}+\frac{c_2}{m^2}\,t^{\mu\nu}_{Q'Q}+\dots}_{\text{transverse part, 18 tensors}}\;+\underbrace{g_1\,\delta^{\mu\nu}+\dots}_{\text{gauge part, 14 tensors}}\right)u(p_i)$$

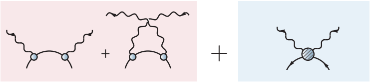


Transverse **Compton FFs** depend on 4 variables, but in T+G basis they scale with **single variable!**



GE, FBS 57 (2016)

At hadronic level: Born terms alone **not** gauge invariant

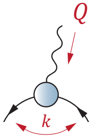


$$\Gamma^{\mu\nu} = \underbrace{\Gamma^{\mu\nu}_{\text{Born}} + \Gamma^{\mu\nu}_{\text{WTI}}}_{\text{gauge invariant}} + \Gamma^{\mu\nu}_{\perp}$$

GE & Fischer, PRD 87 (2013)

Use offshell nucleon-photon vertex, project onto G+T basis
⇒ violation of gauge invariance mostly affects gauge part, **transverse CFFs** only weakly sensitive, still good prediction!

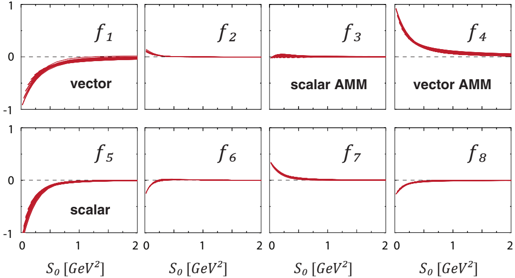
Quark-photon vertex



$$\Gamma^\mu(k,Q) = \left[i\gamma^\mu \, \Sigma_A + 2k^\mu (i\not{k}\, \Delta_A + \Delta_B) \right] + \left[i \sum_{j=1}^{8} f_j \, \tau_j^\mu(k,Q) \right]$$

Ball-Chiu vertex,
determined by WTI,
depends only on
quark propagator
Ball, Chiu, PRD 22 (1980)

Transverse part,
contains dynamics:
VM poles & cuts
Kizilersu et al, PRD 92 (1995),
GE, Fischer, PRD 87 (2013)



τ_1^μ	$= t_{QQ}^{\mu\nu}\, \gamma^\nu$	**vector**
τ_2^μ	$= t_{QQ}^{\mu\nu}\, k\cdot Q\, \frac{i}{2}[\gamma^\nu, \not{k}]$	
τ_3^μ	$= \frac{i}{2}\,[\gamma^\mu, \not{Q}]$	**scalar AMM**
τ_4^μ	$= \frac{1}{6}\,[\gamma^\mu, \not{k}, \not{Q}]$	**vector AMM**
τ_5^μ	$= t_{QQ}^{\mu\nu}\, ik^\nu$	**scalar**
τ_6^μ	$= t_{QQ}^{\mu\nu}\, k^\nu \not{k}$	
τ_7^μ	$= t_{Qk}^{\mu\nu}\, k\cdot Q\, \gamma^\nu$	
τ_8^μ	$= t_{Qk}^{\mu\nu}\, \frac{i}{2}\,[\gamma^\nu, \not{k}]$	

$$t_{ab}^{\mu\nu} := a\cdot b\, \delta^{\mu\nu} - b^\mu a^\nu$$

GE, Acta Phys. Polon. Supp. 7 (2014)

LbL amplitude

Minimal T+G basis:

$$\Gamma = \Gamma_\perp + \Gamma_G$$

$\Gamma_\perp$: physical, transverse part (41 tensors)

Γ_G: vanishes by gauge invariance

- transverse
- no kinematic singularities
- permutation-group singlets
- minimal powers in photon momenta

⇒ FFs have no kinematic singularities or zeros, **only physical poles and cuts**

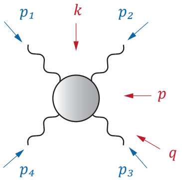
⇒ effectively scale with single variable: **simple**

⇒ broken gauge invariance affects G, not T: even incomplete calculations are **predictive**

Existing examples of such bases:

- ▹ **1** vector boson: scalar or fermion vertex (nucleon-photon, quark-photon, quark-gluon, . . .)
- ▹ **2** vector bosons: HVP, 2-photon currents, Compton scattering
- ▹ **3** vector bosons: three-gluon vertex
- ▹ **4** vector bosons: **LbL, four-gluon vertex??**

Structure of the LbL amplitude



3 independent momenta:

$$p = p_2 + p_3$$
$$q = p_3 + p_1$$
$$k = p_1 + p_2$$

6 Lorentz invariants:

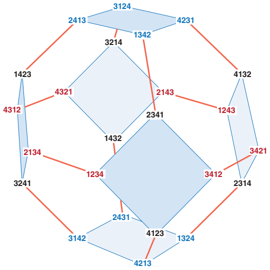
$$p^2, \quad q^2, \quad k^2, \quad p\cdot q, \quad p\cdot k, \quad q\cdot k$$

Bose symmetry:

$$\Gamma^{\mu\nu\rho\sigma}(p,q,k) = \sum_{i=1}^{136} f_i(\dots)\; \tau_i^{\mu\nu\rho\sigma}(p,q,k)$$

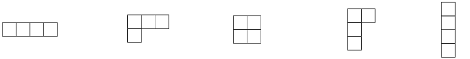
$\stackrel{!}{=}$ **symmetric**

S4 multiplets



- Arrange the 24 permutations of ψ_{1234} into **multiplets:**

Singlet	**Triplets**	**Doublets**	**Antitriplets**	**Antisinglet**
$\mathcal{S}$	$\mathcal{T}_i^+ = \begin{bmatrix} \bullet \\ \bullet \\ \bullet \end{bmatrix}$	$\mathcal{D}_j = \begin{bmatrix} \bullet \\ \bullet \end{bmatrix}$	$\mathcal{T}_i^- = \begin{bmatrix} \bullet \\ \bullet \\ \bullet \end{bmatrix}$	$\mathcal{A}$



- **6 Lorentz invariants** form singlet $\mathcal{S}_0$, doublet $\mathcal{D}$, triplet $\mathcal{T}^+$

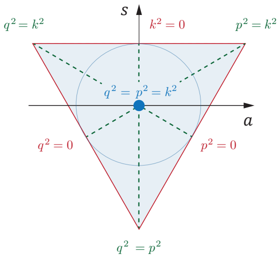
Phase space

- **Singlet:** symmetric variable, carries overall scale:

$$\mathcal{S}_0 = \frac{p^2+q^2+k^2}{4} = \frac{p_1^2+p_2^2+p_3^2+p_4^2}{4}$$

- **Doublet:** $\mathcal{D} = \begin{bmatrix} a \\ s \end{bmatrix}$

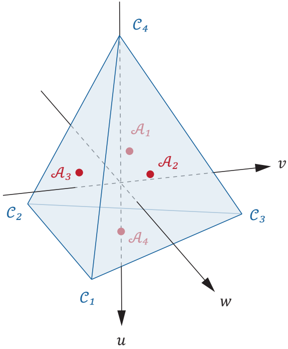
  Mandelstam triangle,
  2-photon poles (pion, scalar, axialvector, ...)



GE, Fischer, Heupel, PRD 92 (2015)

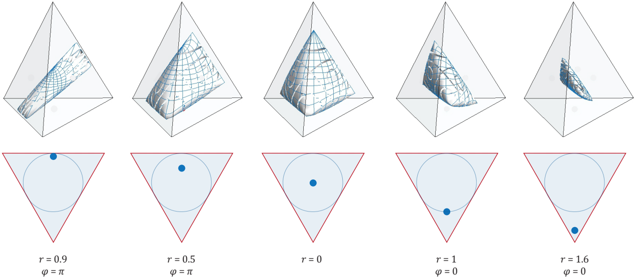
- **Triplet:** $\mathcal{T} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$

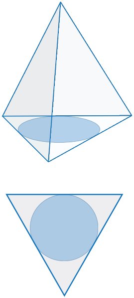
  tetrahedron bounded by $p_i^2 = 0$,
  vector-meson poles



Phase space

Fixed doublet variables $\Longrightarrow$ complicated geometric object inside tetrahedron:



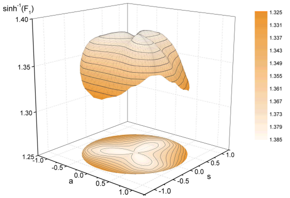


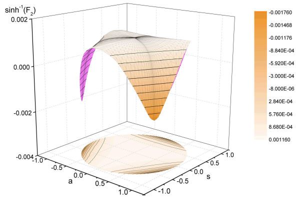
relevant for g-2

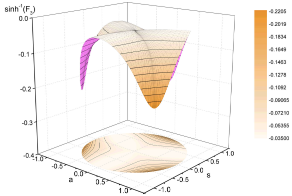
Phase space

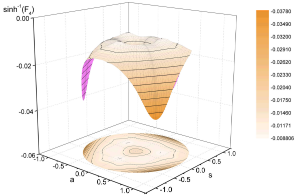
Example: **three-gluon vertex** from its DSE

GE, Williams, Alkofer, Vujinovic, PRD 89 (2014)

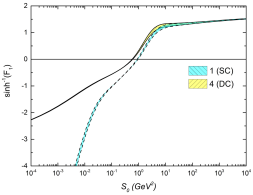








- four tensor structures
- 3 variables:
  1 singlet, 1 doublet
- Variation in **doublet** almost negligible, all four “form factors” scale with **singlet**



Tensor basis

- construct all possible **multiplets** from generic seed elements: 138 elements, but only **136 independent**

n	Seed	#	Multiplet type
0	$\delta^{\mu\nu}\delta^{\rho\sigma}$	3	$\mathcal{S}$, $\mathcal{D}_1$
2	$\delta^{\mu\nu}\, k^\rho\, k^\sigma$	6	$\mathcal{S}$, $\mathcal{D}_1$, $\mathcal{T}_1^+$
	$\delta^{\mu\nu}\, p^\rho\, p^\sigma$	12	$\mathcal{S}$, $\mathcal{D}_1$, $\mathcal{D}_2$, $\mathcal{T}_1^\pm$, $\mathcal{A}$
	$\delta^{\mu\nu}\, p^\rho\, q^\sigma$	12	$\mathcal{S}$, $\mathcal{D}_1$, $\mathcal{T}_1^+$, $\mathcal{T}_2^\pm$
	$\delta^{\mu\nu}\, p^\rho\, k^\sigma$	24	$\mathcal{S}$, $\mathcal{D}_1$, $\mathcal{D}_2$, $\mathcal{T}_1^\pm$, $\mathcal{T}_2^\pm$, $\mathcal{T}_3^\pm$, $\mathcal{A}$
4	$p^\mu\, p^\nu\, p^\rho\, p^\sigma$	3	$\mathcal{S}$, $\mathcal{D}_1$
	$p^\mu\, p^\nu\, q^\rho\, q^\sigma$	6	$\mathcal{S}$, $\mathcal{D}_1$, $\mathcal{T}_1^-$
	$p^\mu\, p^\nu\, k^\rho\, k^\sigma$	10	$\mathcal{S}$, $(\mathcal{D}_1,)$ $\mathcal{D}_2$, $\mathcal{T}_1^\pm$, $\mathcal{A}$
	$p^\mu\, q^\nu\, k^\rho\, k^\sigma$	12	$\mathcal{S}$, $\mathcal{D}_1$, $\mathcal{T}_1^+$, $\mathcal{T}_2^\pm$
	$p^\mu\, p^\nu\, p^\rho\, k^\sigma$	24	$\mathcal{S}$, $\mathcal{D}_1$, $\mathcal{D}_2$, $\mathcal{T}_1^\pm$, $\mathcal{T}_2^\pm$, $\mathcal{T}_3^\pm$, $\mathcal{A}$
	$p^\mu\, p^\nu\, q^\rho\, k^\sigma$	24	$\mathcal{S}$, $\mathcal{D}_1$, $\mathcal{D}_2$, $\mathcal{T}_1^\pm$, $\mathcal{T}_2^\pm$, $\mathcal{T}_3^\pm$, $\mathcal{A}$

Tensor basis

- construct all possible **multiplets** from generic seed elements:
138 elements, but only **136 independent**

$\delta^{\mu\nu}\,\delta^{\rho\sigma}$ $\Longrightarrow$ 3 permutations

$\delta^{\mu\nu}\,q_i^\rho\,q_j^\sigma$ $\Longrightarrow$ 3^2 x 6 = 54 permutations

$q_i^\mu\,q_j^\nu\,q_k^\rho\,q_l^\sigma$ $\Longrightarrow$ 3^4 = 81 permutations

Orthonormalize momenta: $p,\,q,\,k \to n_1,\,n_2,\,n_3$
From three momenta we can define **axialvector,** must appear **in pairs** to ensure positive parity:

$$v^\mu = \varepsilon^{\mu\alpha\beta\gamma}\,n_1^\alpha\,n_2^\beta\,n_3^\gamma$$

$v^\mu\,v^\nu\,v^\rho\,v^\sigma$ $\Longrightarrow$ **1 permutation**

$v^\mu\,v^\nu\,n_i^\rho\,n_j^\sigma$ $\Longrightarrow$ 3^2 x 6 = 54 permutations

$n_i^\mu\,n_j^\nu\,n_k^\rho\,n_l^\sigma$ $\Longrightarrow$ 3^4 = 81 permutations

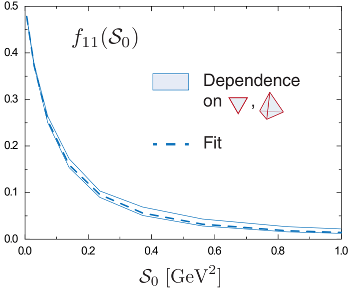
$\delta^{\mu\nu}\,\delta^{\rho\sigma}$ is linearly dependent:

$$\delta^{\mu\nu} = v^\mu\,v^\nu + \sum_{i=1}^{3} n_i^\mu\,n_i^\nu$$

n	Seed	#	Multiplet type
0	$\delta^{\mu\nu}\delta^{\rho\sigma}$	3	$\mathcal{S},\,\mathcal{D}_1$
2	$\delta^{\mu\nu}\,k^\rho\,k^\sigma$	6	$\mathcal{S},\,\mathcal{D}_1,\,\mathcal{T}_1^+$
	$\delta^{\mu\nu}\,p^\rho\,p^\sigma$	12	$\mathcal{S},\,\mathcal{D}_1,\,\mathcal{D}_2,\,\mathcal{T}_1^\pm,\,\mathcal{A}$
	$\delta^{\mu\nu}\,p^\rho\,q^\sigma$	12	$\mathcal{S},\,\mathcal{D}_1,\,\mathcal{T}_1^+,\,\mathcal{T}_2^\pm$
	$\delta^{\mu\nu}\,p^\rho\,k^\sigma$	24	$\mathcal{S},\,\mathcal{D}_1,\,\mathcal{D}_2,\,\mathcal{T}_1^\pm,\,\mathcal{T}_2^\pm,\,\mathcal{T}_3^\pm,\,\mathcal{A}$
4	$p^\mu\,p^\nu\,p^\rho\,p^\sigma$	3	$\mathcal{S},\,\mathcal{D}_1$
	$p^\mu\,p^\nu\,q^\rho\,q^\sigma$	6	$\mathcal{S},\,\mathcal{D}_1,\,\mathcal{T}_1^-$
	$p^\mu\,p^\nu\,k^\rho\,k^\sigma$	10	$\mathcal{S},\,(\mathcal{D}_1,)\,\mathcal{D}_2,\,\mathcal{T}_1^\pm,\,\mathcal{A}$
	$p^\mu\,q^\nu\,k^\rho\,k^\sigma$	12	$\mathcal{S},\,\mathcal{D}_1,\,\mathcal{T}_1^+,\,\mathcal{T}_2^\pm$
	$p^\mu\,p^\nu\,p^\rho\,k^\sigma$	24	$\mathcal{S},\,\mathcal{D}_1,\,\mathcal{D}_2,\,\mathcal{T}_1^\pm,\,\mathcal{T}_2^\pm,\,\mathcal{T}_3^\pm,\,\mathcal{A}$
	$p^\mu\,p^\nu\,q^\rho\,k^\sigma$	24	$\mathcal{S},\,\mathcal{D}_1,\,\mathcal{D}_2,\,\mathcal{T}_1^\pm,\,\mathcal{T}_2^\pm,\,\mathcal{T}_3^\pm,\,\mathcal{A}$

Tensor basis

- construct all possible **multiplets** from generic seed elements: 138 elements, but only **136 independent**
- transversality not yet implemented, but **quark loop** projected on this basis already behaves as expected: **singlet FFs scale with** $\mathcal{S}_0$!



n	Seed	#	Multiplet type
0	$\delta^{\mu\nu}\,\delta^{\rho\sigma}$	3	$\mathcal{S}$, $\mathcal{D}_1$
2	$\delta^{\mu\nu}\,k^\rho\,k^\sigma$	6	$\mathcal{S}$, $\mathcal{D}_1$, $\mathcal{T}_1^+$
	$\delta^{\mu\nu}\,p^\rho\,p^\sigma$	12	$\mathcal{S}$, $\mathcal{D}_1$, $\mathcal{D}_2$, $\mathcal{T}_1^\pm$, $\mathcal{A}$
	$\delta^{\mu\nu}\,p^\rho\,q^\sigma$	12	$\mathcal{S}$, $\mathcal{D}_1$, $\mathcal{T}_1^+$, $\mathcal{T}_2^\pm$
	$\delta^{\mu\nu}\,p^\rho\,k^\sigma$	24	$\mathcal{S}$, $\mathcal{D}_1$, $\mathcal{D}_2$, $\mathcal{T}_1^\pm$, $\mathcal{T}_2^\pm$, $\mathcal{T}_3^\pm$, $\mathcal{A}$
4	$p^\mu\,p^\nu\,p^\rho\,p^\sigma$	3	$\mathcal{S}$, $\mathcal{D}_1$
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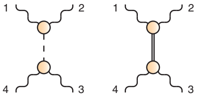
	$p^\mu\,p^\nu\,k^\rho\,k^\sigma$	10	$\mathcal{S}$, $(\mathcal{D}_1,)$ $\mathcal{D}_2$, $\mathcal{T}_1^\pm$, $\mathcal{A}$
	$p^\mu\,q^\nu\,k^\rho\,k^\sigma$	12	$\mathcal{S}$, $\mathcal{D}_1$, $\mathcal{T}_1^+$, $\mathcal{T}_2^\pm$
	$p^\mu\,p^\nu\,p^\rho\,k^\sigma$	24	$\mathcal{S}$, $\mathcal{D}_1$, $\mathcal{D}_2$, $\mathcal{T}_1^\pm$, $\mathcal{T}_2^\pm$, $\mathcal{T}_3^\pm$, $\mathcal{A}$
	$p^\mu\,p^\nu\,q^\rho\,k^\sigma$	24	$\mathcal{S}$, $\mathcal{D}_1$, $\mathcal{D}_2$, $\mathcal{T}_1^\pm$, $\mathcal{T}_2^\pm$, $\mathcal{T}_3^\pm$, $\mathcal{A}$

Transverse basis

- Same argument: 43 elements, but only **41 independent**
- Need to work out **transversality conditions** $p_1^\mu \, \mathcal{M}^{\mu\nu\rho\sigma} = 0, \quad \dots \quad p_4^\sigma \, \mathcal{M}^{\mu\nu\rho\sigma} = 0$ **without** introducing kinematic singularities, then construct **singlets** with lowest momentum powers: hard (both analytically and symbolically)
- Simpler: find **41 tensors** that are transverse, analytic & have lowest mass dimension:

$$\varepsilon_{ab}^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta} \, a^\alpha \, b^\beta \qquad\qquad \Longrightarrow \qquad \varepsilon_{12}^{\mu\nu} \, \varepsilon_{34}^{\rho\sigma}$$
$$t_{ab}^{\mu\nu} = a \cdot b \, \delta^{\mu\nu} - b^\mu \, a^\nu \qquad\qquad\qquad\qquad t_{12}^{\mu\nu} \, t_{34}^{\rho\sigma}$$

Dimension 4,
3 permuations each:
1 singlet, 1 doublet



- To construct singlets, combine them with **momentum multiplets:**

$$\mathcal{S}_1 = \mathcal{S}$$
$$\mathcal{S}_2 = \mathcal{D} \cdot \mathcal{D}$$
$$\mathcal{S}_3 = (\alpha \, \mathcal{D} * \mathcal{D} + \beta \, \mathcal{T} * \mathcal{T}) \cdot \mathcal{D}$$

Ambiguity: two doublets with same mass dimension

	$(2,0)$	$(1,1)$	$(0,2)$	$(3,0)$	$(2,1)$	$(1,2)$	$(0,3)$
Singlet	$\mathcal{D}\cdot\mathcal{D}$		$\mathcal{T}\cdot\mathcal{T}$	$\mathcal{D}\cdot(\mathcal{D}*\mathcal{D})$		$\mathcal{D}\cdot(\mathcal{T}*\mathcal{T})$	$\mathcal{T}\cdot(\mathcal{T}\vee\mathcal{T})$
Doublet	$\mathcal{D}*\mathcal{D}$		$\mathcal{T}*\mathcal{T}$	$(\mathcal{D}\cdot\mathcal{D})\,\mathcal{D}$		$\mathcal{D}*(\mathcal{T}*\mathcal{T})$ $(\mathcal{T}\cdot\mathcal{T})\,\mathcal{D}$	
Triplet		$\mathcal{T}\vee\mathcal{D}$	$\mathcal{T}\vee\mathcal{T}$		$\mathcal{T}\vee(\mathcal{D}*\mathcal{D})$ $(\mathcal{D}\cdot\mathcal{D})\,\mathcal{T}$	$\mathcal{T}\vee(\mathcal{T}\vee\mathcal{D})$	$\mathcal{T}\vee(\mathcal{T}\vee\mathcal{T})$ $(\mathcal{T}\cdot\mathcal{T})\,\mathcal{T}$
Antitriplet		$\mathcal{T}\wedge\mathcal{D}$			$\mathcal{T}\wedge(\mathcal{D}*\mathcal{D})$	$\mathcal{T}\wedge(\mathcal{T}\vee\mathcal{D})$	$\mathcal{T}\wedge(\mathcal{T}\vee\mathcal{T})$
Antisinglet				$\mathcal{D}\wedge(\mathcal{D}*\mathcal{D})$		$\mathcal{D}\wedge(\mathcal{T}*\mathcal{T})$	

Transverse basis

- In total: 7 seed elements produce **41 singlets** with minimal mass dimensions:
  GE, Fischer, Heupel, PRD 92 (2015)

n	Seed element	#	Multiplets	$n=4$	$n=6$	$n=8$	$n=10$	$n=12$
4	$t^{\mu\nu}_{12}\, t^{\rho\sigma}_{34}$	3	$\mathcal{S},\, \mathcal{D}_1$	1	1	1		
	$\varepsilon^{\mu\nu}_{12}\, \varepsilon^{\rho\sigma}_{34}$	3	$\mathcal{S},\, \mathcal{D}_1$	1	1	1		
6	$\varepsilon^{\mu\lambda\alpha}_{1} t^{\alpha\nu}_{22}\, \varepsilon^{\rho\lambda\beta}_{3} t^{\beta\sigma}_{44}$	12	$\mathcal{S},\, \mathcal{D}_1,\, \mathcal{D}_2,\, \mathcal{T}^+_2,\, \mathcal{T}^-_2,\, \mathcal{A}$		1	3	5	3
	$t^{\mu\nu}_{12}\, t^{\rho\lambda}_{33}\, t^{\lambda\sigma}_{44}$	6	$\mathcal{S},\, \mathcal{D}_1,\, \mathcal{T}^+_1$		1	2	3	
	$t^{\mu\nu}_{12}\, t^{\rho\lambda}_{31}\, t^{\lambda\sigma}_{24}$	7	$\mathcal{S},\, \mathcal{T}^+_1,\, \mathcal{T}^-_1$		1	1	3	2
	$\varepsilon^{\mu\nu}_{12}\, \varepsilon^{\rho\lambda}_{31}\, t^{\lambda\sigma}_{24}$	7	$\mathcal{D}_2,\, \mathcal{T}^+_2,\, \mathcal{T}^-_1,\, \mathcal{T}^-_2$			2	5	
8	$t^{\mu\nu}_{12}\, t^{\rho\alpha}_{31}\, t^{\alpha\beta}_{12}\, t^{\beta\sigma}_{24}$	3	$\mathcal{S},\, \mathcal{D}_1,\, \mathcal{T}^+_1$			1	2	
	Total	41		2	5	11	18	5

- 7 equivalent seeds in dispersive approach:
  Colangelo, Hoferichter, Procura, Stoffer, JHEP 09 (2015)

$$\varepsilon^{\mu\nu}_{12}\, \varepsilon^{\rho\sigma}_{34}, \quad t^{\mu\nu}_{12}\, t^{\rho\sigma}_{34}, \quad t^{\mu\nu}_{12}\, t^{\rho\lambda}_{31}\, t^{\lambda\sigma}_{14}, \quad t^{\mu\nu}_{12}\, t^{\rho\lambda}_{31}\, t^{\lambda\sigma}_{24}$$

$$t^{\mu\nu}_{12}\, t^{\rho}_{312}\, t^{\sigma}_{412}, \quad t^{\mu}_{134}\, t^{\nu\alpha\beta}_{2}\, t^{\rho\alpha\lambda}_{3}\, t^{\sigma\beta\lambda}_{4}, \quad (t^{\mu\alpha}_{14}\, t^{\beta\nu}_{32} - t^{\mu\beta}_{13}\, t^{\alpha\nu}_{42})\, t^{\rho\alpha\lambda}_{3}\, t^{\sigma\beta\lambda}_{4}$$

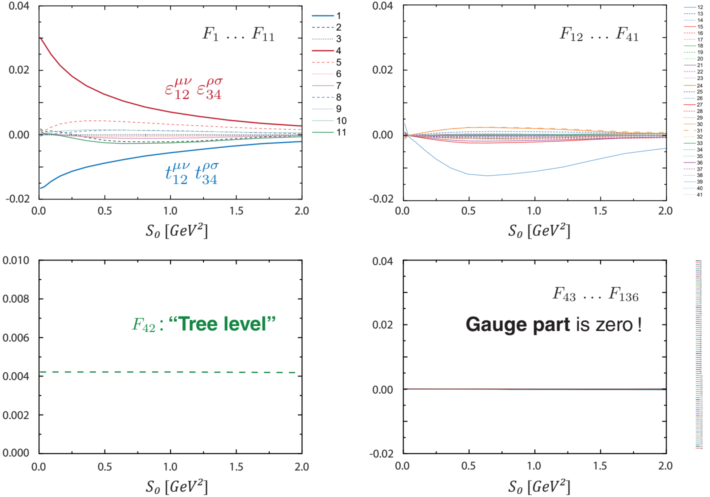
- However, to determine **quark loop** we need **gauge part** too: only poor constraints here

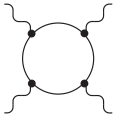
$$\Pi^{\mu\nu}(Q) = \Pi(Q^2)\, t^{\mu\nu}_{QQ} + \widetilde{\Pi}(Q^2)\, \delta^{\mu\nu}$$

$$= \left[\Pi(Q^2) + \frac{\widetilde{\Pi}(Q^2)}{Q^2}\right] t^{\mu\nu}_{QQ} + \frac{\widetilde{\Pi}(Q^2)}{Q^2}\, Q^\mu Q^\nu$$

Quark loop with m_q = const

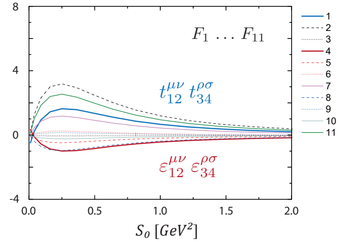
LbL amplitude in NJL model: S_0 dependence for fixed doublet & triplet variables

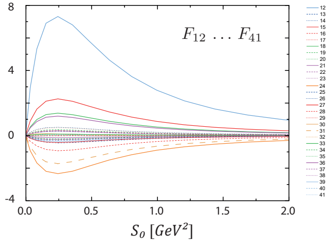


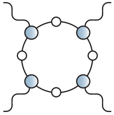


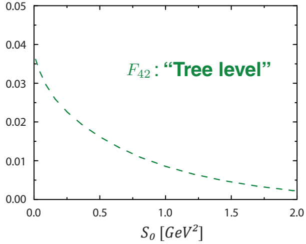
Quark loop from DSE

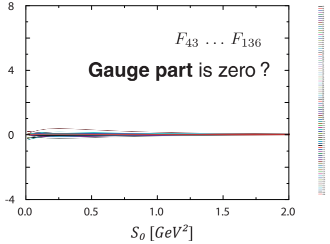
LbL amplitude from DSE: S_0 dependence for fixed doublet & triplet variables







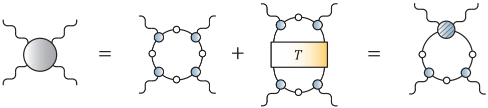




If quark loop breaks gauge invariance, the effects are **small!**

Summary

- Understanding **structure of the LbL amplitude** is important for pinning down g-2
- **Microscopic** decomposition:



  - ▹ revisit transversality constraints to derive **T+G** basis
    ⇒ pin down **quark loop**
  - ▹ calculate **sum of both diagrams** (gauge invariant),
    in tandem with Compton scattering
  - ▹ calculate **two-photon form factors**
    ⇒ missing effects in T-matrix?

Thank you!

- Best DSE values so far:
  Mini-review: GE, Fischer, Heupel, Williams, 1411.7876, AIP Conf. Proc. 1701 (2016)

$$a_\mu^{\text{HVP}} = 676 \times 10^{-10} \qquad a_\mu^{\text{QL}} = 10.7\,(2) \times 10^{-10} \qquad a_\mu^{\text{PS}} = 8.1\,(1.2) \times 10^{-10}$$

Backup slides

Electron vs. muon g-2

$a_e\ [10^{-10}]$

Exp:	11 596 521.81		
QED:	11 596 521.71	(0.09)	Cs
	.81	(0.08)	Rb
EW:	0.00		
Hadronic:	0.02		
SM:	11 596 521.73	(0.09)	Cs
	.83	(0.08)	Rb

$a_\mu\ [10^{-10}]$

Exp:	11 659 208.9	(6.3)
QED:	11 658 471.9	(0.0)
EW:	15.3	(0.2)
Hadronic:		
• VP (LO+HO)	685.1	(4.3)
• LBL	10.5	(2.6)
SM:	11 659 182.8	(4.9)
Diff:	26.1	(8.0)

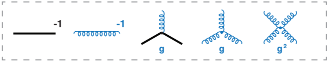
Bijnens, Prades, Mod. Phys. Lett. A22 (2007)
Jegerlehner, Nyffeler, Phys. Rept. 477 (2009)
Hagiwara et al., J. Phys. G 38 (2011)

... to Dyson-Schwinger equations

QCD's classical action:

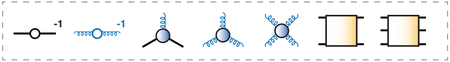
$$S = \int d^4x \left[\bar\psi \left(\partial\!\!\!/ + igA\!\!\!/ + m \right) \psi + \tfrac{1}{4} F^{a}_{\mu\nu} F^{\mu\nu}_{a} \right]$$

=

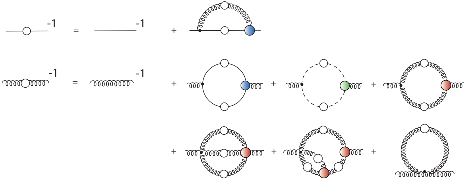


Quantum "effective action":

$$\int \mathcal{D}[\psi, \bar\psi, A]\, e^{-S} = e^{-\Gamma}$$



DSEs = quantum equations of motion:
instead of calculating n-point functions directly,
derive eqs. of motion for them from path integral



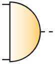
infinitely many coupled eqs.,
in practice truncations:
model / neglect higher
n-point functions to obtain
closed system

For reviews see:

Roberts, Williams, Prog. Part. Nucl. Phys. 33 (1994),
Alkofer, von Smekal, Phys. Rept. 353 (2001)
Fischer, J. Phys. G32 (2006)

Mesons

- The **pion** plays special role in hadron physics:
  quark-antiquark **bound state** ⟺ Goldstone boson of **spontaneous chiral symmetry breaking**



$$= \gamma_5 \left(f_1 + f_2 \not{P} + f_3 \not{q} + f_4 [\not{q}, \not{P}] \right) \otimes \text{Color} \otimes \text{Flavor}$$

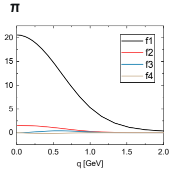
most general Dirac-Lorentz structure,
Lorentz-invariant dressing functions:

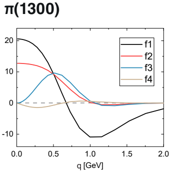
$$f_i = f_i(q^2, q\cdot P, P^2 = -m^2)$$

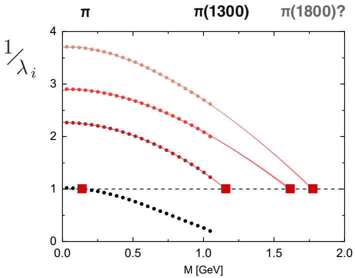
⟹ pion is made of **s waves** and **p waves!**
(relative momentum ~ orbital angular momentum)

- Eigenvalue spectrum of BS kernel:
  Holl, Krassnigg, Roberts, PRC 70 (2004)

$$K\,\psi_i = \lambda_i(P^2)\,\psi_i\,, \qquad \lambda_i \xrightarrow{P^2 \to -m_i^2} 1$$

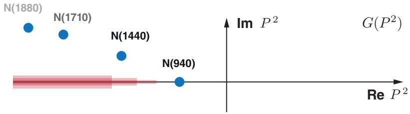




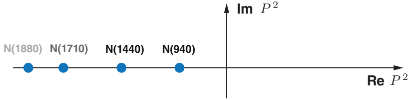


Resonances?

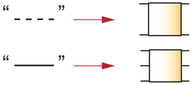
Branch cuts & widths generated by
meson-baryon interactions: Roper $\to N\pi$, etc.



Without them: **bound states without widths**

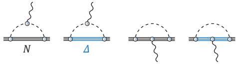


Difficult to implement at **quark-gluon level:**
complicated topologies beyond rainbow-ladder

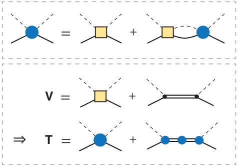


Different phenomenological pictures
how this could happen:

- **'pion-cloud effects'** affect masses and form factors in light-quark region



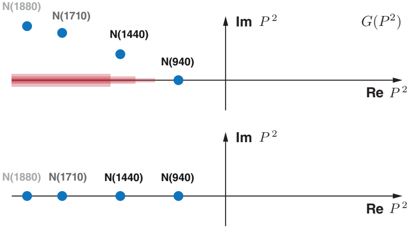
- **dynamical generation of resonances:** start with 'bare' seed, hadronic interactions produce new poles



e.g.
Suzuki et al.,
PRL 104 (2010)

- Three-quark vs. five-quark / molecular components

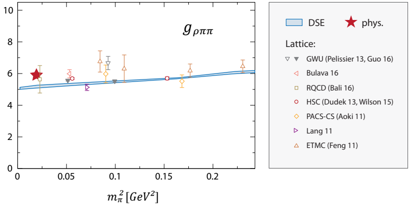
So what does it mean?

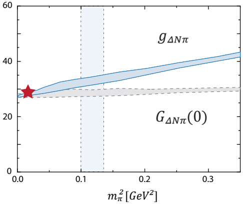


Note: **'bound states without widths'** doesn't mean that $\rho \to \pi\pi$, $\Delta \to N\pi, \dots$ decays are zero!!

Results favor 'mild' scenario:

- spectrum generated by quark-gluon interactions
- meson-baryon effects would merely shift poles into complex plane
- Effects on masses? Scale set by f_π , but pion-cloud affects f_π too so only 'non-trivial effects' visible
- Will be interesting to study **transition form factors**





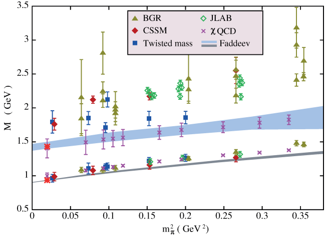
Mader, GE, Blank, Krassnigg, PRD 84 (2011),

GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, 1606.09602

Structure properties

- **Current-mass evolution** of Roper similar to nucleon. Lattice?

  GE, Fischer, Sanchis-Alepuz, 1607.05748



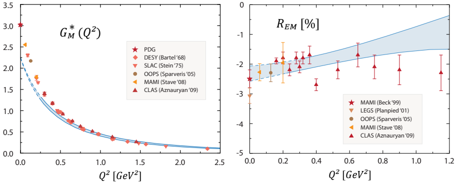
- All signatures of 1st **radial excitation:** partial-wave content, zero crossing

- **Roper transition form factors** in qualitative agreement with experiment

  Segovia et al., PRL 115 (2015)

- $\gamma N \to \Delta$ **transition form factors:**

  GE, Nicmorus, PRD 85 (2012)



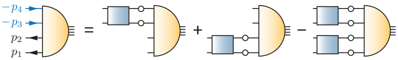
Discrepancies mainly in **magnetic dipole (G_M^*):** "Core + 25% pion cloud"

Electric quadrupole ratio
small & negative, encodes deformation.
No pion cloud necessary: **OAM** from **p waves!**

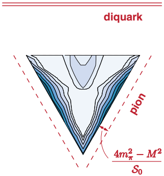
First three-body results similar
Alkofer, GE, Sanchis-Alepuz, Williams, Hyp. Int. 234 (2015)

Tetraquarks *are* resonances

- **Light scalar mesons** σ, κ, a_0, f_0 as **tetraquarks:** solution of four-body equation reproduces mass pattern
  GE, Fischer, Heupel, PLB 753 (2016)

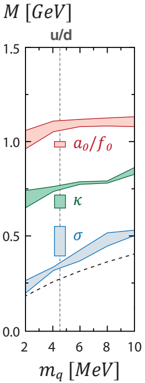


BSE dynamically generates **meson poles** in wave function, drive σ mass from 1.5 GeV to ~350 MeV

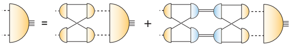


Four quarks rearrange to **"meson molecule"**

Tetraquarks are "dynamically generated **resonances**" (but from the quark level!)

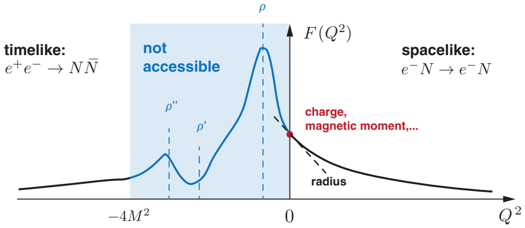


- Similar in **meson-meson / diquark-antidiquark** approximation (analogue of quark-diquark for baryons) Heupel, GE, Fischer, PLB 718 (2012)

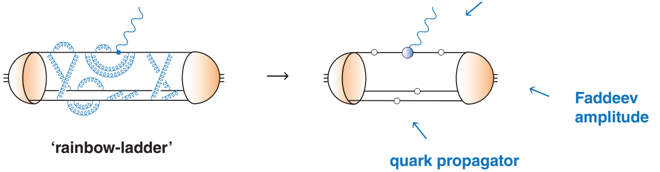


Form factors

Sketch of a generic electromagnetic form factor:

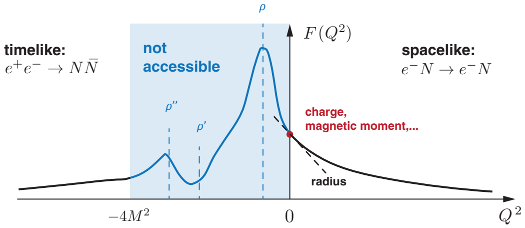


How can we calculate this from the **quark level?**

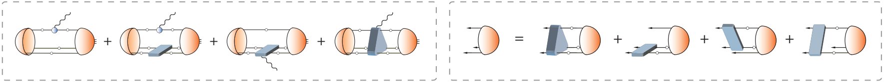


Form factors

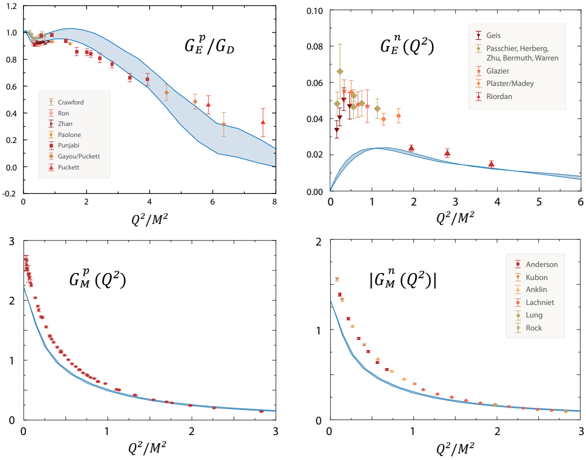
Sketch of a generic electromagnetic form factor:



Microscopic decomposition of current matrix element:
satisfies **electromagnetic gauge invariance,** consistent with baryon's Faddeev equation



Nucleon em. form factors



Three-body results:
all ingredients calculated,
model dependence shown
by bands GE, PRD 84 (2011)

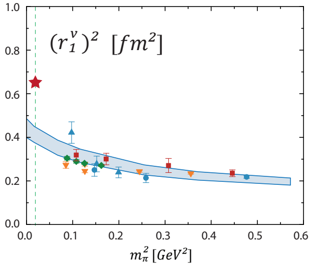
- **electric proton form factor:** consistent with data, possible zero crossing
- **magnetic form factors:** missing pion effects at low Q^2
- Similar for axial & ps. FFs, Δ elastic and $N\to\Delta\gamma$ transition
  GE, Fischer, EPJ A 48 (2012),
  Sanchis-Alepuz et al., PRD 87 (2013),
  Alkofer et al., Hyp. Int. 234 (2015)

$\Rightarrow$ **“quark core without pion-cloud effects”**

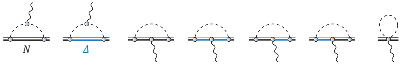
Nucleon em. form factors

Nucleon charge radii:

isovector (p-n) Dirac (F1) radius

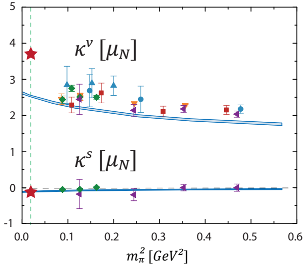


- **Pion-cloud effects** missing (⇒ divergence!), agreement with lattice at larger quark masses.



Nucleon magnetic moments:

isovector (p-n), isoscalar (p+n)



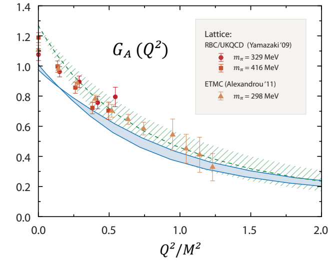
- **But:** pion-cloud **cancels** in κ^s ⇔ **quark core**

Exp: $\kappa^s = -0.12$

Calc: $\kappa^s = -0.12(1)$ **!!**

GE, PRD 84 (2011)

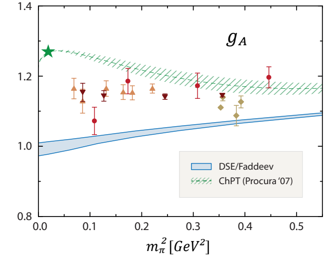
Axial form factors

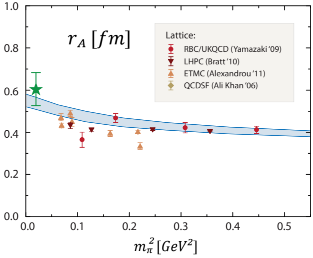


- looks like magnetic form factors:
  missing structure at low $Q^2 \Rightarrow g_A$ **too small**
- **Timelike meson poles:**
  a_1 in G_A, π & $\pi(1300)$ in G_P , $G_{\pi NN}$
- **Goldberger-Treiman relation**
  reproduced for **all** quark masses:

$$G_A(0) = \frac{f_\pi}{M_N}\, G_{\pi NN}(0)$$

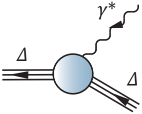
GE & Fischer, EPJ A 48 (2012)





Δ electromagnetic FFs

Almost no experimental information since Δ unstable: $\Delta \to N\pi$



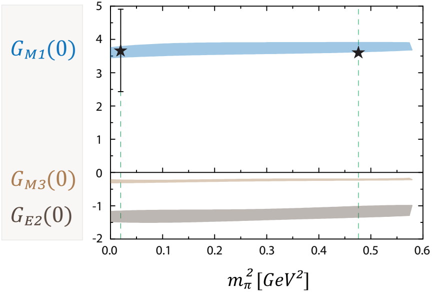
Magnetic moment $\mu_\Delta \sim 3.5$ with large errors (Δ^+).
But Ω^- (spin 3/2, *sss*) is stable w.r.t strong interaction,
magnetic moment $|\mu_\Omega| = 3.6(1)$. Accidental?

$$J^{\mu,\rho\sigma}(P,Q) = i\,\mathbb{P}^{\rho\alpha}(P_f)\left[\left(F_1^\star\,\gamma^\mu - F_2^\star\,\frac{\sigma^{\mu\nu}Q^\nu}{2M_\Delta}\right)\delta^{\alpha\beta} - \left(F_3^\star\,\gamma^\mu - F_4^\star\,\frac{\sigma^{\mu\nu}Q^\nu}{2M_\Delta}\right)\frac{Q^\alpha Q^\beta}{4M_\Delta^2}\right]\mathbb{P}^{\beta\sigma}(P_i)$$

Form factors at Q^2=0:

$G_{E_0}(0) = e_\Delta$ charge
$G_{E_2}(0) = \mathcal{Q}$ electric quadrupole moment

$G_{M_1}(0) = \mu_\Delta$ magnetic dipole moment
$G_{M_3}(0) = \mathcal{O}$ magnetic octupole moment



almost quark-mass independent,
match Ω^- magnetic moment
Nicmorus, GE, Alkofer, PRD 82 (2010)

Three-body results similar (except G_{M_3})
Sanchis-Alepuz, Alkofer, Williams, PRD 87 (2013)