The HVP contribution to a_μ from full lattice QCD

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HPQCD collaboration

HC2NP, Tenerife
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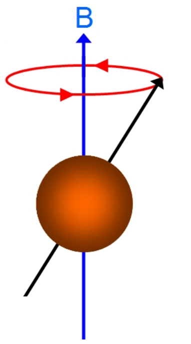
Work with: Bipasha Chakraborty, Jonna Koponen, Peter Lepage and others ..

Using the Darwin (9600 core) Sandybridge/infiniband cluster at Cambridge, part of STFC's DiRAC HPC facility

Muon anomalous magnetic moment

$$\vec{\mu} = g\frac{e}{2m}\vec{S} \qquad a_\mu = \frac{g-2}{2}$$



Measure using polarised muons circulating in E and B fields. At a momentum where $\beta \times E$ terms cancel, difference between precession and cyclotron frequencies:

$$\omega_a = -\frac{e}{m}a_\mu B$$

BNL result:

$$a_\mu^{expt} = 11659208.9(6.3) \times 10^{-10}$$

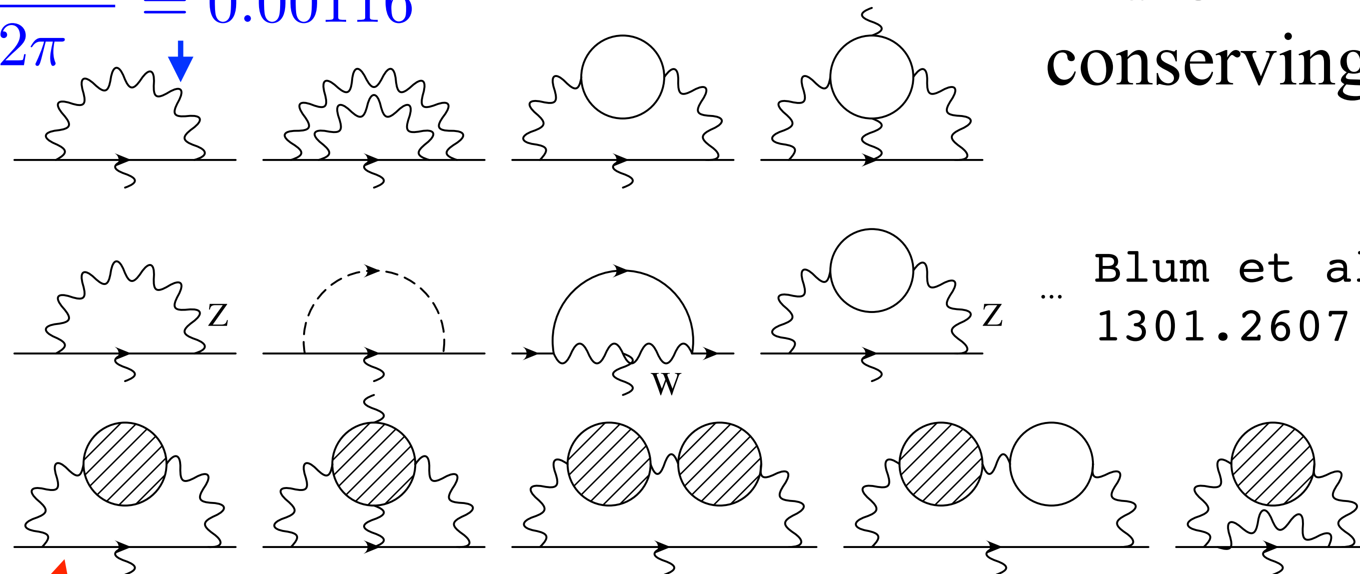
E989 (FNAL) will reduce exptl uncty to 1.6, starting 2017

Standard Model theory expectations

Contributions from QED, EW and QCD interactions. QED dominates. QCD contribs start at α_{QED}^2

$$\frac{\alpha_{QED}}{2\pi} = 0.00116$$

flavour and CP conserving



Blum et al, 1301.2607

LO Hadronic vacuum polarisation (HVP) dominates uncertainty

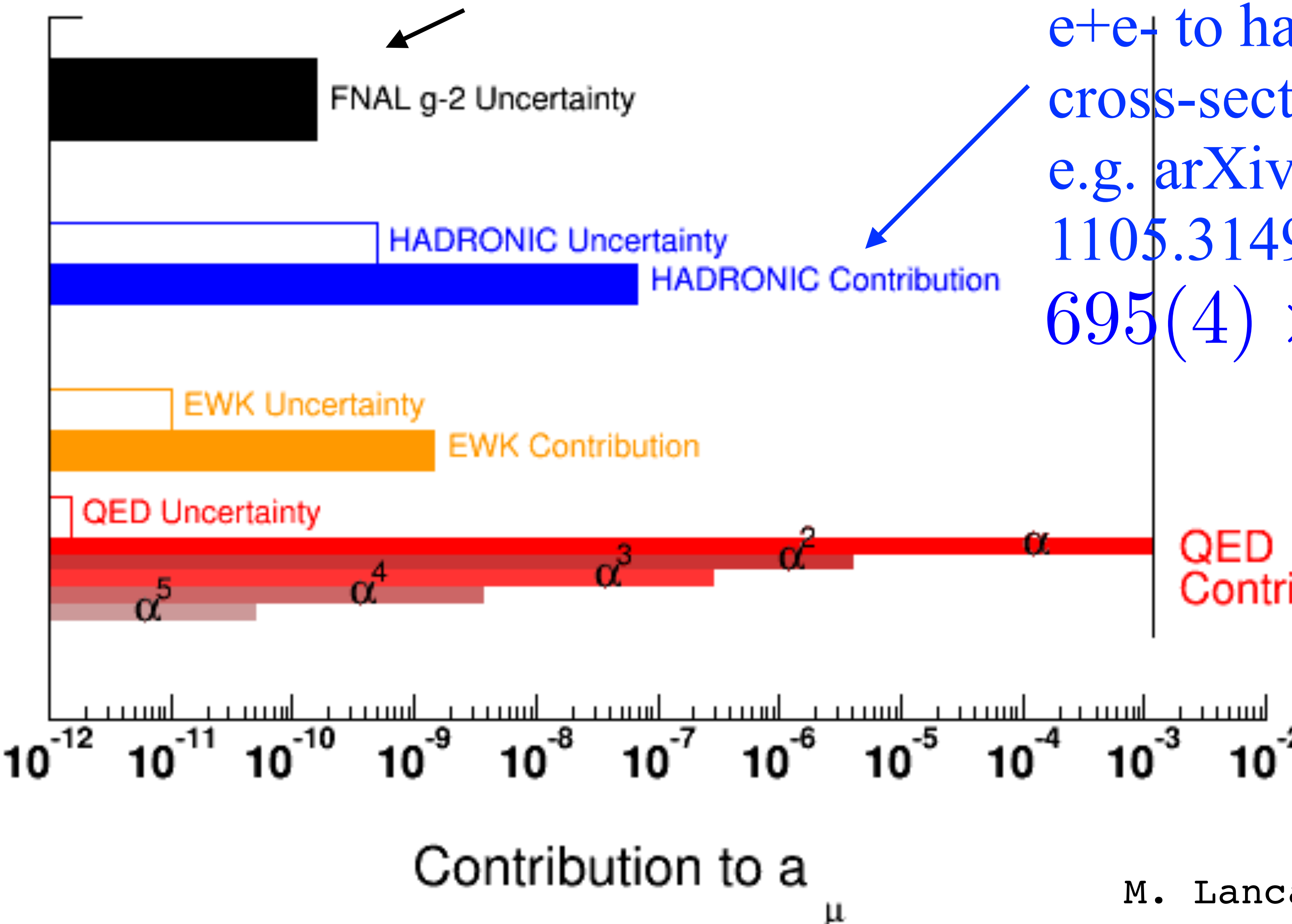
$$a_\mu^{QED} = 11658471.885(4) \times 10^{-10}$$

$$a_\mu^{EW} = 15.4(2) \times 10^{-10}$$

$$a_\mu^{E821} = 11659208.9(6.3) \times 10^{-10}$$

Uncertainty dominated by that from hadronic contribns

planned



value from e+e- to hadrons cross-section e.g. arXiv: 1105.3149

$$695(4) \times 10^{-10}$$

M. Lancaster PPAP2015

Hadronic contributions

$$a_\mu^{expt} - a_\mu^{QED} - a_\mu^{EW} = 721.7(6.3) \times 10^{-10}$$

$$= a_\mu^{HVP} + a_\mu^{HOHVP} + a_\mu^{HLBL} + a_\mu^{new\, physics}$$

Focus on lowest order hadronic vacuum polarisation, so assume:

$$a_\mu^{HLbL} = 10.5(2.6) \times 10^{-10}$$

$$a_\mu^{HOHVP} = -8.85(9) \times 10^{-10}$$ ← NLO+NNLO

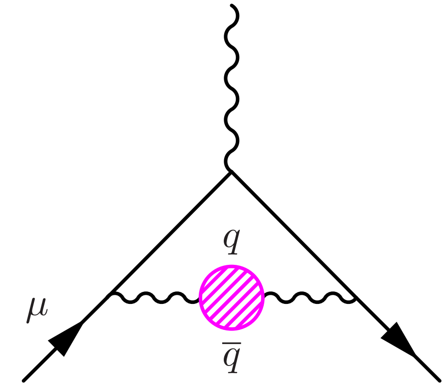
Kurz et al, 1403.6400

$$a_\mu^{HVP, no\, new\, physics} = 719.8(6.8) \times 10^{-10}$$

**compare 1105.3149 - discrepancy $24.9(8.0) \times 10^{-10}$

Lattice calculation of HVP

Analytically continue to Euclidean q^2.



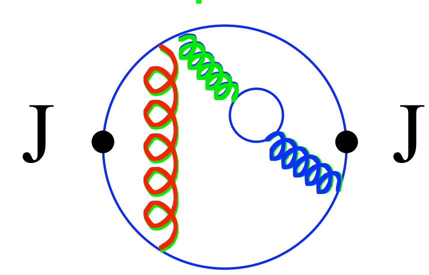
Blum, hep-lat/0212018

$$a_\mu^{HVP,i} = \frac{\alpha}{\pi}\int_0^\infty dq^2 f(q^2)(4\pi\alpha e_i^2)\hat{\Pi}_i(q^2)$$

connected contribution for flavour i

$f(q^2)$ divergent function with scale set by m_μ

$$\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$$



HPQCD method: time-moments of vector JJ correlators give expansion around $q^2=0$

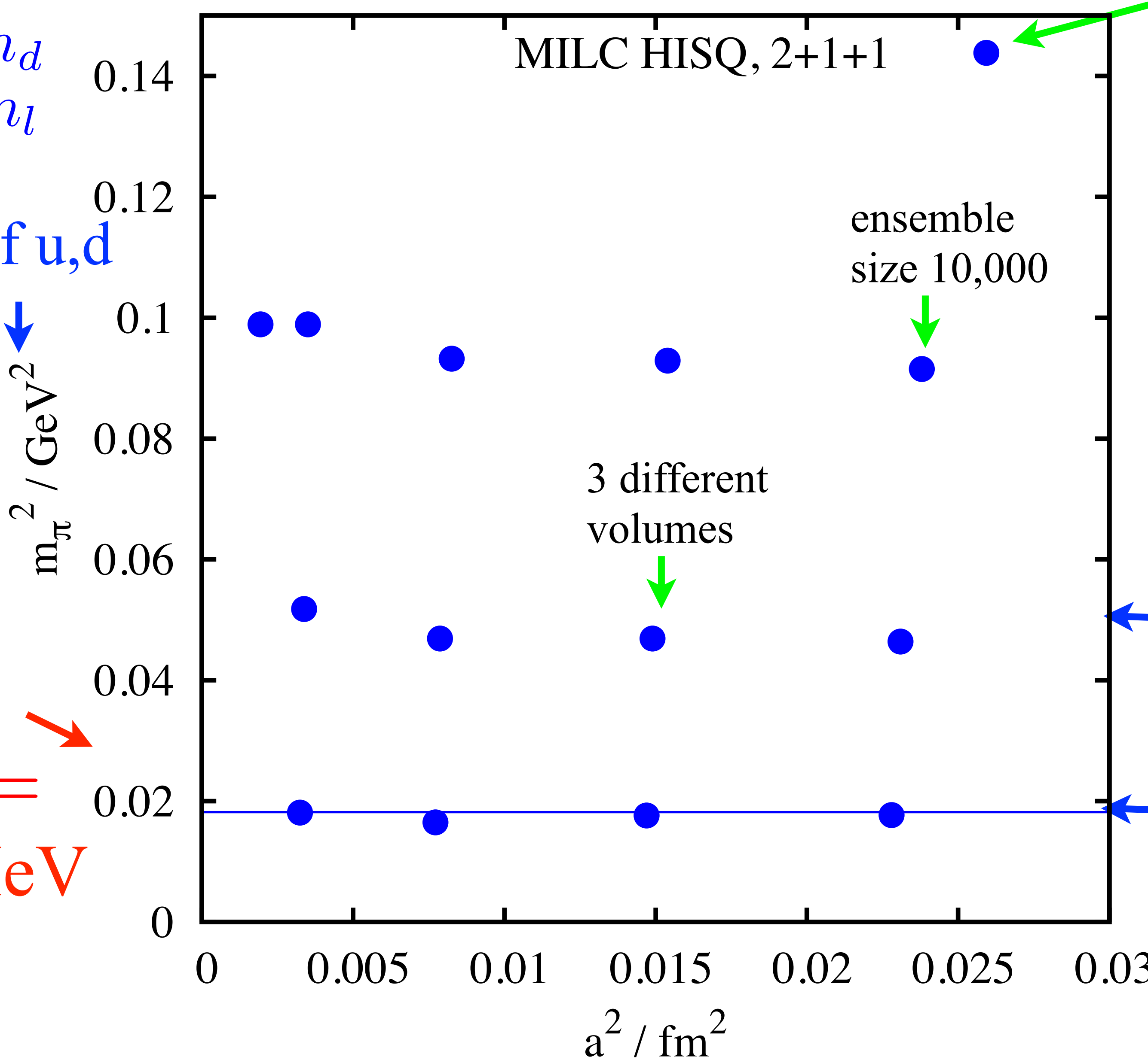
$$G_n \equiv \sum_{t,\vec{x}} t^n Z_V^2 \langle J^j(\vec{x},t) J^j(0)\rangle \qquad \hat{\Pi}(q^2) = \sum_{k=1}^{\infty} q^{2k}\Pi_k$$

$$\Pi_k = (-1)^{k+1}\frac{G_{2k+2}}{(2k+2)!}$$

replace with [2,2] Padé

HPQCD, 1403.1778

Working with staggered quarks in new '2nd generation' calculations - further improved gluon and quark actions



$m_u = m_d = m_l$

mass of u,d quarks

real world $m_{\pi^0} = 135$ MeV

"2nd generation" lattices inc. c quarks in sea

HISQ = Highly improved staggered quarks - very accurate discretisation

E.Follana, et al, HPQCD, hep-lat/0610092.

$m_{u,d} \approx m_s/10$

$m_{u,d} \approx m_s/27$

Volume:

$m_\pi L > 3$

STRANGE contribution

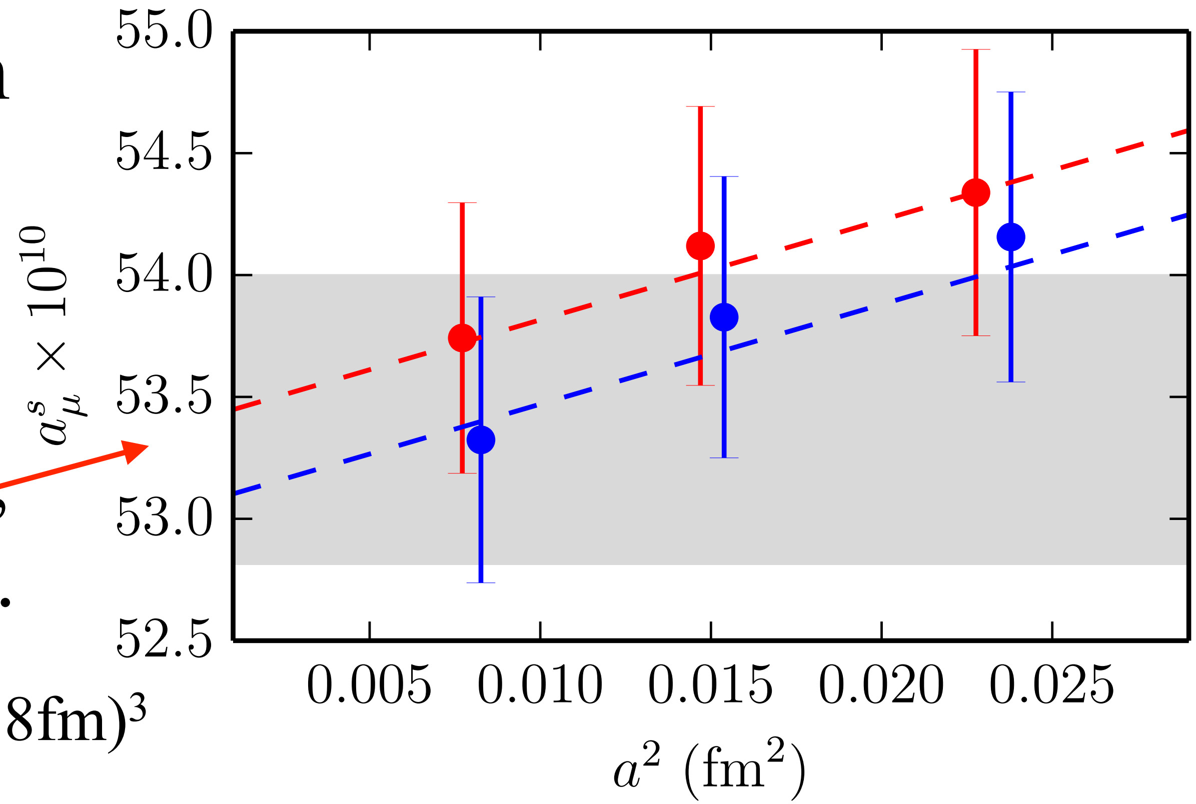
HPQCD 1403.1778

HISQ valence quarks on MILC 2+1+1 HISQ configs. Local J_v - nonpert. Z_v.

multiple a (fixed by w_0), m_l (inc. phys.), volumes.

Tune s from η_s

up to $(5.8\text{fm})^3$

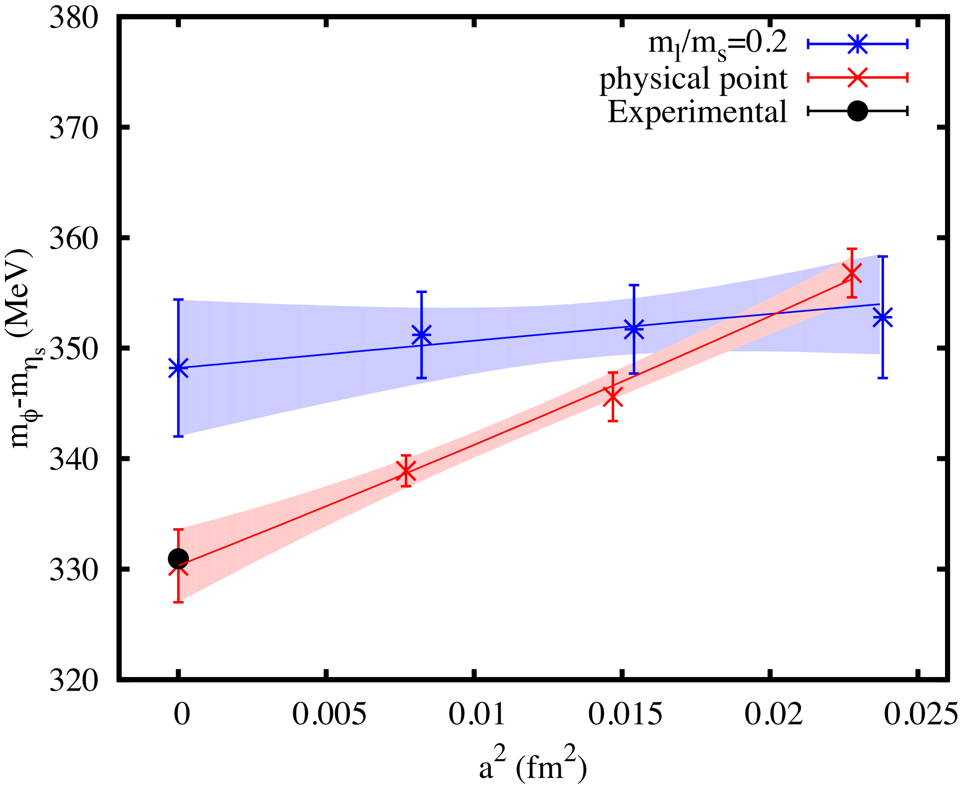


	a_μ^s
Uncertainty in lattice spacing (w_0, r_1):	1.0%
Uncertainty in Z_V:	0.4%
Monte Carlo statistics:	0.1%
$a^2 \to 0$ extrapolation:	0.1%
QED corrections:	0.1%
Quark mass tuning:	0.0%
Finite lattice volume:	$< 0.1\%$
Padé approximants:	$< 0.1\%$
Total:	1.1%

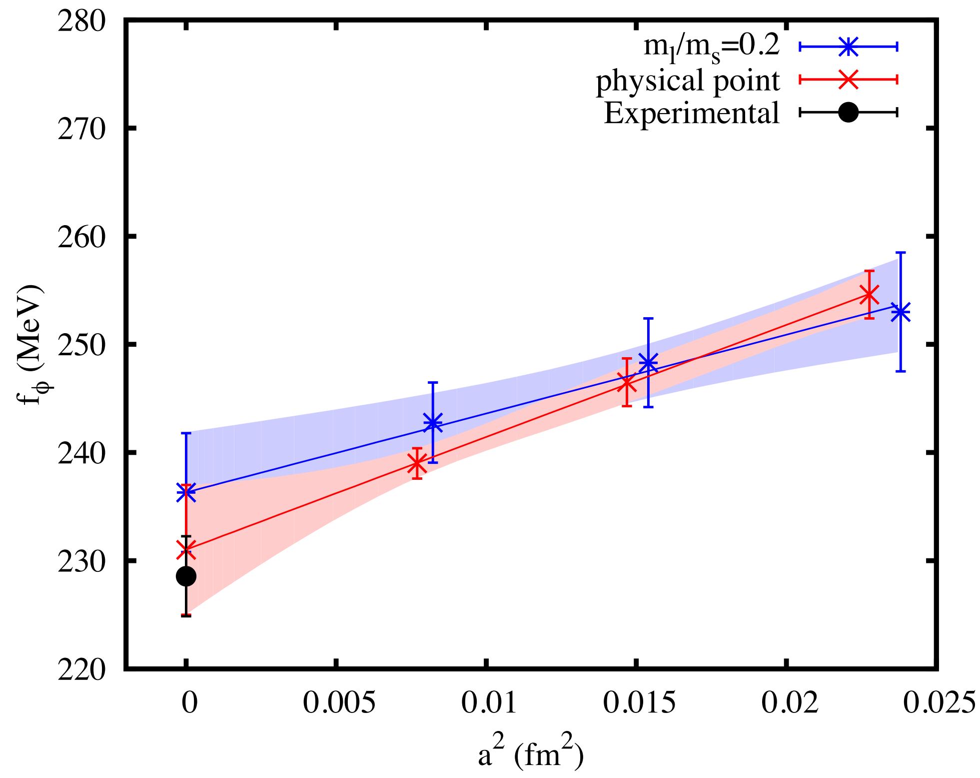
$$a^s_{\mu,\text{lat}} = a^s_\mu \times (1 + c_{a^2}(a\Lambda_{\text{QCD}}/\pi)^2 + c_{\text{sea}}\delta x_{\text{sea}} + c_{\text{val}}\delta x_{\text{val}})$$

$$a_\mu^{HVP,s} = 53.41(59) \times 10^{-10}$$

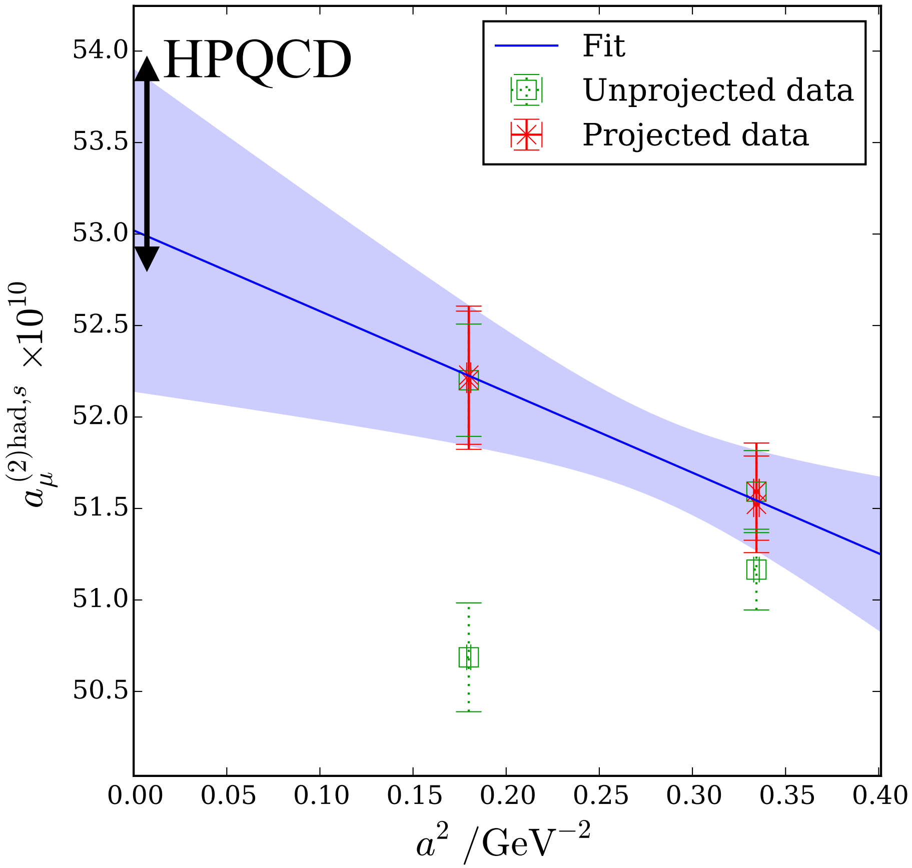
Check mass and decay constant of ϕ from these correlators against expt

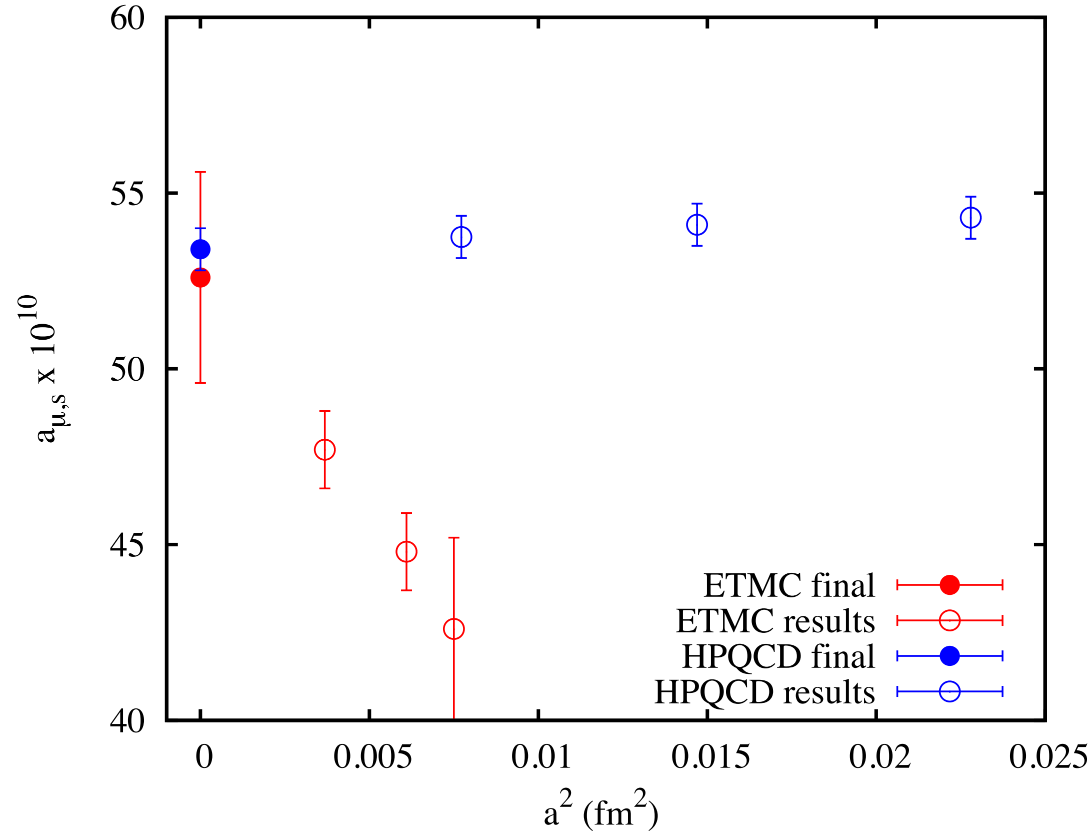


Tests large time behaviour of correlator



New results from other formalisms provide good check





RBC/UKQCD domain wall

ETMC twisted mass

$$a_\mu^{HVP,s} = 53.1(9) \times 10^{-10}$$

1607.01767

$$a_\mu^{HVP,s} = 53(3) \times 10^{-10}$$

From $R_{e^+e^-}$ we estimate $a_\mu^{HVP,s} <\approx 55 \times 10^{-10}$

CHARM contribution

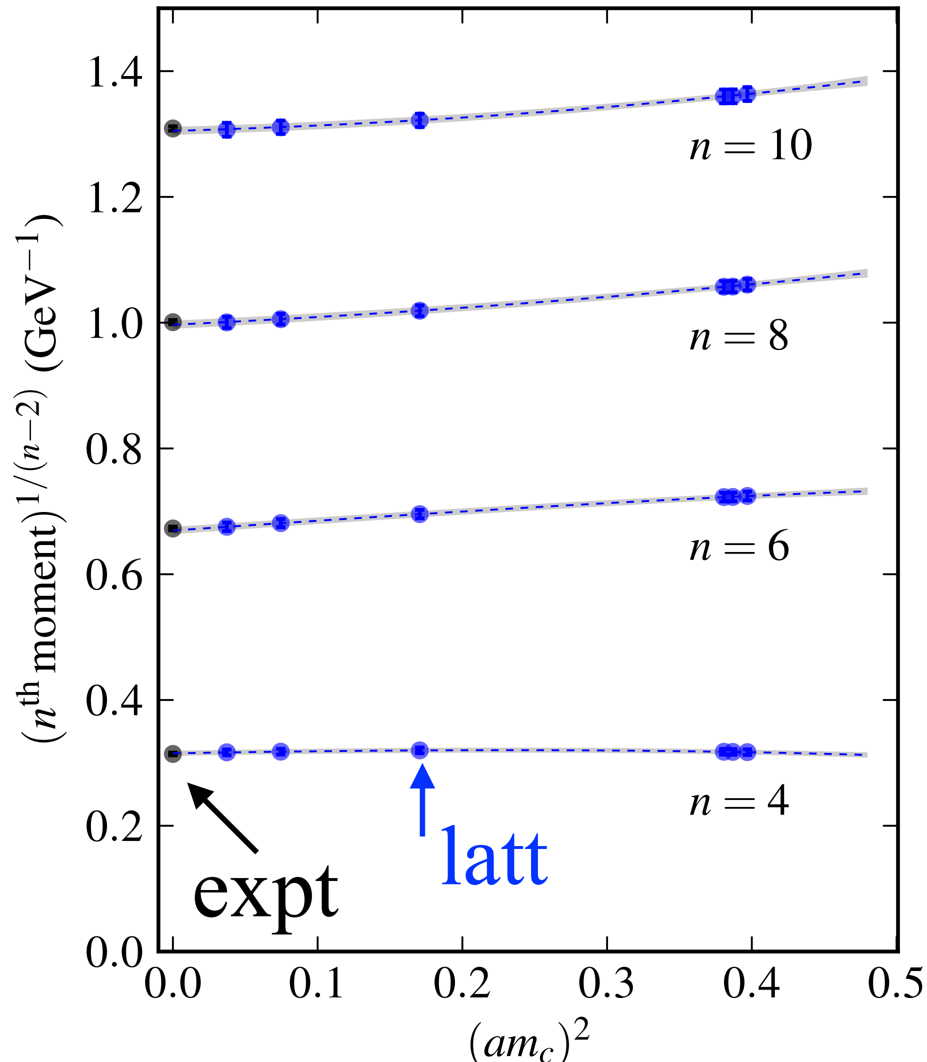
HPQCD 1004.4285, 1208.2855

Part of the set of calculations that gave

$$m_c, M(J/\psi) - M(\eta_c), \Gamma(J/\Psi \to e^+e^-), \Gamma(J/\psi \to \eta_c\gamma)$$

Used HISQ valence quarks on MILC 2+1 asqtad configs. Z_V from contnm QCD pert. th.

Extrapolation to physical point allows us to compare directly to moments from e+e- expt. in charm region



$$a_\mu^{HVP,c} = 14.4(4) \times 10^{-10}$$

HPQCD 1403.1778

preliminary result on 2+1+1 agrees

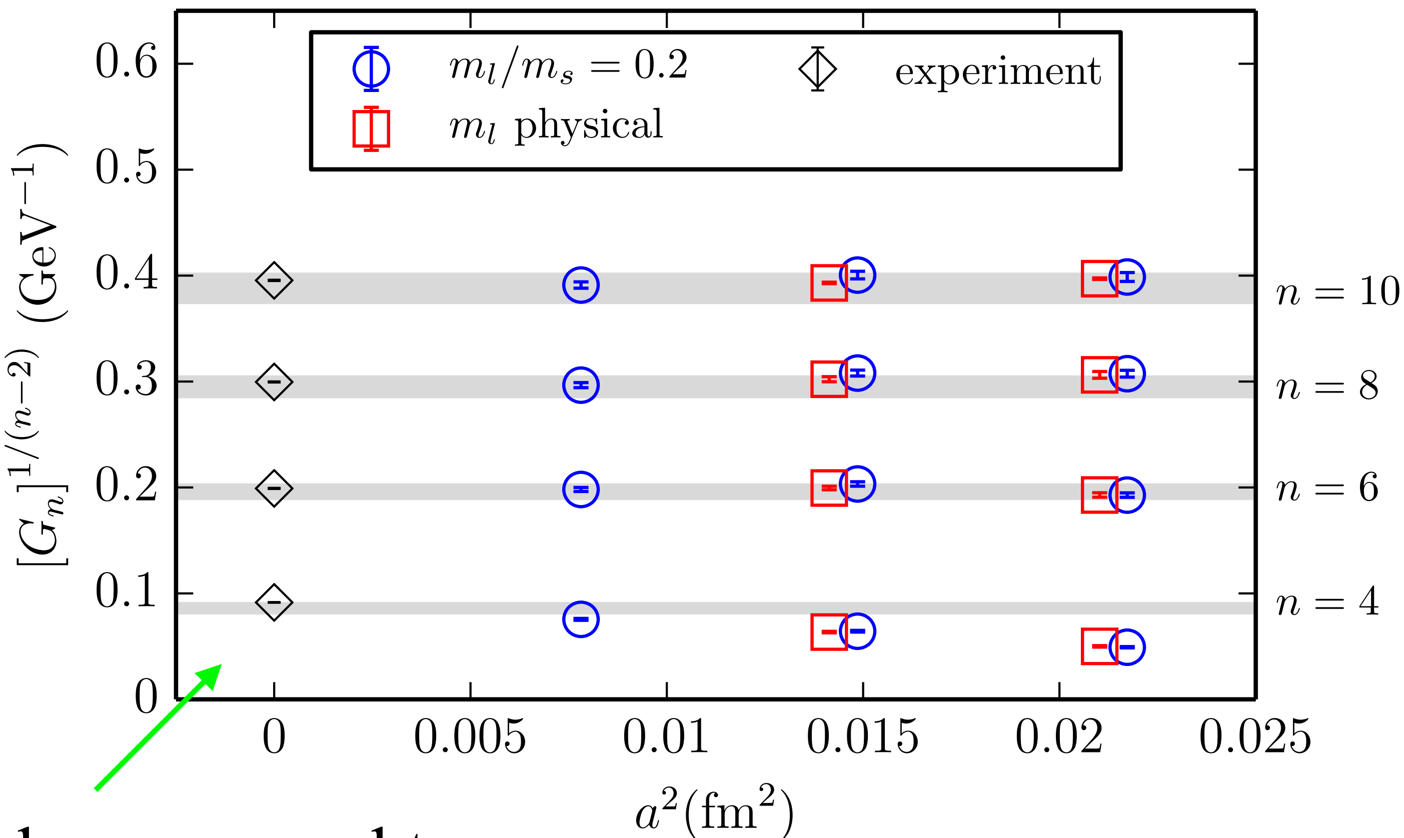
BOTTOM contribution

HPQCD 1110.6887, 1309.5797, 1408.5768

Part of the set of calculations that gave

$m_b, M(\Upsilon) - M(\eta_b), M(\Upsilon') - M(\eta_b'), \Gamma(\Upsilon \to e^+e^-), \Gamma(\Upsilon' \to e^+e^-)$

Used NRQCD valence quarks on MILC 2+1+1 HISQ configs. Z_V from contnm QCD pert. th.



Again, moments can be compared to those extracted from expt.

$a_\mu^{HVP,b} = 0.27(4) \times 10^{-10}$

LIGHT contribution $m_u = m_d$

HPQCD 1601.03071

HISQ valence quarks on MILC 2+1+1 HISQ configs. Use Z_V from s calc.

Multiple a (use w_0), m_l (inc. phys.), volumes (at ml/ms=0.1).

New ingredient since correlators much noisier. Use:

$$G(t) = \begin{cases} G_{\text{data}}(t) & \text{for } t \leq t^* \\ G_{\text{fit}}(t) & \text{for } t > t^* \end{cases}$$

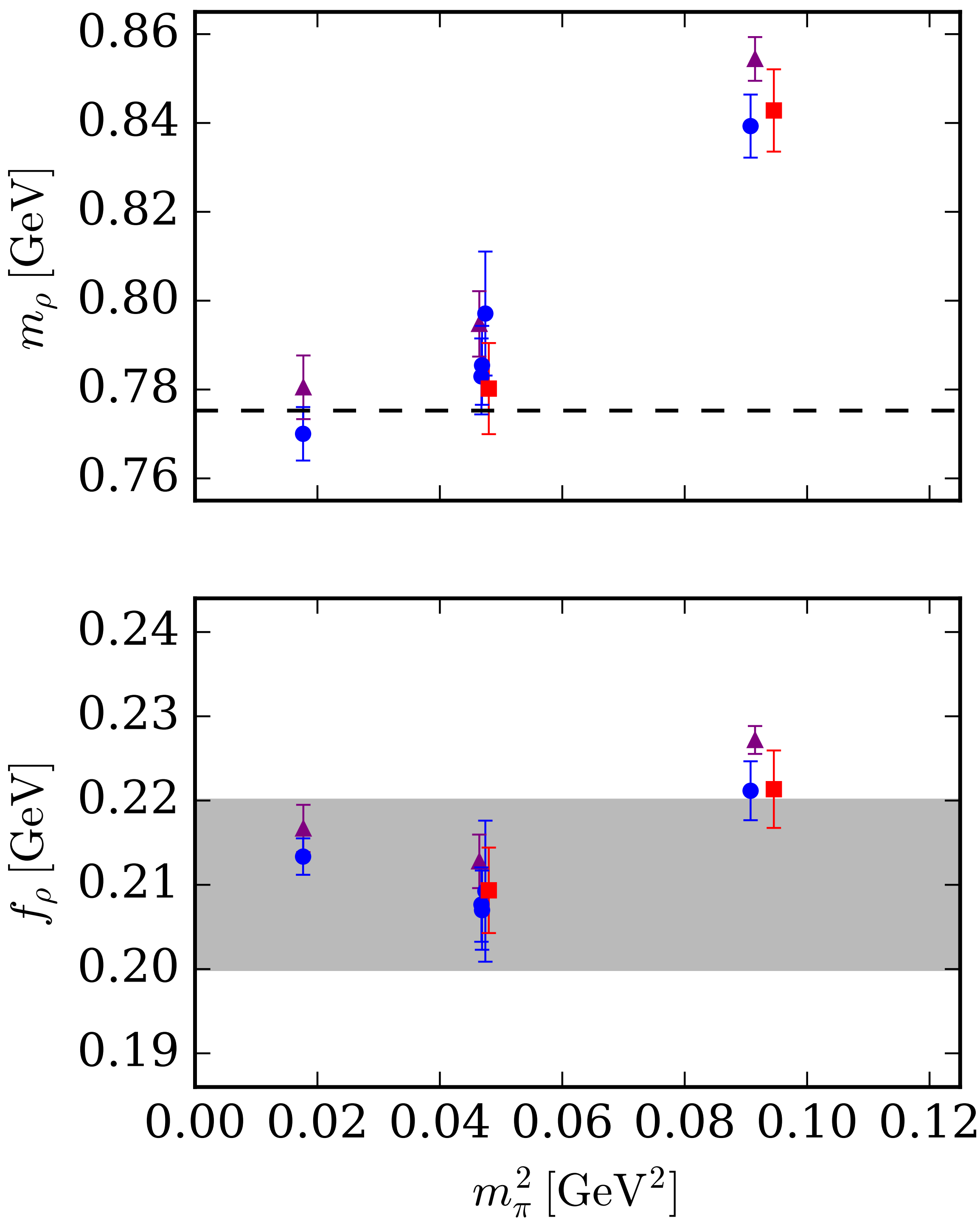
← from Monte Carlo (for G_{data})

← from multi-exponential fit (for G_{fit})

$t^* = 1.5\text{fm} = 6/m_\rho$ so 70% of result from G_{data}

- 80% of result comes from ρ meson pole, so need to understand ρ on lattice

- 10% from $\pi\pi$, sensitive to finite-volume and m_π (so taste-issues for staggered quarks).



Mass and decay constant of the ρ from large time behaviour of u/d vector correlators

Correct key lattice systematics

$$\hat{\Pi}_j^{latt} \to (\hat{\Pi}_j^{latt} - \hat{\Pi}_j^{latt}(\pi\pi)) \left[\frac{m_\rho^{2j,latt}}{m_\rho^{2j,expt}} \right] + \hat{\Pi}_j^{cont}(\pi\pi)$$

Remove lattice $\pi\pi$ using effective theory of ρ, π, γ inc. staggered quark effects and finite vol.

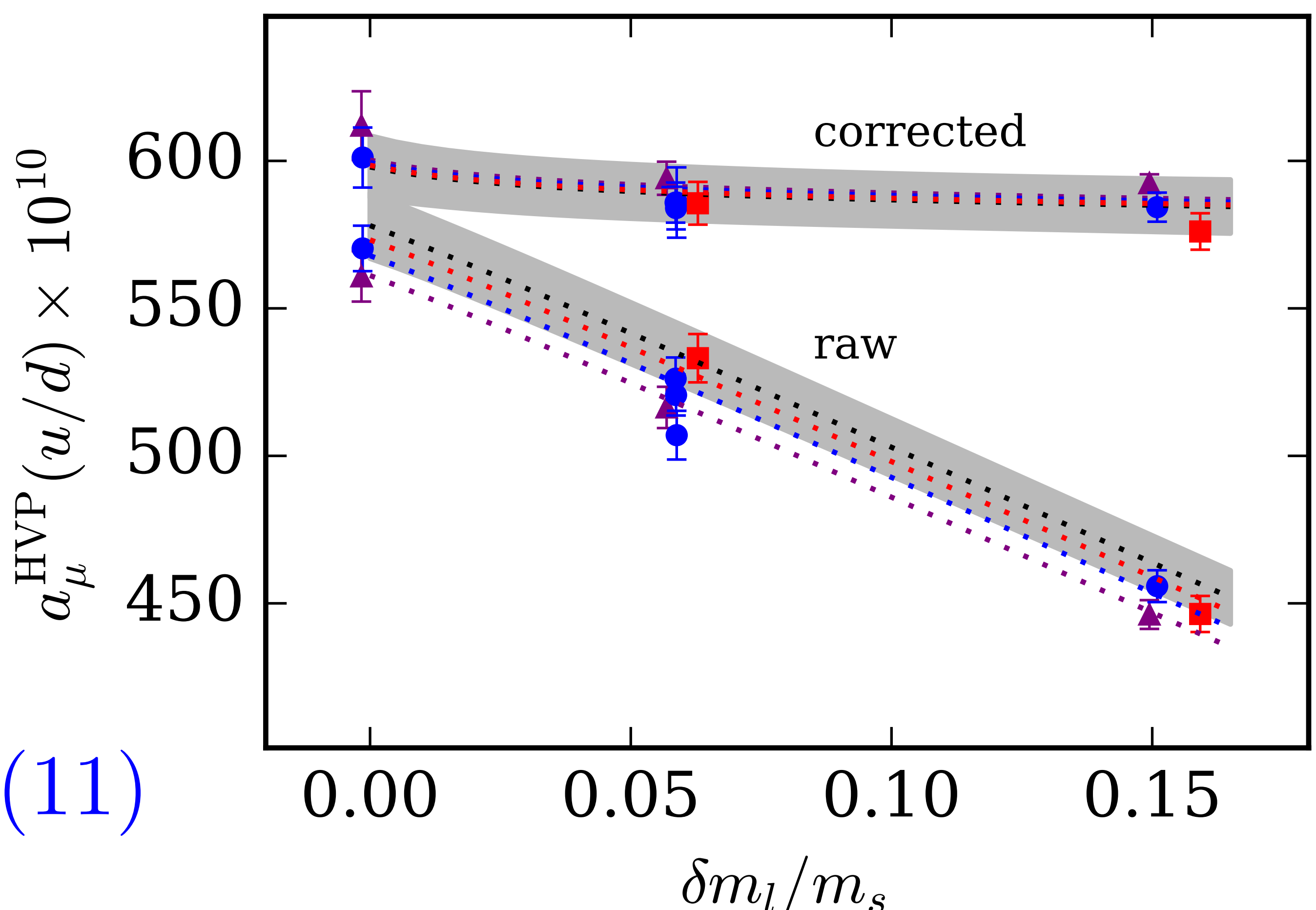
Rescale using exptl m_ρ to reduce m_l dependence

Restore $\pi\pi$ from continuum

Corrections reduce spread of results.

Fit for remaining dependence on a and m_l

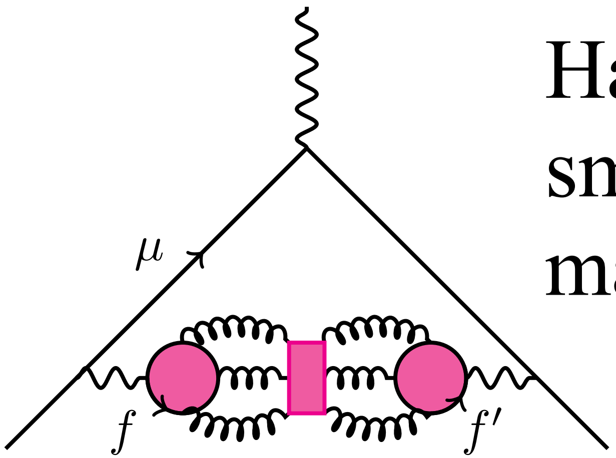
HPQCD 1601.03071



$$a_\mu^{HVP,u/d} = 598(11)$$

Quark-line disconnected contribution

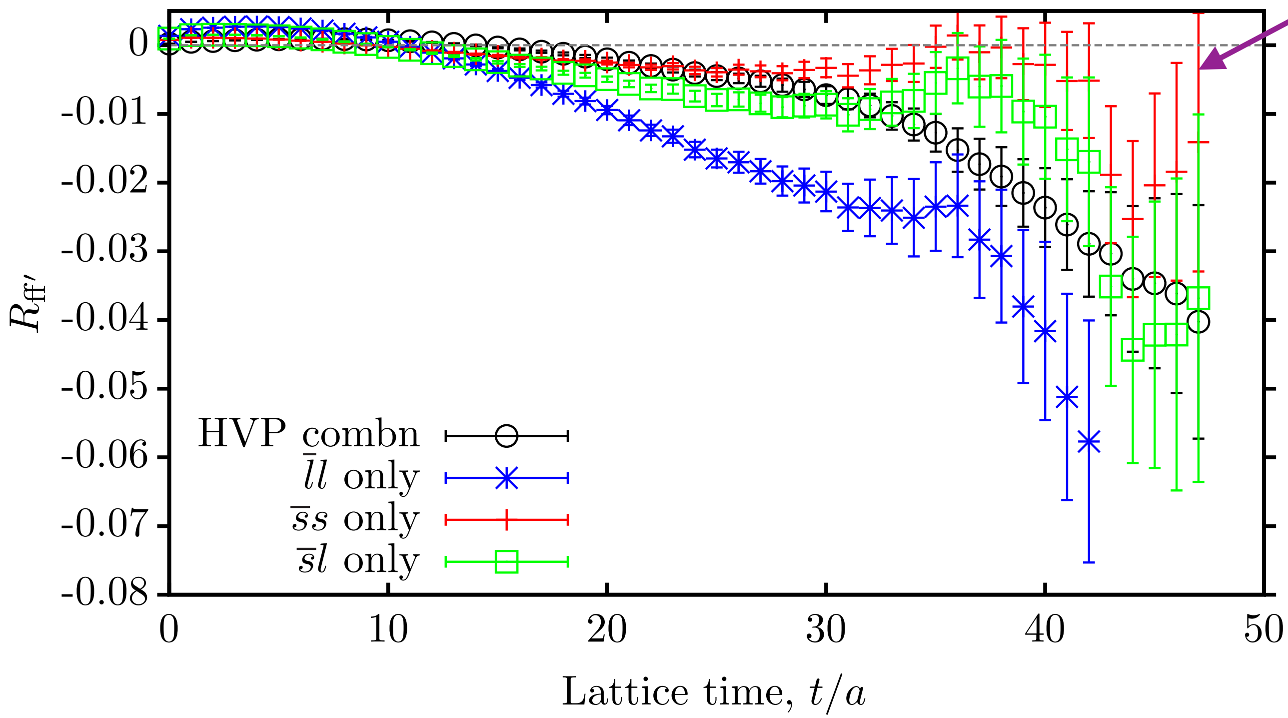
HPQCD/Hadspec 1512.03270



Hard to calculate but small. Suppressed by masses since

$$\sum_{u,d,s} Q_f = 0$$

Simple estimates give ratio of disc. to conn contribution of -1(1)%



Hadspec ratio of disc. to conn. correlators small and contribution further suppressed (by factor 5 by quark charges)

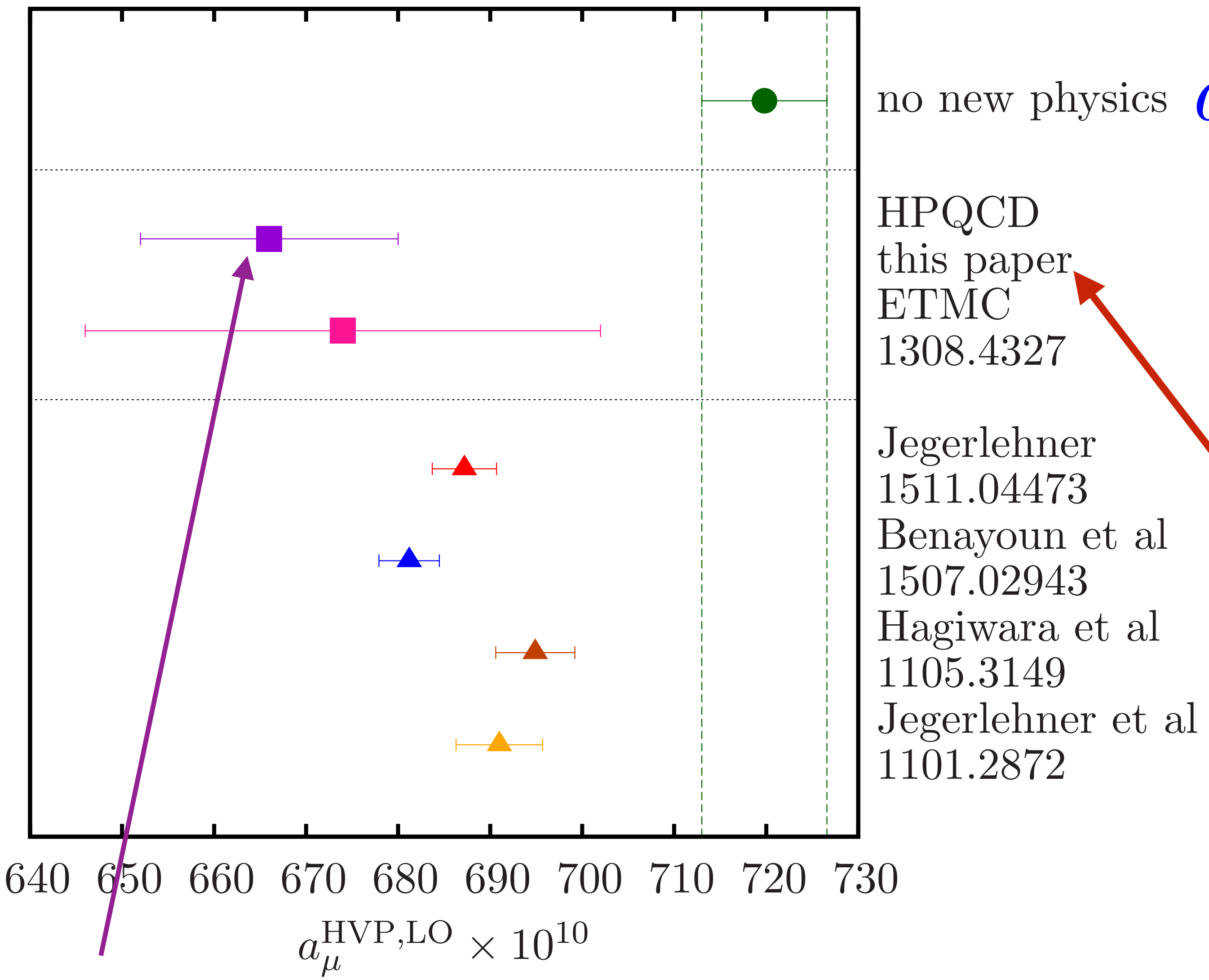
Estimate (after fitting):

$$a_\mu^{HVP,disc} = 0(9) \times 10^{-10}$$

see also RBC/UKQCD 1512.09054:
-9.6(4.0) x 10^{-10}

Conclusion: Combining numbers for a total

HPQCD: 1601.030711



$a_\mu^{\mathrm{HVP,LO}} \times 10^{-10}$

598(11) u/d

53.4(6) s

14.4(4) c

0.27(4) b

Total 666(6)(12)

add syst from disc. diags (1.5%) in quad

3.5σ discrepancy with no new physics

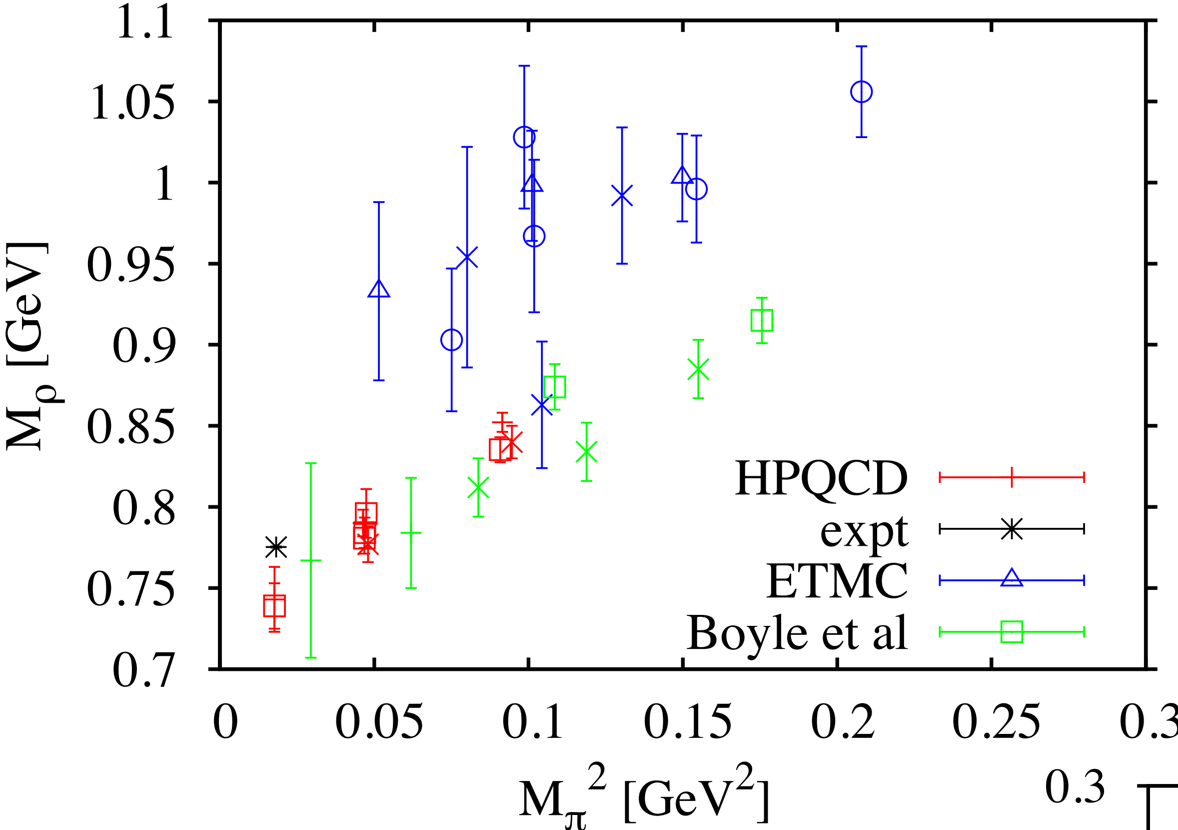
Future focus (with MILC) : improve physical point results and reduce systs from QED, m_u/m_d and disc.

Backup Slides

Error budget for u/d HVP

TABLE III: Error budget for the connected contributions to the muon anomaly a_μ from vacuum polarization of u/d quarks.

	$a_\mu^{\mathrm{HVP,LO}}(u/d)$
QED corrections:	1.0 %
Isospin breaking corrections:	1.0 %
Staggered pions, finite volume:	0.7 %
Noise reduction (t^*):	0.5 %
Valence m_ℓ extrapolation:	0.4 %
Monte Carlo statistics:	0.4 %
Padé approximants:	0.4 %
$a^2 \to 0$ extrapolation:	0.3 %
Z_V uncertainty:	0.4 %
Correlator fits:	0.2 %
Tuning sea-quark masses:	0.2 %
Lattice spacing uncertainty:	< 0.05 %
Total:	1.9 %



Analysis of ρ parameters

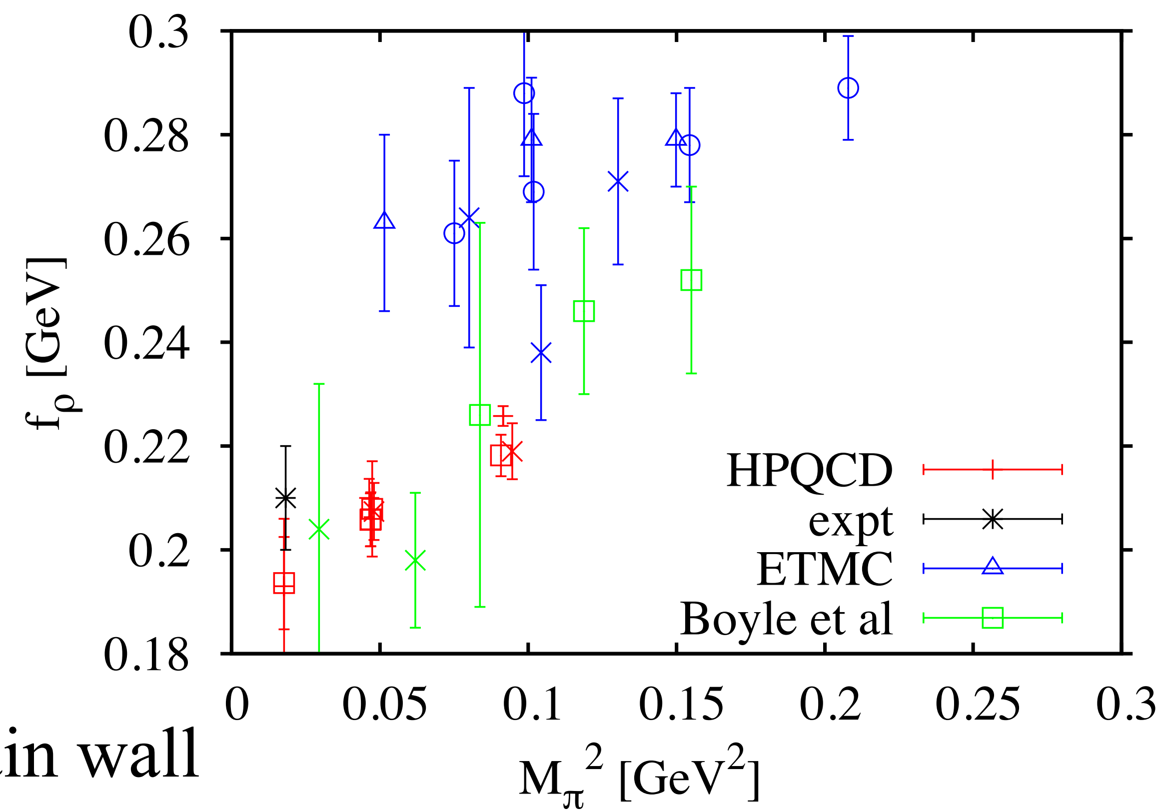
Direct comparison with ETMC (1308.4327) and Boyle et al (1107.1497) possible

ETMC a 0.06-0.08fm
L 2.5- 2.9fm

HPQCD a 0.09-0.15fm
L 2.5-5.8fm

Boyle et al a 0.09-0.14fm
L 2.7-4.6fm

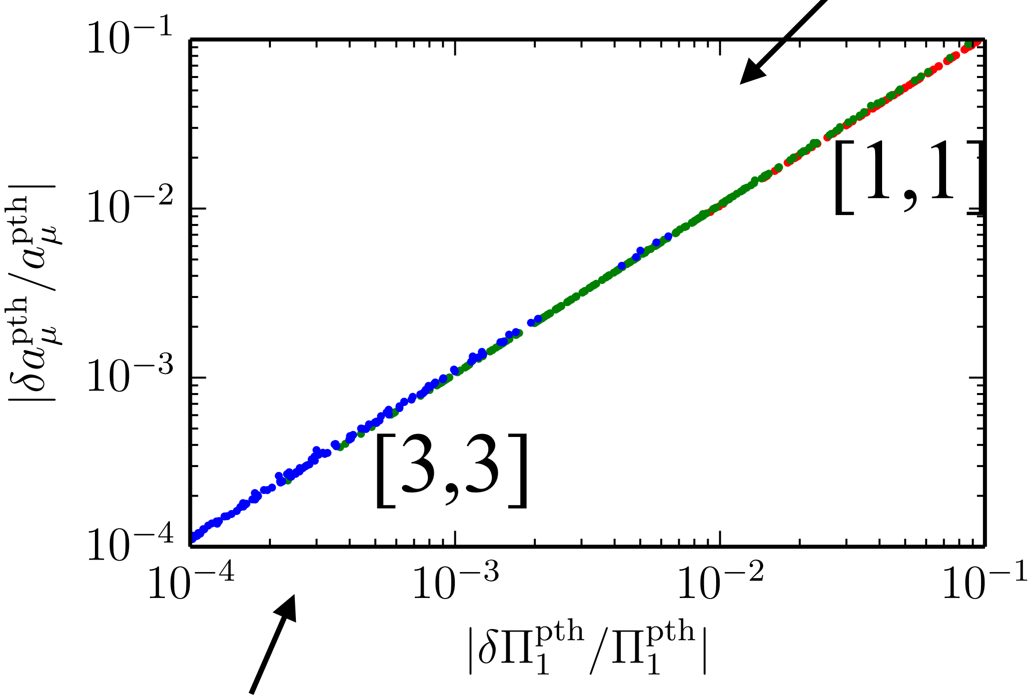
domain wall

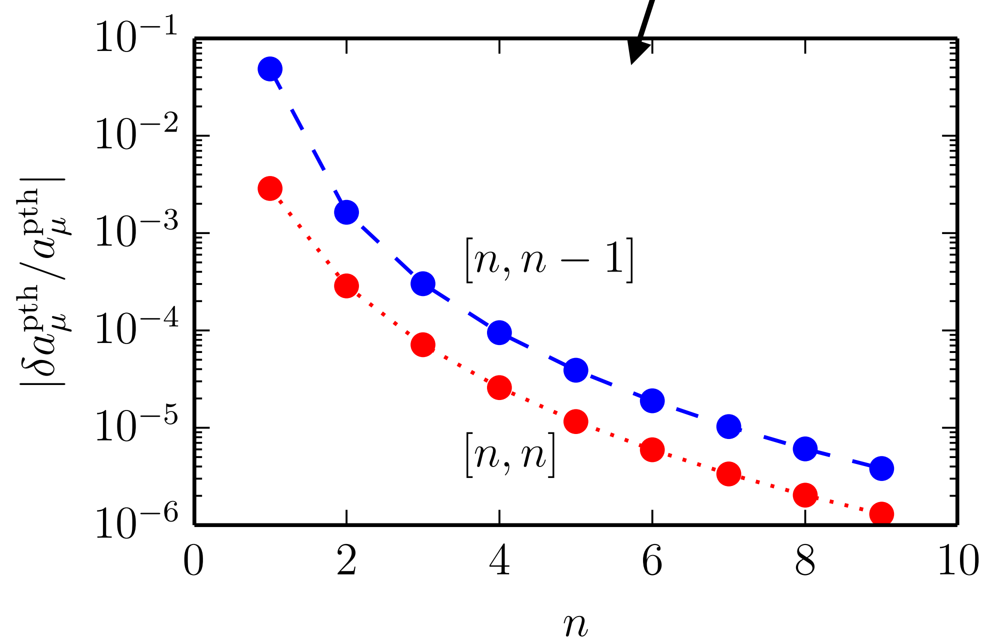


Allows us to reconstruct $\hat{\Pi}(q^2)$ and integrate

Use Pade approximants (ratio of m/n polynomials) rather than Taylor expansion for better large q^2 behaviour.

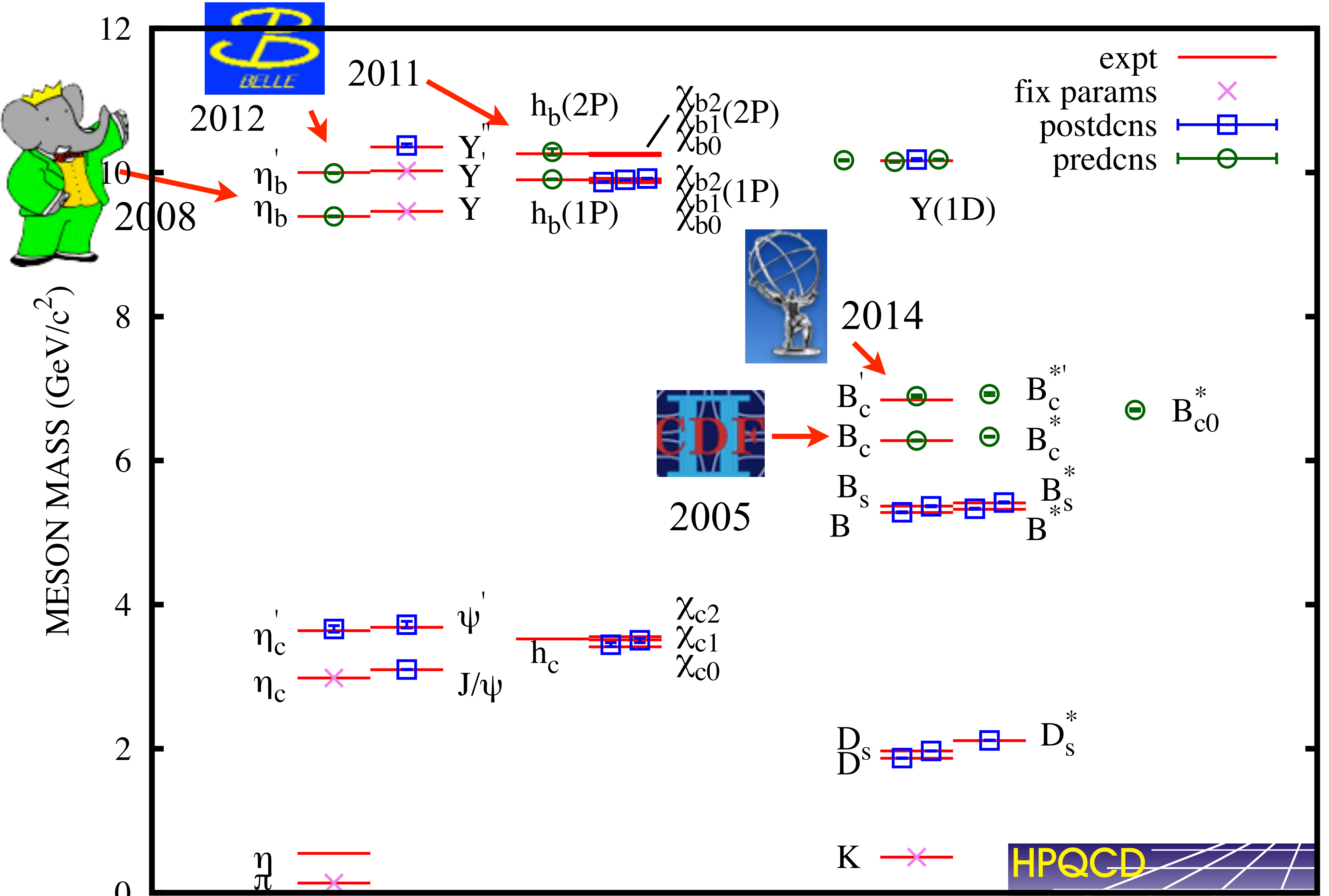
Test Pade approximants in similar scenarios (1-loop quark vacuum polarisation, with noise added)





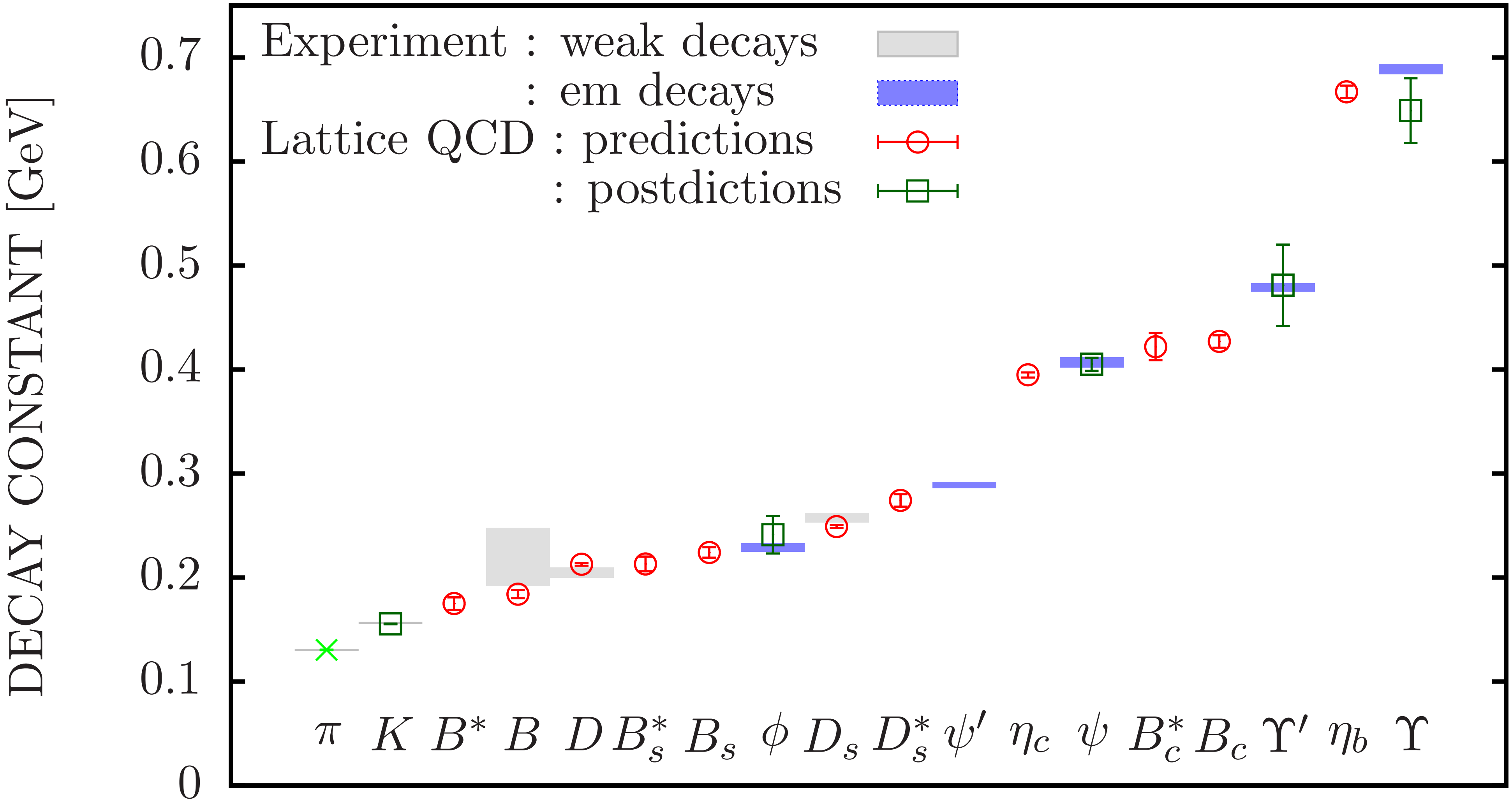
Improved precision allows higher order Pade - we use [2,2]

Keep an eye on the 'big' picture whilst doing this …..



few MeV uncertainties in many cases

Keep an eye on the 'big'picture whilst doing this …..



HPQCD 1503.05762