

Experimental Inputs for the Hadronic Calculations of $(g-2)_\mu$

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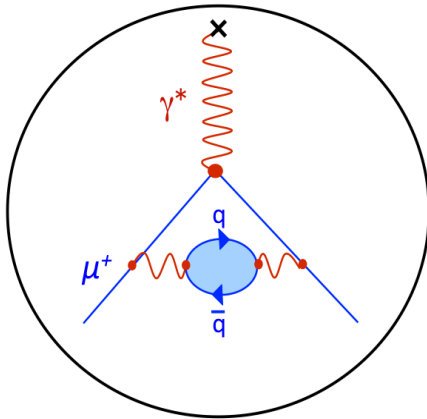
HC₂NP

25th Sept.~30th Sept. 2016 Tenerife



Hadronic Contributions

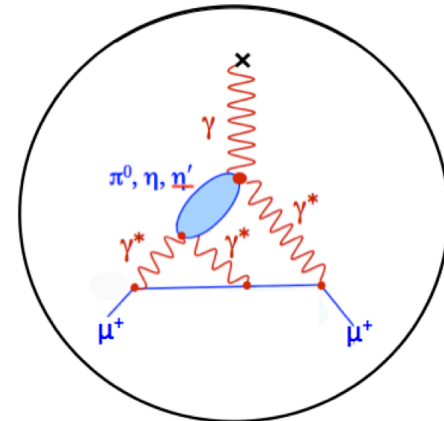
Hadronic Vacuum Polarization



692.3 ± 4.2

[Davier et. al. (2011)]

Hadronic Light-by-Light



11.6 ± 4.0

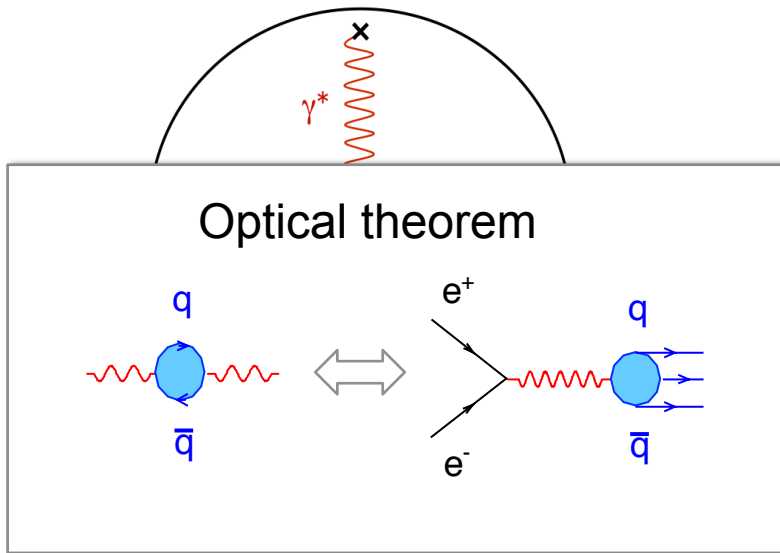
[Jegerlehner, Nyffler (2009)]

10.5 ± 2.6

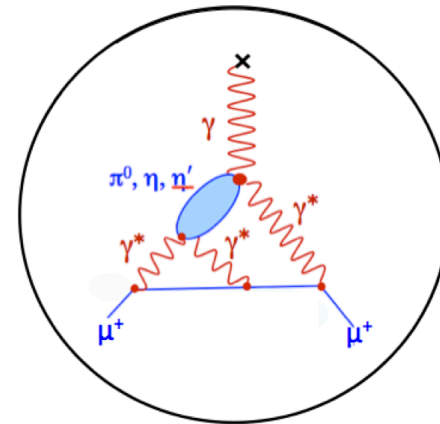
[Prades et al. (2009)]

Hadronic Contributions

Hadronic Vacuum Polarization



Hadronic Light-by-Light



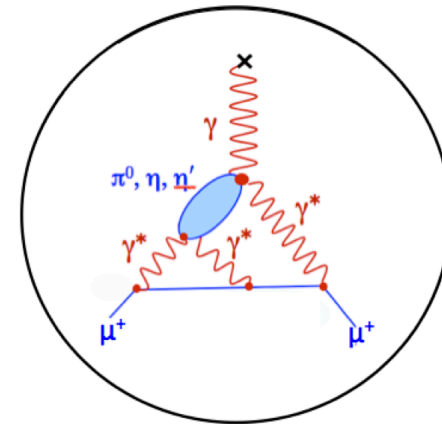
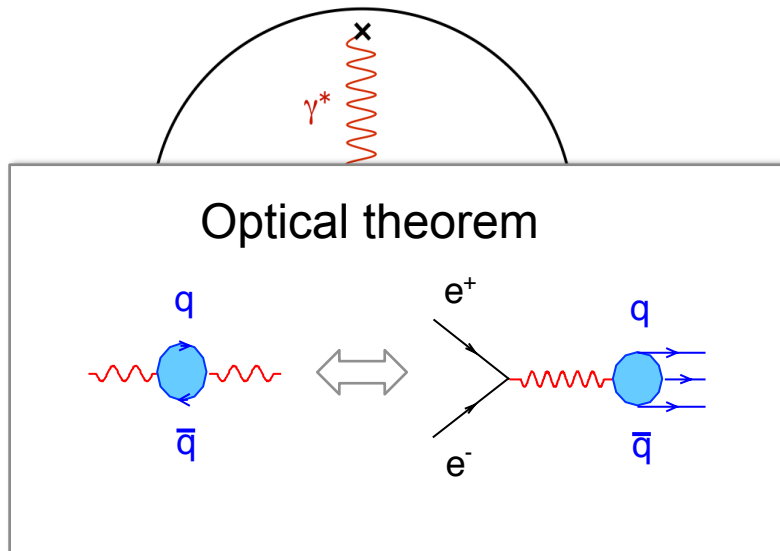
Dispersion integral

$$a_{\mu,LO}^{\text{HVP}} = \frac{1}{4\pi^3} \int_{m_{\pi^0}}^{\infty} ds K(s) \sigma_{\text{had}}(s)$$

Hadronic Contributions

Hadronic Vacuum Polarization

Hadronic Light-by-Light



Dispersion integral

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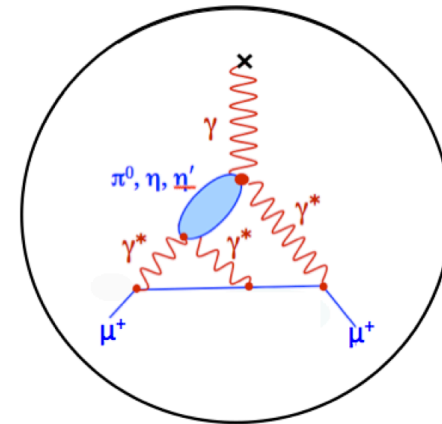
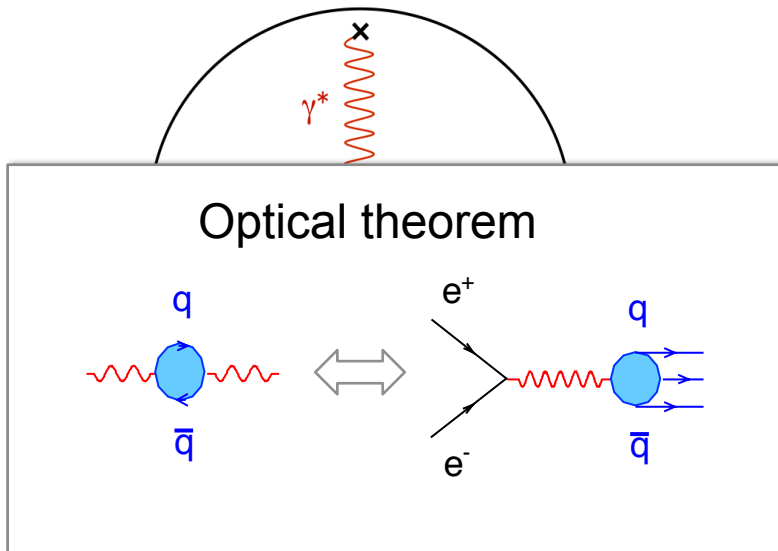
Kernel function $\sim 1/s$

$$\sigma_{\text{had}} = \sigma(e^+e^- \rightarrow \text{hadrons}) \sim 1/s$$

Hadronic Contributions

Hadronic Vacuum Polarization

Hadronic Light-by-Light



Dispersion integral

$$a_{\mu,LO}^{\text{HVP}} = \frac{1}{4\pi^3} \int_{m_{\pi^0}}^{\infty} ds K(s) \sigma_{\text{had}}(s)$$

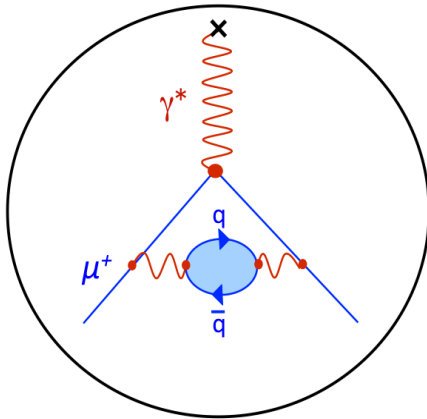
Kernel function $\sim 1/s$

$$\sigma_{\text{had}} = \sigma(e^+e^- \rightarrow \text{hadrons}) \sim 1/s$$

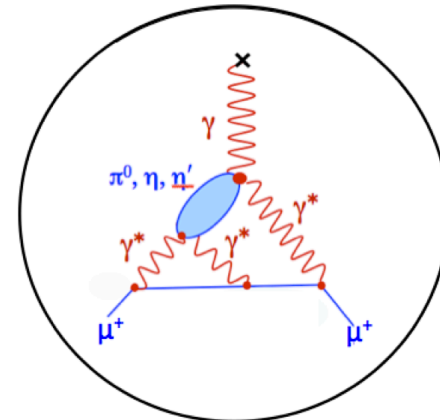
Low energy contributions important!

Hadronic Contributions

Hadronic Vacuum Polarization



Hadronic Light-by-Light



Transition form factors
Helicity amplitude

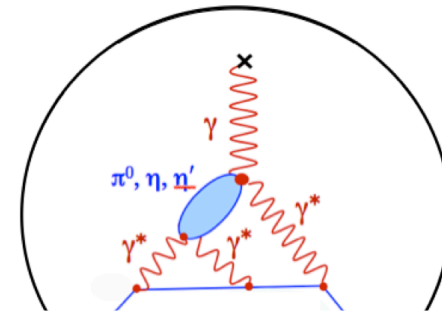
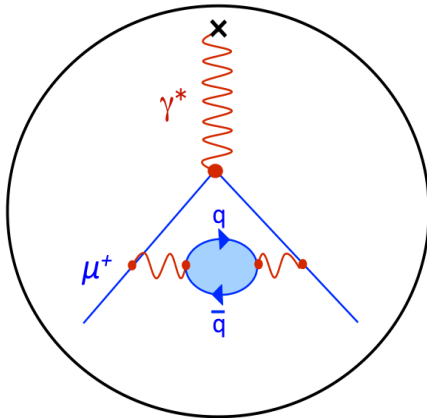


- Only model calculations so far
 - Data-driven approach been developed
- [Colangelo et al '14; Pauk, Vanderhaeghen '14]

Hadronic Contributions

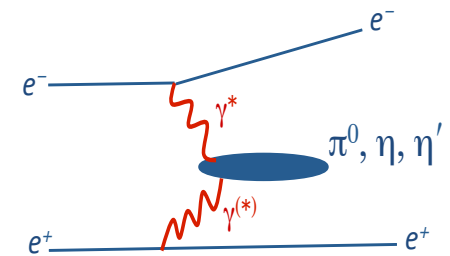
Hadronic Vacuum Polarization

Hadronic Light-by-Light

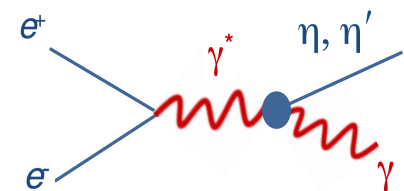
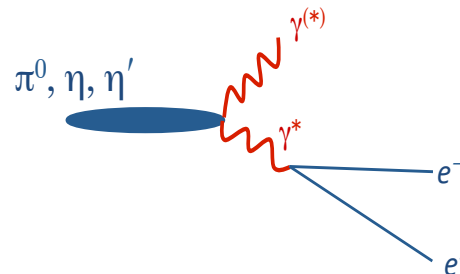


Transition form factors
Helicity amplitude

Spacelike



Timelike



Hadronic Cross Section

$$\sigma_{\text{had}} (e^+ e^- \rightarrow \text{hadrons})$$

Hadronic Cross Section

- **Energy Scan:**
 - CMD & SND at VEPP-2M & VEPP-2000 in Novosibirsk
 - BESIII at BEPCII in Beijing

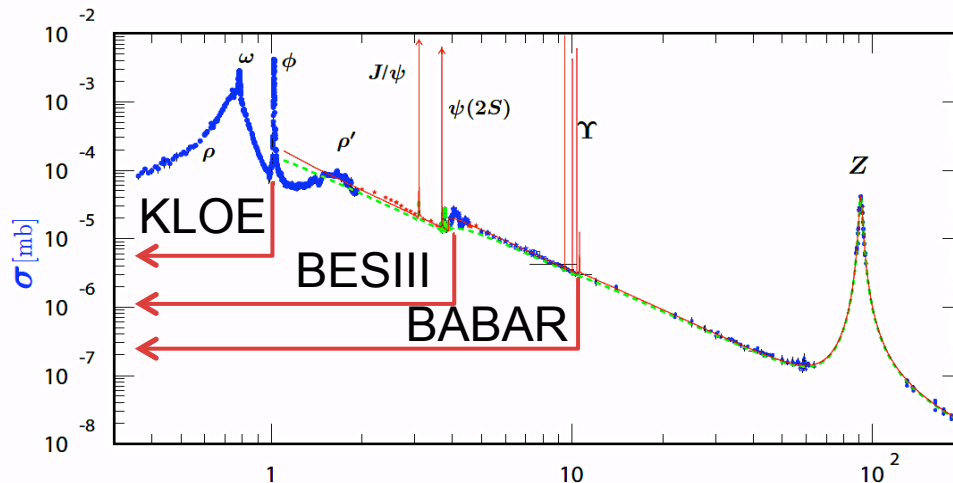
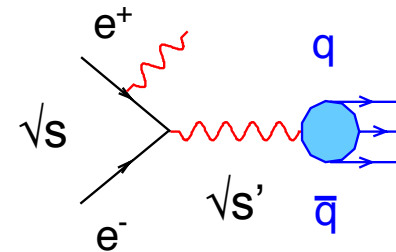
Hadronic Cross Section

- **Energy Scan:**

- CMD & SND at VEPP-2M & VEPP-2000 in Novosibirsk
- BESIII at BEPCII in Beijing

- **Initial State Radiation:**

- KLOE at DAΦNE in Frascati
- BABAR at PEP-II in Stanford
- BESIII at BEPCII in Beijing



- Needs no systematic variation of beam energy
- High statistics thanks to high integrated luminosities

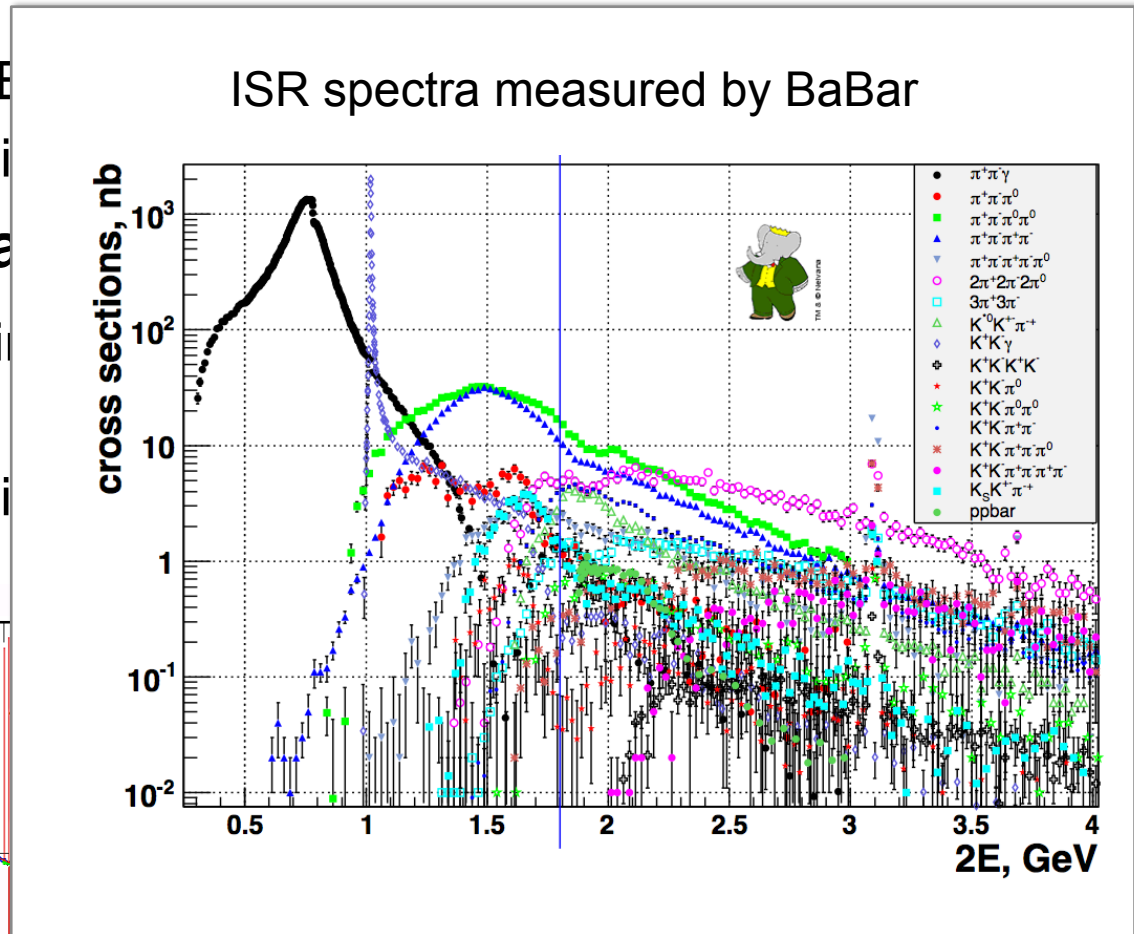
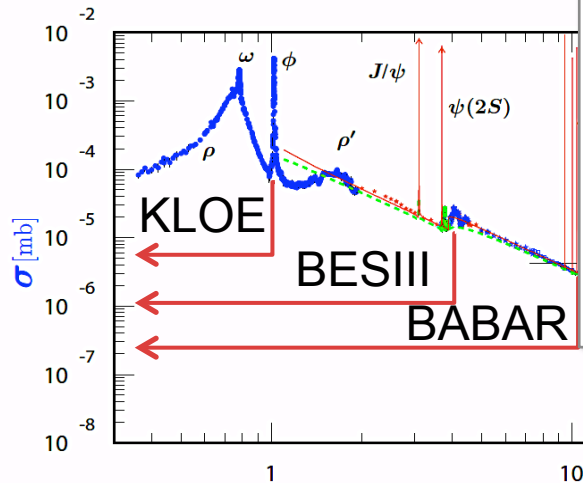
Hadronic Cross Section

- **Energy Scan:**

- CMD & SND at VEPP
- BESIII at BEPCII in China

- **Initial State Radiation**

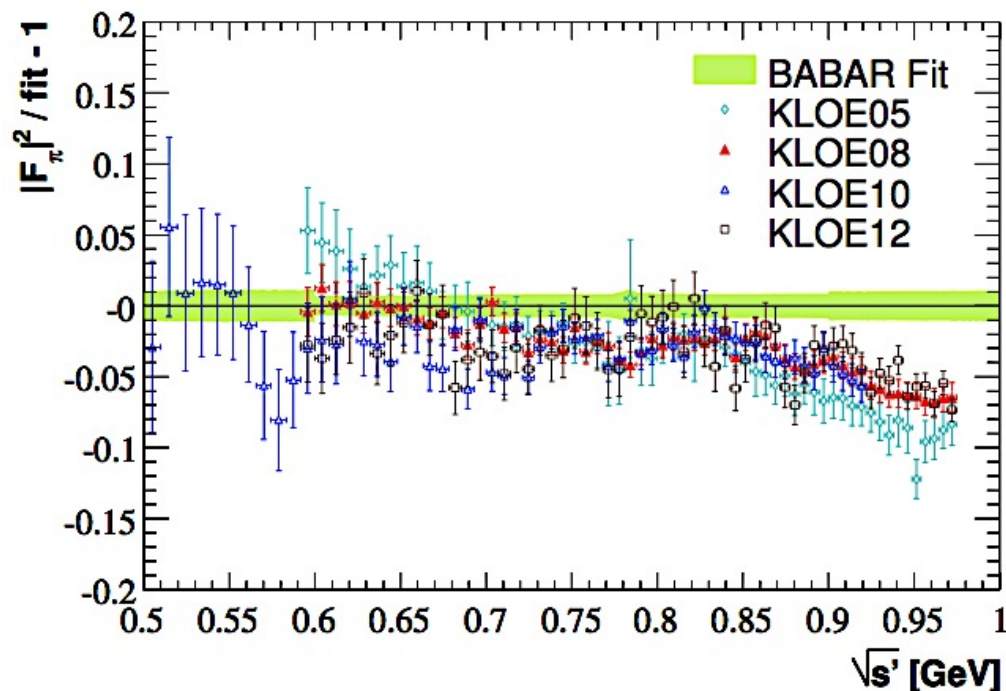
- KLOE at DAΦNE in Italy
- BABAR at PEP-II in SLAC
- BESIII at BEPCII in China



Most Relevant Channel:

$$e^+e^- \rightarrow \pi^+\pi^-$$

- KLOE and BABAR dominate the world average
- Both with uncertainties smaller than 1%
- Relatively large systematic differences, especially above ρ peak
- Knowledge of a_μ^{had} dramatically limited due to this difference



Note: KLOE05 superseded by KLOE08

Beijing Electron Positron Collider-II



Linear Accelerator

Storage Ring

BESIII
Detector

BEPCII: τ -charm factory
Beam energy:
1-2.3 GeV
Design luminosity:
 $1 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$ (April 2016)
Data taking from 2009 to present

Beijing Electron Positron Collider-II

MUC: 9/8 layer RPC, $\sigma_{R\Phi}$: 2 cm

Magnet yoke

TOF: (σ_T)
80 ps / 110 ps

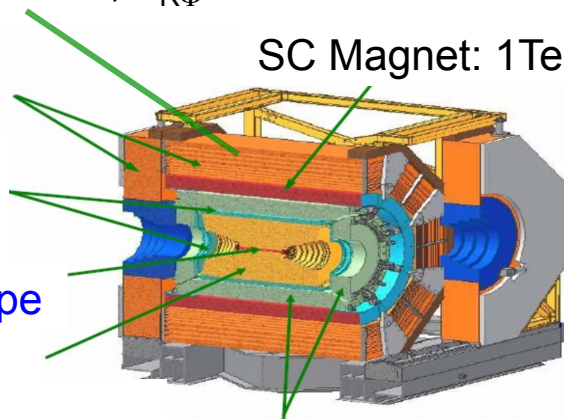
Beam pipe

MDC:

σ_p/p : 0.5% at 1 GeV/c

dE/dx: 6%

SC Magnet: 1 Tesla



CsI calorimeter:

$\Delta E/E$: 2.5% / 5.0% at 1 GeV;
 σ_z : 0.6 cm/ \sqrt{E}

Linear Accelerator

BEPCII: τ -charm factory

Beam energy:

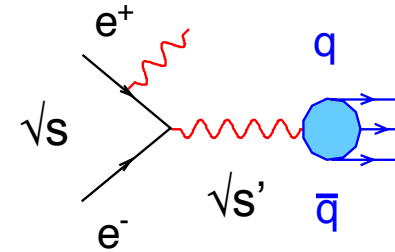
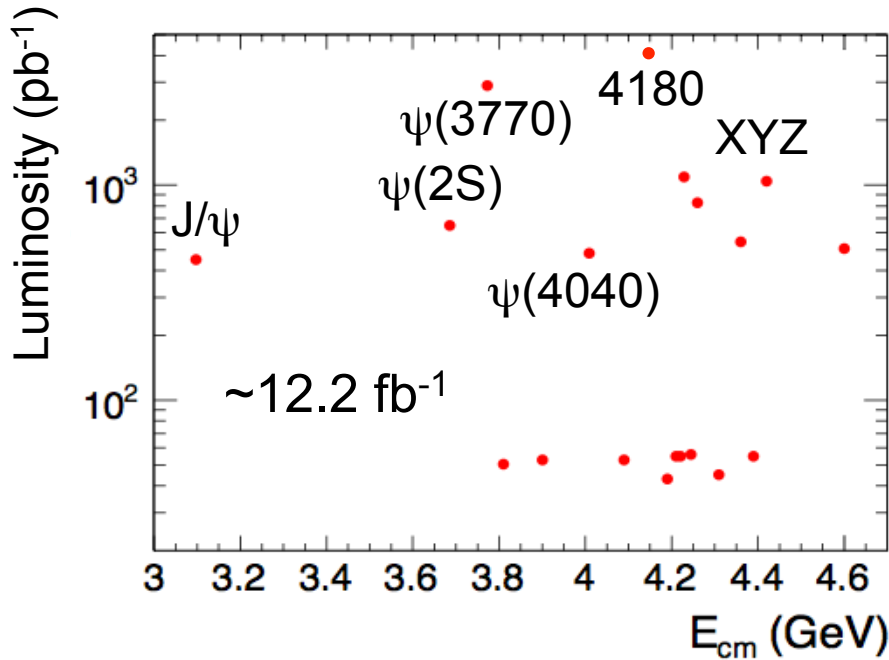
1-2.3 GeV

Design luminosity:

$1 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$ (April 2016)

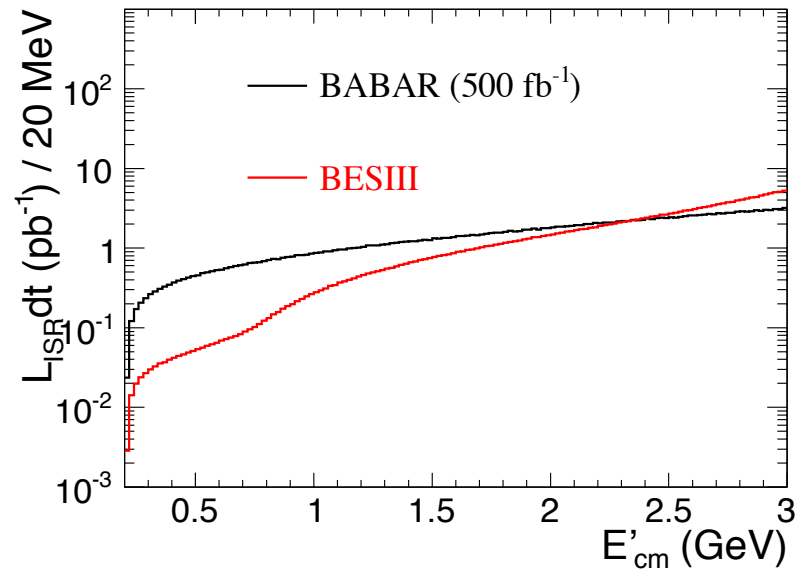
Data taking from 2009 to present

Data Samples for ISR Study



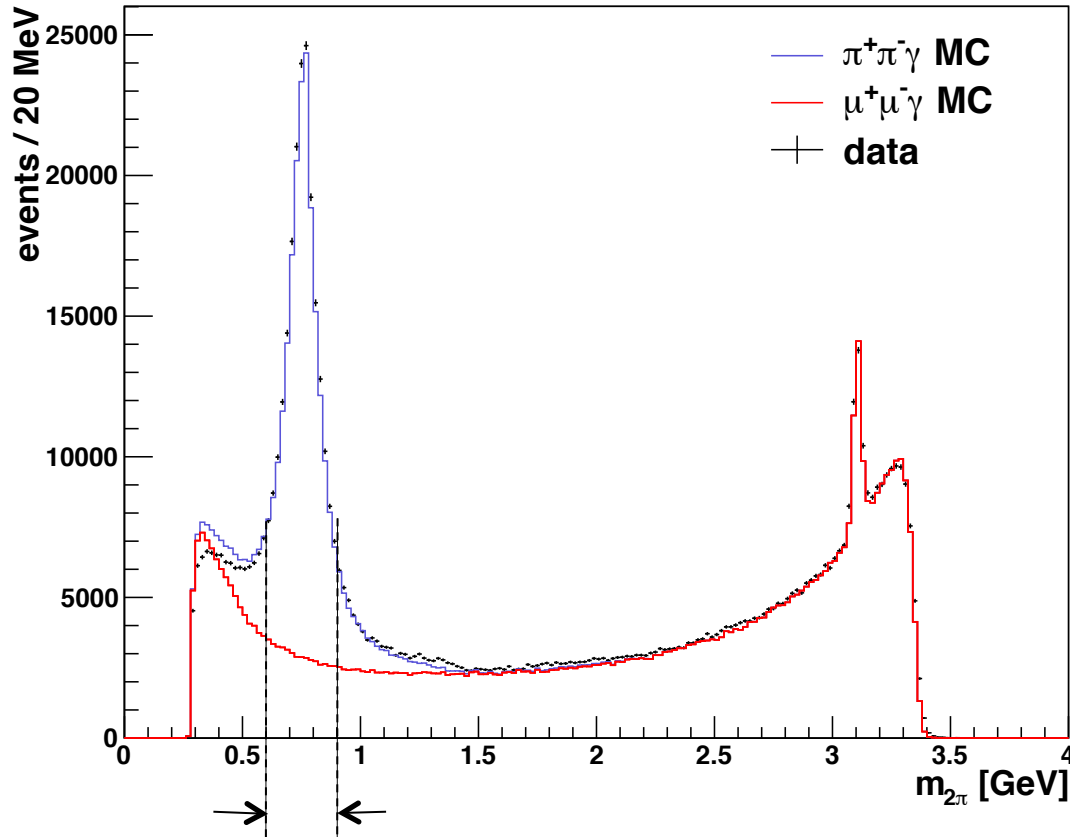
ISR luminosity = $L dt \times H_{\text{rad}}$

Radiator function, well known



$e^+e^- \rightarrow \gamma_{\text{ISR}} \pi^+\pi^-$ at BESIII

Event yield after preliminary selection



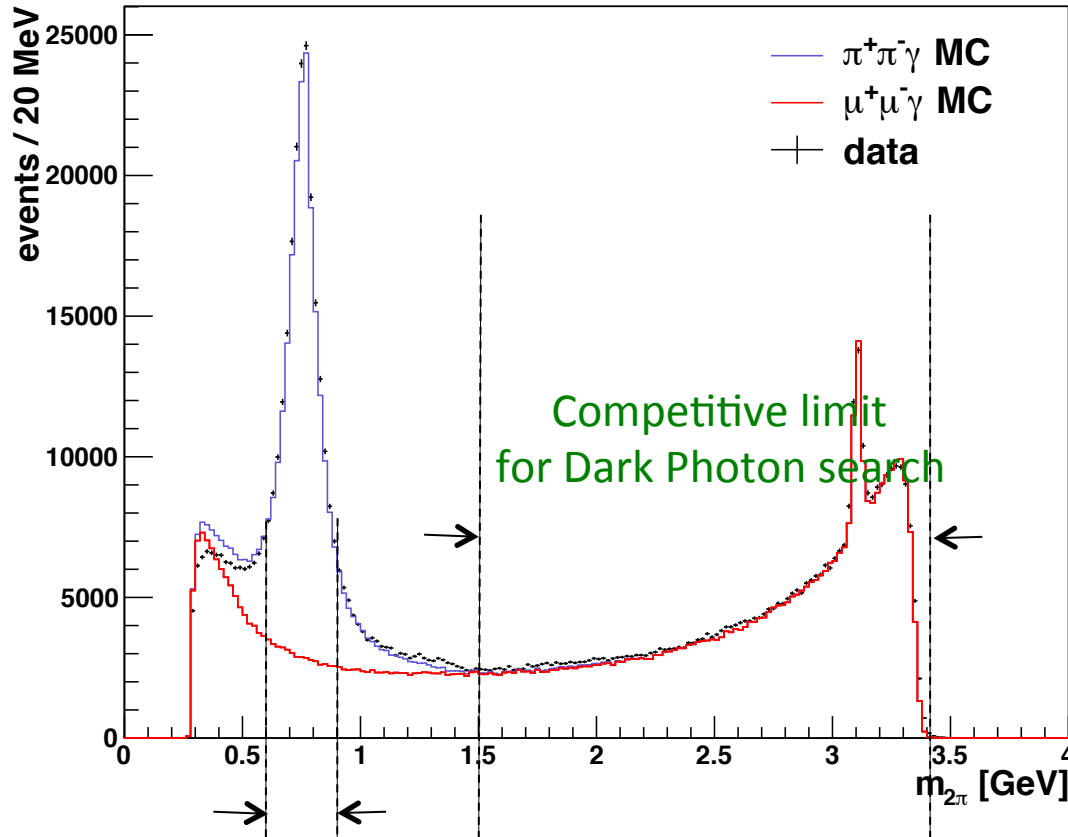
Initial publication
600 – 900 MeV

[Phys. Lett. B753 (2016) 629]

- $\psi(3770)$ data (2.9 fb^{-1})
- Tag ISR photon
- No dedicated background subtraction
- $e^+e^- \rightarrow \gamma\pi^+\pi^-$: large statistics
- $e^+e^- \rightarrow \gamma\mu^+\mu^-$: dominate background
- Data - MC differences visible

$e^+e^- \rightarrow \gamma_{\text{ISR}} \pi^+\pi^-$ at BESIII

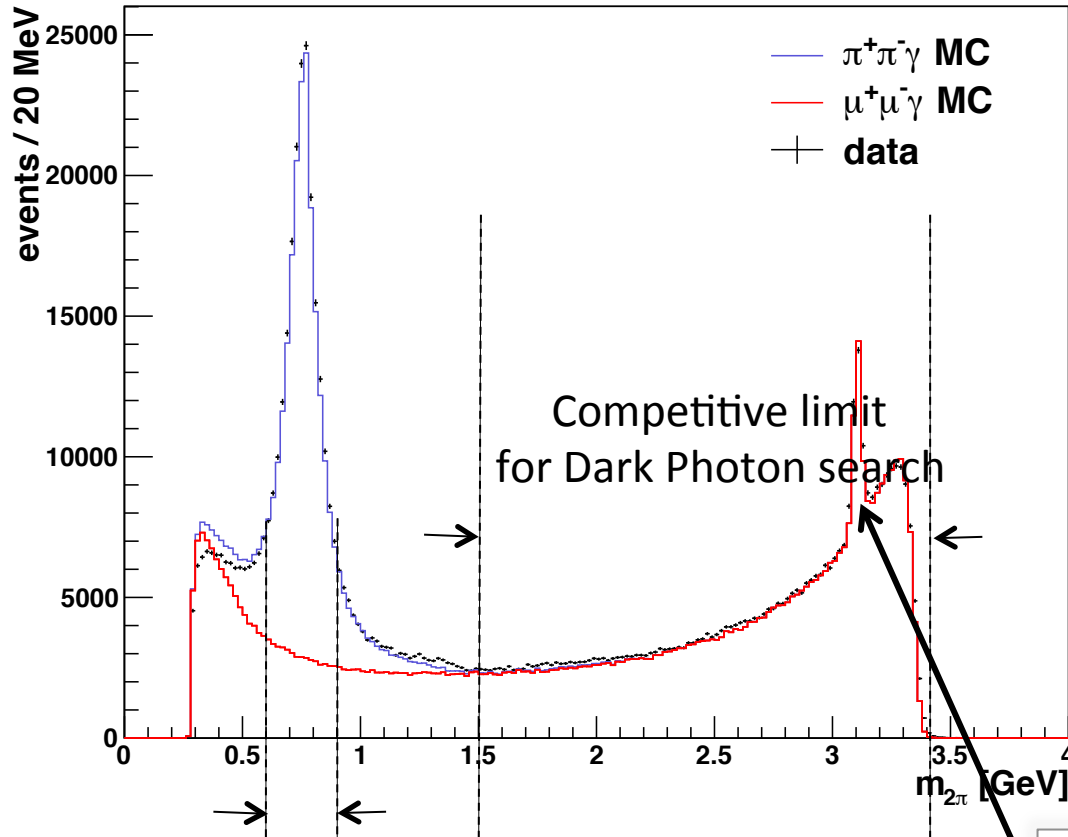
Event yield after preliminary selection



- $\psi(3770)$ data only (2.9 fb^{-1})
- Tag ISR photon
- No dedicated background subtraction
- $e^+e^- \rightarrow \gamma\pi^+\pi^-$: large statistics
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Event yield after preliminary selection

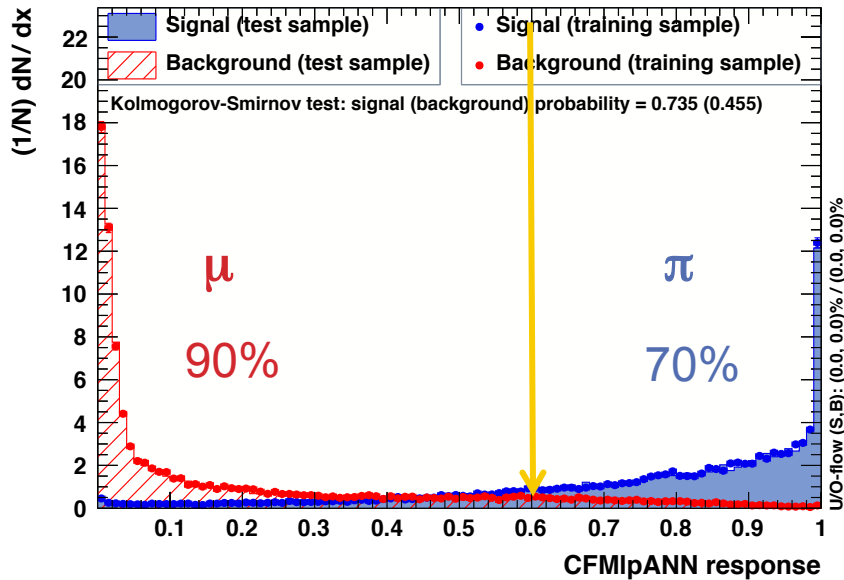


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Initial publication
600 – 900 MeV

World's best measurement
of Γ_{ee} of J/ψ

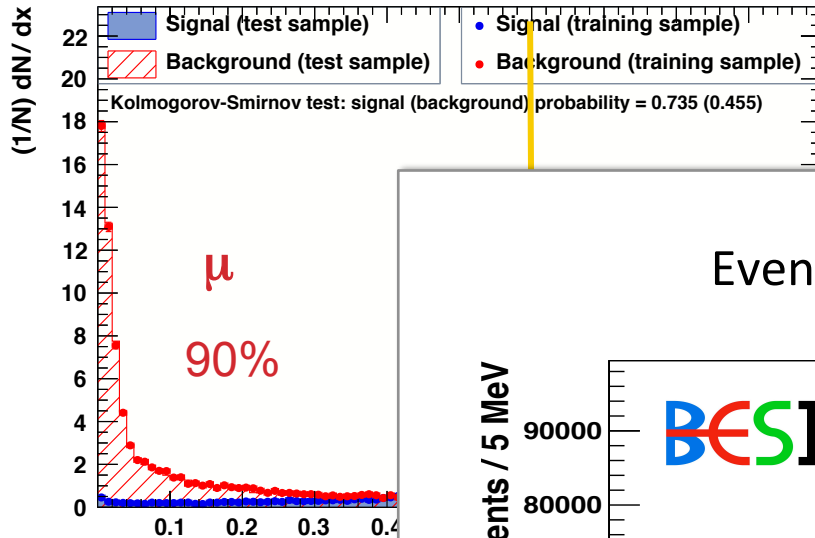
$e^+e^- \rightarrow \gamma_{ISR} \pi^+\pi^-$: π - μ separation



TMVA method (Neural Network):

- Trained using $\gamma\mu\mu$ and $\gamma\pi\pi$ MC events
- Information based on track level
- Efficiency matrix (p, Θ) for data, MC
- Correct for data - MC differences
- Cross checked for different TMVA methods

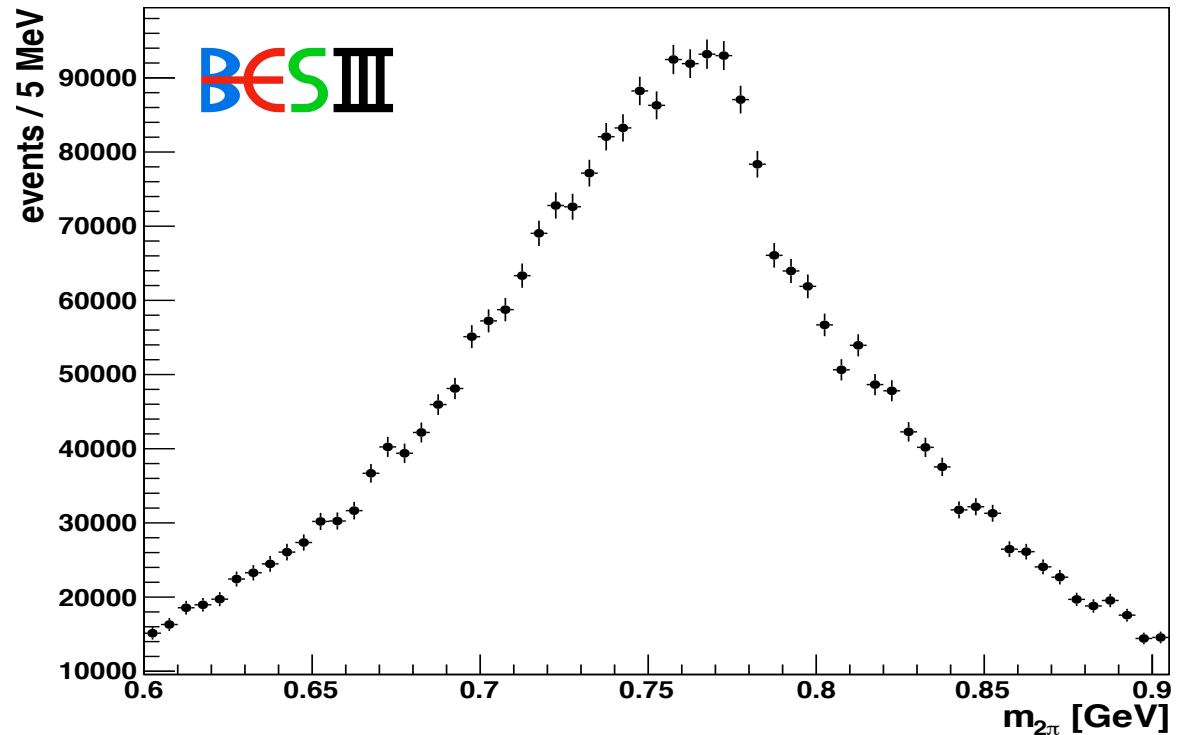
$e^+e^- \rightarrow \gamma_{ISR} \pi^+\pi^-$: π - μ separation



TMVA method (Neural Network):

- Trained using $\gamma\mu\mu$ and $\gamma\pi\pi$ MC events

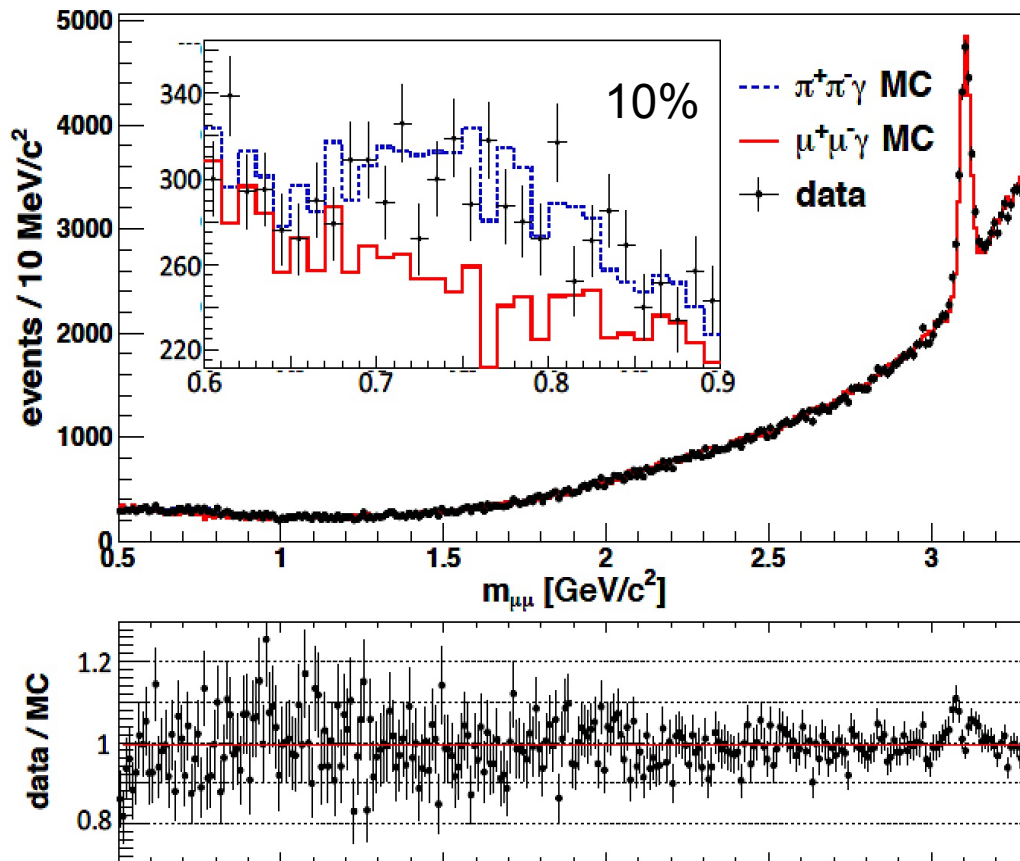
Event yield $\gamma\pi\pi$ after π - μ separation



ods

QED Test: $e^+e^- \rightarrow \gamma\mu^+\mu^-$

Event yield $\gamma\mu\mu$ after π - μ separation and all efficiency corrections



- Background from $\gamma\pi\pi$ small
- PHOKHARA uncertainty $< 0.5\%$
- Luminosity measurement based on Bhabha events, 1.0% accuracy

$$\Delta(\text{MC}/\text{QED-data}) - 1 = (1.0 \pm 0.3_{\text{stat}} \pm 0.9_{\text{syst}}) \%$$

- Excellent agreement with QED
- Accuracy on 1% level as needed to be competitive !

$e^+e^- \rightarrow \pi^+\pi^-$ Cross section

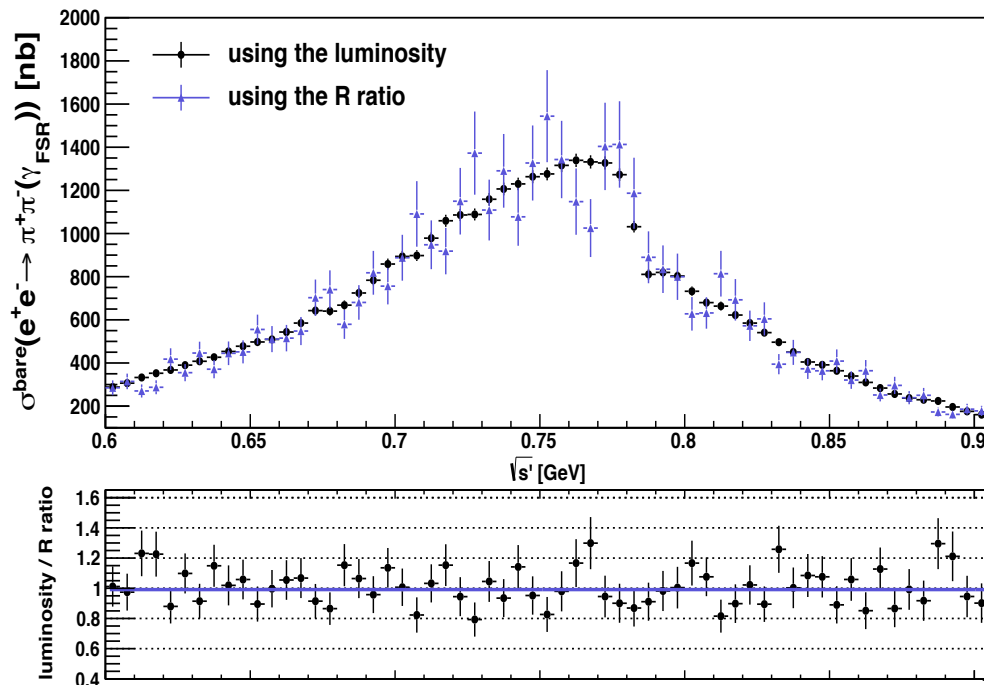
2 normalization methods:

- Normalization to L_{int} (obtained from Bhabha events)

$$\sigma_{bare}(e^+e^- \rightarrow \pi^+\pi^-) = \frac{N_{\pi\pi\gamma}}{L_{int} \cdot H_{rad} \cdot \delta_{vac} \cdot (1 + \delta_{FSR})}$$

- Normalization to $\gamma\mu\mu$ events, i.e. R ratio ($\gamma\pi\pi/\gamma\mu\mu$)

$L_{int}, H_{rad}, \delta_{vac}$ cancel in ratio



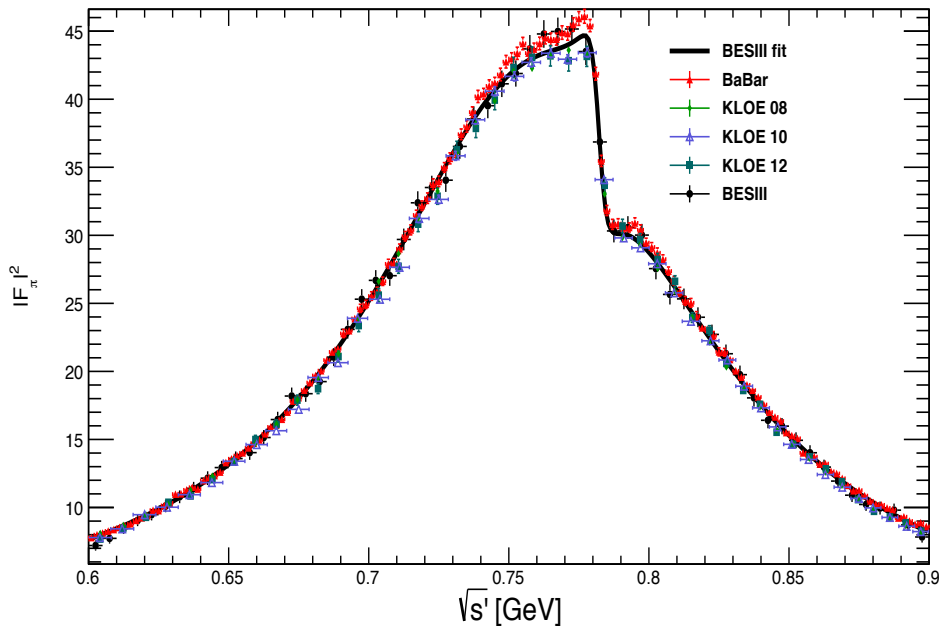
Good agreement between two methods

**luminosity / R ratio -1
= (0.85 ± 1.68) %**

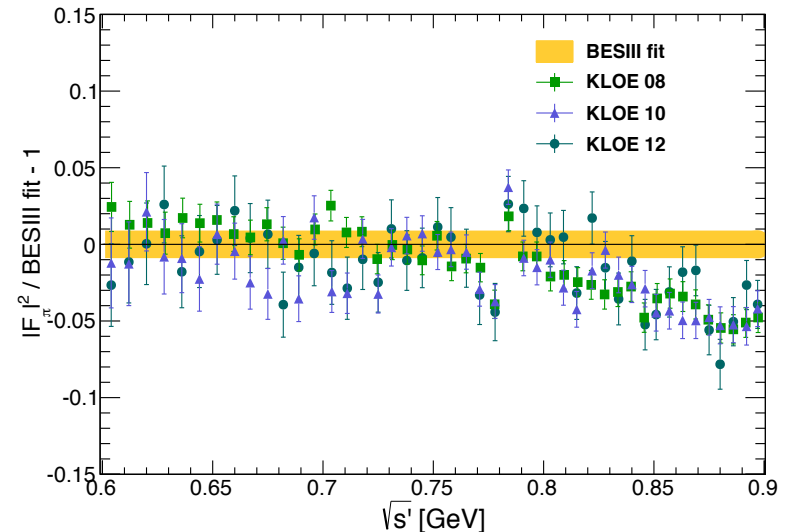
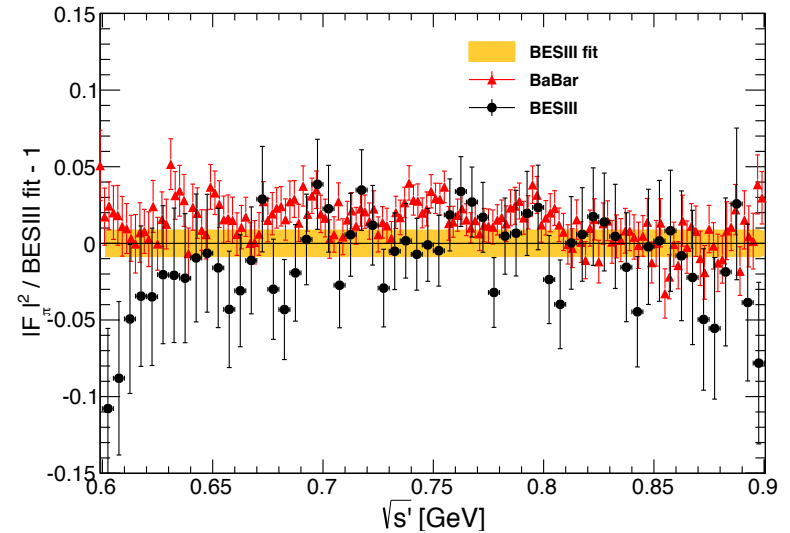
limited by low $\gamma\mu\mu$ statistics

Compare with Existing Data

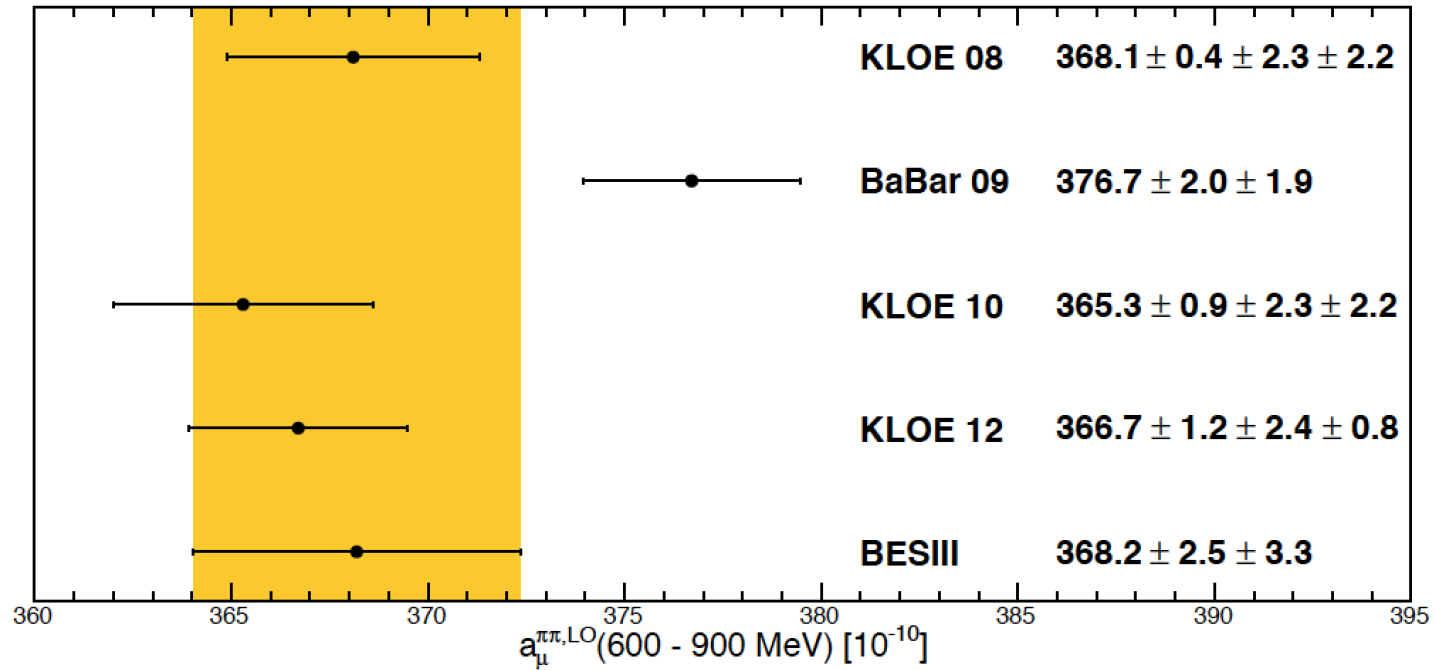
Pion Form Factor F_π



- Gounaris and Sakurai parameterization
- 0.9 % accuracy (dominated by theory)
- Normalization to luminosity \times radiator function

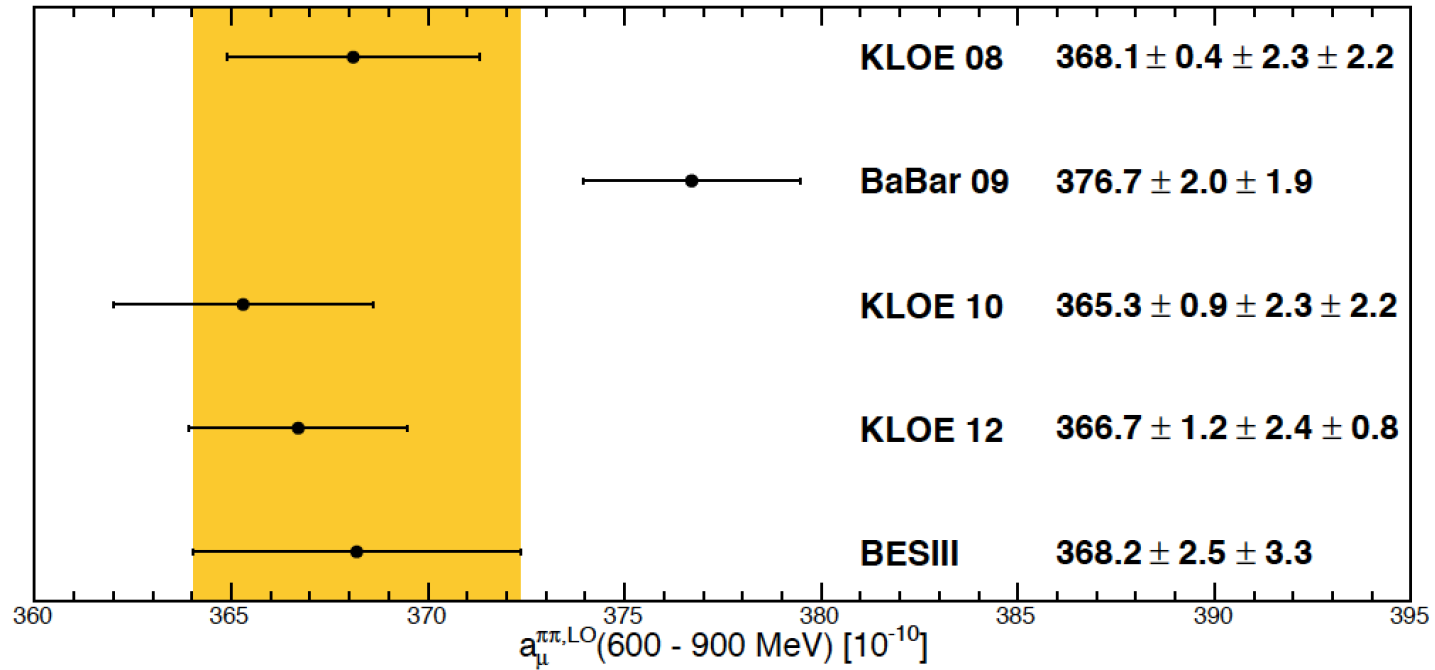


Impact on a_{μ}^{HVP}

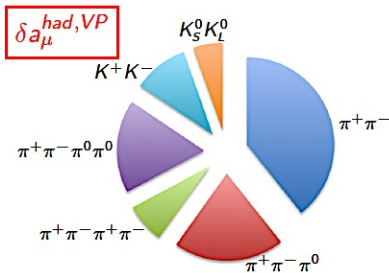


Deviation on $(g-2)_{\mu}$ between experimental and SM: 3-4 sigma

Impact on a_μ^{HVP}



Deviation on $(g-2)_\mu$ between experimental and SM: 3-4 sigma



Study of $\pi^+\pi^-\pi^0$ and $\pi^+\pi^-\pi^0\pi^0$ processes undergoing at BESIII

Energy Scan from 2.0 to 4.6 GeV

World's best measurement from BES/BESII with 5% ~ 8% total uncertainty (statistical uncertainty: 3% ~ 5%)

BESIII: aim at systematic accuracy: 3.0%

151 energy points $>10^5$ hadronic events each \rightarrow statistical error negligible

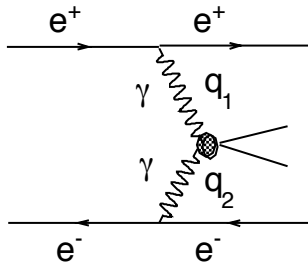
Energy region	Energy points	Note
2.400~3.400	4	Mini-scan
3.800~4.590	104	Fine-scan heavy charm resonant
2.000~3.080	21	R&QCD-scan
3.050~3.120	16	J/ ψ -scan
3.542~3.600	5	τ -scan
3.650,3.671	2	$\psi(3686)$ -scan

Reducing the uncertainty of $\alpha_{em}(M_Z^2)$

\rightarrow A new quantity of electroweak precision fits

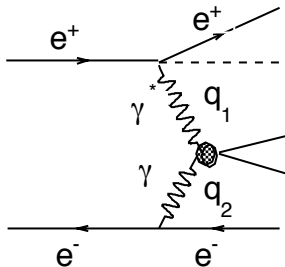
Meson Transition Form Factor $|F(Q^2)|$

Spacelike Transition FFs



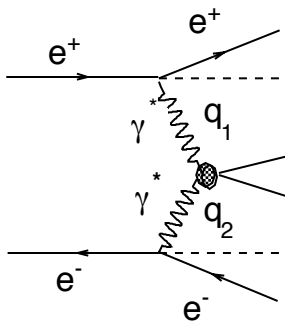
Untag:

- Only tag the hadron products, P_t -balance
- $Q_i^2 \sim 0 \text{ GeV}^2$, quasi-real photon



Single tag:

- Tag the hadron products
- Tag only one lepton, missing momentum direction
- $Q_1^2 \sim 0 \text{ GeV}^2$, $Q_1^2 = -q_2^2 \text{ GeV}^2$; highly virtual photon



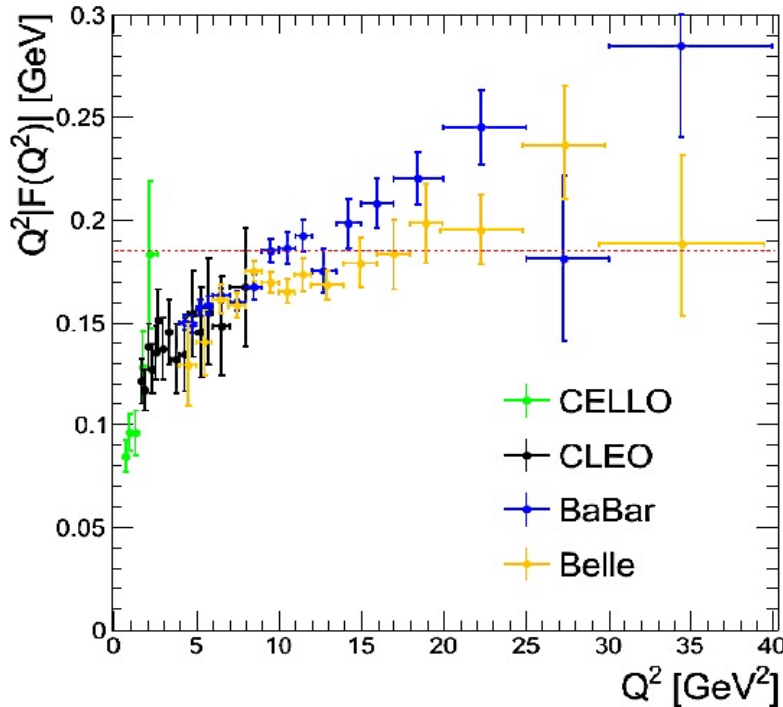
Double tag:

- Tag the hadron products
- Tag both leptons
- Both photons are virtual

Input for data-driven approach

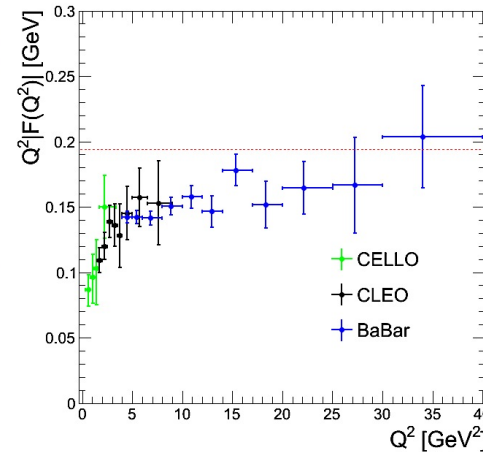
Existing Data on Spacelike TFFs

$$e^+e^- \rightarrow e^+e^- \pi^0$$

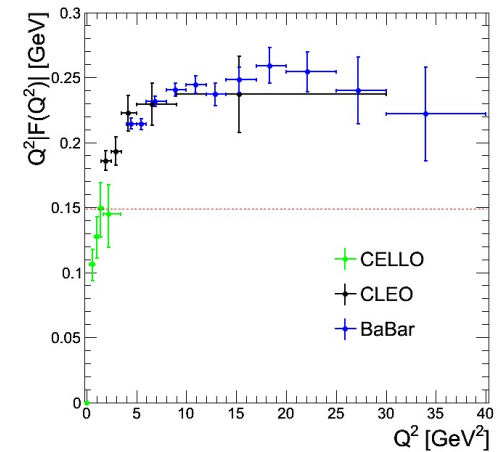


[CELLO: Z. Phys. C 49 401 (1991)]
 [CLEO: Phys. Rev. D57 33 (1998)]
 [BaBar: Phys. Rev. D80 052002 (2009)]
 [Belle: Phys. Rev. D86 092007 (2012)]

$$e^+e^- \rightarrow e^+e^- \eta$$



$$e^+e^- \rightarrow e^+e^- \eta'$$



- Recent results from BABAR and BELLE:
 $Q^2 > 4 \text{ GeV}^2$
- CLEO: $Q^2 > 1.5 \text{ GeV}^2$
- CELLO: $Q^2 < 1.5 \text{ GeV}^2$, very poor accuracy

Low Q^2 range not covered/precise

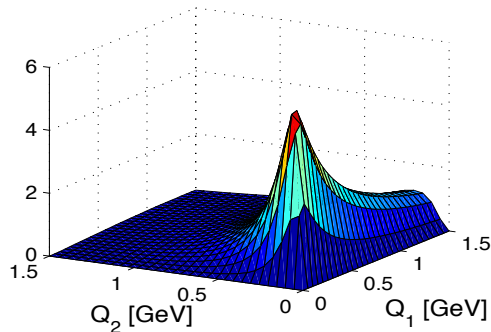
Relevant Q^2 Region

$$a_{\mu}^{\text{HLBL};\pi^0} = \int_0^{\infty} dQ_1 \int_0^{\infty} dQ_2 \sum_i w_i(Q_1, Q_2) f_i(Q_1, Q_2)$$

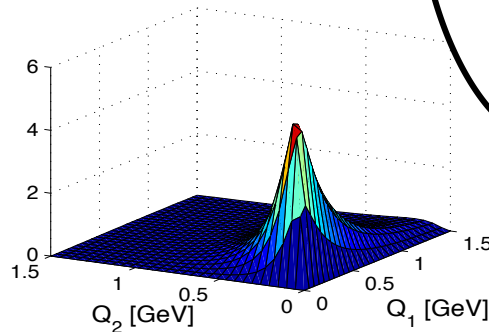
Form factor dependent

Universal weight functions

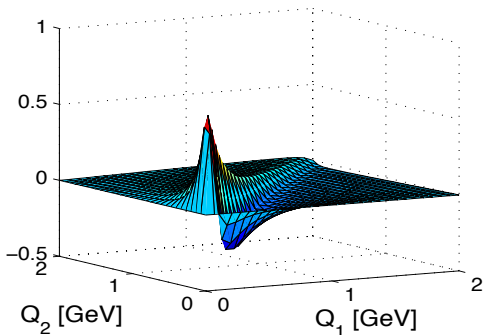
$w_f(Q_1, Q_2)$



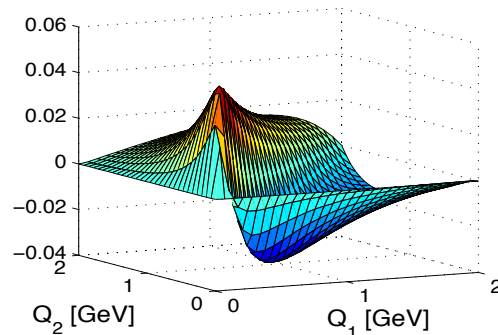
$w_{g_1}(M_V, Q_1, Q_2)$



$w_{g_2}(M_{\pi}, Q_1, Q_2)$



$w_{g_2}(M_V, Q_1, Q_2)$

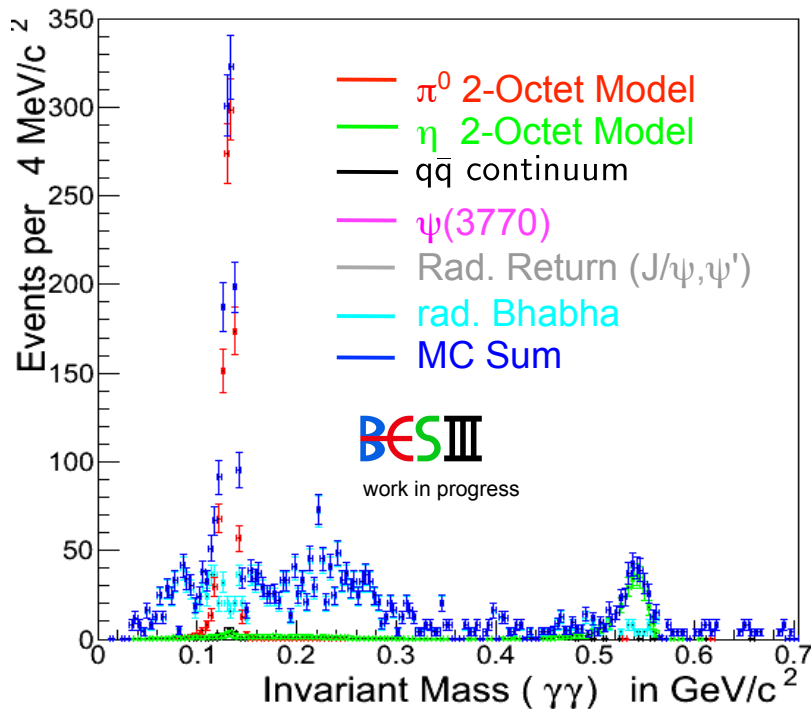


Relevant Q^2 region:
1.5 GeV^2

$e^+e^- \rightarrow e^+e^- \pi^0$ at BESIII

L_{int} : 927 pb $^{-1}$ Tagged lepton: e^-

MC only, part of full statistics

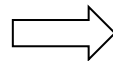


Event Selection:

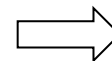
- Exactly one lepton candidate
- At least two, max four photons
- Helicity angle $\cos \theta_H > 0.8$
- Kinematic cuts to reject ISR background
- Cut on angle of missing momentum

Strategy:

Count
 π^0 yield in
bins of Q^2

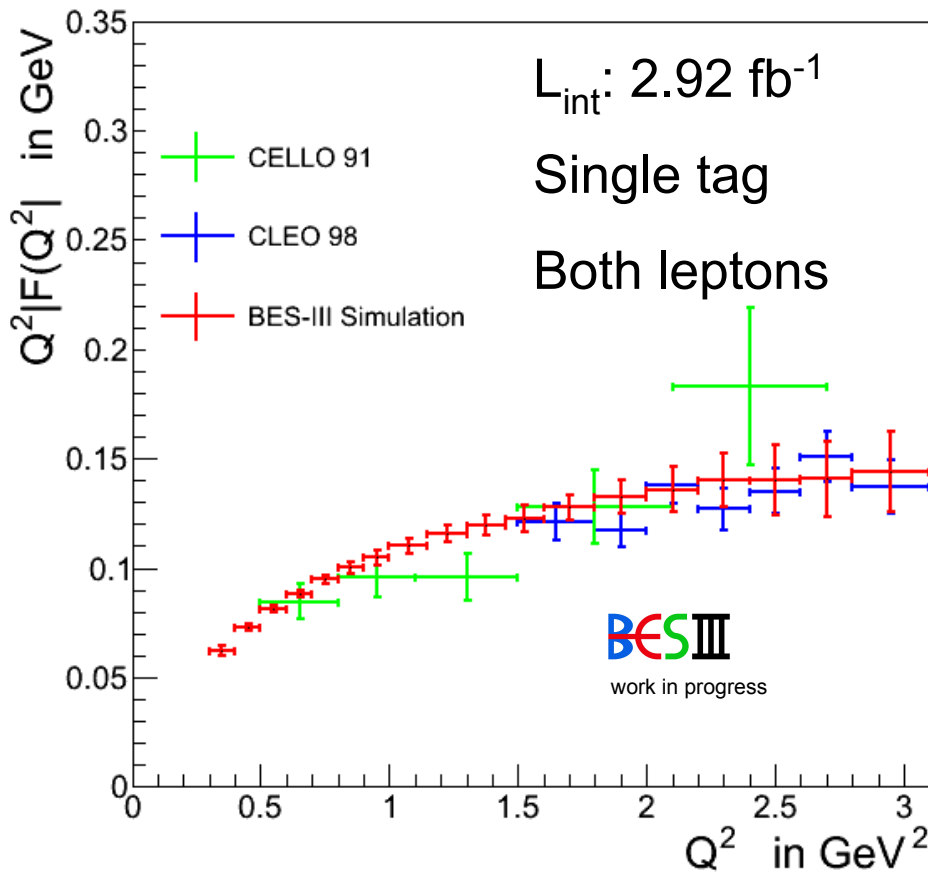


$d\sigma/dQ^2$



Form factor
 $F(Q^2)$

Spacelike transition FFs: π^0



[CELLO: Z. Phys. C 49 401 (1991)]
[CLEO: Phys. Rev. D57 33 (1998)]

MC only, red error bars
corresponding to BESIII statistics

Extract TFF for:

$$0.3 \leq Q^2[\text{GeV}^2] \leq 3.1$$

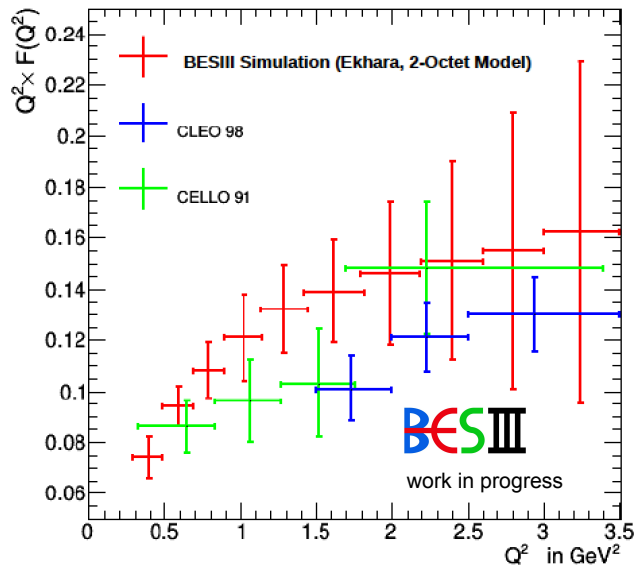
Significantly improves and
extends data set below $Q^2 =$
 1.5 GeV^2

Input for $(g-2)_\mu!$

Spacelike transition FFs: η / η'

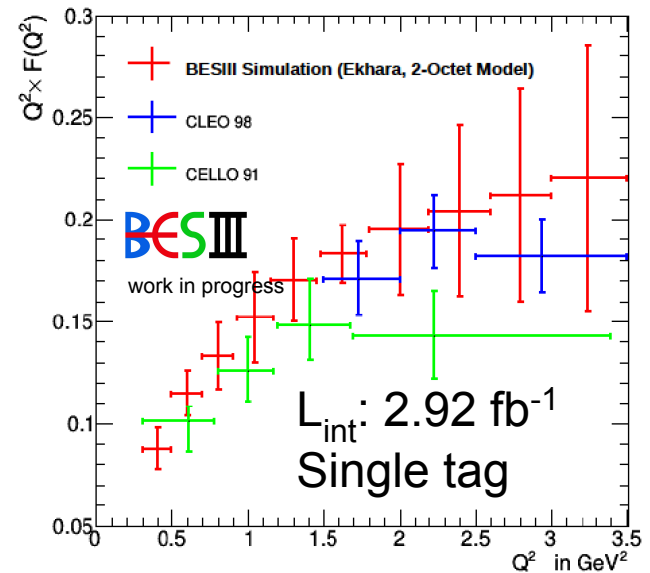
MC only, red error bars corresponding to BESIII statistics

$$F_{\eta, \gamma, \gamma^*}(Q^2)$$



$$\eta \rightarrow \pi^+ \pi^- \pi^0 \rightarrow \pi^+ \pi^- \gamma \gamma$$

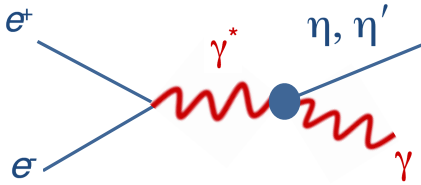
$$F_{\eta', \gamma, \gamma^*}(Q^2)$$



$$\eta' \rightarrow \pi^+ \pi^- \eta \rightarrow \pi^+ \pi^- \gamma \gamma$$

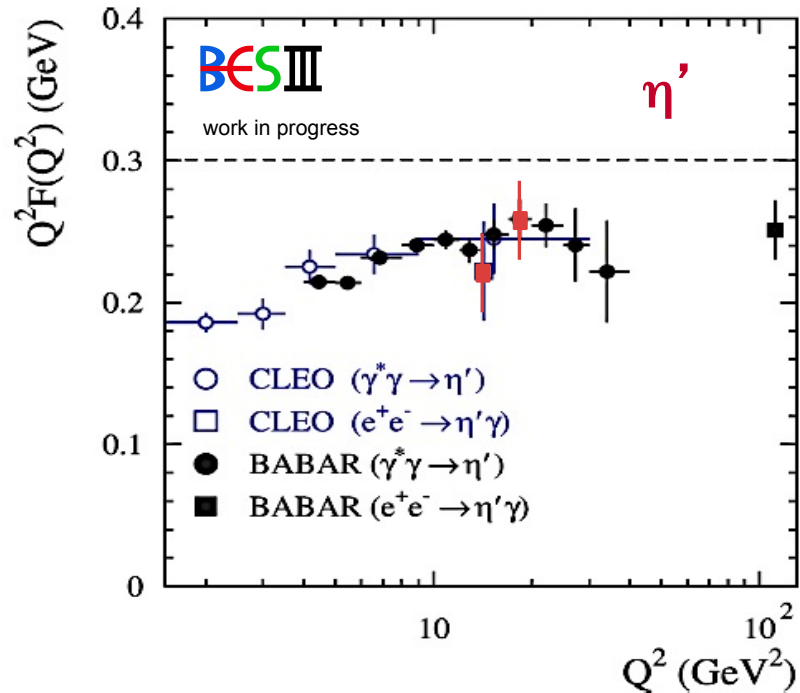
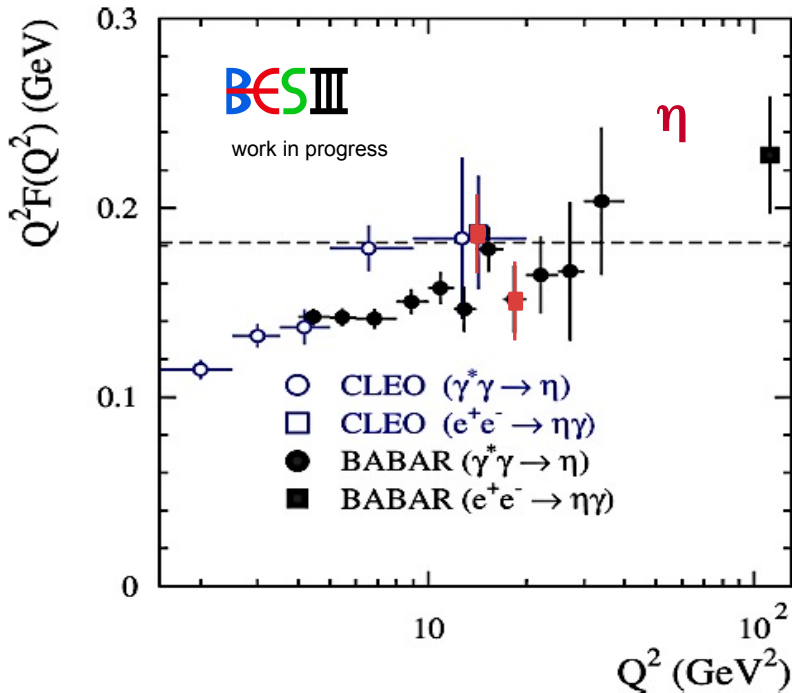
- Results competitive to previous measurement
- More data and more decay modes \rightarrow order of magnitude improvement

Timelike transition FFs



η and η' timelike TFF

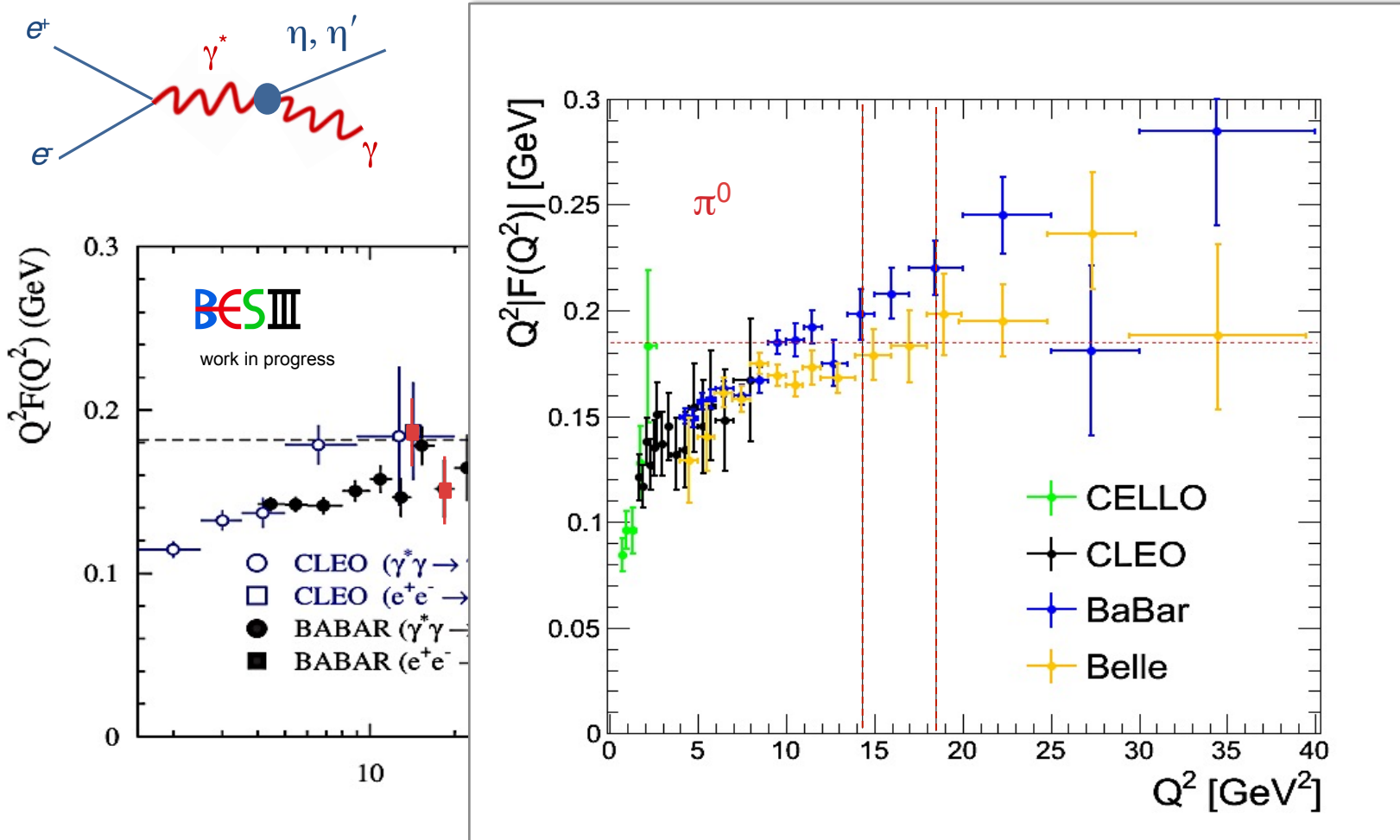
$$\sigma_{e^+e^- \rightarrow \eta^{(\prime)}\gamma}(q^2) = \frac{2\pi^2\alpha^3}{3} \left(1 - \frac{m_{\eta'}^2}{s}\right)^3 |F_{\eta'}(q^2)|^2$$



- $\langle Q^2 \rangle$: 18.5 GeV², 5.3 fb⁻¹ from 4.0-4.6 GeV
- $\langle Q^2 \rangle$: 14.2 GeV², 2.9 fb⁻¹ at 3.773 GeV

Position: arbitrary
Error: Corresponding to statistics

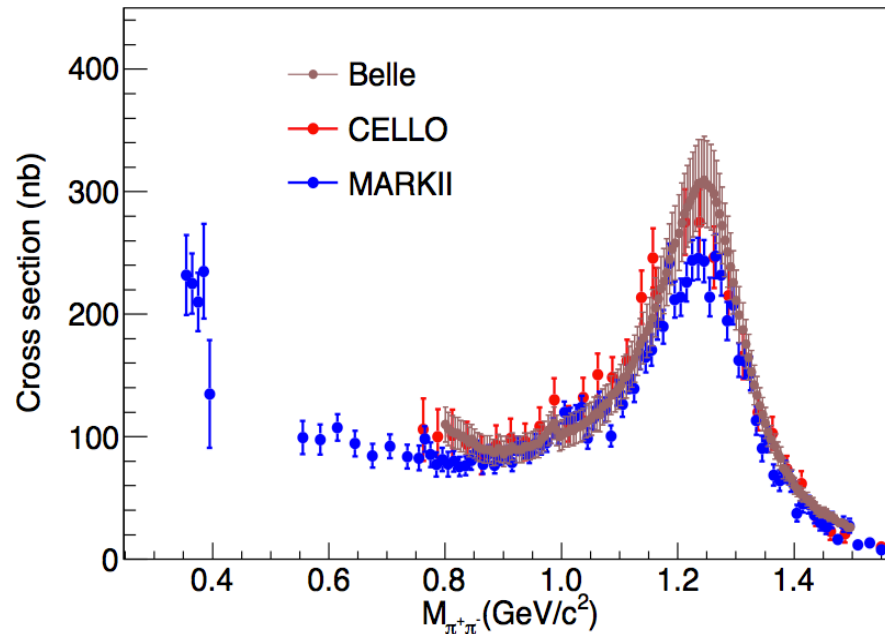
Timelike transition FFs



- $\langle Q^2 \rangle$: 18.5 GeV^2 , 5.3 fb^{-1} from 4.0-4.6 GeV
- $\langle Q^2 \rangle$: 14.2 GeV^2 , 2.9 fb^{-1} at 3.773 GeV

$\gamma \gamma^* \rightarrow \pi^+ \pi^-$ at BESIII

- Previous measurements:
 - All in two real photon case: $\gamma\gamma \rightarrow \pi^+\pi^-$
 - In low mass region, only measurement come from MarkII

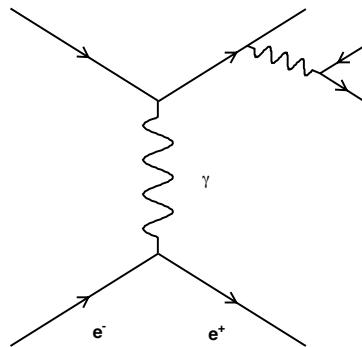


$\gamma \gamma^* \rightarrow \pi^+ \pi^-$ at BESIII

Background dominated by $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ events:

1. Cross section about 6 times larger the signal process
2. μ/π ID needed

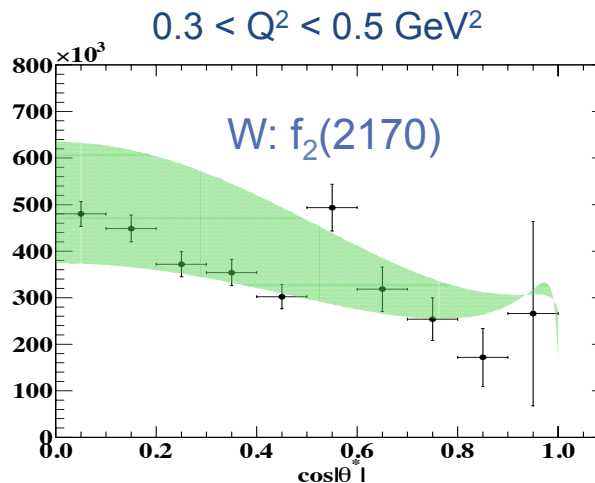
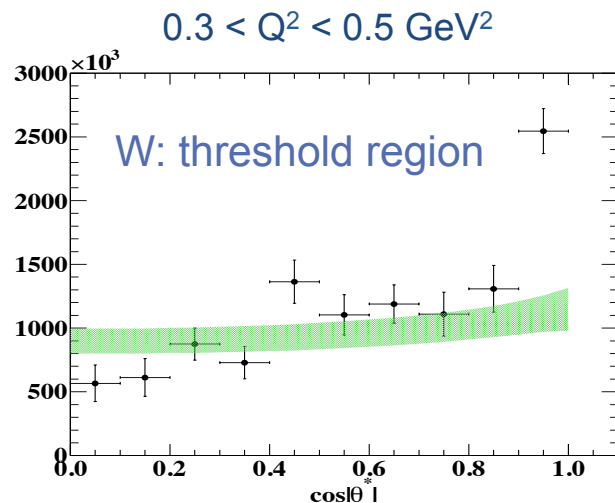
Background from vector mesons:



subtract use fit curve

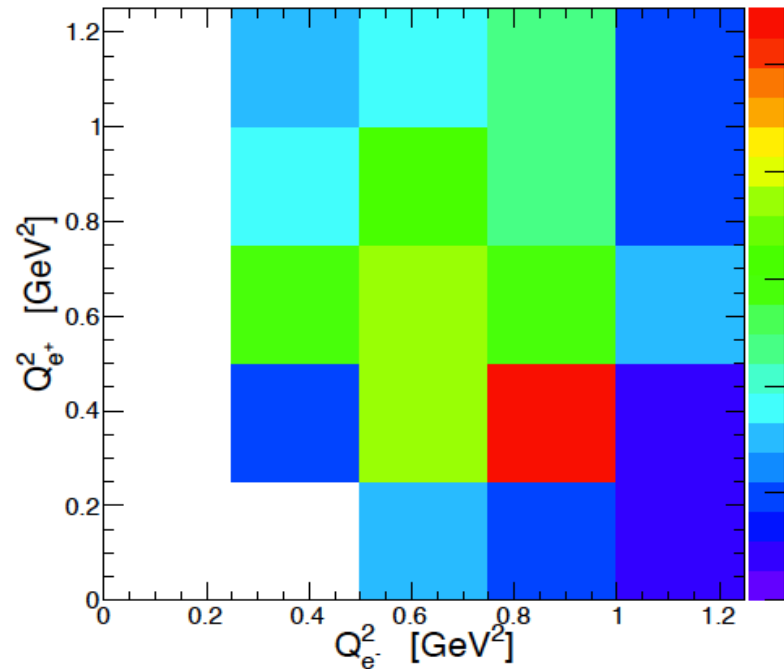
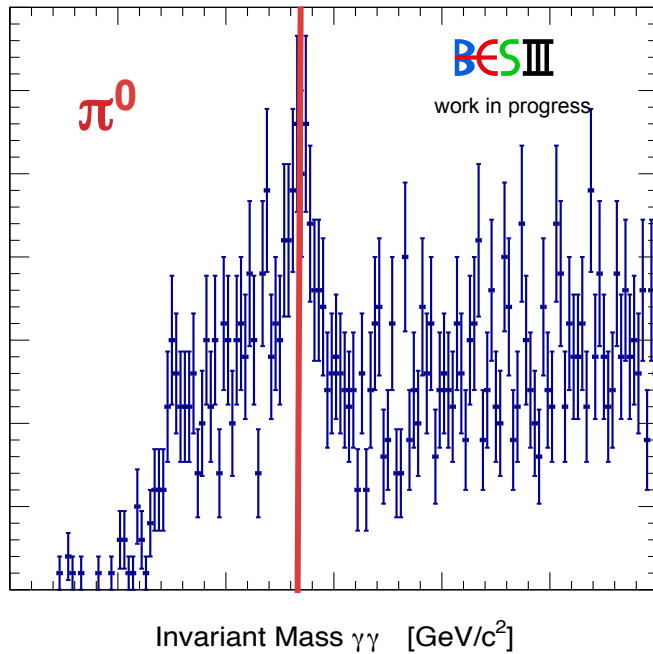
$\gamma \gamma^* \rightarrow \pi^+ \pi^-$ at BESIII

- First single tag measurement of $\pi^+ \pi^-$ channel
- Q^2 : 0.1-3 GeV^2
- W : threshold-1.5 GeV/c^2
 - Obvious $f_2(1270)$ resonance
 - Can measure from threshold
- First complete coverage of the helicity angle of pion system



Spacelike transition FFs

Exploratory first double tag measurement: $\gamma^* \gamma^* \rightarrow \pi^0$



- Preliminary study shows feasible in most of the parameter space
- Further background suppression using multivariate analysis tool

Conclusion and Outlook

- Important results (to be expected) from BESIII for SM prediction of $(g-2)_\mu$
 - HVP: precision inclusive and exclusive measurements
 - $\pi^+\pi^-$, $\pi^+\pi^-\pi^0$, and $\pi^+\pi^-\pi^0\pi^0$ cross section
 - Inclusive hadronic cross section
 - HLbL: spacelike form factors measurement in relevant region
 - Form factors at low Q^2 region
 - First measurement of single tag $\gamma\gamma\rightarrow\pi^+\pi^-$
 - Doubly off-shell form factor
- Reduction of factor of 2 of the uncertainty of a_μ^{had} in reach

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THANK YOU FOR YOUR ATTENTION!