SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

PRISMA

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

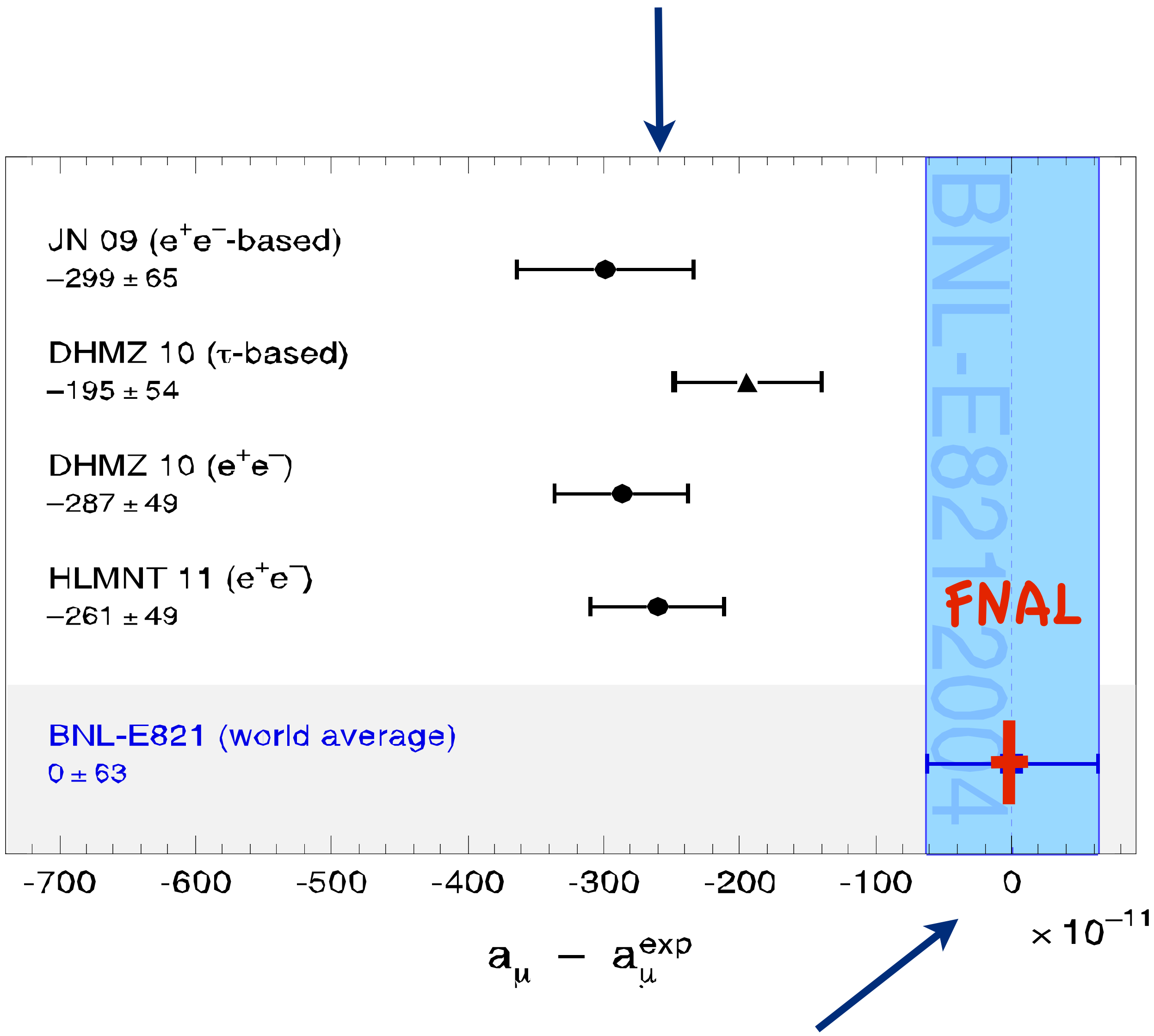
Hadronic Light-by-Light corrections to $(g-2)_\mu$

Marc Vanderhaeghen

HC2NP 2016, September 26 - 30, 2016, Puerto de la Cruz, Tenerife

$(g-2)_\mu$: theory vs experiment

SM predictions for a_μ



BNL-E821 measurement of a_μ

a_μ^{exp} = (11 659 208.9 ± 6.3) x 10^{-10}

$a_\mu^{exp} - a_\mu^{SM}$ =

(26.1 ± 5.0 $_{th}$ ± 6.3 $_{exp}$) x 10^{-10}

Hagiwara et al. (2011)

3 - 4 σ deviation from SM value !

Errors or new physics ?

New FNAL, J-PARC experiments

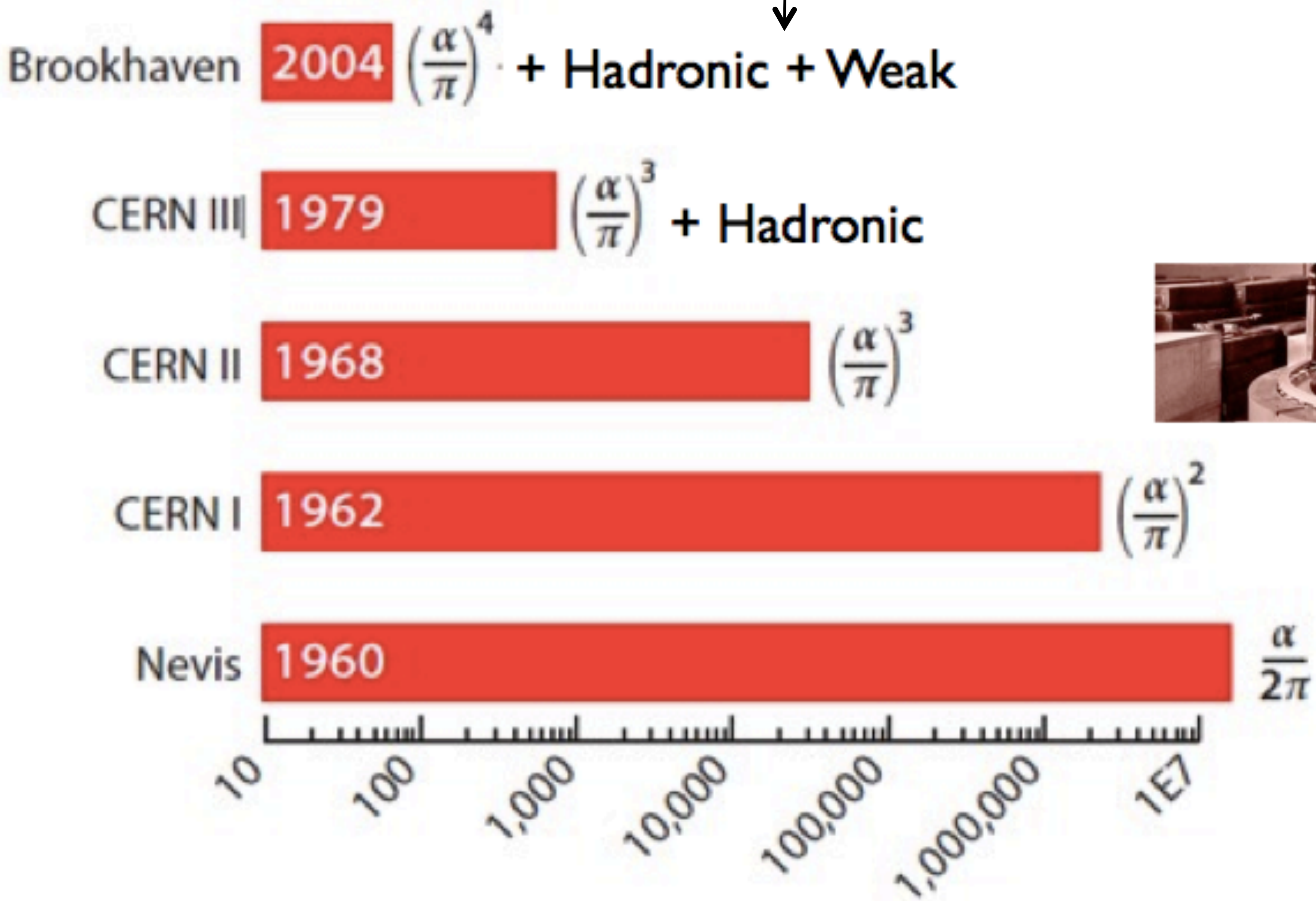
δa_μ^{FNAL} = 1.6 x 10^{-10} M. Lancaster

factor 4 improvement in exp. error

-> Improve theory !

$(g\text{-}2)_\mu$: history of relevant corrections

Contribution (theory) resolved

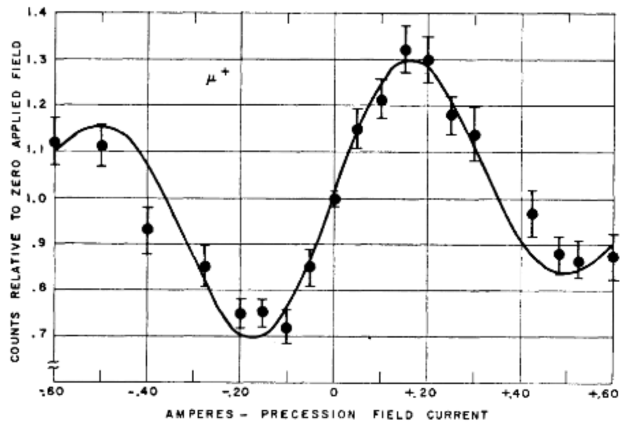


Uncertainty of measurement in 10^{-11}

Brookhaven

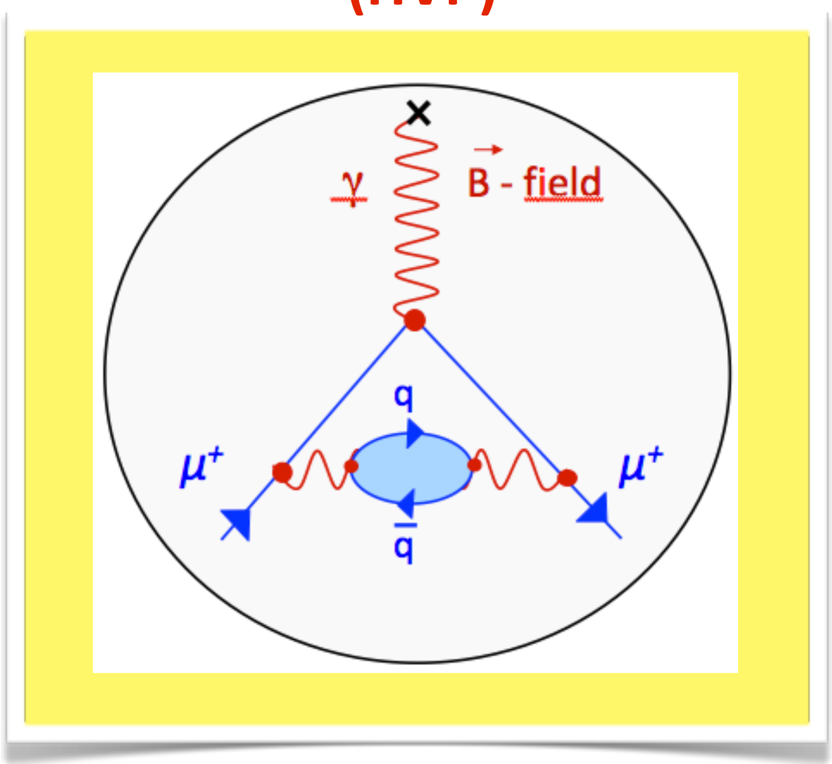
CERN I

Nevis



strong contributions to $(g-2)_\mu$

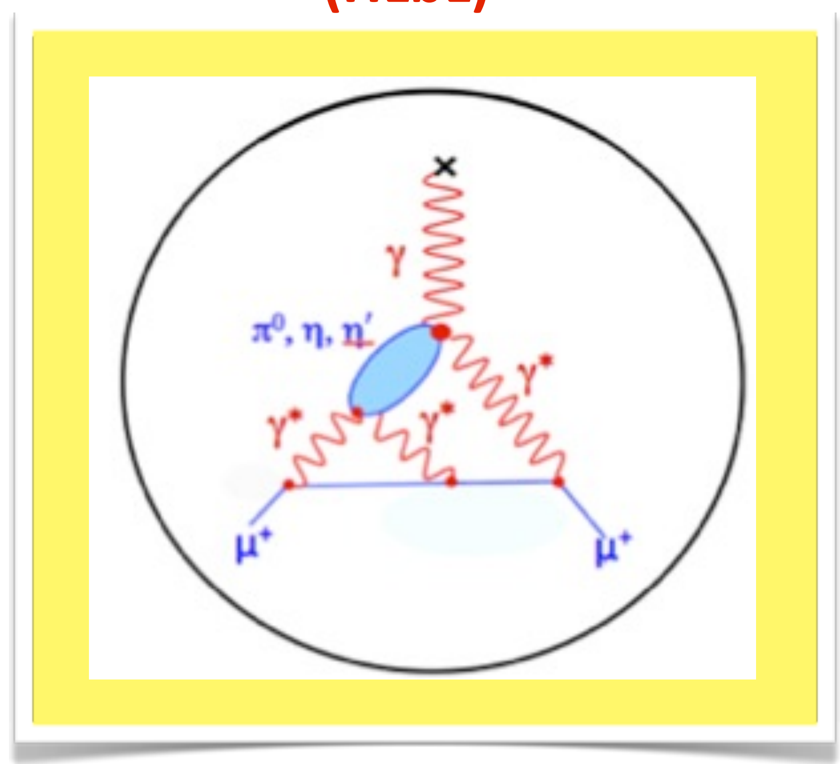
hadronic vacuum polarization (HVP)



$a_\mu^{\text{l.o. had, VP}} = (692.3 \pm 4.2) \times 10^{-10}$

Teubner et al. (2011)

hadronic light-by-light scattering (HLbL)



$a_\mu^{\text{had, LbL}} = (10.5 \pm 2.6) \times 10^{-10}$

$= (10.2 \pm 3.9) \times 10^{-10}$

Prades, de Rafael, Vainshtein (2009)

Jegerlehner, Nyffeler (2009)

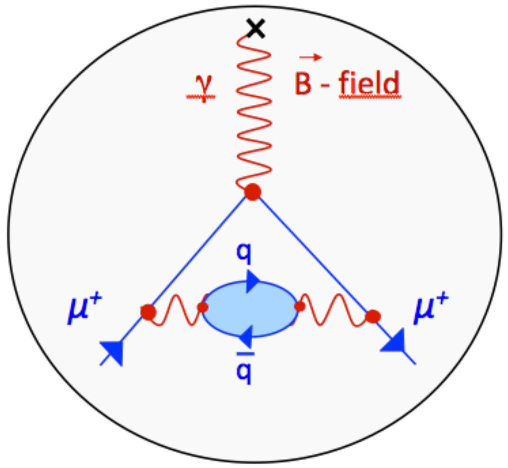
Jegerlehner (2015)

New FNAL and J-Parc $(g-2)_\mu$ expt. : $\delta a_\mu^{exp} = 1.6 \times 10^{-10}$

HVP determined by cross section measurements of e^+e^- -> hadrons

measurements of meson transition form factors required as input to reduce uncertainty

HVP corrections to $(g\text{-}2)_\mu$

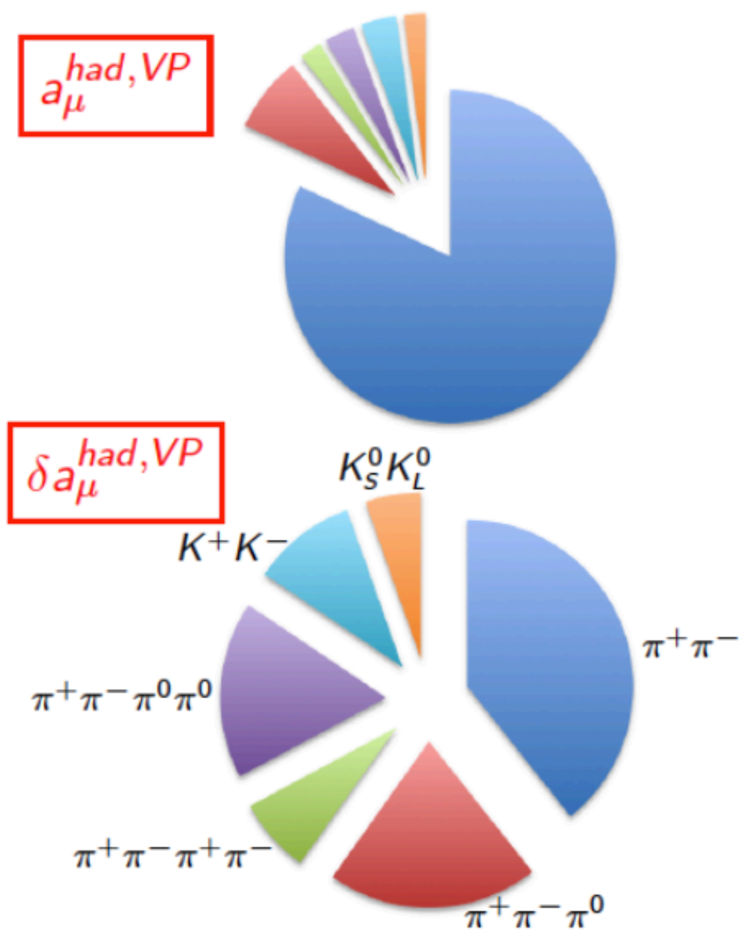


Optical theorem and analyticity allow to relate HVP contribution to $(g\text{-}2)_\mu$ with $\sigma_{had} = \sigma(e^+e^- \rightarrow \text{hadrons})$

$$a_\mu^{had,VP} = \frac{1}{4\pi^3} \int_{4 m_\pi^2}^{\infty} ds\, K(s)\, \sigma_{had}$$

known Kernel function

Hadronic cross section



Future improvement of a_μ^{had} ?

1st priority:
Clarify situation regarding π+π− (KLOE vs. BABAR puzzle)

2nd priority:
Measure 3π, 4π channels

Ongoing ISR analyses BESIII, BEPC-II collider

3rd priority:
KK and higher multiplicities

σ_{had}: Energy range up to 3 GeV essential !

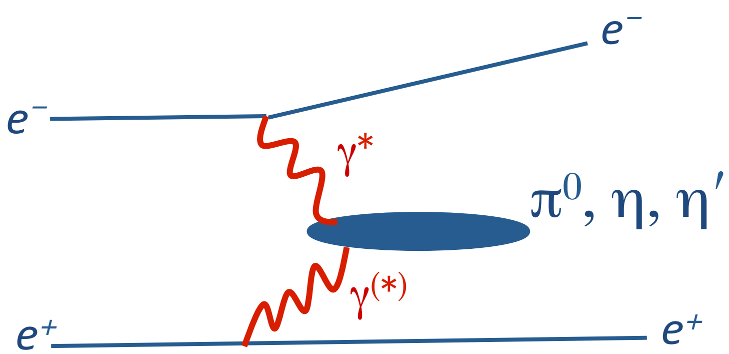
Y. Guo

aim: reduction of current error by factor of 2

theory developments: - update ππ **I. Caprini**
- lattice QCD **A. Juettner, Ch. Davies**

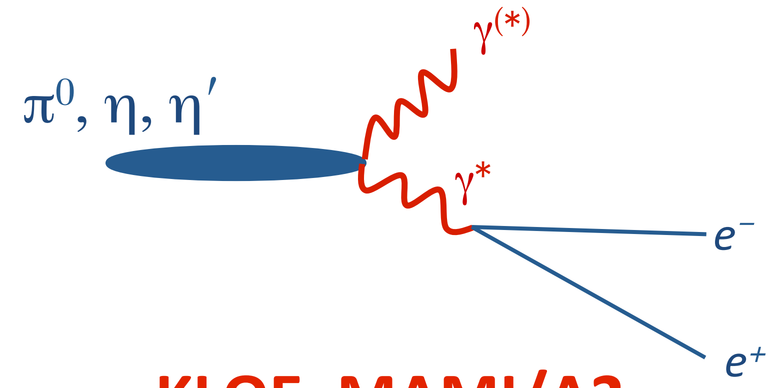
hadronic LbL corrections to $(g\text{-}2)_\mu$

experimental input: meson transition FFs, $\gamma^* \gamma^*$ -> multi-meson states, meson Dalitz decays

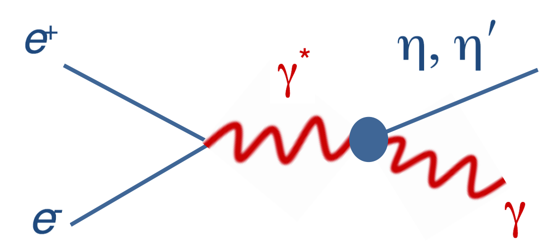


CLEO, BaBar, Belle, BESIII, ...

Y. Guo



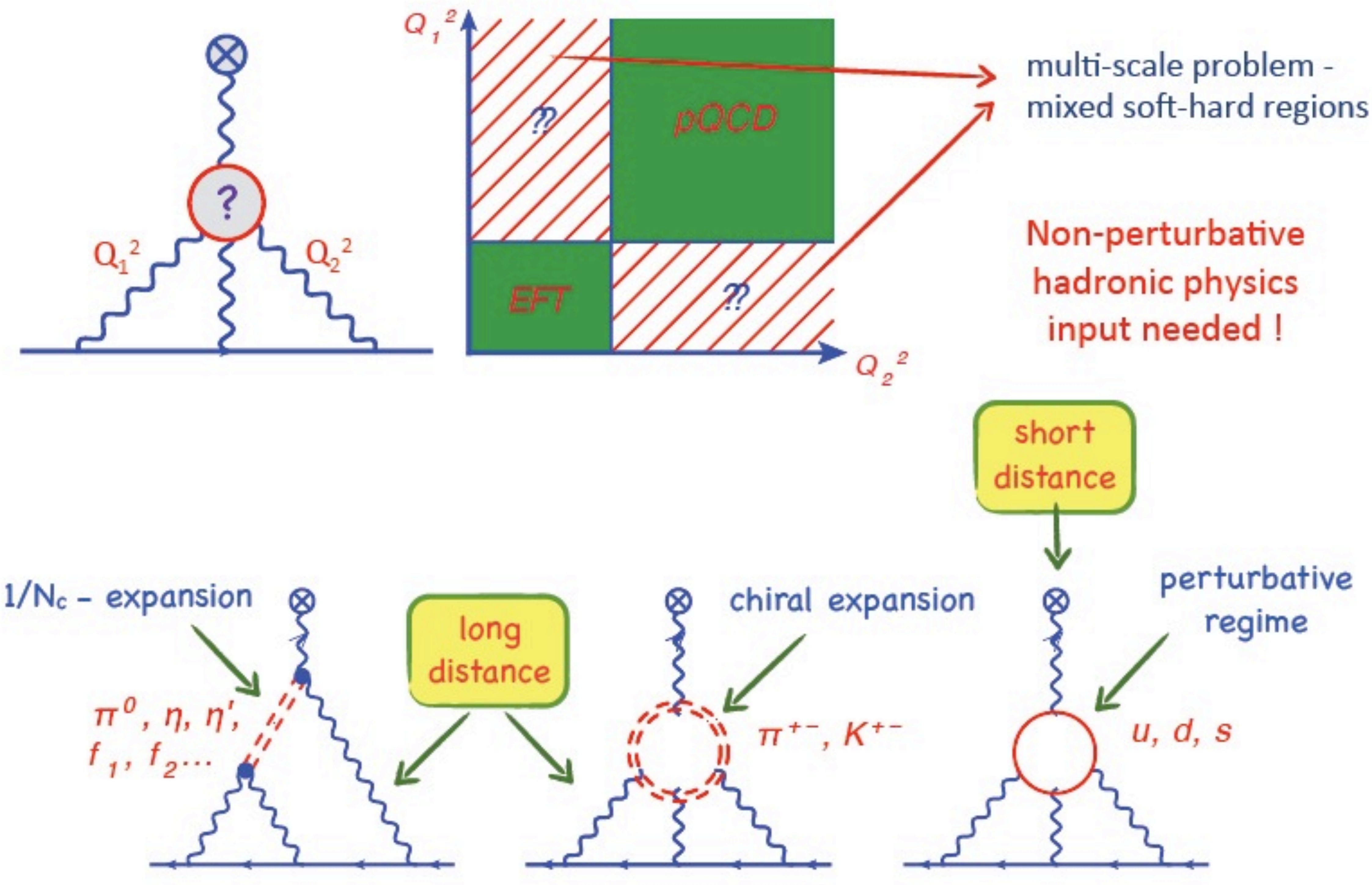
KLOE, MAMI/A2, BESIII, ...

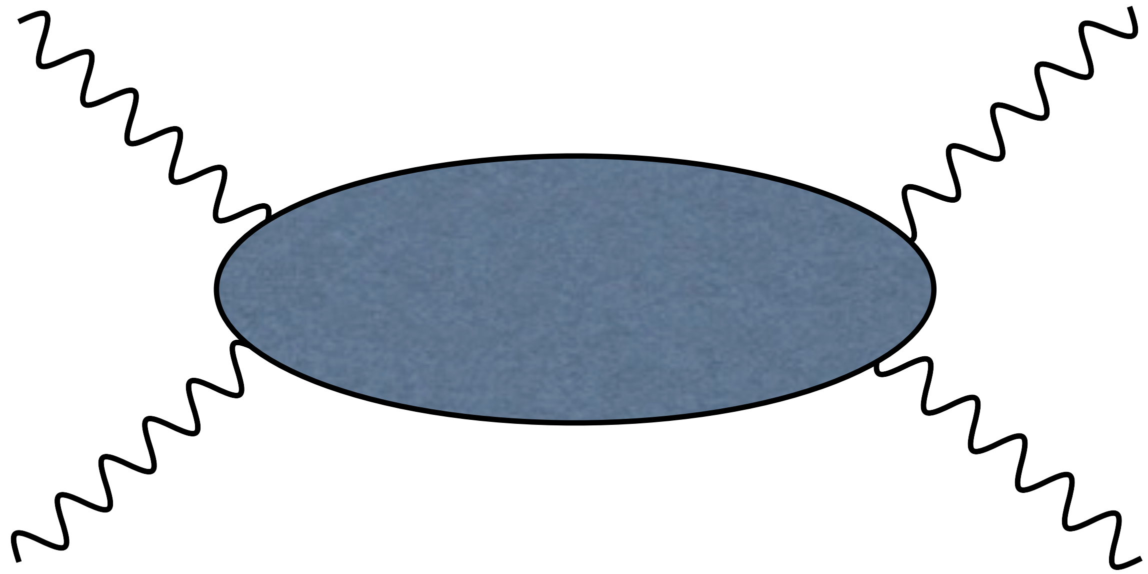


SND, CMD-2, BESIII, ...

theory deveolpments:
- sum rules, dispersion relations — this talk, G. Colangelo
- lattice QCD
- Dyson-Schwinger — G. Eichmann
- phenomenology, modeling — P. Sanchez Puertas, A. Zhevlakov

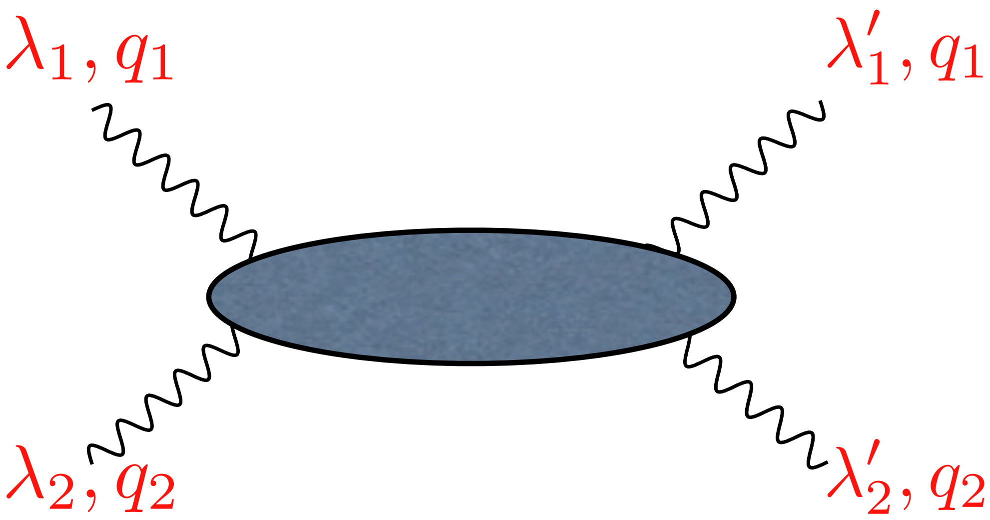
hadronic LbL corrections to $(g\text{-}2)_\mu$: relevant contributions





what is known about hadronic LbL scattering ?

Theory: sum rules for LbL scattering (I)



$$\gamma^*(\lambda_1, q_1) + \gamma^*(\lambda_2, q_2) \to \gamma^*(\lambda'_1, q_1) + \gamma^*(\lambda'_2, q_2)$$

kinematical invariants:

$$s = (q_1 + q_2)^2, \qquad u = (q_1 - q_2)^2$$

$$\nu \equiv \frac{s-u}{4}, \quad Q_1^2 \equiv -q_1^2, \quad Q_2^2 \equiv -q_2^2$$

helicity amplitudes: $M_{\lambda'_1 \lambda'_2, \lambda_1 \lambda_2}(\nu, Q_1^2, Q_2^2)$ $\quad \lambda = 0, \pm 1$

discrete symmetries: → 8 independent amplitudes:

$$P: \quad M_{\lambda'_1 \lambda'_2, \lambda_1 \lambda_2} = M_{-\lambda'_1 -\lambda'_2, -\lambda_1 -\lambda_2}$$

$$T: \quad M_{\lambda'_1 \lambda'_2, \lambda_1 \lambda_2} = M_{\lambda_1 \lambda_2, \lambda'_1 \lambda'_2}$$

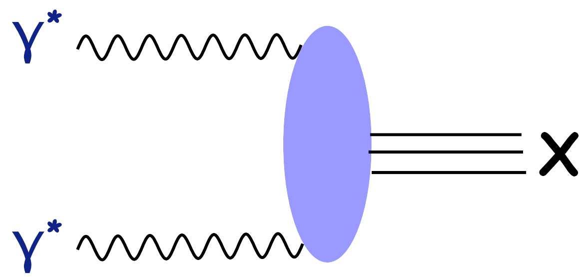
$M_{++,++}, \; M_{+-,+-}, \; M_{++,--},$ **T**

$M_{00,00}, \; M_{+0,+0}, \; M_{0+,0+}, \; M_{++,00}, \; M_{0+,-0}$ **T and L**

sum rules for LbL scattering (II)

Unitarity: link to $\gamma^* \gamma^* \to$ X cross sections

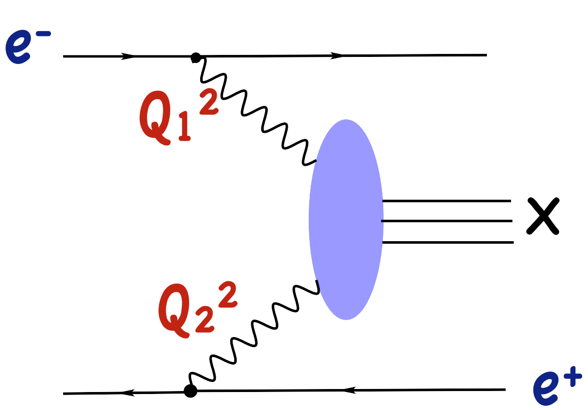
$$W_{\lambda'_1 \lambda'_2, \lambda_1 \lambda_2} \equiv \mathrm{Im}\, M_{\lambda'_1 \lambda'_2, \lambda_1 \lambda_2}$$



$$\begin{aligned}
W_{++,++} + W_{+-,+-} &\equiv 2\sqrt{X}\,(\sigma_0 + \sigma_2) = 2\sqrt{X}\,(\sigma_\parallel + \sigma_\perp) \equiv 4\sqrt{X}\,\sigma_{TT},\\
W_{++,++} - W_{+-,+-} &\equiv 2\sqrt{X}\,(\sigma_0 - \sigma_2) \equiv 4\sqrt{X}\,\tau^a_{TT},\\
W_{++,--} &\equiv 2\sqrt{X}\,(\sigma_\parallel - \sigma_\perp) \equiv 2\sqrt{X}\,\tau_{TT},\\
W_{00,00} &\equiv 2\sqrt{X}\,\sigma_{LL},\\
W_{+0,+0} &\equiv 2\sqrt{X}\,\sigma_{TL},\\
W_{0+,0+} &\equiv 2\sqrt{X}\,\sigma_{LT},\\
W_{++,00} + W_{0+,-0} &\equiv 4\sqrt{X}\,\tau_{TL},\\
W_{++,00} - W_{0+,-0} &\equiv 4\sqrt{X}\,\tau^a_{TL}.
\end{aligned}$$

$$X \equiv \nu^2 - Q_1^2 Q_2^2$$

Experiment: e⁻ e⁺ → e⁻ e⁺ X cross sections

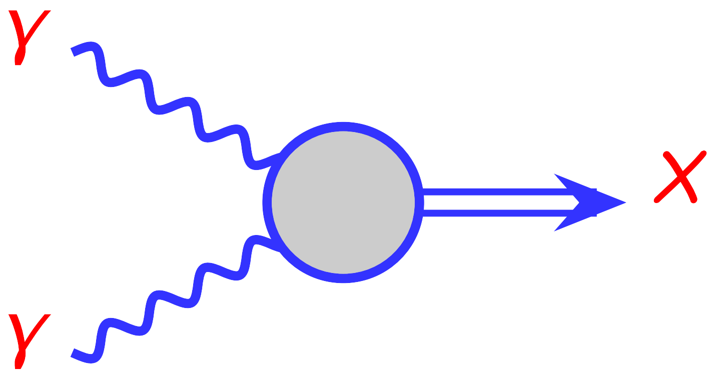


$$\begin{aligned}
d\sigma = &\frac{\alpha^2}{16\pi^4 Q_1^2 Q_2^2}\,\frac{2\sqrt{X}}{s(1-4m^2/s)} \cdot \frac{d^3\vec{p}_1{}'}{E'_1} \cdot \frac{d^3\vec{p}_2{}'}{E'_2}\\
&\times \Big\{4\,\rho_1^{++}\,\rho_2^{++}\,\sigma_{TT} + \rho_1^{00}\,\rho_2^{00}\,\sigma_{LL} + 2\,\rho_1^{++}\,\rho_2^{00}\,\sigma_{TL} + 2\,\rho_1^{00}\,\rho_2^{++}\,\sigma_{LT}\\
&+2\,(\rho_1^{++}-1)\,(\rho_2^{++}-1)\,\left(\cos 2\tilde{\phi}\right)\,\tau_{TT} + 8\left[\frac{(\rho_1^{00}+1)\,(\rho_2^{00}+1)}{(\rho_1^{++}-1)\,(\rho_2^{++}-1)}\right]^{1/2}\left(\cos\tilde{\phi}\right)\,\tau_{TL}\\
&+h_1 h_2\, 4\left[(\rho_1^{00}+1)\,(\rho_2^{00}+1)\right]^{1/2}\,\tau^a_{TT} + h_1 h_2\, 8\left[(\rho_1^{++}-1)\,(\rho_2^{++}-1)\right]^{1/2}\left(\cos\tilde{\phi}\right)\,\tau^a_{TL}\Big\}
\end{aligned}$$

lepton beam polarization ($h_1 h_2$)

ρ's, ϕ : kinematical quantities

sum rules for LbL scattering (III)

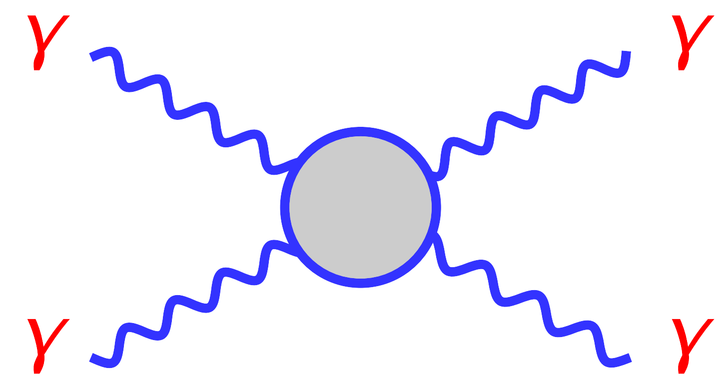


micro-causality

unitarity

LETs

dispersion theory



imaginary part of the amplitude - photon-photon fusion into leptons and hadrons:

real part of the amplitude - low-energy structure of the elastic LbL scattering:

$$\mathrm{Im} f^{(-)}(s) = -\frac{s}{8}[\sigma_2(s) - \sigma_0(s)]$$

$$\mathrm{Im} f^{(+)}(s) = -\frac{s}{8}[\sigma_{tot}(s)]$$

Sum rules

$$\mathcal{L}^{(8)} = c_1 (F_{\mu\nu}F^{\mu\nu})^2 + c_2 (F_{\mu\nu}\tilde{F}^{\mu\nu})^2$$

Euler, Heisenberg (1936)

$$\int_{s_0}^{\infty} \frac{ds}{s} [\sigma_2(s) - \sigma_0(s)] = 0$$

$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} \frac{ds}{s^2} [\sigma_{\parallel}(s) \pm \sigma_{\perp}(s)]$$

sum rules for LbL scattering (IV)

3 superconvergent relations:

Pascalutsa, Vdh (2010)

Pascalutsa, Pauk, Vdh (2012, 2014)

helicity difference sum rule

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} \left[\sigma_0 - \sigma_2\right]_{Q_2^2=0}$$

for $Q^2 = 0$: GDH sum rule

Gerasimov, Moulin (1975), Brodsky, Schmidt (1995)

sum rules involving longitudinal photons

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_\parallel + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^{a}\right]_{Q_2^2=0}$$

$$0 = \int_{s_0}^{\infty} ds \left[\frac{\tau_{TL}}{Q_1 Q_2}\right]_{Q_2^2=0}$$

SRs involving LbL low-energy constants:

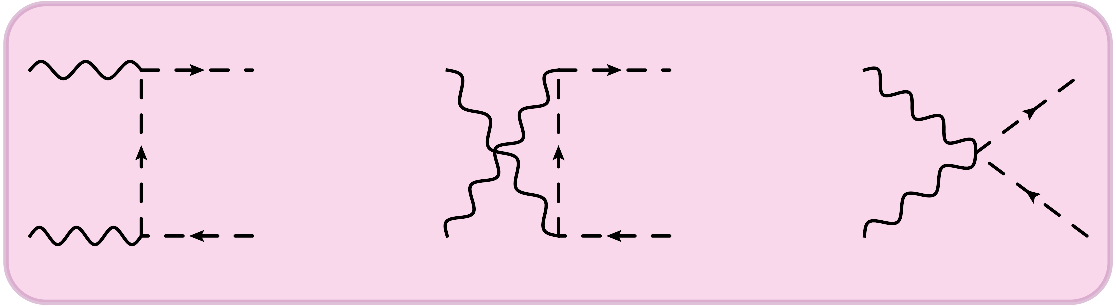
$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} \frac{ds}{s^2} \left[\sigma_\parallel(s) \pm \sigma_\perp(s)\right]$$

+ 6 new LECs at next order

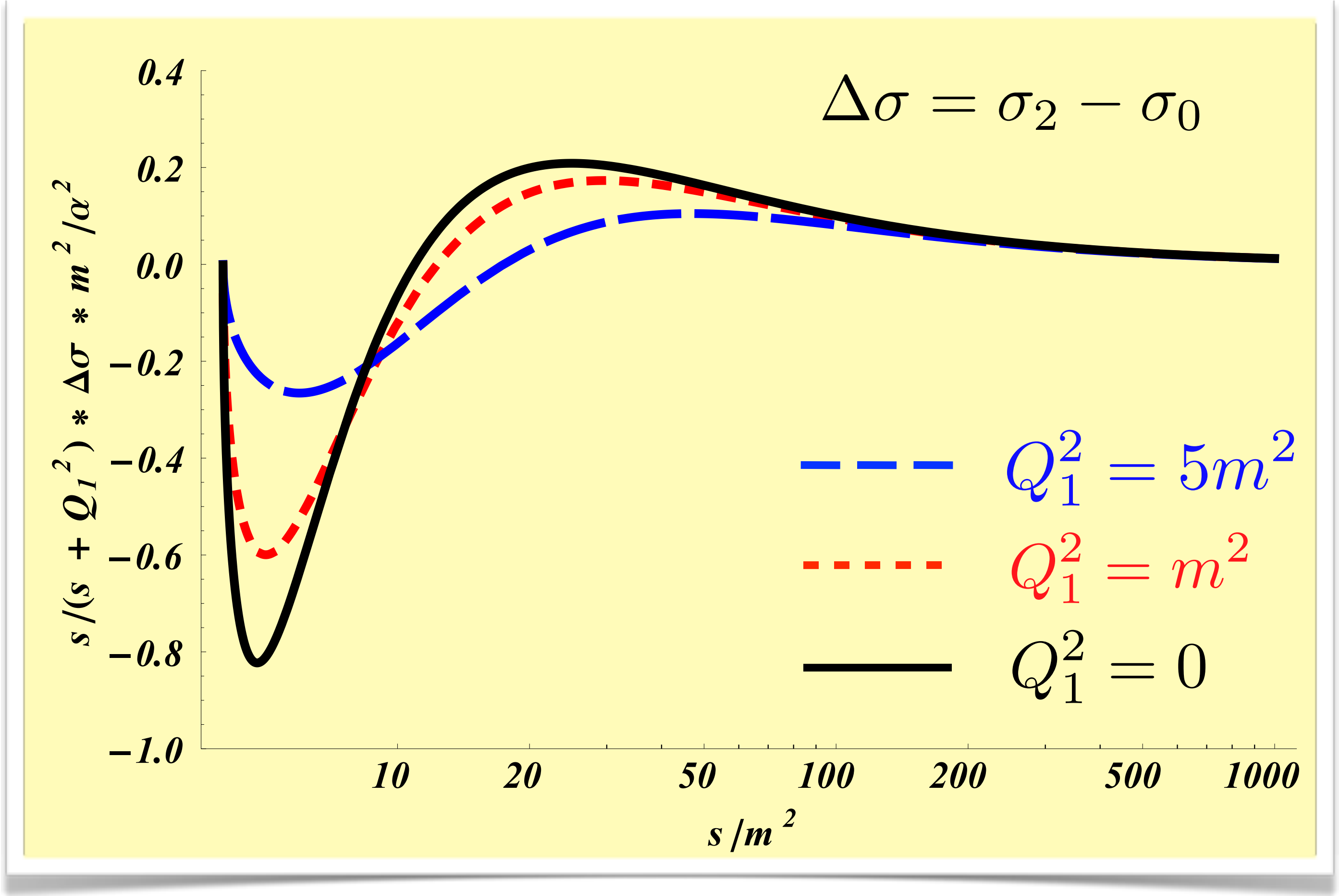
sum rules for LbL scattering (V)

➡ sum rules have been tested in perturbative QFT both at tree-level and 1-loop level

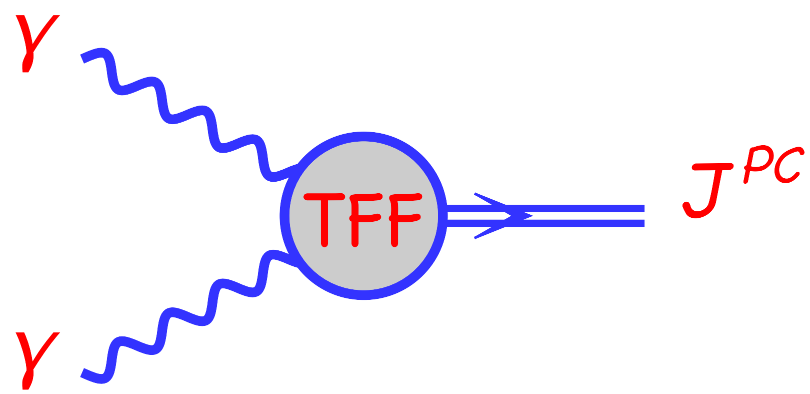
scalar QED



$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} \left[\sigma_0 - \sigma_2\right]_{Q_2^2=0}$$



meson production in $\gamma\gamma$ collisions (I)



- two-photon state: produced meson has C=+1

- both photons are real: J=1 final state is forbidden (Landau-Yang theorem);

the main contribution comes from

J=0: 0^{-+} (pseudoscalar) and 0^{++} (scalar)

and J=2: 2^{++} (tensor)

– the SRs hold separately for channels of given intrinsic quantum numbers: isoscalar and isovector mesons, $c\bar{c}$ states

– input for the absorptive part of the SRs: γγ-hadrons response functions, can be expressed in terms of γγ→M transition form factors

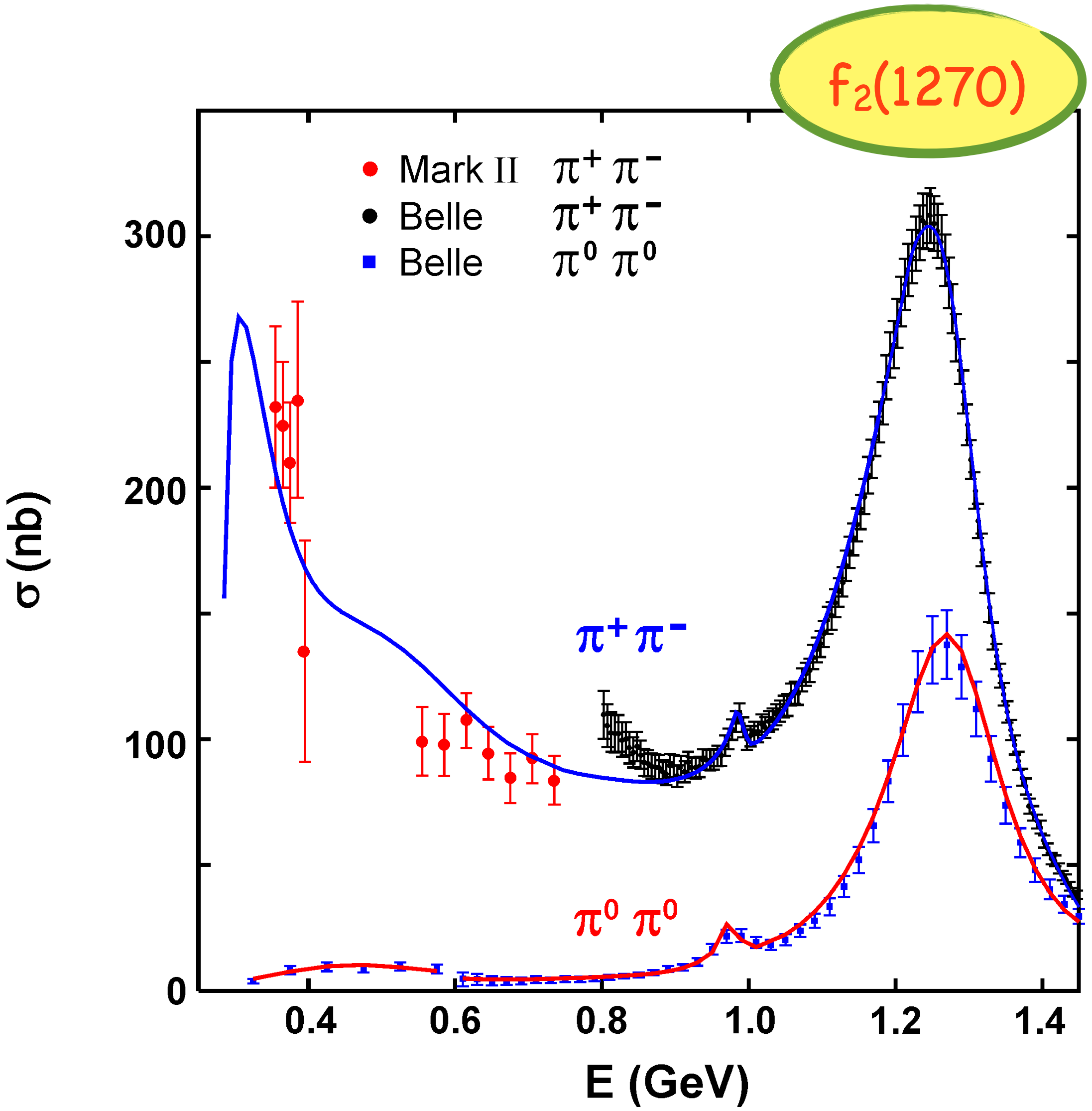
$$\sigma_{\Lambda}^{\gamma\gamma\to M}(s) \approx (2J+1)16\pi^2 \frac{\Gamma_{\gamma\gamma}}{m_M}\delta(s-m_M^2)$$

meson contribution to the cross-section in the narrow-resonance approximation

$$\Gamma_{\gamma\gamma}(\mathcal{P}) = \frac{\pi\alpha^2}{4} m^3 \, |F_{\mathcal{M}\gamma^*\gamma^*}(0,0)|^2$$

two-photons decay rate for the meson

meson production in $\gamma\gamma$ collisions (II)



Dai, Pennington (2014)

dominant features:

- Born terms for $\pi^+ \pi^-$
- s-wave rescattering
- tensor resonance $f_2(1270)$

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} [\sigma_0 - \sigma_2]_{Q_2^2=0}$$

the I=0 channel

SR for $Q_1^2 = 0$

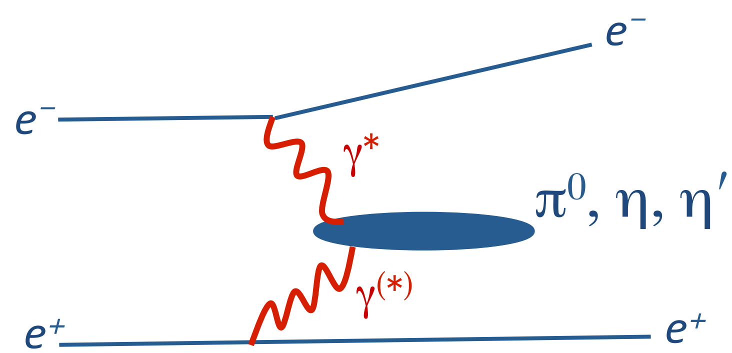
	$\int \frac{ds}{s}(\sigma_2 - \sigma_0)$ [nb]	c_1 [10^{-4}GeV^{-4}]	c_2 [10^{-4}GeV^{-4}]
η	-191 ± 10	0	0.65 ± 0.03
η'	-300 ± 10	0	0.33 ± 0.01
$f_0(980)$	-19 ± 5	0.020 ± 0.005	0
$f_0'(1370)$	-91 ± 36	0.049 ± 0.019	0
$f_2(1270)$	449 ± 52	0.141 ± 0.016	0.141 ± 0.016
$f_2'(1525)$	7 ± 1	0.002 ± 0.000	0.002 ± 0.000
$f_2(1565)$	56 ± 11	0.012 ± 0.002	0.012 ± 0.002
Sum	-89 ± 66	0.22 ± 0.03	1.14 ± 0.04

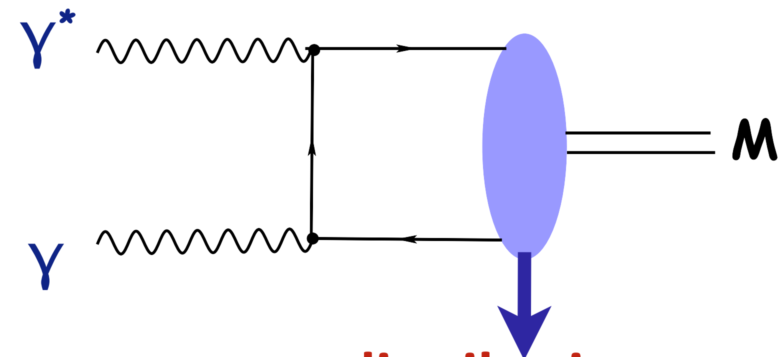
dominant contribution to c_2 comes from η, η' and $f_2(1270)$

dominant contribution to c_1 comes from $f_2(1270)$

Pascalutsa, Pauk, Vdh (2012)

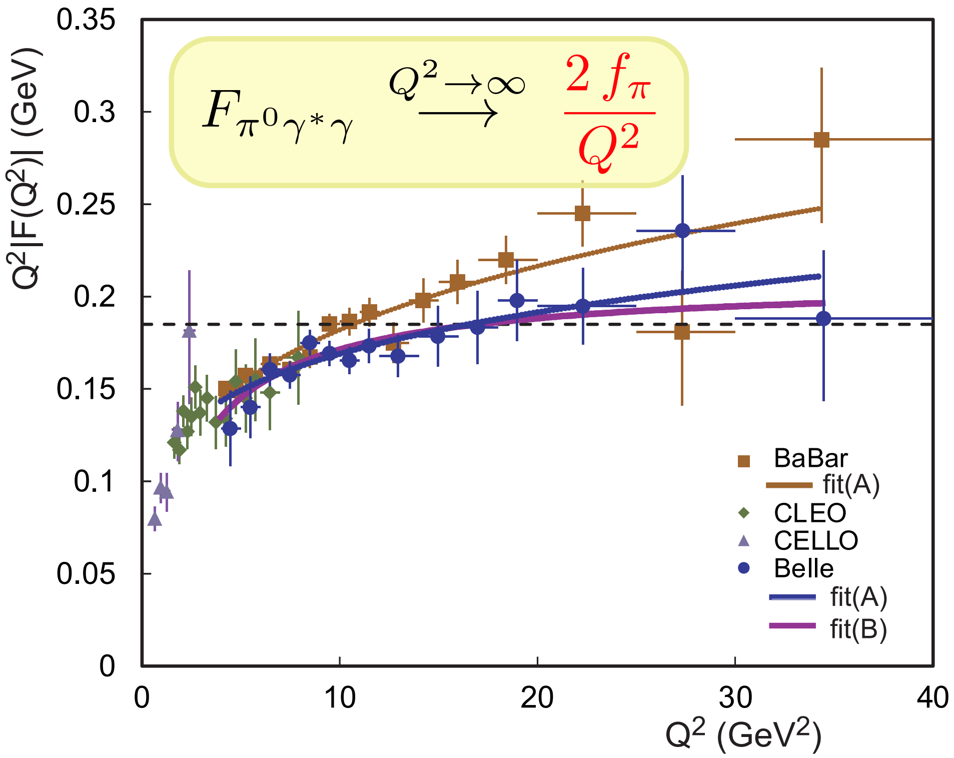
$\gamma^*\gamma^* \to$ M processes: meson transition form factors (TFFs)

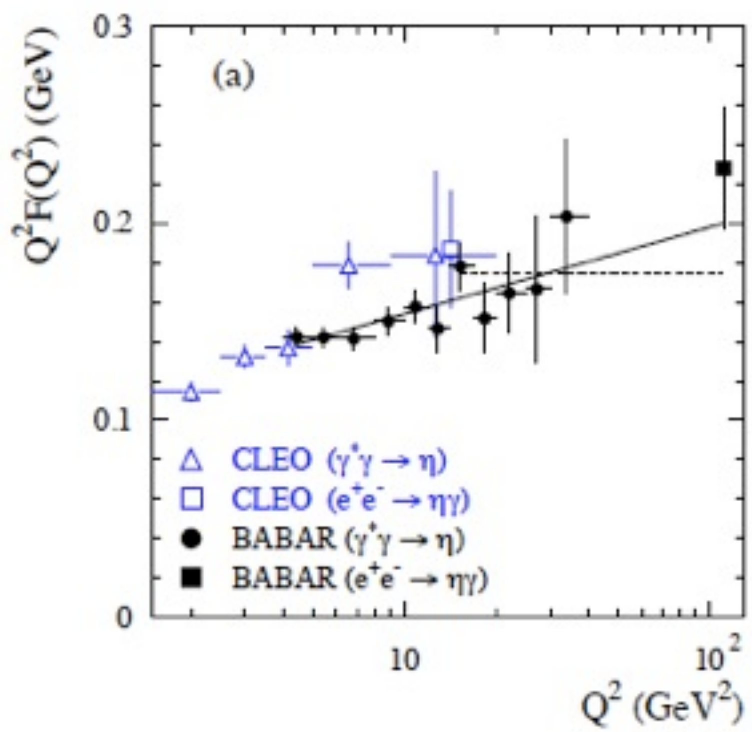


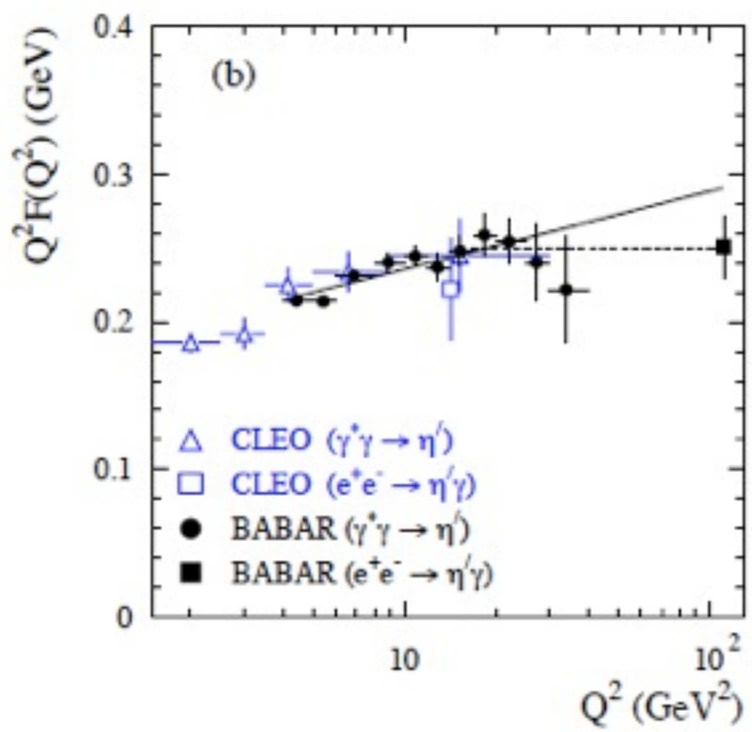


meson distribution amplitude

$\gamma^*\gamma \to \pi^0$ | $\gamma^*\gamma \to \eta$ | $\gamma^*\gamma \to \eta'$





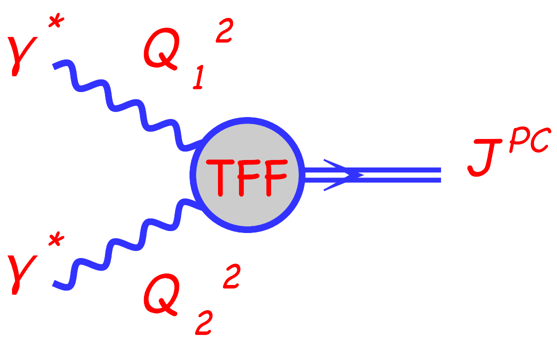


➡ theory: - dispersive analyses (Bonn/Jülich groups)

- Padé fit analyses   P. Sanchez Puertas

➡ experiment: new data 0.3 $GeV^2 < Q^2 <$ 3 GeV^2 from BES-III under analysis   Y. Guo

heavier meson TFFs



- one photon is virtual Q_1^2, second is quasi-real $Q_2^2 \simeq 0$:
- axial-vector mesons 1^{++} are allowed
- $f_1(1285)$, $f_1(1420)$ transition FFs constrained from LEP (L3) data

$$0 = \int_{s_0}^{\infty} ds \, \frac{1}{(s+Q_1^2)^2} \left[\sigma_{\parallel} + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a \right]_{Q_2^2=0}$$

	m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int \frac{ds}{s^2} \sigma_{\parallel}(s)$ [nb / GeV^2]	$\int ds \left[\frac{1}{s} \frac{\tau_{TL}^a}{Q_1 Q_2} \right]_{Q_i^2=0}$ [nb / GeV^2]	$\int ds \left[\frac{1}{s^2} \sigma_{\parallel} + \frac{1}{s} \frac{\tau_{TL}^a}{Q_1 Q_2} \right]_{Q_i^2=0}$ [nb / GeV^2]
$f_1(1285)$	1281.8 ± 0.6	3.5 ± 0.8	0	-93 ± 21	-93 ± 21
$f_1(1420)$	1426.4 ± 0.9	3.2 ± 0.9	0	-50 ± 14	-50 ± 14
$f_0(980)$	980 ± 10	0.29 ± 0.07	20 ± 5	0	20 ± 5
$f_0'(1370)$	$1200 - 1500$	3.8 ± 1.5	48 ± 19	0	48 ± 19
$f_2(1270)$	1275.1 ± 1.2	3.03 ± 0.35	138 ± 16	$\gtrsim 0$	138 ± 16
$f_2'(1525)$	1525 ± 5	0.081 ± 0.009	1.5 ± 0.2	$\gtrsim 0$	1.5 ± 0.2
$f_2(1565)$	1562 ± 13	0.70 ± 0.14	12 ± 2	$\gtrsim 0$	12 ± 2
Sum					76 ± 36

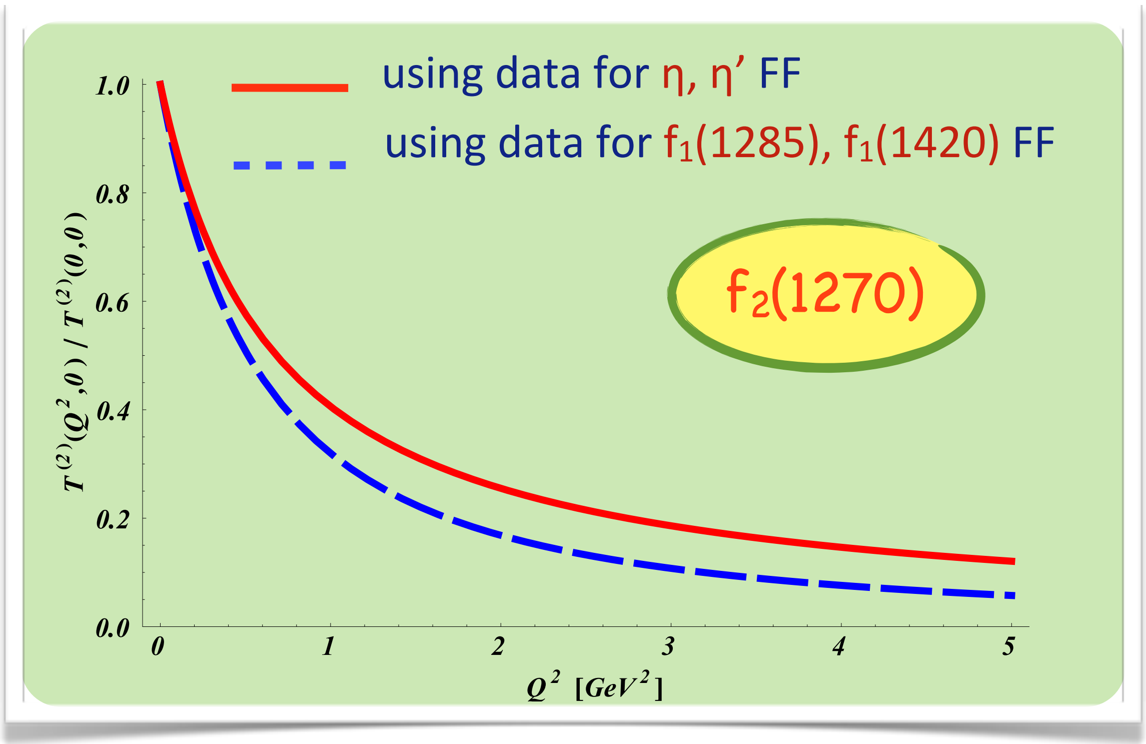
for $Q_1^2=0$

Pascalutsa, Pauk, Vdh (2012)

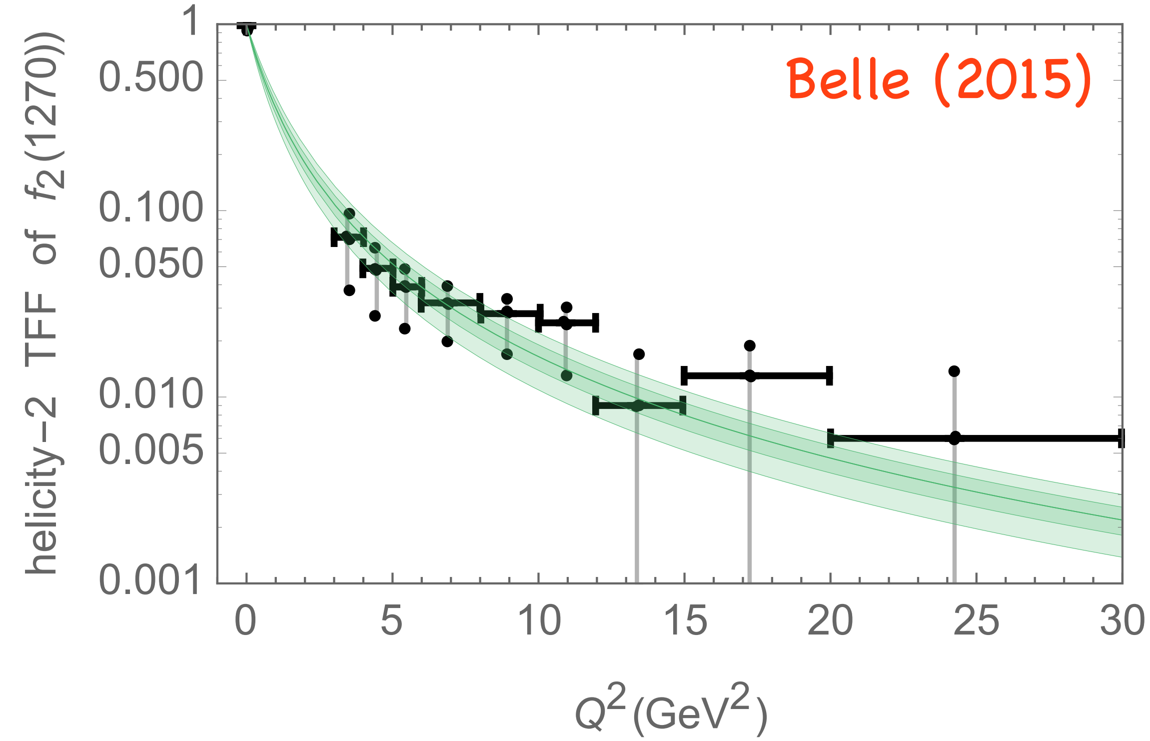
- dominant features: $f_1(1285)$, $f_1(1420)$, $f_2(1270)$
- sum rules allow to constrain so far unmeasured contributions, e.g. $\gamma^* \gamma^* \to$ tensor TFFs

comparison for $f_2(1270)$ TFFs with new Belle data

$f_2(1270)$ helicity-2 TFF from LbL sum rules

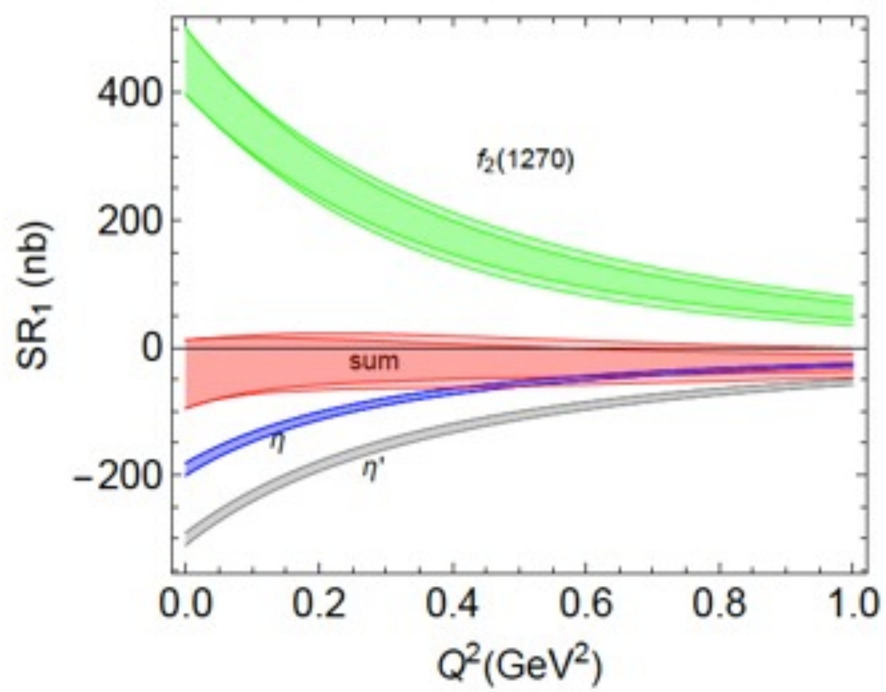


$f_2(1270)$ helicity-2 TFF from data



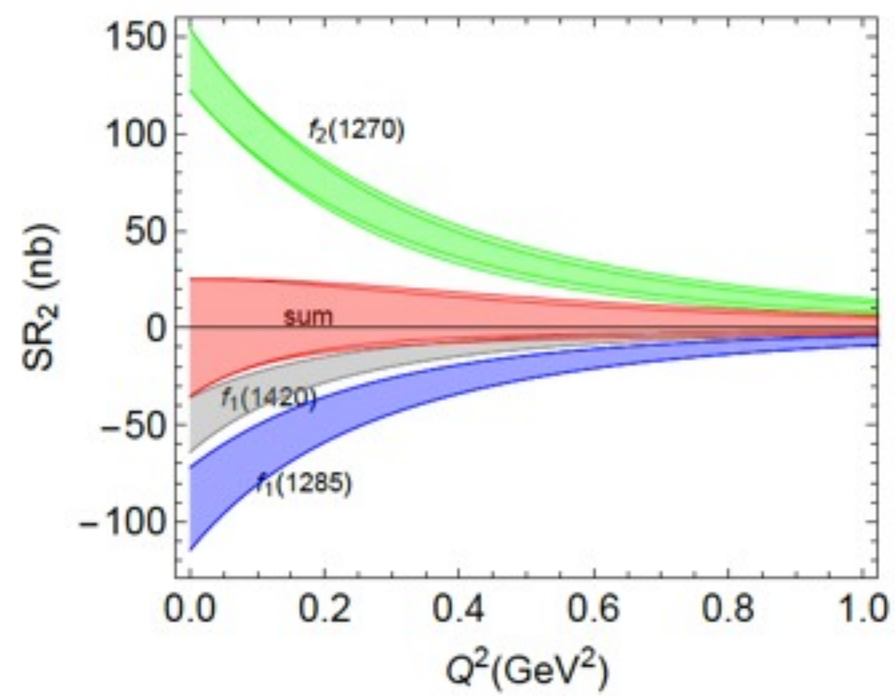
$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} [\sigma_0 - \sigma_2]_{Q_2^2=0}$$

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_\parallel + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a \right]_{Q_2^2=0}$$

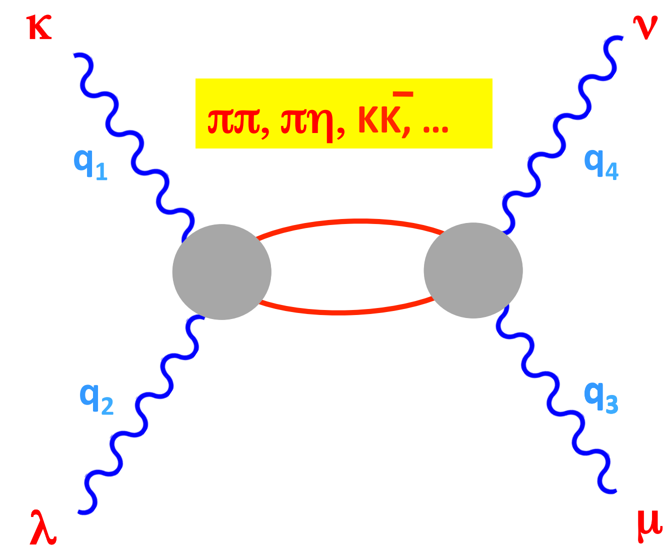


Danilkin, Vdh (2016)

sum rules well saturated below 1 GeV^2



multi-meson production in $\gamma^*\gamma^*$ collisions

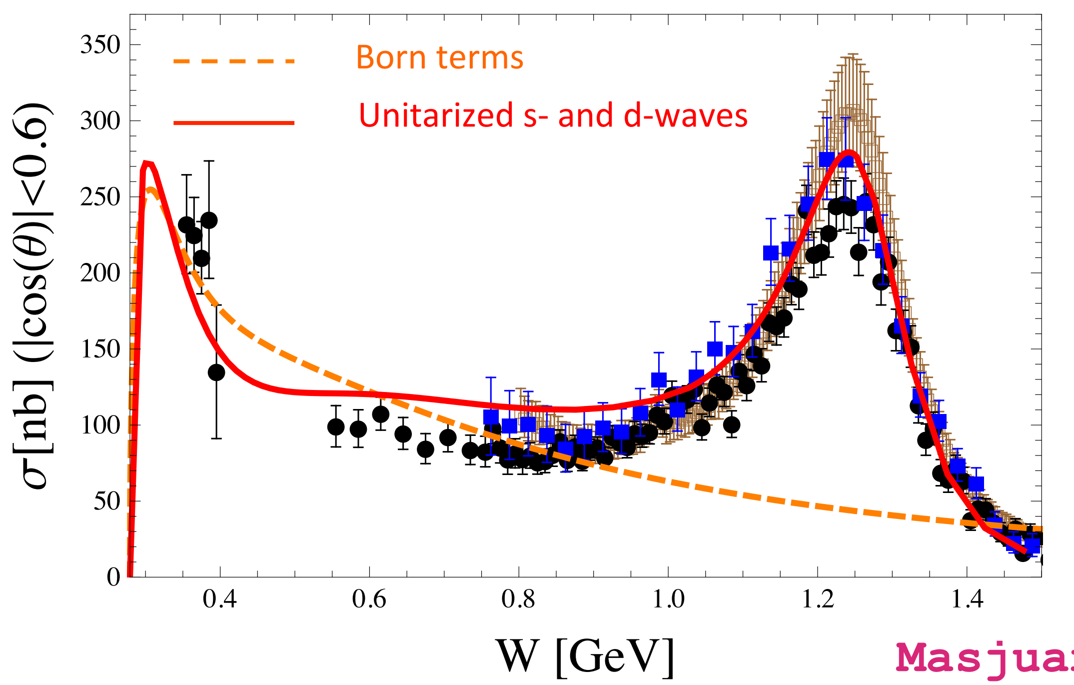


- **dispersive analyses** for $\gamma\gamma \to \pi\pi$, $\gamma^*\gamma \to \pi\pi$

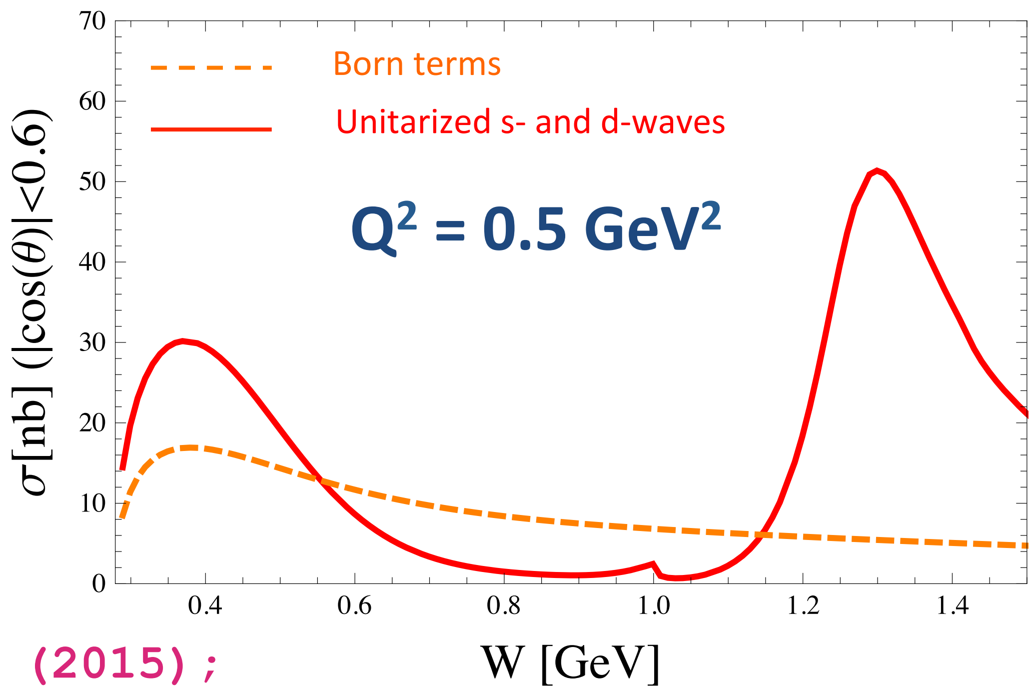
  Dai, Pennington (2014); Moussallam (2013); ...

- related approaches:
  - Roy Steiner — Hoferichter, Phillips, Schat (2011)
  - unitarized ChPT — Oller, Roca (2008);...
  - coupled channel — Danilkin, Lutz, Leupold, Terschlusen (2013);...

$\gamma\gamma \to \pi^+\pi^-$



$\gamma^*\gamma \to \pi^+\pi^-$



Masjuan, Vdh (2015);
Danilkin (in progress)

new BES-III under analysis, first comparison with theory — **Y. Guo**

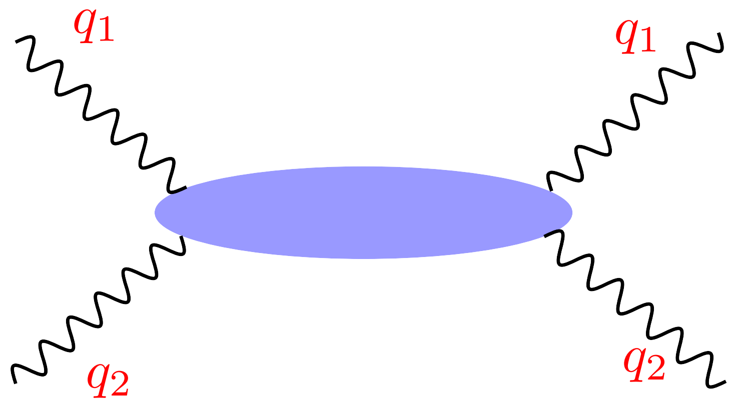
lattice calculation of forward $\gamma^*\gamma^*$ scattering

Green, Gryniuk, von Hippel, Meyer, Pascalutsa (2015)

➡ Euclidean correlator for LbL scattering

$$\Pi^E_{\mu_1\mu_2\mu_3\mu_4}(P_4;P_1,P_2) \equiv \int d^4X_1\, d^4X_2\, d^4X_4 e^{-i\sum_a P_a\cdot X_a}\langle J_{\mu_1}(X_1)J_{\mu_2}(X_2)J_{\mu_3}(0)J_{\mu_4}(X_4)\rangle_{\mathrm{E}}$$

➡ forward amplitude for two transverse (T) γ^*



$$q_1^2 = -Q_1^2, \quad q_2^2 = -Q_2^2, \quad \nu = q_1\cdot q_2$$

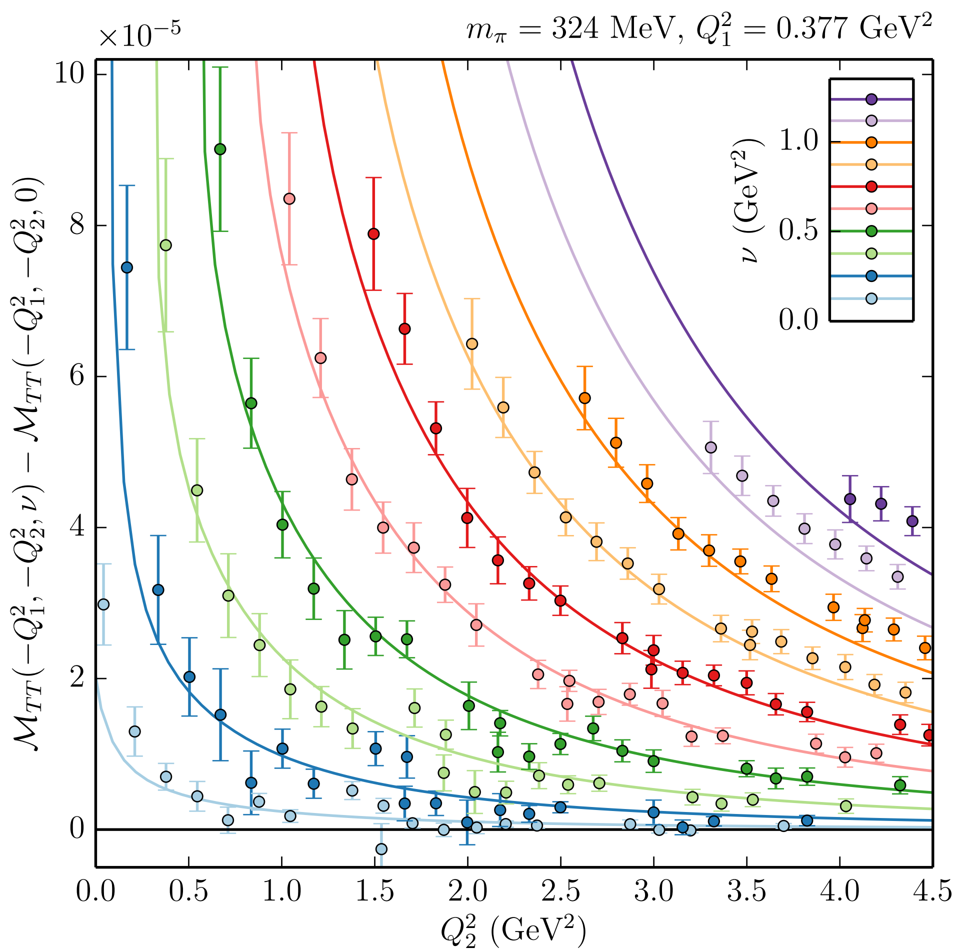
$$\mathcal{M}_{\mathrm{TT}}(-Q_1^2,-Q_2^2,-Q_1\cdot Q_2) = \frac{e^4}{4} R^E_{\mu_1\mu_3} R^E_{\mu_2\mu_4} \Pi^E_{\mu_1\mu_3\mu_4\mu_2}(-Q_2;-Q_1,Q_1)$$

R^E : transverse projectors

➡ comparison with dispersive sum rule evaluation

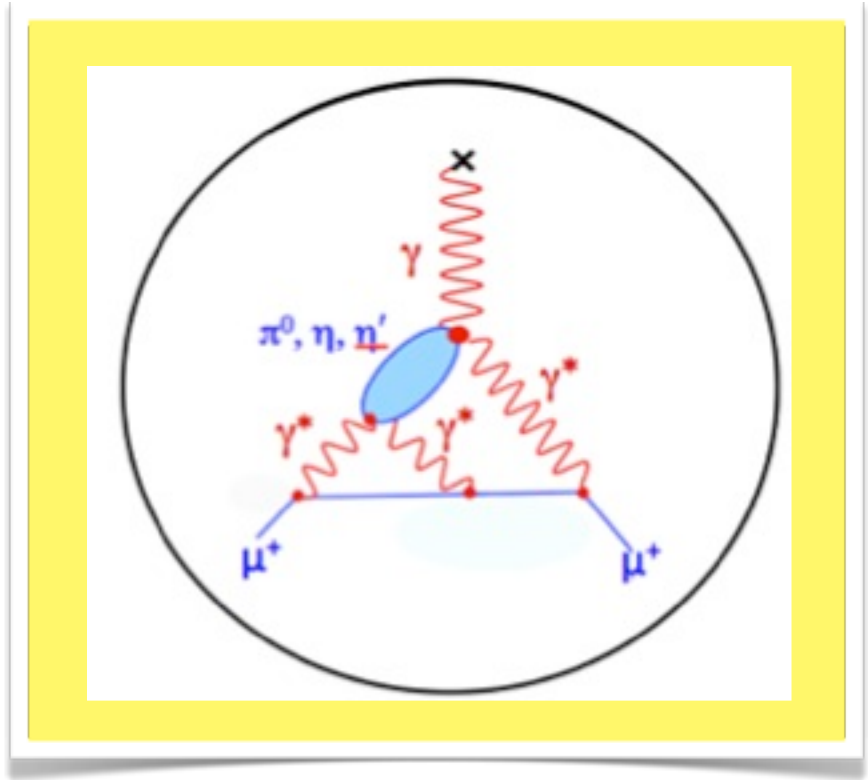
$$\mathcal{M}_{\mathrm{TT}}(-Q_1^2,-Q_2^2,\nu) - \mathcal{M}_{\mathrm{TT}}(-Q_1^2,-Q_2^2,0)$$
$$= \frac{2\nu^2}{\pi}\int_{\nu_0}^{\infty} d\nu' \frac{\sqrt{\nu'^2 - Q_1^2Q_2^2}}{\nu'(\nu'^2-\nu^2-i\epsilon)}(\sigma_0+\sigma_2)(\nu')$$

2-flavor QCD, quark connected contribution →



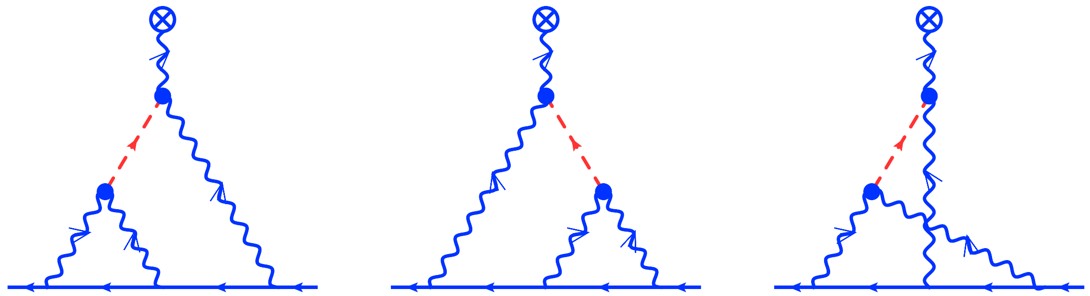
promising consistency between lattice and dispersive estimates

next steps: disconnected, lattice evaluation of a_μ from Π^E



how to estimate the HLbL contribution to a_μ ?

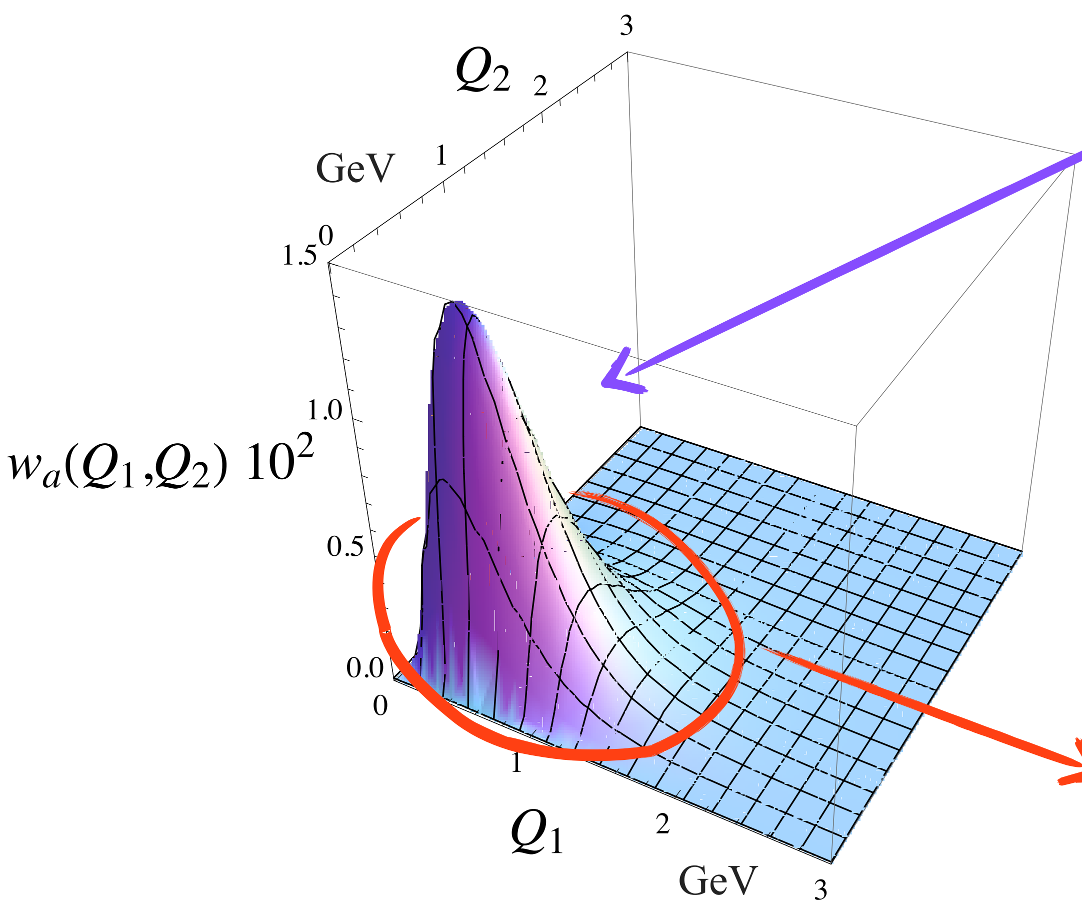
single meson contributions to a_μ (I)



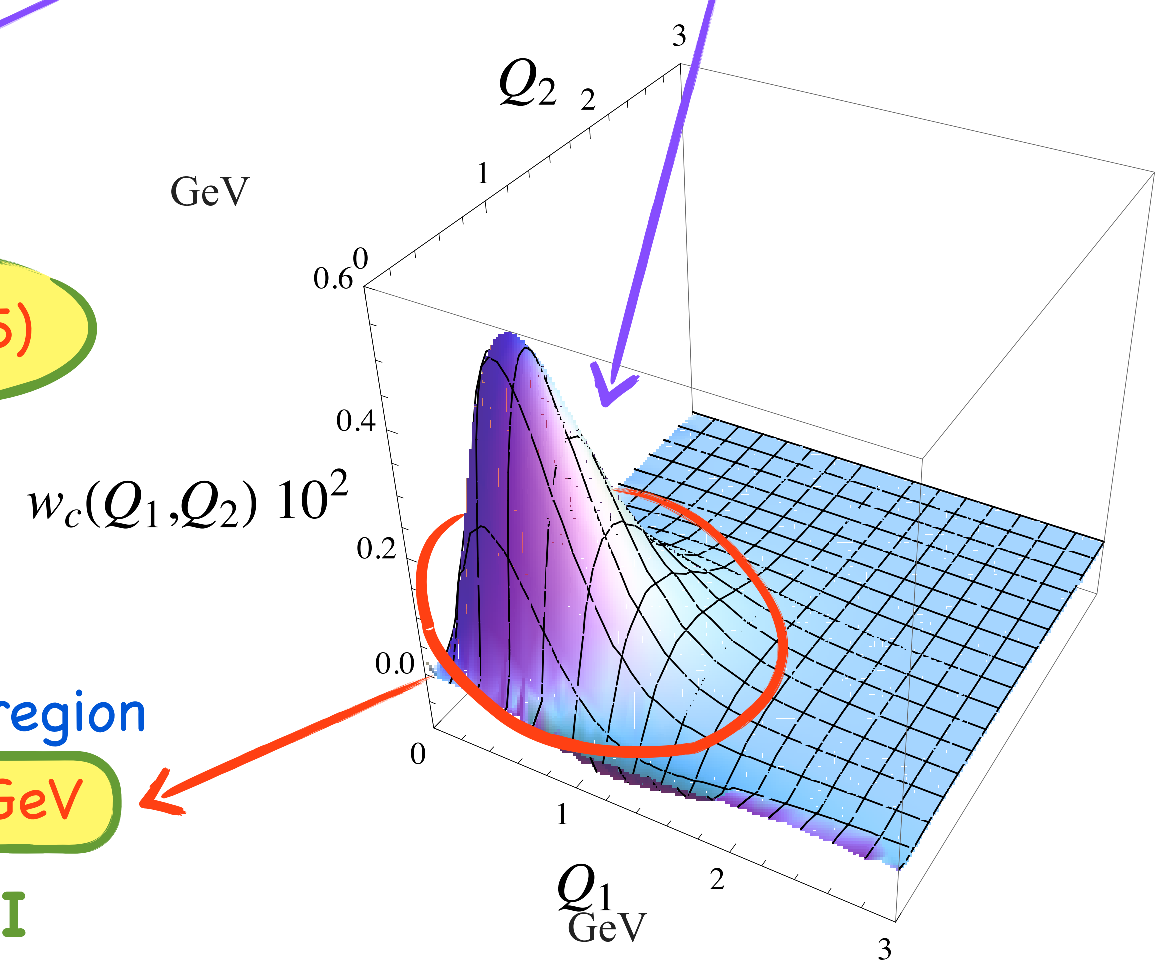
for π^0 : **Knecht, Nyffeler (2002)**

extended in many works

$$a_\mu^{LbL} = \frac{\alpha}{(2\pi)^2} \frac{\Lambda_{A1}^6 \Lambda_{A2}^6 \tilde{\Gamma}_{\gamma\gamma}(A)}{m\, m_A^5} \int dQ_1 \int dQ_2 \left[2 w_a(Q_1, Q_2) + w_c(Q_1, Q_2) \right]$$



f₁(1285)



dominating region

$Q_1 \sim Q_2 \sim 1$ GeV

BES III

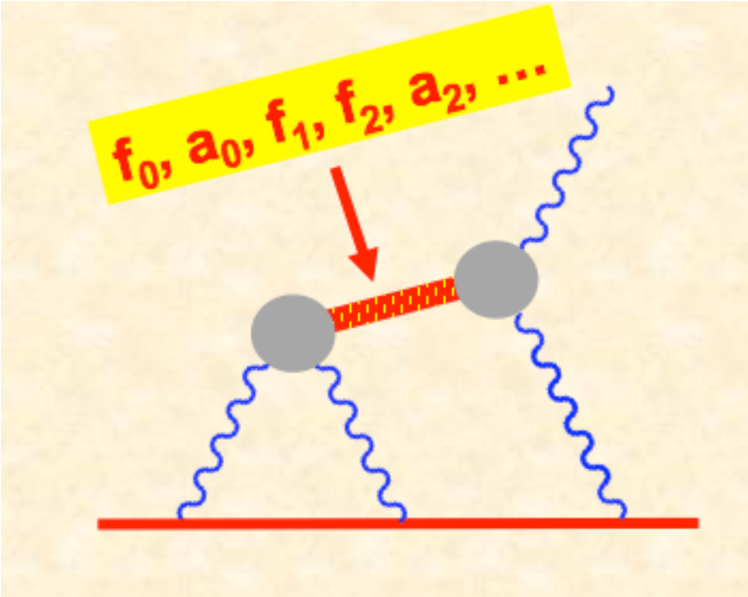
single meson contributions to a_μ (II)

- **axial-vector meson** re-evaluation was reported in 2 works

  - implementation of Landau-Yang theorem

  constraint leads to difference with previous results

- **tensor mesons** evaluated for first time



a_μ in units 10^{-11}

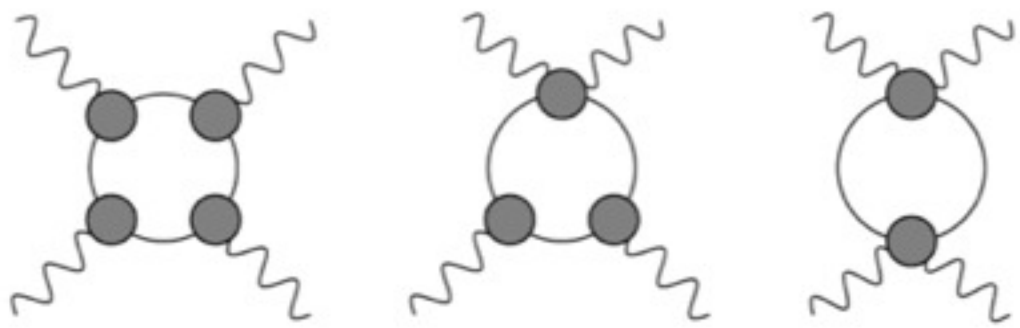
	pseudo-scalars	axial-vectors	scalars	tensors
BPP	85 ± 13	2.5 ± 1.0	-7 ± 2	-
HKS	82.7 ± 6.4	1.7 ± 1.7	-	-
MV	114 ± 10	22 ± 5	-	-
KN	83 ± 12	-	-	-
J	93.9 ± 12.4	~ 7	-6.0 ± 1.2	-
this work	-	6.4 ± 2.0	$-(0.9 \sim 3.1) \pm 0.8$	1.1 ± 0.1

Jegerlehner

Pauk, Vdh (2013)

HLbL to a_μ : present status and outlook

- estimate for **pion loop** contribution (with full VMD FF) **Bijnens et al. (2014)**



$a_\mu^{LbL\ \pi\text{-loop}} = (-\ 2.0 \pm 0.5) \times 10^{-10}$

integrating momenta in loop up to 1 GeV

- Total HLbL [a_μ in units 10^{-11}] **Jegerlehner (2015)**

Contribution	HKS	BPP	KN	MV	PdRV	N/JN	
π^0, η, η'	82.7±6.4	85±13	83±12	114±10	114±13	99±16	
π, K loops	−4.5±8.1	−19±13	–	0±10	−19±19	−19±13	
axial vectors	1.7±1.7	2.5±1.0	–	22±5	15±10	22±5	→ 7.5 ± 2.7
scalars	–	−6.8±2.0	–	–	−7±7	−7±2	
quark loops	9.7±11.1	21±3	–	–	2.3	21±3	
total	89.6±15.4	83±32	80±40	136±25	105±26	116±39	→ 102 ± 39

updated HLbL
$a_\mu = (102 \pm 39) \times 10^{-11}$

- how to improve on the present calculations ?

- spacelike doubly-virtual measurement of π^0 TFF at BESIII ($Q_1^2, Q_2^2 \sim 0.5 - 1\ GeV^2$) **Y. Guo**

- dispersive analyses for $\pi\pi$ loop contribution to a_μ **Colangelo, Hoferichter, Procura, Stoffer (2014,2015) Colangelo**

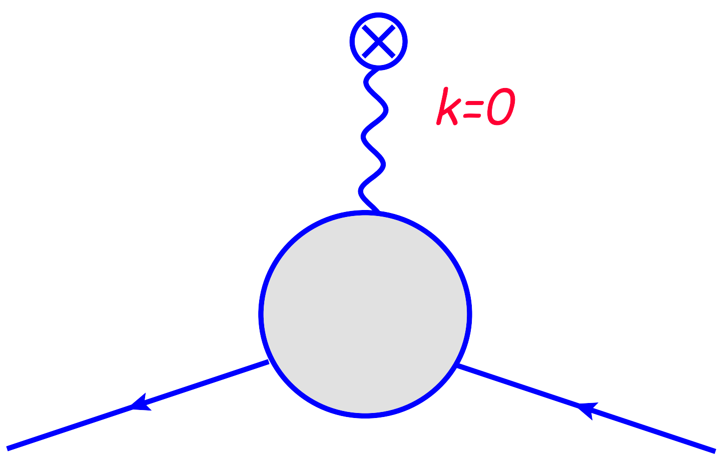
- dispersive analysis for a_μ **Pauk, Vdh (2014)**

dispersive analysis for a_μ (I)

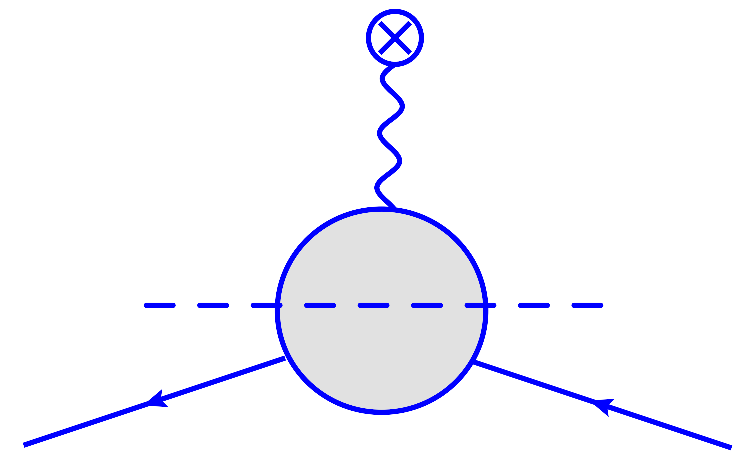
➡ **dispersion formalism** directly for a_μ Pauk, Vdh (2014)

$$a_\mu = \lim_{k \to 0} F_2(k^2, (p+k)^2, p^2)$$

$$F_2(0) = \frac{1}{2\pi i} \int \frac{dk^2}{k^2} \mathrm{Abs}\, F_2(k^2)$$



$$a_\mu = F_2(0)$$



weighting functions (entire)

$$F_2(k^2) = e^6 \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda} (-1)^{\lambda+\lambda_1+\lambda_2+\lambda_3} \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} L_{\lambda_1\lambda_2\lambda_3\lambda}(p, p', q_1, k-q_1-q_2, q_2)$$

analytic structure ⟶

$$\times \frac{\Pi_{\lambda_1\lambda_2\lambda_3\lambda}(q_1, k-q_1-q_2, q_2, k)}{q_1^2 q_2^2 (k-q_1-q_2)^2 \left[(p+q_1)^2 - m^2\right]\left[(p+k-q_2)^2 - m^2\right]}$$

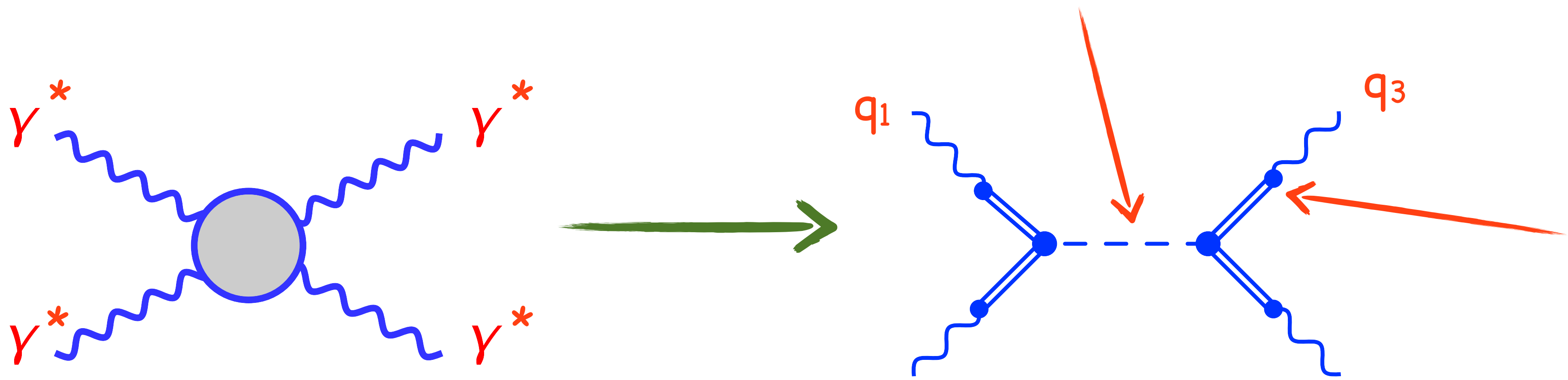
$$\Pi_{\lambda_1\lambda_2\lambda_3\lambda_4}(q_1, q_2, q_3) = \epsilon^\mu(q_1, \lambda_1)\epsilon^\nu(q_2, \lambda_2)\epsilon^\lambda(q_3, \lambda_3)\epsilon^\rho(q_4, \lambda_4)\Pi_{\mu\nu\lambda\rho}(q_1, q_2, q_3)$$

dispersive analysis for a_μ (II)

➡ **proof of principle:** pole contributions

π^0 - pole at $(q_i+q_j)^2 = M^2$

analytical structure of LbL amplitude

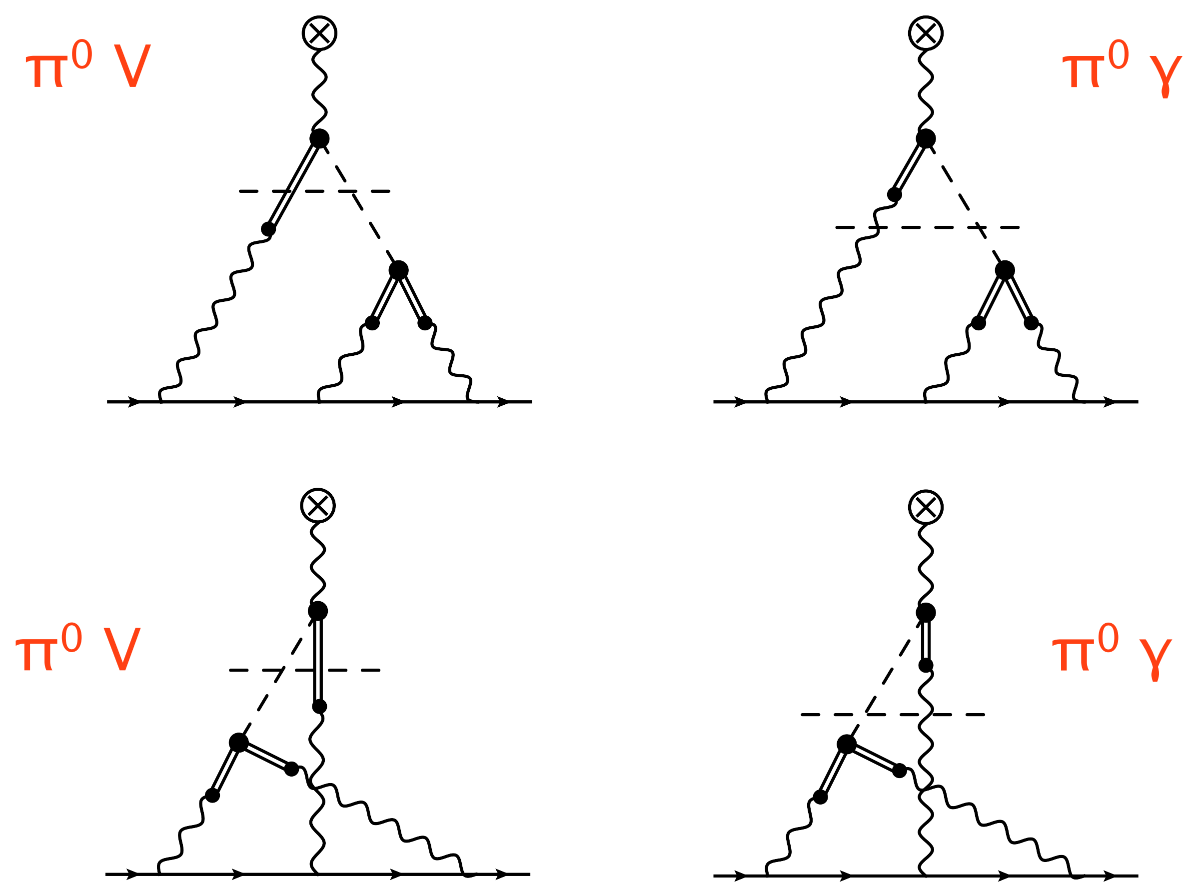


vector - poles at $q_i^2 = \Lambda^2$

➡ **2-particle cuts**

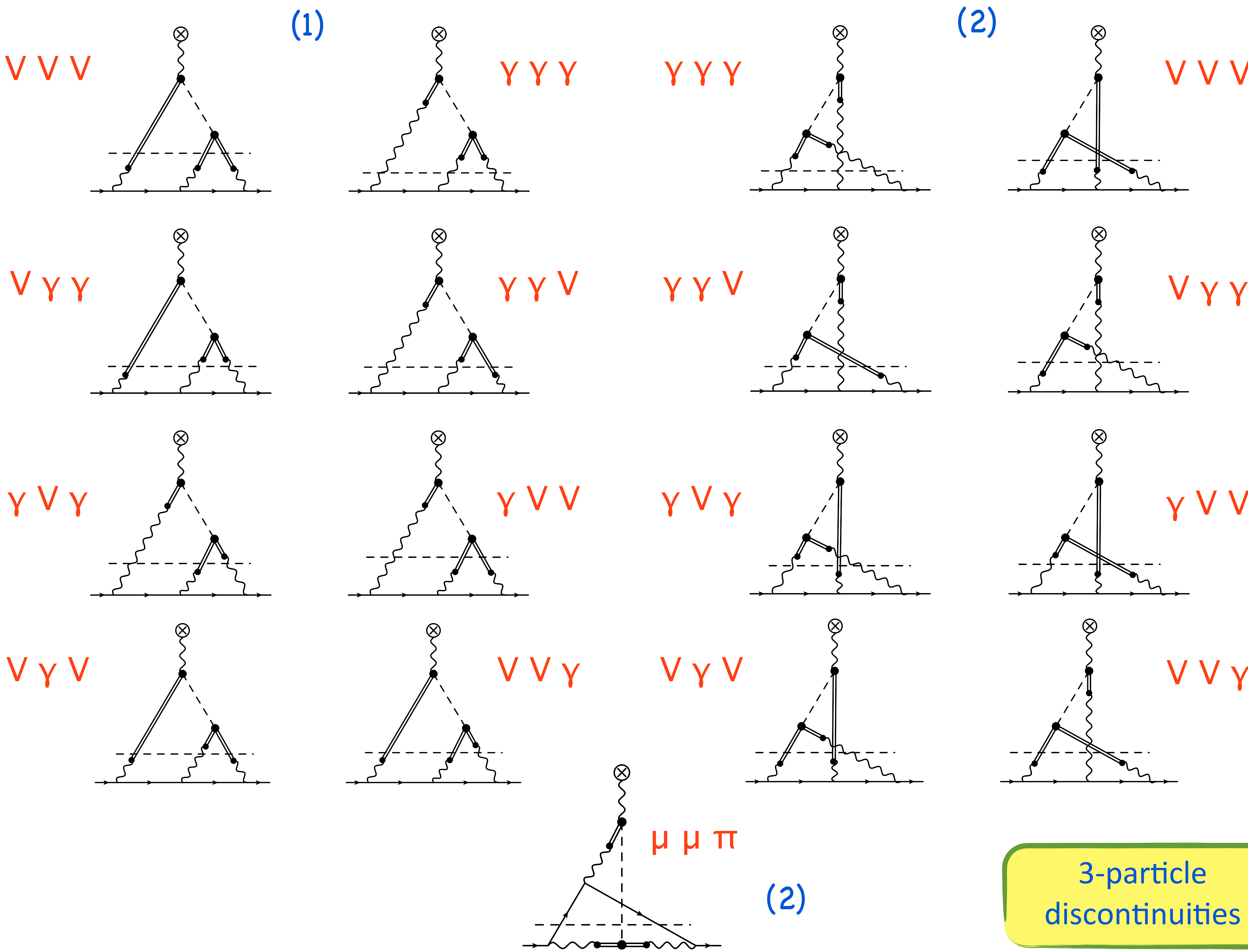
(1)

(2)



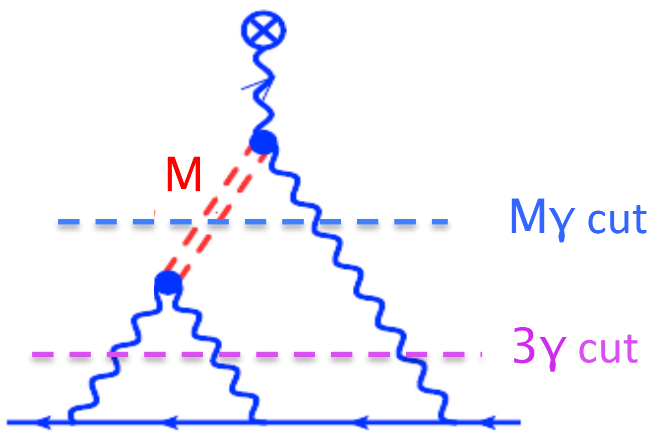
2-particle discontinuities

dispersive analysis for a_μ (III)



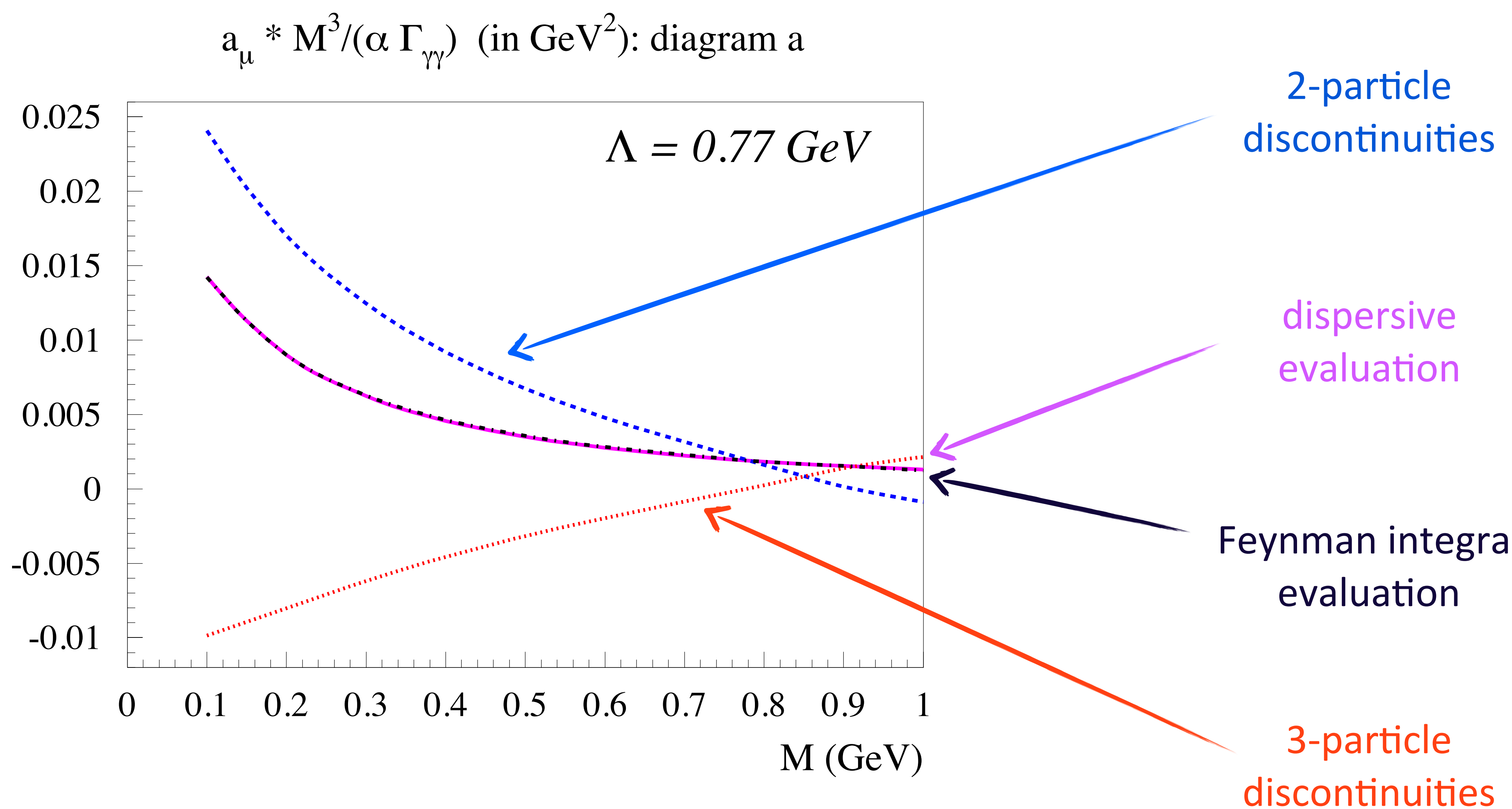
3-particle discontinuities

dispersive analysis for a_μ (IV)



reconstruction of a_μ from dispersion integral: **proof of principle**

$$F_2^{(i)}(0) = \frac{1}{2\pi i}\int_{M^2}^{\infty}\frac{dt}{t}\,\mathrm{Disc}_t^{(2)} F_2^{(i)}(t) + \frac{1}{2\pi i}\int_{0}^{\infty}\frac{dt}{t}\,\mathrm{Disc}_t^{(3)} F_2^{(i)}(t)$$



Pauk, Vdh (2014)

exact agreement between direct 2-loop and dispersive calculation found

Summary and outlook

HLbL: new model independent theoretical tools for $\gamma^* \gamma^* \to X$

- sum rules, dispersive frameworks for meson transition FFs**:**

-> allow to include experimental constraints (new data from Belle, BESIII)

- new evaluation of **heavier meson contributions**

-> $a_\mu = \sim 7 \times 10^{-11}$ (factor 3 smaller than previous estimates)

- **pioneering new lattice QCD** calculations for HLbL:

-> promising agreement with sum rule estimates found

new dispersion relation frameworks for **HLbL** to $\mathbf{a_\mu}$:

-> require close collaboration with experiment (spacelike, timelike, meson decays)

data driven approach also in HLbL

goal: realistic error estimate on a_μ / reduce to 20 x 10^{-11} (20 % of HLbL)