The muon $g-2$ in the standard model

(a theory overview)

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OUTLINE

- Introduction
- Theory I: QED contributions
- Theory II: Weak contributions
- Theory III: Strong interactions
- Summary - Conclusions

Introduction

Linear response of a charged lepton to an external electromagnetic field

$$\langle \ell; p'|J_\rho(0)|\ell; p\rangle \equiv \bar{\mathrm{u}}(p')\Gamma_\rho(p',p)\mathrm{u}(p)$$

$$= \bar{\mathrm{u}}(p')\left[F_1(k^2)\gamma_\rho + \frac{i}{2m_\ell}F_2(k^2)\sigma_{\rho\nu}k^\nu - F_3(k^2)\gamma_5\sigma_{\rho\nu}k^\nu + F_4(k^2)(k^2\gamma_\rho - 2m_\ell k_\rho)\gamma_5\right]\mathrm{u}(p)$$

(Lorentz invariance + conservation of the electromagnetic current J_ρ)

$F_1(k^2) \rightarrow$ Dirac form factor , $F_1(0) = 1$

$F_2(k^2) \rightarrow$ Pauli form factor $\rightarrow F_2(0) = a_\ell$

$F_3(k^2) \rightarrow \not{P}, \not{T}$, electric dipole moment $\rightarrow F_3(0) = d_\ell/e_\ell$

$F_4(k^2) \rightarrow \not{P}$, anapole moment

$$G_E(k^2) = F_1(k^2) + \frac{k^2}{4m_\ell^2}F_2(k^2),\ G_M(k^2) = F_1(k^2) + F_2(k^2)$$

$$\boldsymbol{\mu}_\ell = g_\ell\left(\frac{e_\ell}{2m_\ell c}\right)\mathrm{s}\,,\ \mathrm{s} = \hbar\frac{\boldsymbol{\sigma}}{2} \qquad g_\ell = g_\ell^{\mathrm{Dirac}} \times G_M(0)$$

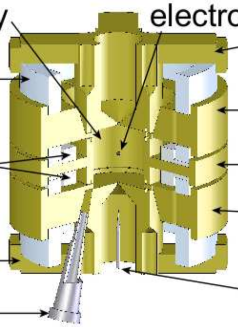
At tree level, $F_1 = 1,\ F_2 = F_3 = F_4 = 0$, $g_\ell = g_\ell^{\mathrm{Dirac}} \equiv 2$

The *anomalous* magnetic moment a_ℓ is induced at loop level $\left(a_\ell \equiv \frac{g_\ell - g_\ell^{\mathrm{Dirac}}}{g_\ell^{\mathrm{Dirac}}}\right)$

a_ℓ probes the contributions of quantum loops from SM and BSM degrees of freedom

In this talk SM only. For BSM, see e.g. talk by H. Stöckinger-Kim or
D. Stöckinger, in *Lepton Dipole Moments*; A. Czarnecki, W. J. Marciano, Phys. Rev. D 64, 013014 (2001)

a_e and a_μ are experimentally measured to very high precision:



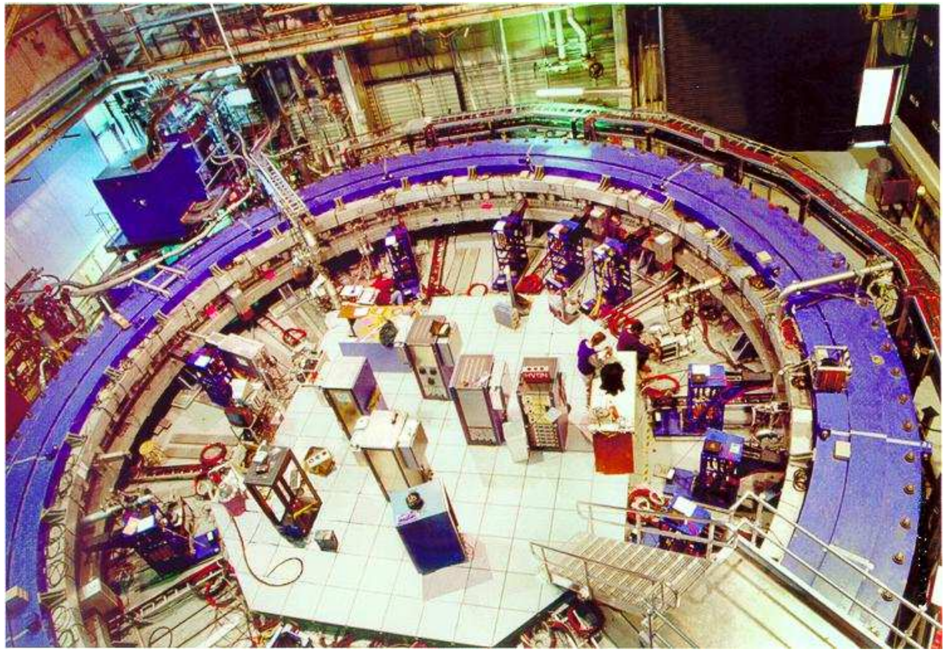
$$a_e^{\text{exp}} = 1\,159\,652\,180.73(0.28) \cdot 10^{-12}$$

$$\Delta a_e^{\text{exp}} = 2.8 \cdot 10^{-13}$$ [0.24ppb]

D. Hanneke et al, Phys. Rev. Lett. 100, 120801 (2008)

$$\tau_\mu = (2.19703 \pm 0.00004) \times 10^{-6} \text{ s}$$

$$\gamma \sim 29.3, p \sim 3.094 \text{ GeV/c}$$



$$a_\mu^{\text{exp}} = 116\,592\,089(63) \cdot 10^{-11}$$

$$\Delta a_\mu^{\text{exp}} = 6.3 \cdot 10^{-10}$$ [0.54ppm]

G. W. Bennett et al, Phys Rev D 73, 072003 (2006)

Note: $\tau_\tau = (290.6 \pm 1.1) \times 10^{-15}\,\text{s}$

$-0.052 < a_\tau^{exp} < +0.013\ (95\%\ \text{CL})$ $[e^+e^- \to e^+e^-\tau^+\tau^-]$ DELPHI, Eur. Phys. J. C 35, 159 (2004)

theory: $a_\tau = 117721(5) \cdot 10^{-8}$

S. Eidelman, M. Passera, Mod. Phys. Lett. A 22, 159 (2007)

S. Narison, Phys Lett B 513 (2001); err. B 526, 414 (2002)

Theory I: QED (a_e and a_μ)

QED contributions : loops with only photons and leptons

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

$A_1^{(2n)} \longrightarrow$ mass-independent (universal) contributions (one-flavour QED)

$A_2^{(2n)}(m_\ell/m_{\ell'}),\ A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''}) \longrightarrow$

mass-dependent (non-universal) contributions (multi-flavour QED)

– a_ℓ is finite (no renormalization needed) and dimensionless

– QED is decoupling

– Massive degrees of freedom with $M \gg m_\ell$ contribute to a_ℓ through powers of m_ℓ^2/M^2 times logarithms (*)

– Light degrees of freedom with $m \ll m_\ell$ give logarithmic contributions to a_ℓ, e.g. $\ln(m_\ell^2/m^2)$ $\left(\pi^2 \ln \frac{m_\mu}{m_e} \sim 50\right)$

(*) also applies to BSM physics to the extent that it is decoupling!

QED prediction ?

$\longrightarrow$ requires an input for the fine structure constant α that matches the experimental accuracy on a_ℓ

– For the muon

$$\frac{\Delta a_\mu}{a_\mu} = 0.54\text{ppm} \;\rightarrow\; \frac{\Delta\alpha}{\alpha} \sim 0.54\text{ppm}$$

- quantum Hall effect

$$\alpha^{-1}[qH] = 137.036\,00300(270) \qquad [19.7\text{ppb}]$$

P. J. Mohr, B. N. Taylor, D. B. Newell, Rev. Mod. Phys. 80, 633 (2008)

Actually, traditionnally, the measurement of a_e was used to extract the value of α

(Assumes the SM to be correct in this case, or, at least, that BSM physics below the experimental accuracy)

– For the electron

$$\frac{\Delta a_e}{a_e} = 0.24\text{ppb} \rightarrow \frac{\Delta\alpha}{\alpha} \sim 0.24\text{ppb} \rightarrow \Delta\alpha \lesssim 2 \cdot 10^{-12}$$

- atomic recoil velocity through photon absorption

$$\alpha^2 = \frac{2R_\infty}{c} \cdot \frac{M_{\text{atom}}}{m_e} \cdot \frac{h}{M_{\text{atom}}}$$

$$\frac{\Delta R_\infty}{R_\infty} = 5 \cdot 10^{-12} \qquad \Delta\left(\frac{M_{\text{Rb}}}{m_e}\right) = 4.4 \cdot 10^{-10}$$

$$\alpha^{-1}[Cs\,02] = 137.036\,0001(11) \qquad [7.7\text{ppb}]$$

A. Wicht, J. M. Hensley, E. Sarajilic, S. Chu, Phys. Scr. T102, 82 (2002)

$$\alpha^{-1}[Rb\,06] = 137.035\,998\,84(91) \qquad [6.7\text{ppb}]$$

P. Cladé et al, Phys. Rev. A 74, 052109 (2006)

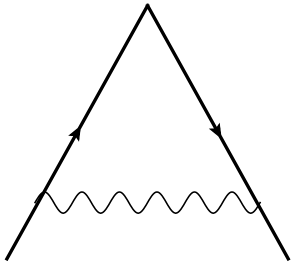
$$\alpha^{-1}[Rb\,08] = 137.035\,999\,45(62) \qquad [4.6\text{ppb}]$$

M. Cadoret et al, Phys. Rev. Lett. 101, 230801 (2008)

$$\alpha^{-1}[Rb\,11] = 137.035\,999\,037(91) \qquad [0.66\text{ppb}]$$

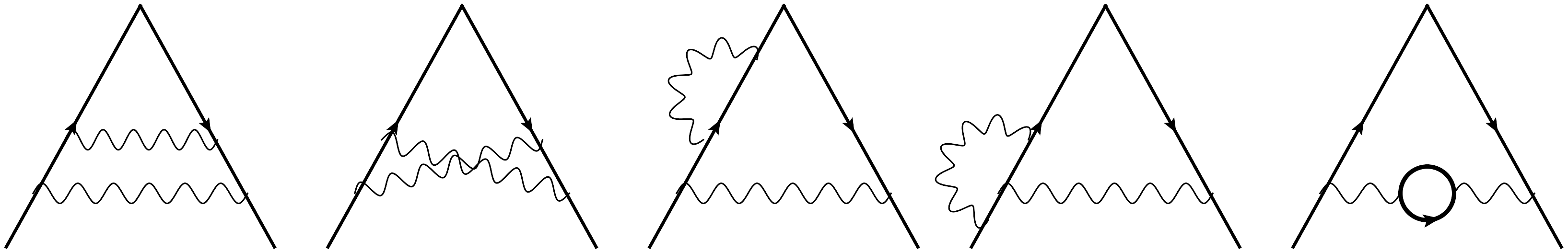
R. Bouchendira, P. Cladé, S. Ghelladi-Khélifa, F. Nez, F. Biraben, Phys. Rev. Lett. 106, 080801 (2011)

Analytic expressions for $A_1^{(2)}$, $A_1^{(4)}$, $A_1^{(6)}$, $A_2^{(4)}$, $A_2^{(6)}$, $A_3^{(6)}$ known



$$A_1^{(2)} = \frac{1}{2}$$

[J. Schwinger, Phys. Rev. 73, 416L (1948)]



$$A_1^{(4)} = \frac{3}{4}\zeta(3) - \frac{\pi^2}{2}\ln 2 + \frac{\pi^2}{12} + \frac{197}{144} = -0.328\,478\,965\,579\,193...$$

C. M. Sommerfield, Phys. Rev. 107, 328 (1957); Ann. Phys. 5, 26 (1958)

A. Petermann, Helv. Phys. Acta 30, 407 (1957)

$$A_2^{(4)}(m_\ell/m_{\ell'}) = \frac{1}{3}\int_{4m_{\ell'}^2}^{\infty} dt\,\sqrt{1-\frac{4m_{\ell'}^2}{t}}\,\frac{t+2m_{\ell'}^2}{t^2}\int_0^1 dx\,\frac{x^2(1-x)}{x^2+(1-x)\,\frac{t}{m_\ell^2}}$$

H. Suura and E. Wichmann, Phys. Rev. 105, 1930 (1955)
A. Petermann, Phys. Rev. 105, 1931 (1955)
H. H. Elend, Phys. Lett. 20, 682 (1966); Err. Ibid. 21, 720 (1966)
M. Passera, Phys. Rev. D 75, 013002 (2007)

$$\begin{aligned} A_2^{(4)}(m_\ell/m_{\ell'}) &= \frac{1}{3}\ln\left(\frac{m_\ell}{m_{\ell'}}\right) - \frac{25}{36} + \frac{\pi^2}{4}\frac{m_{\ell'}}{m_\ell} - 4\left(\frac{m_{\ell'}}{m_\ell}\right)^2 \ln\left(\frac{m_\ell}{m_{\ell'}}\right) \\ &\quad +3\left(\frac{m_{\ell'}}{m_\ell}\right)^2 + \mathcal{O}\left[\left(\frac{m_{\ell'}}{m_\ell}\right)^3\right],\ m_\ell \gg m_{\ell'} \end{aligned}$$

M. A. Samuel and G. Li, Phys. Rev. D 44, 3935 (1991)

$$A_2^{(4)}(m_\mu/m_e) = 1.094\,258\,312\,0(83)$$

$$m_\mu/m_e = 206.768\,2843(52)$$

P. J. Mohr, B. N. Taylor, D. B. Newell, CODATA 2010, Rev. Mod. Phys. 84, 1527 (2012);
arXiv:1203.5425v1[physics.atom-ph]

$$A_2^{(4)}(m_\ell/m_{\ell'}) = \frac{1}{3}\int_{4m_{\ell'}^2}^{\infty} dt \sqrt{1-\frac{4m_{\ell'}^2}{t}}\,\frac{t+2m_{\ell'}^2}{t^2}\int_0^1 dx\,\frac{x^2(1-x)}{x^2+(1-x)\,\frac{t}{m_\ell^2}}$$

H. Suura and E. Wichmann, Phys. Rev. 105, 1930 (1955)
A. Petermann, Phys. Rev. 105, 1931 (1955)
H. H. Elend, Phys. Lett. 20, 682 (1966); Err. Ibid. 21, 720 (1966)
M. Passera, Phys. Rev. D 75, 013002 (2007)

$$\begin{aligned} A_2^{(4)}(m_\ell/m_{\ell'}) &= \frac{1}{45}\left(\frac{m_\ell}{m_{\ell'}}\right)^2 + \frac{1}{70}\left(\frac{m_\ell}{m_{\ell'}}\right)^4 \ln\left(\frac{m_\ell}{m_{\ell'}}\right) \\ &+\frac{9}{19600}\left(\frac{m_\ell}{m_{\ell'}}\right)^4 + \mathcal{O}\left[\left(\frac{m_\ell}{m_{\ell'}}\right)^6 \ln\left(\frac{m_\ell}{m_{\ell'}}\right)\right], \quad m_{\ell'} \gg m_\ell \end{aligned}$$

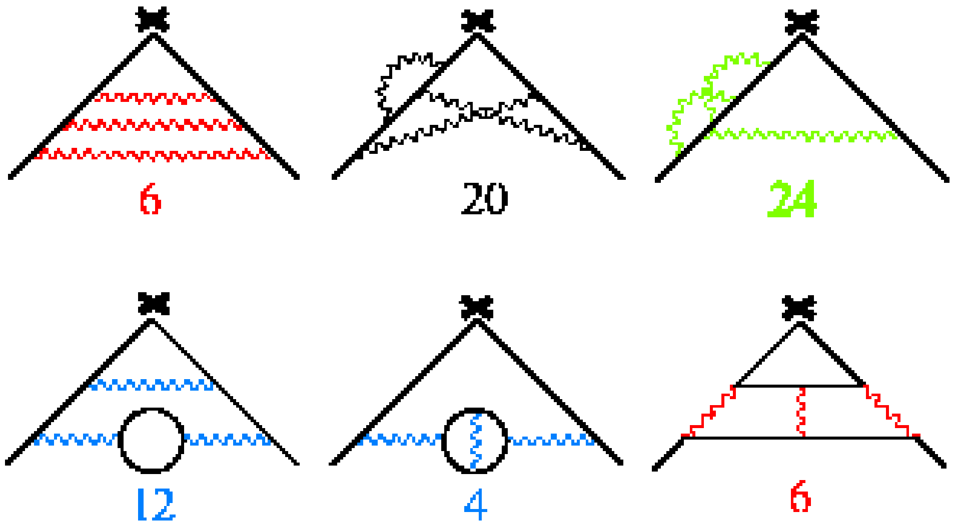
B.E. Lautrup and E. de Rafael, Phys. Rev. 174, 1835 (1965)
M. A. Samuel and G. Li, Phys. Rev. D 44, 3935 (1991)

$$\begin{aligned} A_2^{(4)}(m_e/m_\mu) &= 5.197\,386\,67(26)\cdot 10^{-7} \\ A_2^{(4)}(m_e/m_\tau) &= 1.837\,98(34)\cdot 10^{-9} \\ A_2^{(4)}(m_\mu/m_\tau) &= 7.8079(15)\cdot 10^{-5} \end{aligned}$$

$$m_\mu/m_\tau = 5.946\,49(54)\cdot 10^{-2} \qquad m_e/m_\tau = 2.875\,92(26)\cdot 10^{-4}$$

P. J. Mohr, B. N. Taylor, D. B. Newell, CODATA 2010, Rev. Mod. Phys. 84, 1527 (2012);
arXiv:1203.5425v1[physics.atom-ph]

order $(\alpha/\pi)^3$: 72 diagrams



$$A_1^{(6)} = \frac{87}{72}\pi^2\zeta(3) - \frac{215}{24}\zeta(5) + \frac{100}{3}\left[\left(a_4 + \frac{1}{24}\ln^4 2\right) - \frac{1}{24}\pi^2\ln^2 2\right] - \frac{239}{2160}\pi^4$$

$$+\frac{139}{18}\zeta(3) - \frac{298}{9}\pi^2\ln 2 + \frac{17101}{810}\pi^2 + \frac{28259}{5184} \qquad [a_p = \sum_1^\infty 1/(2^n n^p)]$$

S. Laporta, E. Remiddi, Phys. Lett. B265, 182 (1991); B356, 390 (1995); B379, 283 (1996)

S. Laporta, Phys. Rev. D 47, 4793 (1993); Phys. Lett. B343, 421 (1995)

$$A_1^{(6)} = 1.181\,241\,456\ldots$$

order $(\alpha/\pi)^4$: 891 diagrams

only a few diagrams are known analytically $\longrightarrow$ numerical evaluation

Automated generation of diagrams

Systematic numerical evaluation of multi-dimensional integrals over Feynman-parameter space

$$A_1^{(8)} = -1.912\,98(84)$$

$$A_2^{(8)}(m_e/m_\mu) = 9.161\,970\,703(373) \cdot 10^{-4} \quad A_2^{(8)}(m_e/m_\tau) = 7.429\,24(118) \cdot 10^{-6}$$

$$A_3^{(8)}(m_e/m_\mu, m_e/m_\tau) = 7.468\,7(28) \cdot 10^{-7}$$

$$A_2^{(8)}(m_\mu/m_e) = 132.685\,2(60) \quad A_2^{(8)}(m_\mu/m_\tau) = 0.042\,34(12)$$

$$A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.062\,72(4)$$

Independent check of mass-dependent contributions

A. Kataev, Phys. Rev. D 86, 013019 (2012)

A. Kurz, T. Liu, P. Marquard, M. Steinhauser, Nucl. Phys. B 879, 1 (2014)

A. Kurz, T. Liu, P. Marquard, A. V. Smirnov, V. A. Smirnov, M. Steinhauser, Phys. Rev. D 92, 073019 (2015)]

Agreement at the level of accuracy required by present and future experiments for a_μ

a_e: $A_1^{(8)}$ remains unchecked so far!

order $(\alpha/\pi)^4$: 891 diagrams

only some diagrams are known analytically $\longrightarrow$

numerical evaluation of Feynman-parametrized loop integrals

$$
\begin{aligned}
A_1^{(8)} &= -1.434(138) && \text{[Kinoshita and Lindquist (1990)]}\\
&= -1.557(70) && \text{[Kinoshita (1995)]}\\
&= -1.4092(384) && \text{[Kinoshita (1997)]}\\
&= -1.5098(384) && \text{[Kinoshita (2001)]}\\
&= -1.7366(60) && \text{[Kinoshita (2005)]}\\
&= -1.7260(50) && \text{[Kinoshita (2005)]}\\
&= -1.7283(35) && \text{[Kinoshita and Nio, Phys. Rev. D 73, 013003(2006)]}\\
&= -1.9144(35) && \text{[Aoyama et al., Phys. Rev. Lett. 99, 110406 (2007)]} \leftarrow\\
&= -1.9106(20) && \text{[Aoyama et al., Phys. Rev. Lett. 109, 111807 (2012)]}\\
&= -1.91298(84) && \text{[Aoyama et al., Phys. Rev. D 91, 033006 (2015)]}
\end{aligned}
$$

order $(\alpha/\pi)^5$: 12 672 diagrams...

6 classes, 32 gauge invariant subsets

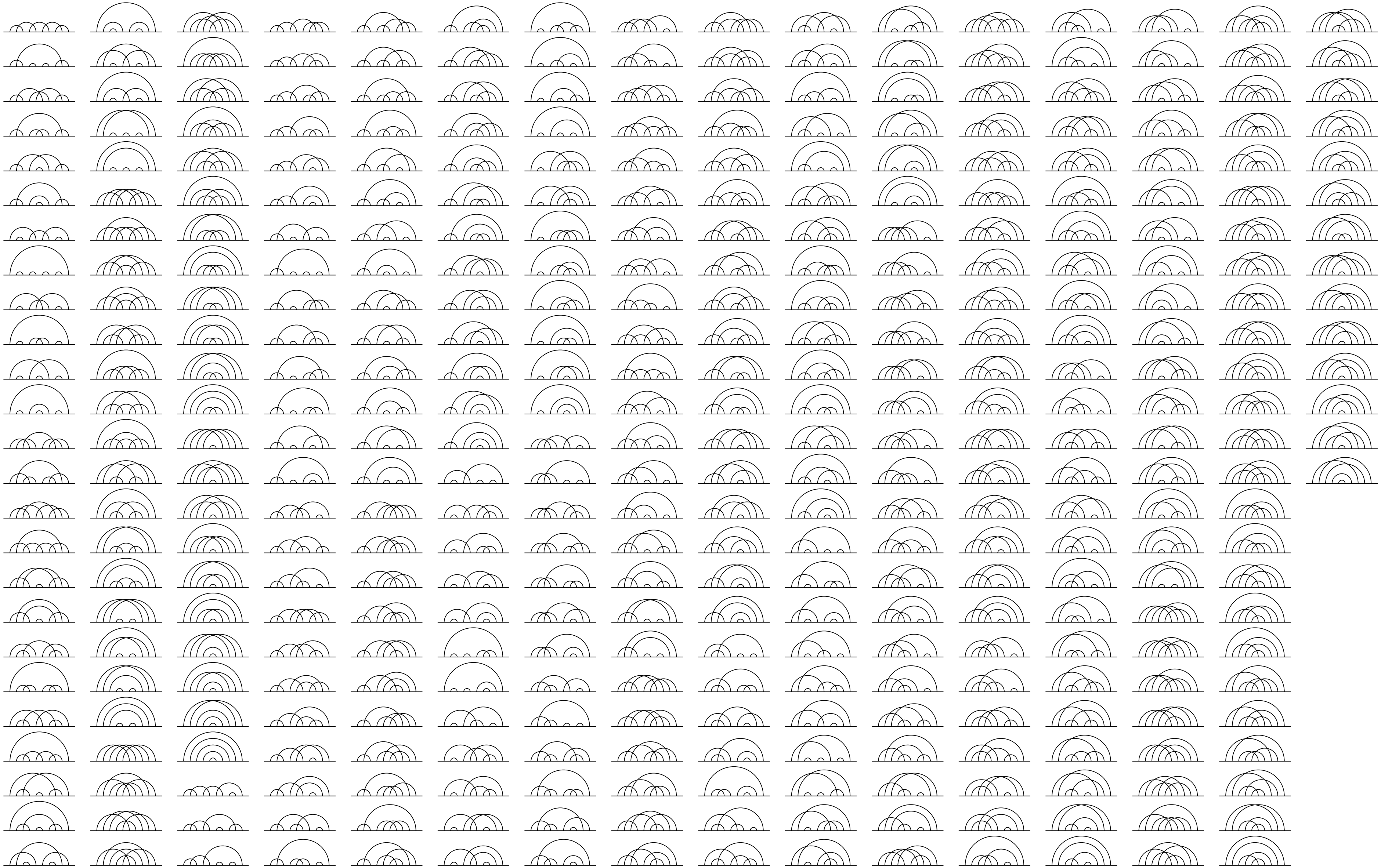
Five of these subsets are known analytically

S. Laporta, Phys. Lett. B 328, 522 (1994)
J.-P. Aguilar, D. Greynat, E. de Rafael, Phys. Rev. D 77, 093010 (2008)

Complete numerical results have been published

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 78, 053005 (2008); D 78, 113006 (2008); D 81, 053009 (2010); D 82, 113004 (2010); D 83, 053002 (2011); D 83, 053003 (2011); D 84, 053003 (2011); D 85, 033007 (2012); Phys. Rev. Lett. 109, 111807 (2012); Phys. Rev. Lett. 109, 111808 (2012)

No systematic cross-checks even for mass-dependent contributions



T. Aoyama et al., Phys. Rev. D 91, 033006 (2015)

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5$$

	$\ell = e$	$\ell = \mu$
$C_\ell^{(2)}$	0.5	0.5
$C_\ell^{(4)}$	$-0.328\,478\,444\,00\ldots$	$0.765\,857\,425(17)$
$C_\ell^{(6)}$	$1.181\,234\,017\ldots$	$24.050\,509\,96(32)$
$C_\ell^{(8)}$	$-1.9096(20)$	$130.879\,6(63)$
$C_\ell^{(10)}$	$9.16(58)$	$753.29(1.04)$

n	1	2	3	4	5
$(\alpha/\pi)^n$	$2.32\ldots \cdot 10^{-3}$	$5.39\ldots \cdot 10^{-6}$	$1.25\ldots \cdot 10^{-8}$	$2.91\ldots \cdot 10^{-11}$	$6.76\ldots \cdot 10^{-14}$

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$C_\ell^{(10)}$	$9.16(58)$	$753.29(1.04)$

n	1	2	3	4	5
$(\alpha/\pi)^n$	$2.32\ldots\cdot 10^{-3}$	$5.39\ldots\cdot 10^{-6}$	$1.25\ldots\cdot 10^{-8}$	$2.91\ldots\cdot 10^{-11}$	$6.76\ldots\cdot 10^{-14}$

$\Delta C_e^{(8)} \cdot (\alpha/\pi)^4 \sim 1.0 \cdot 10^{-13}$ $\quad \Delta C_e^{(10)} \cdot (\alpha/\pi)^5 \sim 0.4 \cdot 10^{-13}$ $\quad \Delta a_e^{\mathrm{exp}} = 2.8 \cdot 10^{-13}$

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$C_\ell^{(2)}$	0.5	0.5
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$C_\ell^{(8)}$	$-1.9096(20)$	$130.879\,6(63)$
$C_\ell^{(10)}$	$9.16(58)$	$753.29(1.04)$

n	1	2	3	4	5
$(\alpha/\pi)^n$	$2.32\ldots \cdot 10^{-3}$	$5.39\ldots \cdot 10^{-6}$	$1.25\ldots \cdot 10^{-8}$	$2.91\ldots \cdot 10^{-11}$	$6.76\ldots \cdot 10^{-14}$

$$\Delta C_\mu^{(4)} \cdot (\alpha/\pi)^2 \sim 0.9 \cdot 10^{-13} \qquad \Delta C_\mu^{(6)} \cdot (\alpha/\pi)^3 \sim 0.04 \cdot 10^{-13}$$

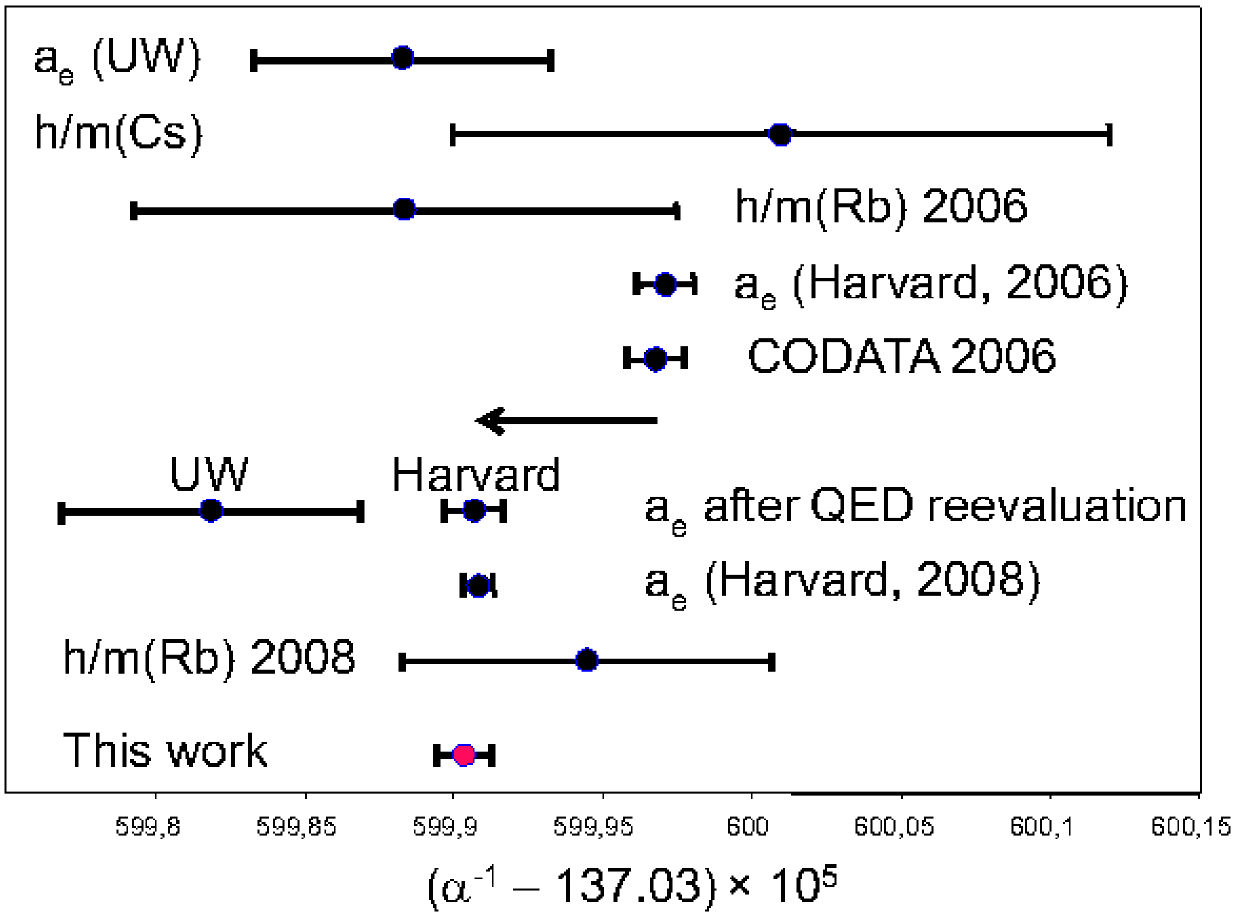
$$\Delta C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 1.8 \cdot 10^{-13} \qquad \Delta C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.7 \cdot 10^{-13} \qquad \Delta a_\mu^{\text{exp}} = 6.3 \cdot 10^{-10}$$

$$C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 3.8 \cdot 10^{-9} \qquad C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.5 \cdot 10^{-10}$$

$$a_e^{\rm QED} = 1\,159\,652\,180.07(6)_{\alpha^4}(4)_{\alpha^5}(77)_{\alpha(Rb11)} \cdot 10^{-12} \qquad a_e^{\rm exp} - a_e^{\rm QED} = 0.67(82) \cdot 10^{-12}$$

$$\alpha[a_e(HV\,08)] = 137.035\,999\,172\,2(68)_{\alpha^4}(46)_{\alpha^5}(19)_{\rm had}(331)_{\rm exp} \qquad [0.25\text{ppb}]$$

Aoyama et al., Phys. Rev. Lett. 109, 111807 (2012)



R. Bouchendira, P. Cladé, S. Ghelladi-Khélifa, F. Nez, F. Biraben, Phys. Rev. Lett. 106, 080801 (2011)

$$a_\mu^{\mathrm{QED}}(Rb) = 1\,165\,847\,189.51(9)_{\mathrm{mass}}(19)_{\alpha^4}(7)_{\alpha^5}(77)_{\alpha(Rb11)} \cdot 10^{-12}$$

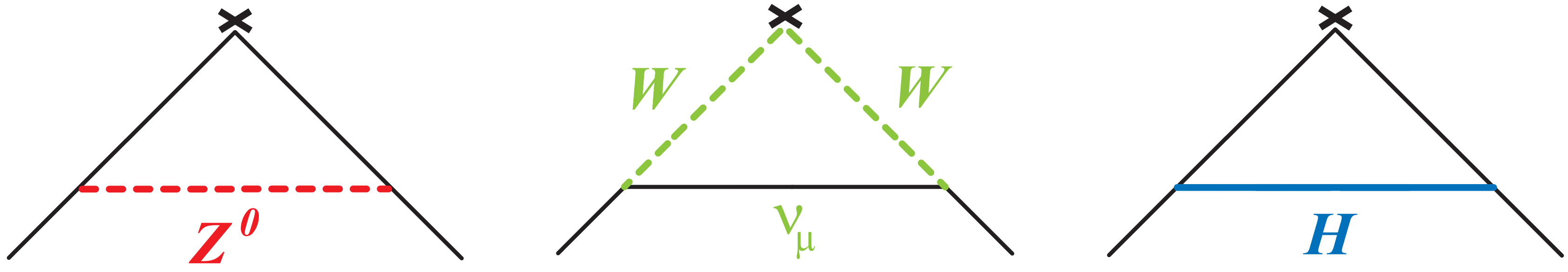
$$a_\mu^{\mathrm{QED}}(a_e) = 1\,165\,847\,188.46(9)_{\mathrm{mass}}(19)_{\alpha^4}(7)_{\alpha^5}(30)_{\alpha(a_e)} \cdot 10^{-12}$$

Aoyama et al., Phys. Rev. Lett. 109, 111807 (2012)

$$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{QED}} = 737.0(6.3) \cdot 10^{-10}$$

Theory II: Weak interactions

- Weak contributions : W, Z,... loops



$$a_\mu^{\text{weak(1)}} = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left[\frac{5}{3} + \frac{1}{3} \left(1 - 4 \sin^2 \theta_W \right)^2 + \mathcal{O}\left(\frac{m_\mu^2}{M_Z^2} \log \frac{M_Z^2}{m_\mu^2} \right) + \mathcal{O}\left(\frac{m_\mu^2}{M_H^2} \log \frac{M_H^2}{m_\mu^2} \right) \right]$$
$$= 19.48 \times 10^{-10}$$

W.A. Bardeen, R. Gastmans and B.E. Lautrup, Nucl. Phys. B46, 315 (1972)

G. Altarelli, N. Cabbibo and L. Maiani, Phys. Lett. 40B, 415 (1972)

R. Jackiw and S. Weinberg, Phys. Rev. D 5, 2473 (1972)

I. Bars and M. Yoshimura, Phys. Rev. D 6, 374 (1972)

M. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D 6, 2923 (1972)

Two-loop bosonic contributions

$$a_\mu^{\text{weak(2);b}} = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} \cdot \left[-5.96 \ln \frac{M_W^2}{m_\mu^2} + 0.19 \right] = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left(\frac{\alpha}{\pi} \right) \cdot (-79.3)$$

A. Czarnecki, B. Krause and W. J. Marciano, Phys. Rev. Lett. 76, 3267 (1996)

Two-loop fermionic contributions

A. Czarnecki, W.J. Marciano, A. Vainshtein, Phys. Rev. D 67, 073006 (2003). Err.-ibid. D 73, 119901 (2006)

M. K., S. Peris, M. Perrottet, E. de Rafael, JHEP11, 003 (2002)

$$a_\mu^{\text{weak}} = (154 \pm 1) \cdot 10^{-11}$$
$$a_e^{\text{weak}} = (0.0297 \pm 0.0005) \cdot 10^{-12}$$

Recent update: $a_\mu^{\text{weak}} = (153.6 \pm 1.0) \cdot 10^{-11}$

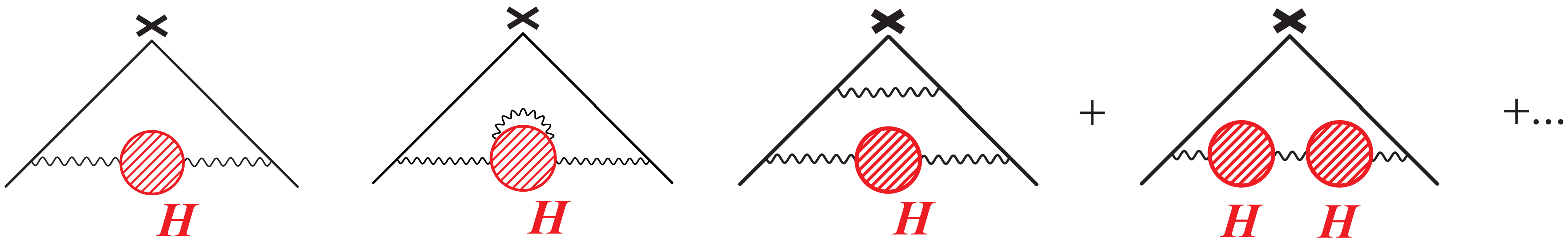
C. Gnendiger, D. Stöckinger, H. Stöckinger-Kim, Phys. Rev. D 88, 053005 (2013)

$$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{QED}} - a_\mu^{\mathsf{weak}} = 721.65(6.38) \cdot 10^{-10}$$

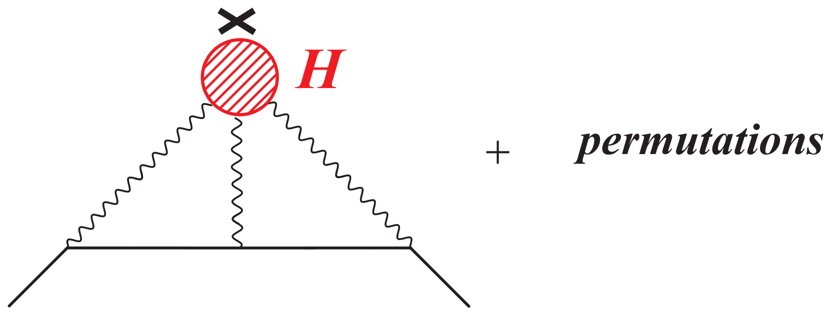
Theory III: Strong interactions

- Hadronic contributions : quark and gluon loops

$$a_\ell^{\text{had}} = a_\ell^{\text{HVP-LO}} + a_\ell^{\text{HVP-HO}} + a_\ell^{\text{HLxL}}$$



Lowest-order hadronic vacuum polarization (HVP-LO) | Higher-order hadronic vacuum polarization (HVP-HO)



Hadronic light-by-light scattering (HLxL)

Hadronic vacuum polarization

- Occurs first at order $\mathcal{O}(\alpha^2)$

- Can be expressed as

$$a_\ell^{\text{HVP-LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4M_\pi^2}^{\infty} \frac{dt}{t} K(t) R^{\text{had}}(t) \quad K(t) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)\frac{t}{m_\ell^2}}$$

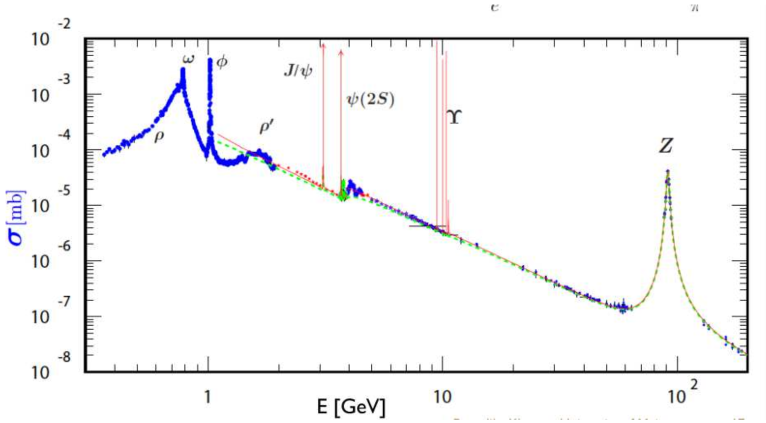
C. Bouchiat, L. Michel, J. Phys. Radium 22, 121 (1961)
L. Durand, Phys. Rev. 128, 441 (1962); Err.-ibid. 129, 2835 (1963)
M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)

- $K(s) > 0$ and $R^{\text{had}}(s) > 0 \Longrightarrow a_\ell^{\text{HVP-LO}} > 0$

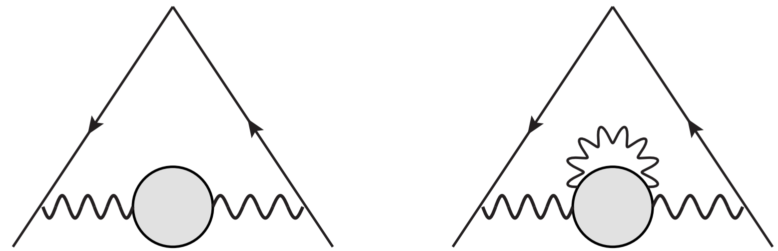
- $K(s) \sim m_\ell^2/(3s)$ as $s \to \infty \Longrightarrow$ the (non perturbative) low-energy region dominates

Hadronic vacuum polarization

- Can be evaluated using available experimental data



$\longrightarrow$



- Some order $\mathcal{O}(\alpha^3)$ corrections included

- exchange of virtual photons between final state hadrons

- some radiative exclusive modes, e.g. $\pi^0\gamma$

$$a_\mu^{\pi^0\gamma}(600\,\mathrm{MeV} - 1030\,\mathrm{MeV}) = 4.4(1.9)\cdot 10^{-10}$$

- The two most recent determinations are in good agreement (being based on the same data sets, this should not be a surprise) and give a relative precision of 0.6%

Latest (published) results

$$a_\mu^{\text{HVP-LO}} = 692.3 \pm 4.2 \cdot 10^{-10}$$ [M. Davier et al., Eur. Phys. J. C 71, 1515 (2011)]

$$a_\mu^{\text{HVP-LO}} = 694.9 \pm 4.3 \cdot 10^{-10}$$ [K. Hagiwara et al., J. Phys. G 38, 085003 (2011)]

$$a_e^{\text{HVP-LO}} = 1.866(11) \cdot 10^{-12}$$ [D. Nomura, T. Teubner, Nucl. Phys. B 867, 236 (2013)]

$$a_\mu^{\text{HVP-NLO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^3 \int_{4M_\pi^2}^{\infty} \frac{dt}{t} K^{(2)}(t) R^{\text{had}}(t)$$

J. Calmet, S. Narison, M. Perrottet, E. de Rafael, Phys. Lett. B 61, 283 (1976)

B. Krause, Phys. Lett. B 390, 392 (1997)

$$a_\mu^{\text{HVP-NLO}} = -9.84 \pm 0.07 \cdot 10^{-10}$$ [K. Hagiwara et al., J. Phys. G 38, 085003 (2011)]

$$a_e^{\text{HVP-NLO}} = -0.2234(14) \cdot 10^{-12}$$ [D. Nomura, T. Teubner, Nucl. Phys. B 867, 236 (2013)]

$$a_\mu^{\text{HVP-NNLO}} = 1.24 \pm 0.01 \cdot 10^{-10}$$ [A. Kurz et al., Phys. Lett. B 734, 144 (2014)]

$$a_e^{\text{HVP-NNLO}} = 0.028(1) \cdot 10^{-12}$$

Hadronic vacuum polarization

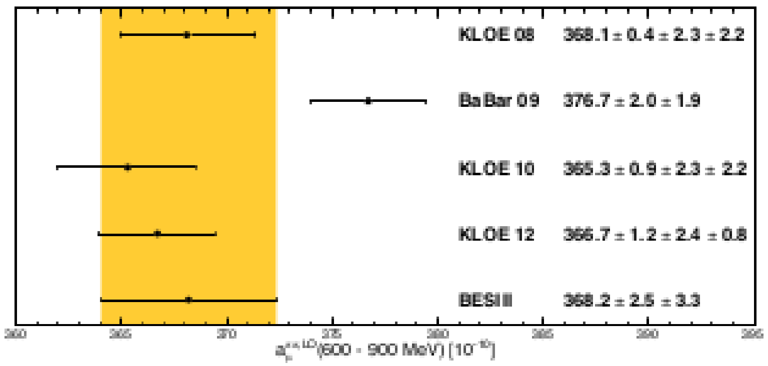
- Some tension between, for instance, the high-precision data collected in the region of the ρ resonance by BaBar and KLOE/KLOE-2

Experiment	$a_\mu^{\mathrm{HVP-LO}\,2\pi}(600 - 900\ \mathrm{MeV})$
BaBar	376.7(2.0)(1.9)
KLOE 08	368.9(0.4)(2.3)(2.2)
KLOE 10	366.1(0.9)(2.3)(2.2)
KLOE 12	366.7(1.2)(2.4)(0.8)

- These tensions need to be resolved in order to achieve higher precision

$\longrightarrow$ new data (KLOE-2, BaBar, VEPP-2000, BESIII,...)

Experiment	$a_\mu^{\mathrm{HVP-LO}\,2\pi}(600 - 900\ \mathrm{MeV})$	
BESIII	368.2(2.5)(3.3)	[BESIII Coll., Phys. Lett. B 753, 629 (2016)]



$\longrightarrow$ talk by Y. Guo

Hadronic vacuum polarization

- Update including new data available since 2011

$$a_\mu^{\text{HVP-LO}} = 686.99 \pm 4.21 \cdot 10^{-10}$$
$$a_\mu^{\text{HVP-NLO}} = -9.934 \pm 0.091 \cdot 10^{-10}$$
$$a_\mu^{\text{HVP-NNLO}} = 1.226 \pm 0.012 \cdot 10^{-10}$$

$$a_e^{\text{HVP-LO}} = 1.8464(121) \cdot 10^{-12}$$
$$a_e^{\text{HVP-NLO}} = -0.2210(14) \cdot 10^{-12}$$
$$a_e^{\text{HVP-NNLO}} = 0.0279(2) \cdot 10^{-12}$$

F. Jegerlehner, arXiv:1511.04473 [hep-ph]

Hadronic vacuum polarization

• Possibility to extract HVP from Bhabha scattering?

C. M. Carloni-Calame, M. Passera, L. Trentadue, G.Venanzoni, Phys. Lett. B 476, 325 (2015)

• Constraints on the pion electromagnetic form factor from analyticity and unitarity

H. Leutwyler, arXiv-ph/0212324

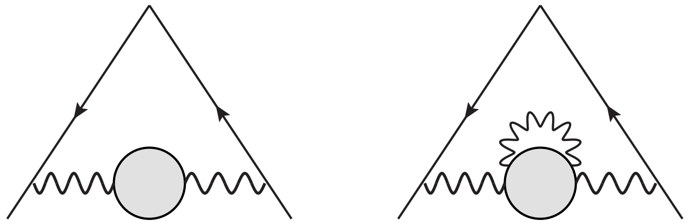
B. Ananthanarayan, I. Caprini, D. Das, I. S. Imsong, Phys. Rev. D 93, 116007 (2016)

⟶ talk by I. Caprini

• Alternative for the (near?) future: Lattice QCD:
several contributions at recent ICHEP and LATTICE conferences

⟶ talks by C. Davies and A. Jüttner

comparison with data at the sub-percent level:
isospin breaking effects (radiative corrections)

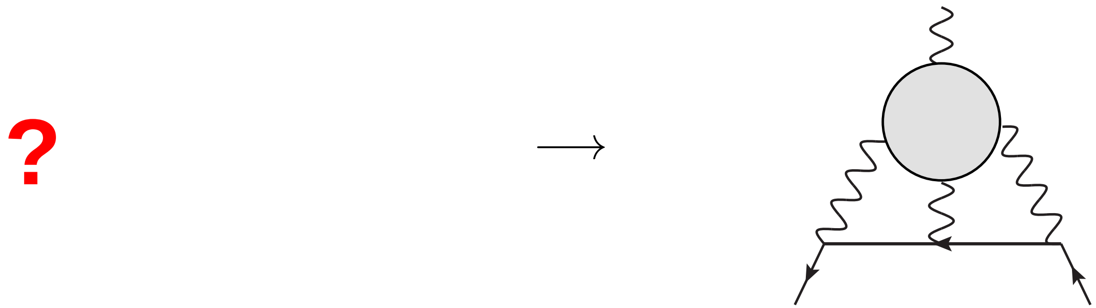


(Experimentalists don't live in the theoretician's paradise)

Hadronic light-by-light: the really complicated thing

- occurs at order $\mathcal{O}(\alpha^3)$

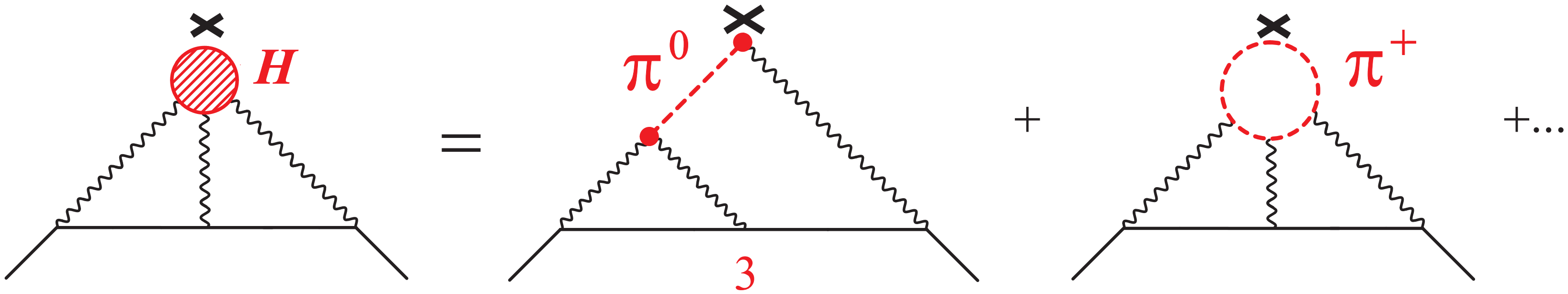
- not related, as a whole, to an experimental observable...



- Involves the fourth-rank vacuum polarization tensor

$$\text{F.T.}\ \langle 0|T\{VVVV\}|0\rangle \longrightarrow \Pi_{\mu\nu\rho\sigma}(q_1,q_2,q_3,q_4) \quad q_1+q_2+q_3+q_4=0$$

- Many identifiable contributions...



Hadronic light-by-light

- Need some organizing principle: ChPT, large-N_c (turns out to be most relevant in practice)

E. de Rafael, Phys. Lett. B 322, 239 (1994)

$$a_\mu^{\rm HLxL} = N_c \left(\frac{\alpha}{\pi}\right)^3 \frac{N_c}{F_\pi^2} \frac{m_\mu^2}{48\pi^2} \left[\ln^2 \frac{M_\rho}{M_\pi} + c_\chi \ln \frac{M_\rho}{M_\pi} + \kappa\right] + \mathcal{O}(N_c^0)$$

M. Knecht, A. Nyffeler, Phys. Rev. D 65, 073034 (2002)
M. Knecht, A. Nyffeler, M. Perrottet, E. de Rafael, Phys. Rev. Lett. 88, 071802 (2002)
M. J. Ramsey-Musolf, M. B. Wise, Phys. Rev. Lett. 89, 041601 (2002)]
J. Prades, E. de Rafael, A. Vainshtein, Glasgow White Paper (2008)

- Impose QCD short-distance properties K. Melnikov, A. Vainshtein, Phys. Rev. D, 113006 (2004)

- Present estimates rely mainly on two model-dependent calculation

$$a_\mu^{\rm HLxL} = +(8.3 \pm 3.2) \cdot 10^{-10}$$

J. Bijnens, E. Pallante, J. Prades, Phys. Rev. Lett. 75, 1447 (1995) [Err.-ibid. 75, 3781 (1995)]; Nucl. Phys. B 474, 379 (1995); Nucl. Phys. B 626, 410 (2002)

$$a_\mu^{\rm HLxL} = +(89.6 \pm 15.4) \cdot 10^{-11}$$

M. Hayakawa, T. Kinoshita, A. I. Sanda, Phys. Rev. Lett. 75, 790 (1995); Phys. Rev. D **54**, 3137 (1996)
M. Hayakawa, T. Kinoshita, Phys. Rev. D 57, 365 (1998) [Err.-ibid. 66, 019902(E) (2002)

that turn out to be positive [M.K. and A. Nyffeler, Phys. Rev. D 65, 073034 (2002)]

Hadronic light-by-light

Recent (partial) reevaluations

$a_\mu^{\text{HLxL}} = (10.5 \pm 2.6) \cdot 10^{-10}$ [J. Prades, E. de Rafael, A. Vainshtein, arXiv:0901.0306]

"best estimate"

$a_\mu^{\text{HLxL}} = (11.5 \pm 4.0) \cdot 10^{-10}$ [A. Nyffeler, Phys. Rev. D 79, 073012 (2009)]

more conservative estimate

$a_e^{\text{HLxL}} = (0.035 \pm 0.010) \cdot 10^{-12}$ [J. Prades, E. de Rafael, A. Vainshtein, in *Lepton Dipole Moments*] units: 10^{-11}

Contribution	BPP	HKS	KN	MV	BP	PdRV	N/JN
π^0, η, η'	85 ± 13	82.7 ± 6.4	83 ± 12	114 ± 10	–	114 ± 13	99 ± 16
π, K loops	-19 ± 13	-4.5 ± 8.1	–	–	–	-19 ± 19	-19 ± 13
π, K l. + subl. in Nc	–	–	–	0 ± 10	–	–	–
axial vectors	2.5 ± 1.0	1.7 ± 1.7	–	22 ± 5	–	15 ± 10	22 ± 5
scalars	-6.8 ± 2.0	–	–	–	–	-7 ± 7	-7 ± 2
quark loops	21 ± 3	9.7 ± 11.1	–	–	–	2.3	21 ± 3
total	83 ± 32	89.6 ± 15.4	80 ± 40	136 ± 25	110 ± 40	105 ± 26	116 ± 39

BPP: J. Bijnens, E. Pallante, J. Prades, Phys. Rev. Lett. 75 (1995) 1447 [Erratum-ibid. 75 (1995) 3781]; Nucl. Phys. B 474 (1996) 379; [Erratum-ibid. 626 (2002) 410]

HKS: M. Hayakawa, T. Kinoshita, A. I. Sanda, Phys. Rev. Lett. 75 (1995) 790; Phys. Rev. D 54 (1996) 3137

KN: M. Knecht, A. Nyffeler, Phys. Rev. D 65 (2002) 073034

MV: K. Melnikov, A. Vainshtein, Phys. Rev. D 70 (2004) 113006

BP: J. Bijnens, J. Prades, Acta Phys. Polon. B 38 (2007) 2819; J. Prades, Nucl. Phys. Proc. Suppl. 181-182 (2008) 15; J. Bijnens, J. Prades, Mod. Phys. Lett. A 22 (2007) 767

BdRV: J. Prades, E. de Rafael, A. Vainshtein, arXiv:0901.0306 [hep-ph]

N/NJ: A. Nyffeler, Phys. Rev. D 79, 073012 (2009); F. Jegerlehner, A. Nyffeler, Phys. Rep. (2009)

Recent reevaluation of single meson exchanges

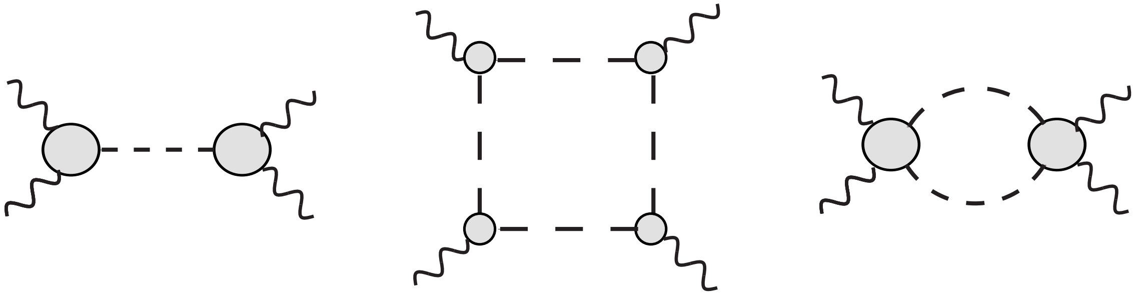
$a_\mu(f_1, f_1') = 6.4(2.0) \cdot 10^{-11}$ $a_\mu(f_0, f_0', a_0) = (-1 \text{ to } -4) \cdot 10^{-11}$ $a_\mu(f_2, f_2', a_2, a_2') = 1.1(0.1) \cdot 10^{-11}$

[V. Pauk, M. Vanderhaeghen, arXiv:0401.0832 [hep-ph]]

Hadronic light-by-light: the really complicated thing

- More recently: dispersive approaches

– for $\Pi_{\mu\nu\rho\sigma}$ $\longrightarrow$ talk by G. Colangelo



$$\Pi = \Pi^{\pi^0,\eta,\eta' \text{ poles}} + \Pi^{\pi^\pm,K^\pm \text{ loops}} + \Pi^{\pi\pi} + \Pi^{\text{residual}}$$

[G. Colangelo, M. Hoferichter, M. Procura, P. Stoffer, JHEP09, 091 (2014); arXiv:1506.01386 [hep-ph]]

Needs input from data (transition form factors,...) $\longrightarrow$ talk by P. Sanchez-Puertas

[G. Colangelo, M. Hoferichter, B. Kubis, M. Procura, P. Stoffer, Phys. Lett. B 738, 6 (2014)]

[A. Nyffeler, arXiv:1602.03398 [hep-ph]]

– for $F_2^{\text{HLxL}}(k^2)$ $\longrightarrow$ talk by M. Vanderhaeghen

only pion pole with VMD form factor (two-loop graph) reconstructed this way so far

[V. Pauk and M. Vanderhaeghen, Phys. Rev. D 90, 113012 (2014) [arXiv:1409.0819 [hep-ph]]]

Open issues:

- how will short-distance constraints be imposed?
- how will Π^{residual} be estimated? Cf. axial vectors (leading in large-N_c) $\to 3\pi$ channel

Hadronic light-by-light: the really complicated thing

- Other recent approaches

— Dyson-Schwinger/Bethe-Salpeter equations $\longrightarrow$ talk by G. Eichmann

— Non-local quark model $\longrightarrow$ talk by A. Zhevaklov

Goal: evaluation of HLxL with a reliable uncertainty of $\sim 10\%$

Summary - Conclusions

- The anomalous magnetic moments of the electron and of the muon are among the most precisely measured observables of the standard model

$$a_e^{\text{exp}} = 1\,159\,652\,180.73(0.28) \cdot 10^{-12} \quad [0.24\text{ppb}]$$
$$a_\mu^{\text{exp}} = 116\,592\,089(63) \cdot 10^{-11} \quad [0.54\text{ppm}]$$

• The anomalous magnetic moments of the electron and of the muon are among the most precisely measured observables of the standard model

$$\begin{aligned} a_e^{\text{exp}} &= 1\,159\,652\,180.73(0.28) \cdot 10^{-12} \quad [0.24\text{ppb}] \\ a_\mu^{\text{exp}} &= 116\,592\,089(63) \cdot 10^{-11} \quad [0.54\text{ppm}] \end{aligned}$$

• At present, the standard model value for a_μ misses the experimental determination by about 3 to 3.5 standard deviations:

$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (27.4 \pm 8.0) \cdot 10^{-10}$ $[3.4\sigma]$ for $a_\mu^{\text{HLxL}} = (10.5 \pm 2.6) \cdot 10^{-10}$, $a_\mu^{\text{HVP-LO}} = 692.3 \pm 4.2 \cdot 10^{-10}$

$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (23.7 \pm 8.6) \cdot 10^{-10}$ $[2.8\sigma]$ for $a_\mu^{\text{HLxL}} = (11.6 \pm 4.0) \cdot 10^{-10}$, $a_\mu^{\text{HVP-LO}} = 694.9 \pm 4.3 \cdot 10^{-10}$

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- It is not obvious to find a straightforward explanation for this persistent discrepancy:

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (28.7 \pm 8.0) \cdot 10^{-10} \, (\sim 2 \cdot a_\mu^{\text{weak}}, \sim a_\mu^{\text{QED}}(\alpha^4), \sim 3 \cdot a_\mu^{\text{HLxL}}, \ldots)$$

$$\Delta a_\mu^{\text{exp}} = 6.3 \cdot 10^{-10}$$

- The anomalous magnetic moments of the electron and of the muon are among the most precisely measured observables of the standard model

$$a_e^{\text{exp}} = 1\,159\,652\,180.73(0.28) \cdot 10^{-12} \quad [0.24\text{ppb}]$$
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$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (28.7 \pm 8.0) \cdot 10^{-10}$$

Higher order QED effects?

$$A_2^{(12)}(m_\mu/m_e) \sim A_2^{(6)}(m_\mu/m_e; \text{LxL}) \left[\frac{2}{3} \ln \frac{m_\mu}{m_e} - \frac{5}{9}\right]^3 \cdot 10 \sim 0.6 \cdot 10^4 \longrightarrow \delta a_\mu^{\text{QED}} \sim 1 \cdot 10^{-12}$$

Higher order QCD effects?

$$a_\mu^{\text{HVP-NNLO}} = (1.24 \pm 0.01) \cdot 10^{-10}$$

A. Kurz et al., Phys. Lett. B 734, 144 (2014)

$$a_\mu^{\text{HLxL-HO}} \sim (0.3 \pm 0.2) \cdot 10^{-10}$$

G. Colangelo et al., Phys. Lett. B 735, 90 (2014)

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$$a_e^{\text{exp}} = 1\,159\,652\,180.73(0.28) \cdot 10^{-12} \quad [0.24\text{ppb}]$$
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$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (25.0 \pm 8.6) \cdot 10^{-10} \ [2.9\sigma] \quad \text{for } a_\mu^{\text{HLxL}} = (11.6 \pm 4.0) \cdot 10^{-10}, \ a_\mu^{\text{HVP-LO}} = 694.9 \pm 4.3 \cdot 10^{-10}$$

- It is not obvious to find a straightforward explanation for this persistent discrepancy:

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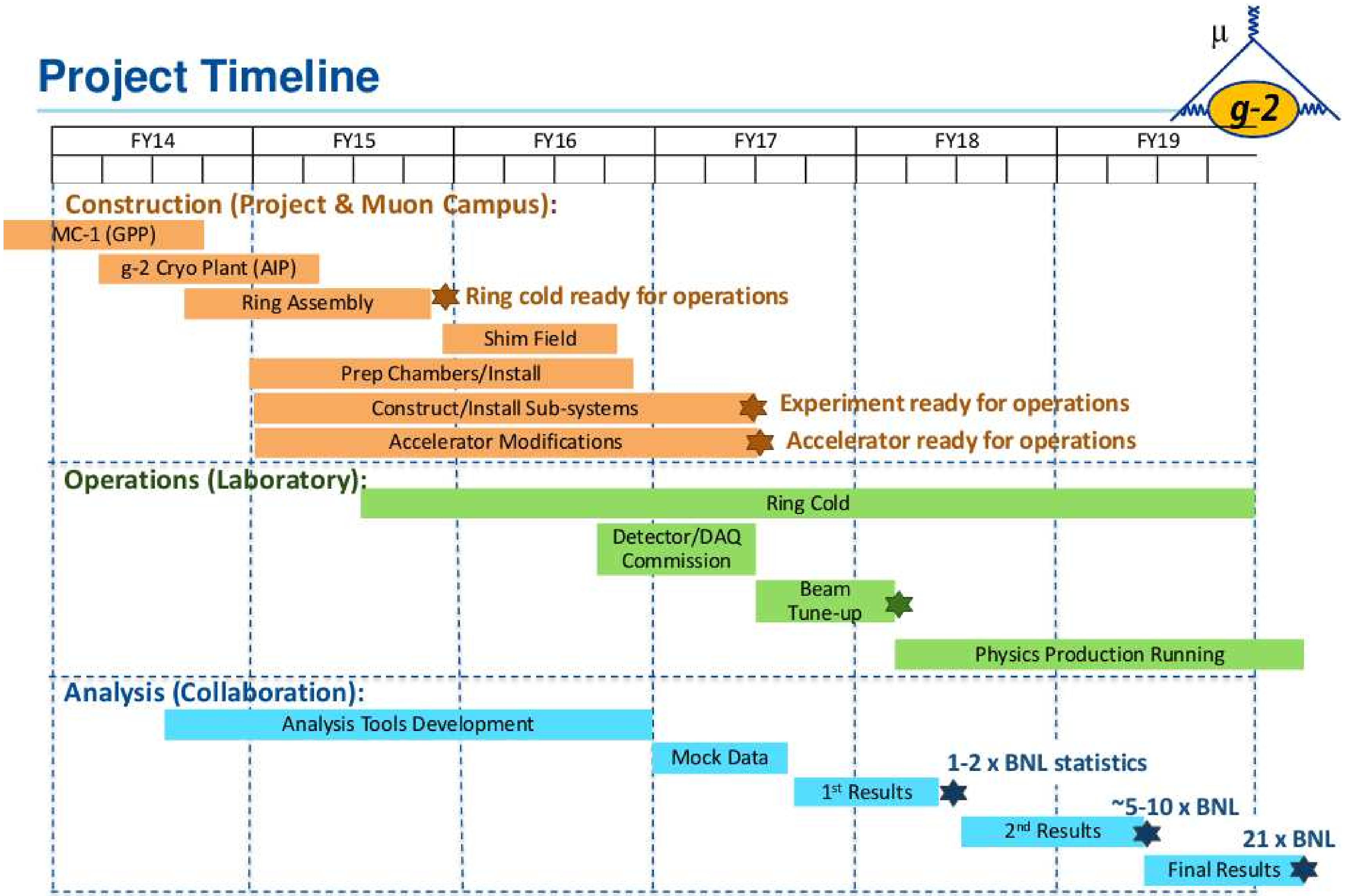
Manifestation of BSM degrees of freedom?

- Two new experiments, with the aim of reducing the experimental uncertainty by a factor of 4, are being prepared, at FNAL and at J-PARC (first results expected in ~ 2 years)

$\longrightarrow$ talk by M. Lancaster

Project Timeline



Fermilab

C. Polly, E. Swanson, ICHEP, Aug. 2016

- Two new experiments, with the aim of reducing the experimental uncertainty by a factor of 4, are being prepared, at FNAL and at J-PARC (first results expected in ~ 2 years)

- New high-precision data from VEPP-2000, BESIII,...

- New input from theory+lattice

- a_e is expected to be about $(m_\mu/m_e)^2 \sim 40000$ times less sensitive to BSM effects that a_μ. But it is known with much better (~ 2300) precision... Possibilities to observe BSM effects through a_e

G. F. Giudice, P. Paradisi, M. Passera, JHEP 1211, 113 (2012)

- Needs improvements on the determinations of a_e [from 0.24ppb to 0.06ppb], of R_∞, m_e/m_{u},...

$$\alpha^2 = \frac{2R_\infty}{c} \frac{M_{\mathrm{at}}}{m_{\mathrm{u}}} \frac{m_{\mathrm{u}}}{m_e} \frac{h}{M_{\mathrm{at}}}$$

that are within reach on a timescale similar to the one of the new $(g-2)_\mu$ experiments.

F. Terranova, G. M. Tino, arXiv:1312.2346 [hep-ex]

Thanks for your attention!