

Long-range hadronic effects and precision tests of SM

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Outline

- Context: precise tests of SM with electron scattering
- Long-range effects from 2γ -box
 - Charge radius and beam normal spin asymmetry
- Long-range effects from $PV2\gamma$ -box
 - Superconvergence relation in ChPT
 - Estimates for the $PV2\gamma$ correction
- Conclusions

Test of SM with running of weak mixing angle

Weak mixing angle: very central role in the EW sector

$$\begin{pmatrix} \gamma \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} B^0 \\ W^0 \end{pmatrix}$$

Tree level: fixed by boson masses and SU(2)/U(1) couplings

$$\sin^2 \theta_W = 1 - M_W^2/M_Z^2 = g'^2/(g^2 + g'^2)$$

Upon renormalization: weak mixing angle is scale-dependent

$$\sin^2 \theta_W \rightarrow \sin^2 \theta_{\text{eff}}(Q)$$

The running is a unique prediction of the SM;

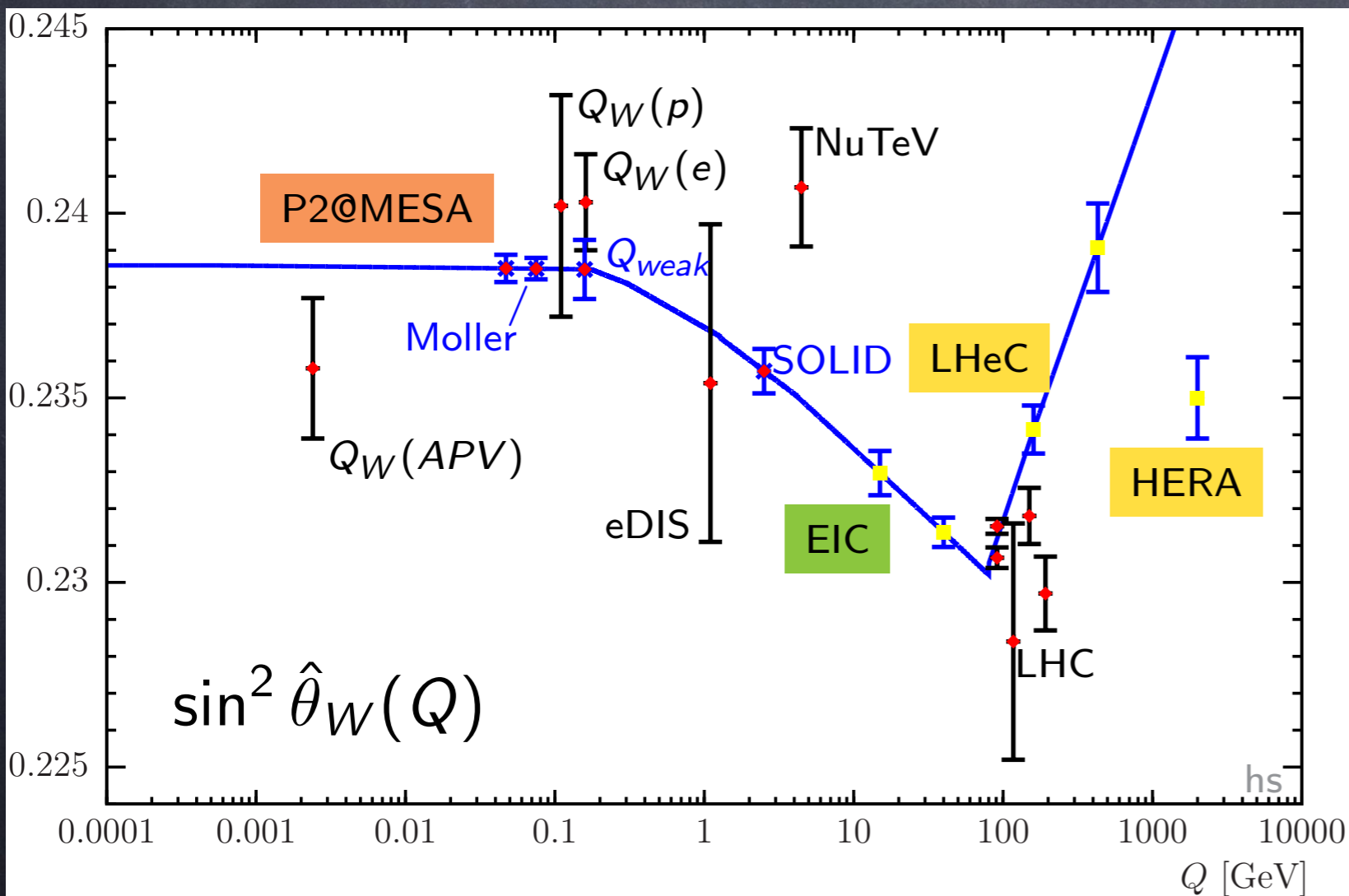
A theory with a different content will predict a different running;

WMA - a good way to test the SM and New Physics

Test of SM with running of weak mixing angle

SM running: confirmed qualitatively (not yet quantitatively)

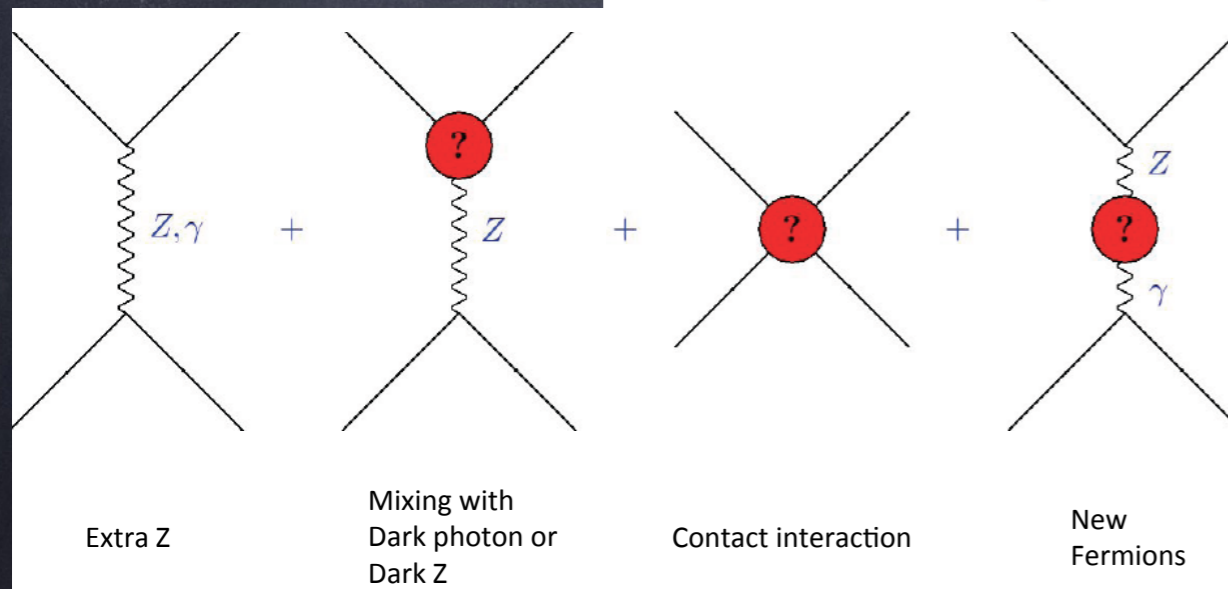
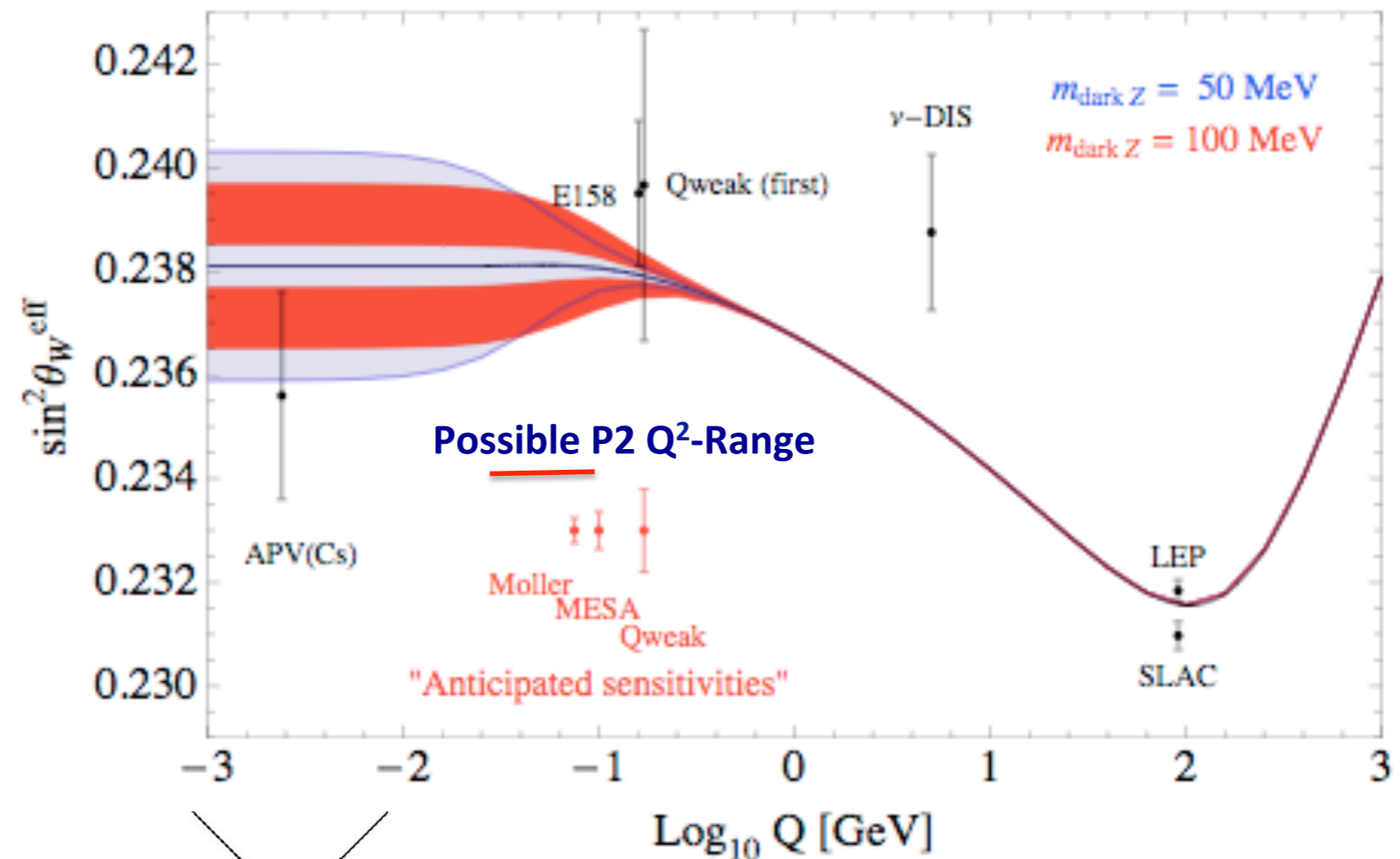
Existing and planned measurements



- Atomic PV (Cs)
- Neutrino scattering
- LEP and SLC (Z-pole)
- Møller scattering
- Q_{weak} (under analysis)
- ATLAS (under analysis)
- MOLLER (planned)
- MESA P2 (planned)
- MESA C12 (proposed)
- DIS SOLID (planned)
- APV with Yb, Dy (planned)
- Future colliders

A theory with a different content will predict different running

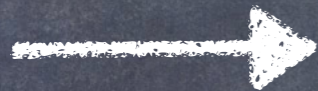
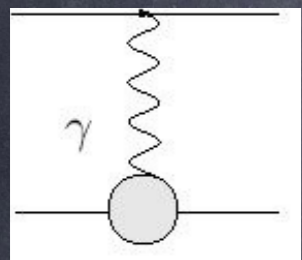
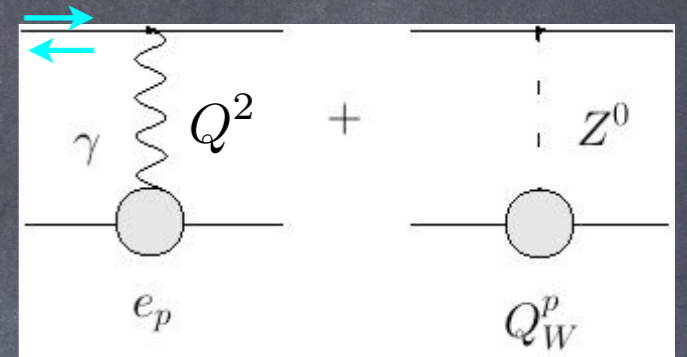
Running $\sin^2 \theta_W$ and Dark Parity Violation



Bill Marciano

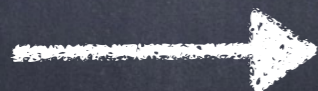
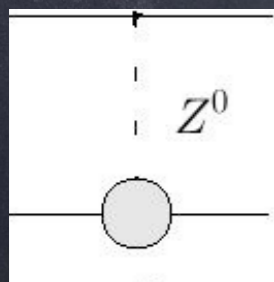
Weak Charge of the Proton from PVES

- Elastic e-p scattering with polarized e^- beam



$$\frac{e^2}{Q^2} [1 - \{R_p^2, \mu_p\} Q^2 + O(Q^4)]$$

Talk by Keith



$$-\frac{G_F}{2\sqrt{2}} [Q_W^p + \{R_{p,n}^2, \mu_{p,n}, G_{E,M}^s, G_A\} Q^2 + O(Q^4)]$$

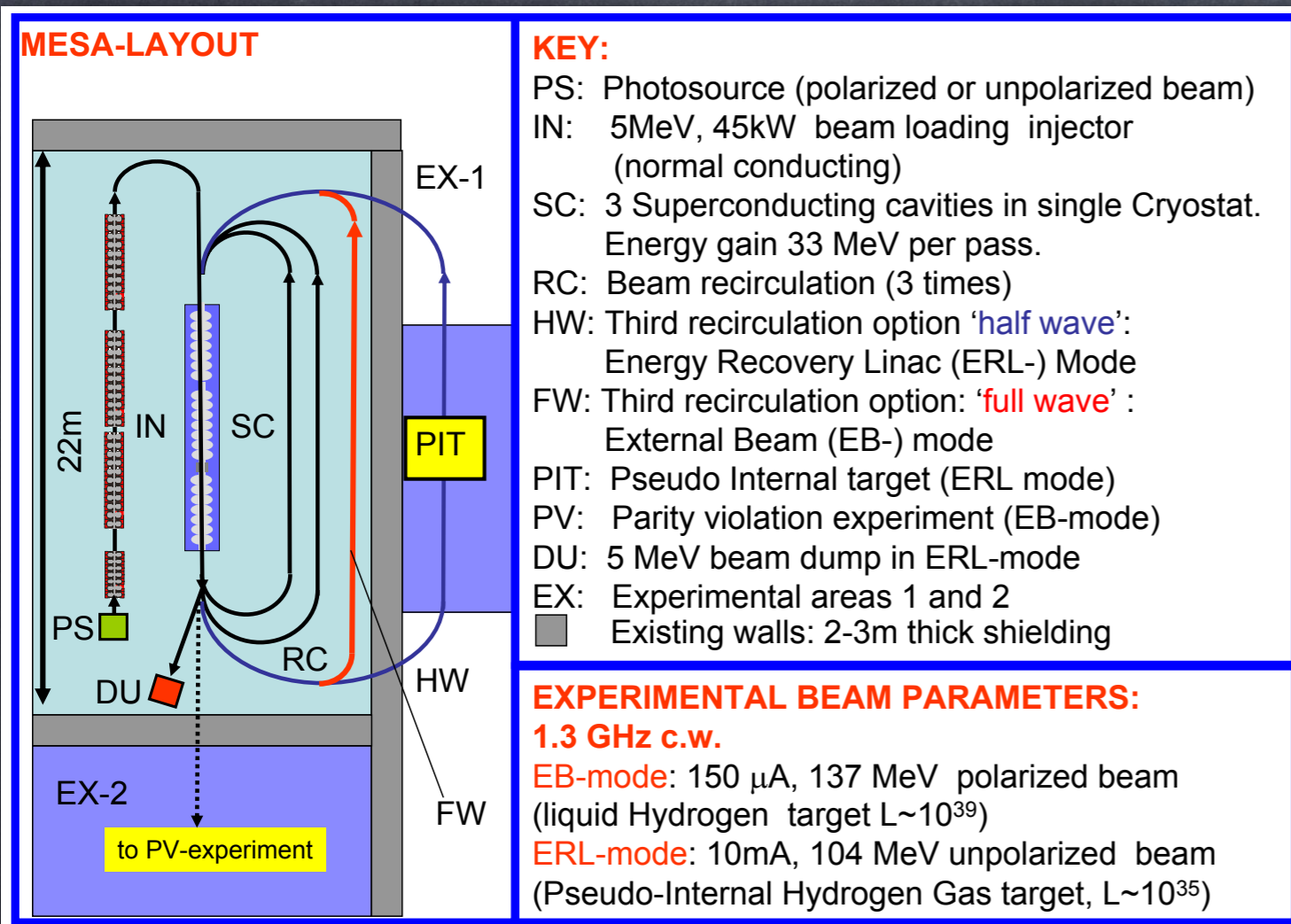
low Q^2 , $\epsilon \rightarrow 1$



$$A^{PV}(\epsilon, Q^2) = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha} [Q_W^p + B(Q^2) Q^2]$$

Talk by Ross

WMA determination with MESA/P2



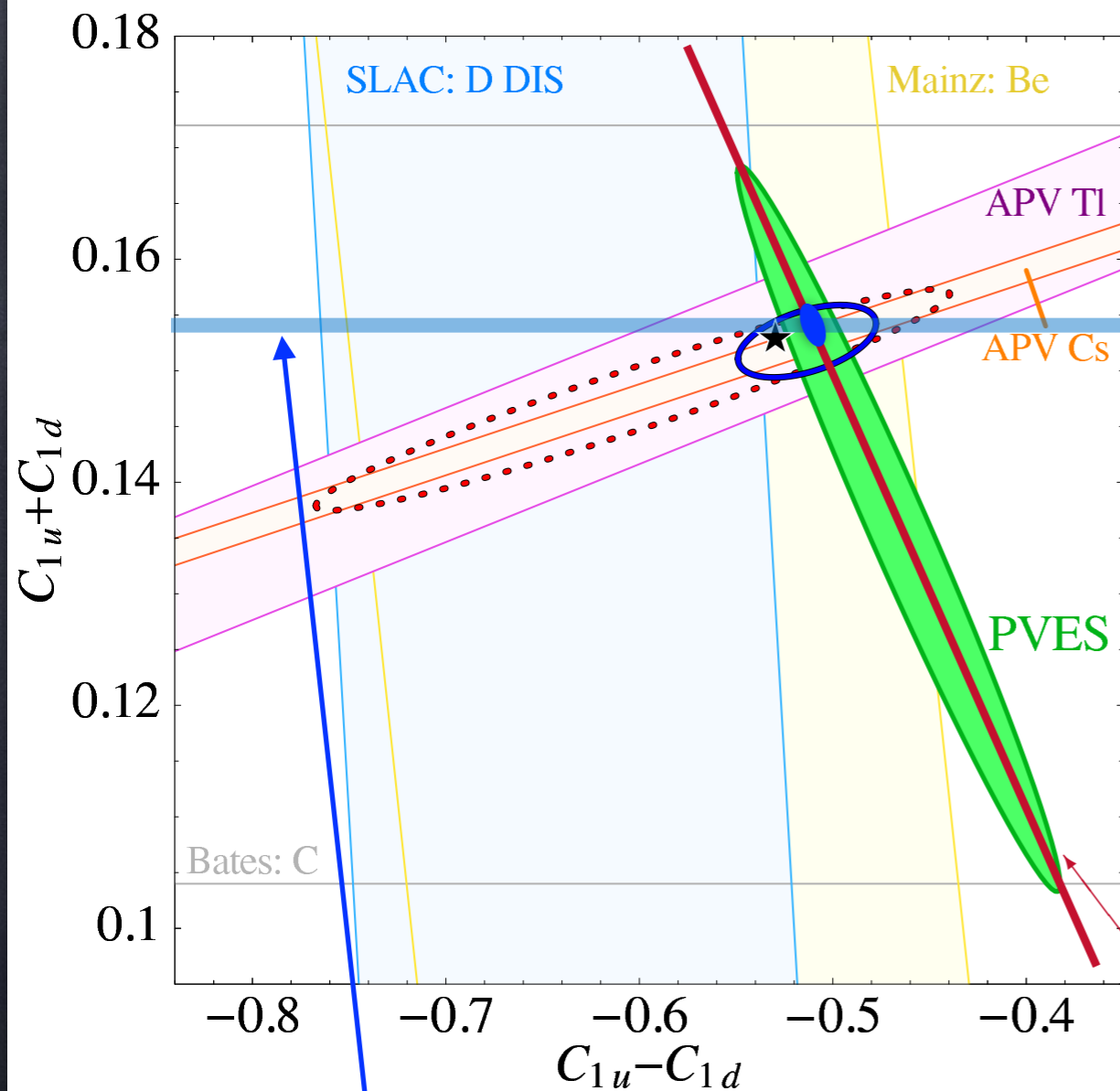
- $E = 155 \text{ MeV}, 150 \mu\text{A}$
- Scattering angle $20^\circ \pm 10^\circ$
- $Q^2 = 0.0045 \text{ GeV}^2$
- Polarization $(85 \pm 0.5)\%$
- Pol. flip few 1000/sec
- 60cm Liquid H target
- Asymmetry $A = -29 \text{ ppb}$
- $\delta A/A = 1.5\%$

Requirements to the beam:
1-2 o.o.m. improvement w.r.t. MAMI

Beam Quantity	Achieved at MAMI	Contribution to $\delta(A_{PV})$	Required for MESA
Energy	0.04 eV	< 0.1 ppb	fulfilled
Position	3 nm	5 ppb	0.13 nm
Angle	0.5 nrad	3 ppb	0.06 nrad
Intensity	14 ppb	4 ppb	0.36 ppb

Timeline: Accelerator commissioning: 2018
Data taking: 2020

Impact of MESA (H and C12) on SM tests



A more general approach
for extensions of the Standard Model:
model independent coupling constants,
effective low-energy 4-fermion interaction

$$C_{1f}: A_e \otimes V_f, C_{2f}: V_e \otimes A_f$$

SM prediction (black star):

$$C_{1f} = -I_f + 2Q_f \sin^2 \theta_W$$

$$(C_{1u} - C_{1d} = -1 + 2 \sin^2 \theta_W,$$

$$C_{1u} + C_{1d} = \frac{2}{3} \sin^2 \theta_W)$$

$$Q_W(p) = -2(2C_{1u} + C_{1d})$$

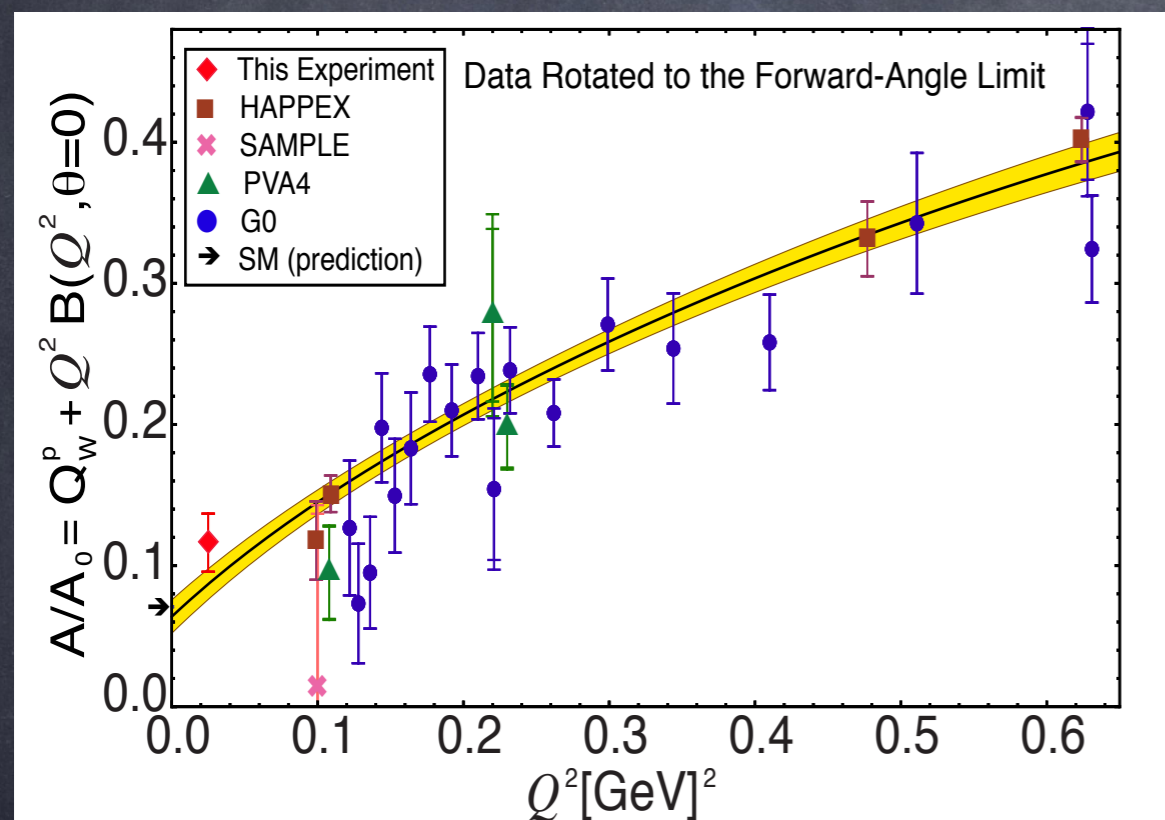
$$\text{Mainz P2: } \Delta Q_W(p) = \pm 0.0097 \text{ (2.1 \%)}$$

$$\text{MESA C12: } \Delta Q_W(\text{C12}) = 18\Delta(C_{1u} + C_{1d}) = \pm 0.0086 \text{ (0.3\%)}$$

Theory uncertainties

$$A^{PV}(\epsilon, Q^2) = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \left[Q_W^p + B(Q^2)Q^2 \right]$$

- $B(Q^2)$ – take from somewhere else (PVES, lattice, ...)

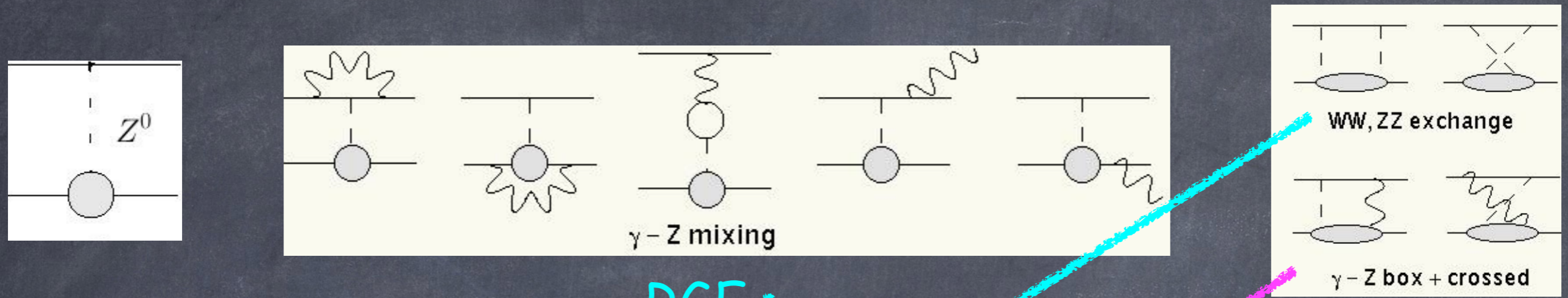


Young, Carlini, Thomas, Roche, PRL 2007;
Androic et al. [Qweak Coll.], PRL 2013

- Rationale: go to the lowest Q^2 – asymmetry directly measures the weak charge

How is this picture modified by the radiative corrections?

1-loop radiative corrections to Z-exchange



RGE

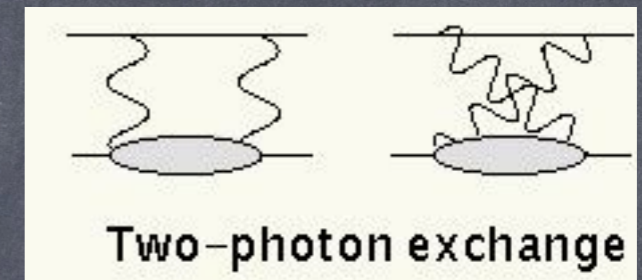
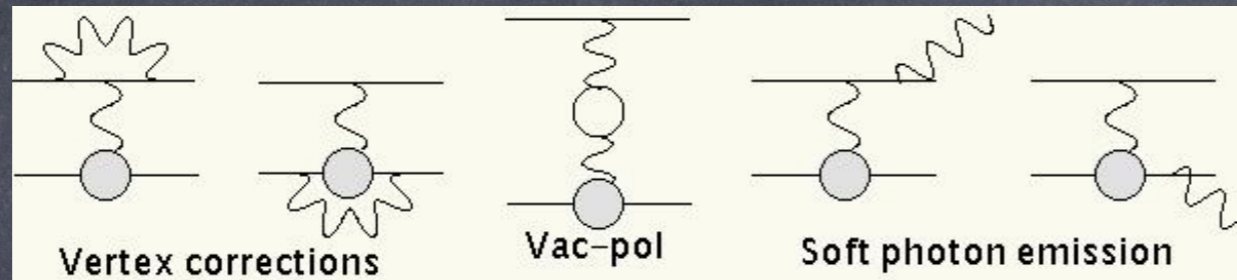
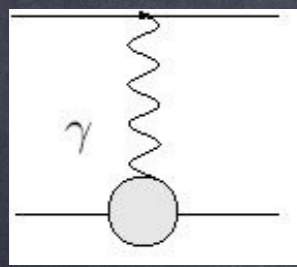
$$\rightarrow -\frac{G_F}{2\sqrt{2}} [Q_W^p + \delta Q_W^p(\mu) + \square_{\gamma Z}(E) + O(Q^2)]$$

γZ -box: hadron structure- and energy dependent

Marciano & Sirlin; Erler, Kurylov, Ramsey-Musolf; MG & Horowitz

In presence of 1-loop RC's the Z-exchange amplitude is **not modified essentially** as function of Q^2 (at low Q^2); γZ -box shifts the apparent value of the weak charge.

1-loop radiative corrections to γ -exchange



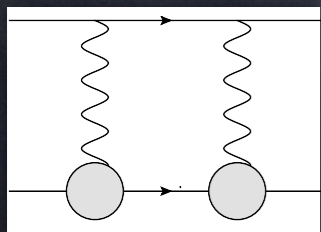
2 γ -exchange: inclusive off-shell hadronic states, arbitrary kinematics

$$T_{2\gamma} = \int \frac{d^4q}{(2\pi)^4} \frac{\ell_{\mu\nu} W^{\mu\nu}}{q^2 q'^2 [(k-q)^2 - m_e^2]} \quad W^{\mu\nu} = \int dx e^{iqx} \langle N' | T[J^\nu(x) J^\mu(0)] | N \rangle$$

Two current correlator: can't calculate from first principles in QCD

Elastic box: IR divergent, UV finite, calculable with known form factors

$$W_{el}^{\mu\nu} \sim \langle N' | J^\nu | N \rangle \langle N | J^\mu | N \rangle$$



a long history in the literature:

Mo, Tsai; Maximon, Tjon; Feshbach, McKinley; Blunden, Melnitchouk, Tjon; Kobushkin, Borysiuk; Tomalak, Vanderhaeghen; ...

Elastic box correction δ_{RC}^{el} is subtracted at the observables level

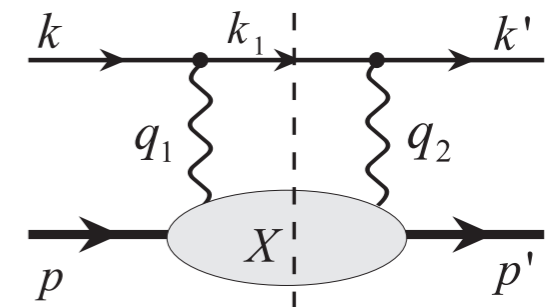
Inelastic 2γ -exchange

Cannot calculate in arbitrary kinematics!

In forward kinematics:
optical theorem + dispersion relation

$$W^{\mu\nu} \sim 2M\omega \sigma_{\gamma p}^{tot}(\omega) g^{\mu\nu} + \dots$$

$$2\text{Im}T_{2\gamma} = e^4 \int \frac{d^3\vec{k}_1}{(2\pi)^3 2E_1} \frac{\ell_{\mu\nu} \cdot \text{Im}W^{\mu\nu}}{(q_1^2 + i\epsilon)(q_2^2 + i\epsilon)}$$



Collinear log enhancement

Brown 1970; Gorchtein 2007, 2014

$$\frac{e^2}{Q^2} \left[1 - \{R_p^2, \mu_p\} Q^2 + \delta_{RC}^{elastic} + \frac{\alpha}{\pi} Q^2 C_{2\gamma}(E) \ln \frac{4E^2}{Q^2} + O(Q^2) \right],$$

Sum rule for the coeff. $C_{2\gamma}$

$$C_{2\gamma}(E) = \frac{1}{4\pi^2 \alpha} \int_{\nu_\pi}^{\infty} \frac{d\omega}{\omega} \sigma_{\gamma p}^{tot}(\omega) f(\omega, E)$$

generates a long-range potential (shorter than Coulomb);
essentially modifies the low- Q^2 asymptotics!

Numerical impact for charge radius extraction

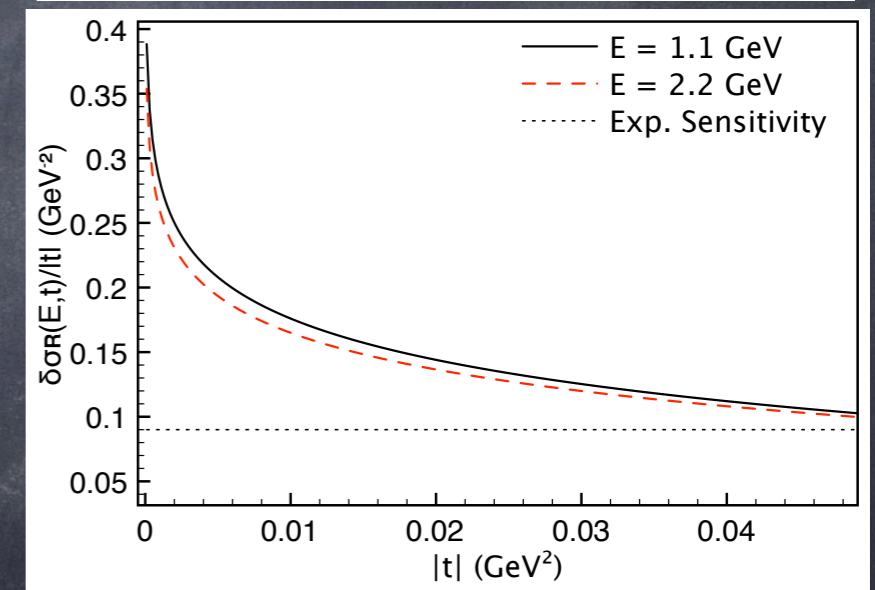
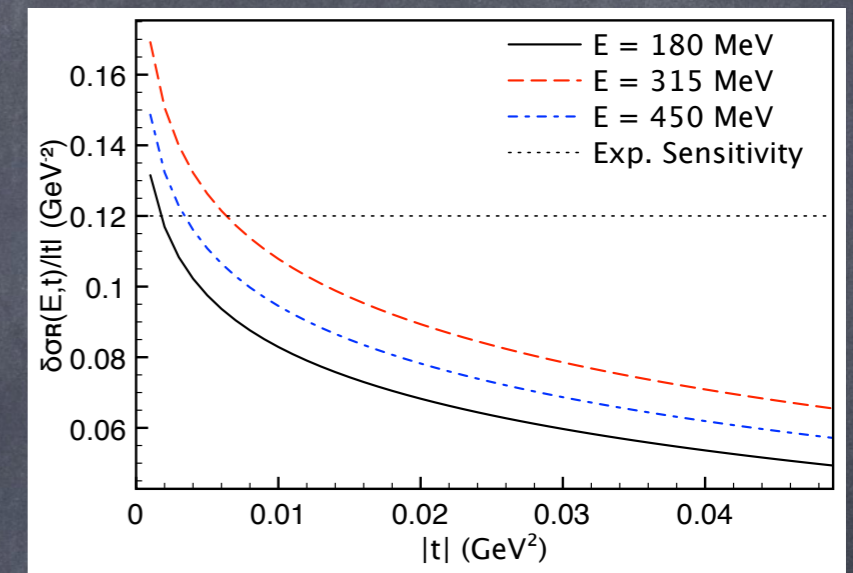
$$\sigma_R - \delta\sigma_{RC}^{el} = 1 - Q^2 R_p^2/3 + (\alpha/\pi)Q^2 C_{2\gamma}(E) \ln(4E^2/Q^2) + \dots$$

A1@MAMI: $R_p = 0.879(8)$ fm

$$\frac{\sigma_R - 1}{Q^2} = -\frac{R_p^2}{3} \left(1 \pm 2\frac{\delta R_p}{R_p}\right) = -6.61(12) \text{ GeV}^{-2}$$

PRad@JLab: higher E, lower Q^2 , R_p below 1%

$$\frac{\sigma_R - 1}{Q^2} = -\frac{R_p^2}{3} \left(1 \pm 2\frac{\delta R_p}{R_p}\right) = -6.61(9) \text{ GeV}^{-2}$$



Log Q^2 dependence affects the extraction of the radius;
But the log term is exactly calculable!

2γ-exchange correction to the weak charge

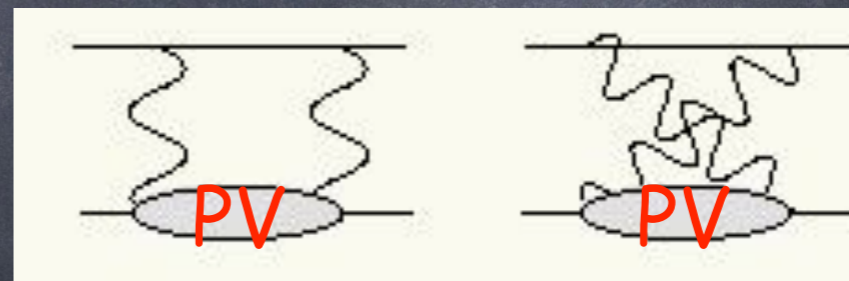
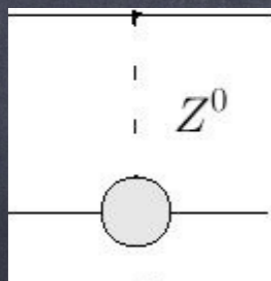
2γ-box ~ 1-3% of the charge radius; does it matter for the Q_W^p ?

$$Q_W^p \rightarrow Q_W^p + Q_W^p \frac{\alpha}{\pi} Q^2 C_{2\gamma}(E) \ln \frac{4E^2}{Q^2} \quad \text{part of the } B(Q^2) \text{ term!}$$

What if the 2γ-box contributed to the PV amplitude?

"Long-range parity-nonconserving interactions", Flambaum 1992

"PV-odd van der Waals forces", Khriplovich, Zhizhimov, 1982



$$-\frac{G_F}{2\sqrt{2}} [Q_W^p + \dots] + \frac{e^2}{Q^2} \frac{\alpha}{\pi} Q^2 C_{2\gamma}^{PV} \ln \frac{4E^2}{Q^2}$$

Dangerous for the weak charge definition!

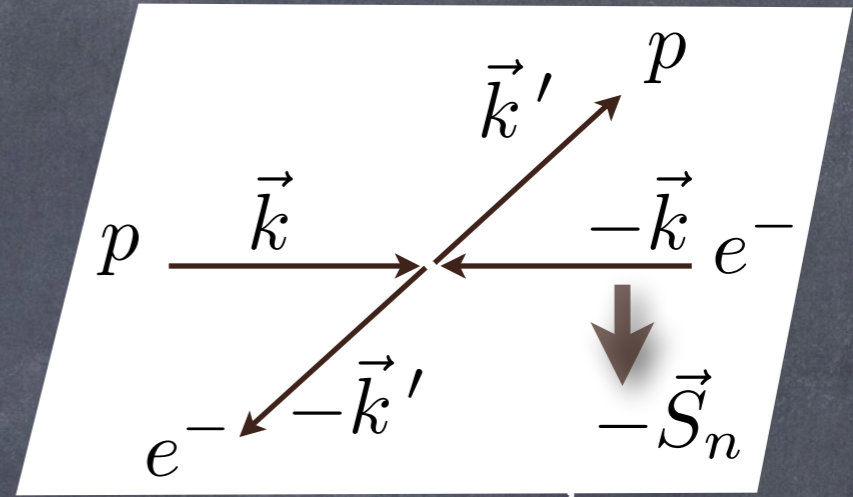
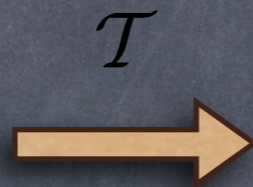
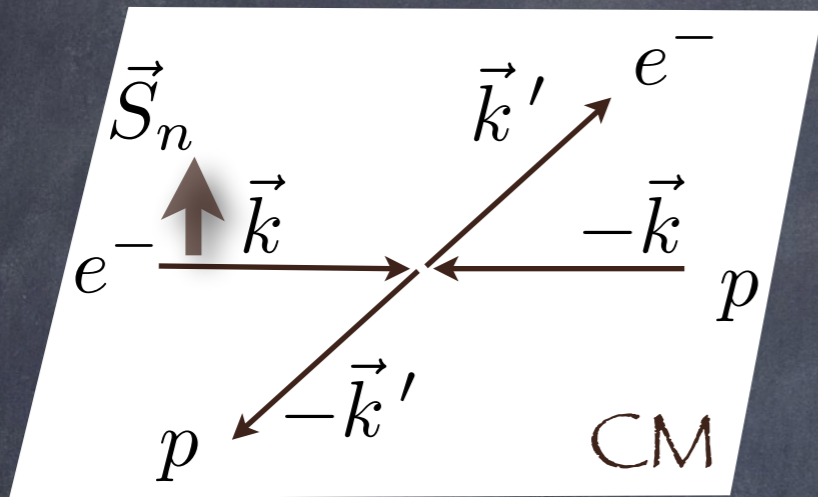
$$Q_W^p \rightarrow Q_W^p + \frac{4\sqrt{2}\alpha^2}{G_F} C_{2\gamma}^{PV}(E) \ln \frac{4E^2}{Q^2}$$

Two questions to ask:

1. are these collinear log calculations reliable?
2. is this catastrophic scenario for the weak charge realized?

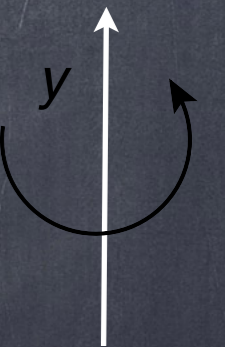
How well do we understand these collinear logarithms?

Beam normal spin asymmetry:
collinear logs are measurable and dominate

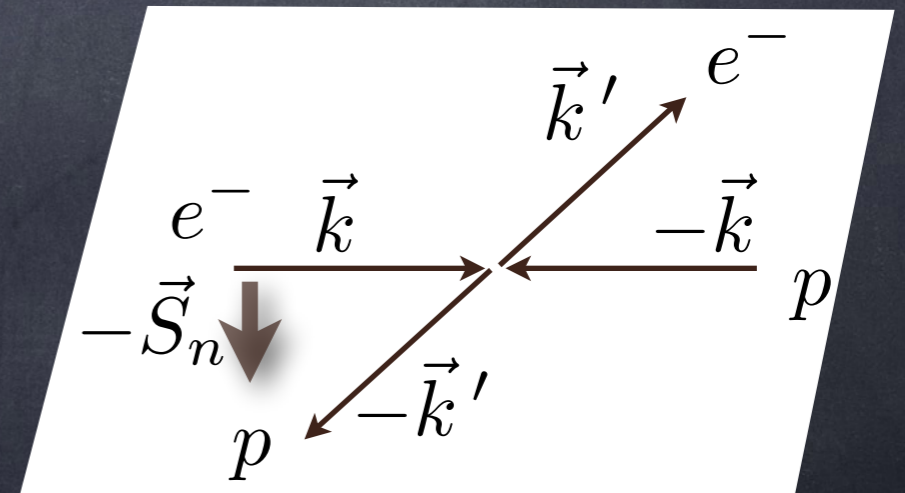


180° rotation around y-axis

$$T(S_n, \vec{k}, \vec{k}') \rightarrow \eta_1 T^*(-S_n, -\vec{k}, -\vec{k}') \rightarrow \eta_1 \eta_2 T^*(-S_n, \vec{k}, \vec{k}')$$



Mismatch between time-reversed states
is due to imaginary part of the amplitude
(in absence of CP- and CPT-violation)

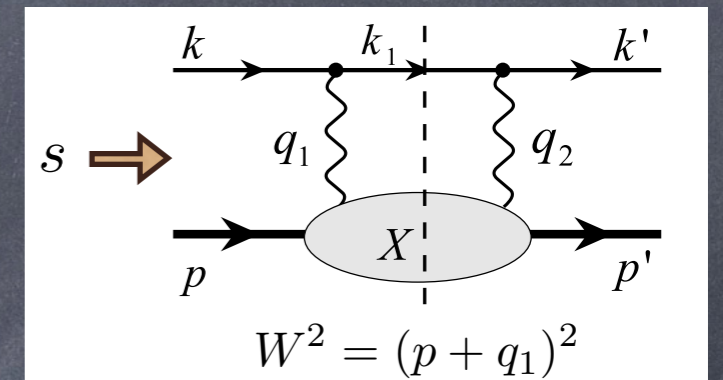


Elastic e-p scattering in presence of two-photon exchange

$$T_{ep} = T_{1\gamma} + T_{2\gamma} + \dots$$

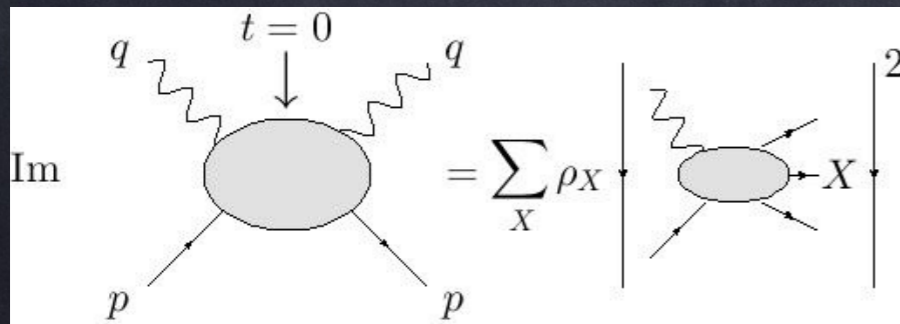
$$B_n = \frac{T_{1\gamma}^* 2\text{Im}T_{2\gamma}}{|T_{1\gamma}|^2}$$

B_n in forward kinematics



$$\text{Im}T_{2\gamma} = e^4 \int \frac{d^3\vec{k}_1}{2E_1(2\pi)^3} \frac{\bar{u}(k')\gamma_\nu(k_1 + m_e)\gamma_\mu u(k)}{Q_1^2 Q_2^2} \text{Im}W^{\mu\nu}(W^2, Q_1^2, Q_2^2, t)$$

Forward spin-independent Compton tensor – from Optical Theorem



$$W^{\mu\nu} = 2\pi \left[-g^{\mu\nu} F_1^{\gamma\gamma} + \frac{P^\mu P^\nu}{(P \cdot q_1)} F_2^{\gamma\gamma} \right]$$

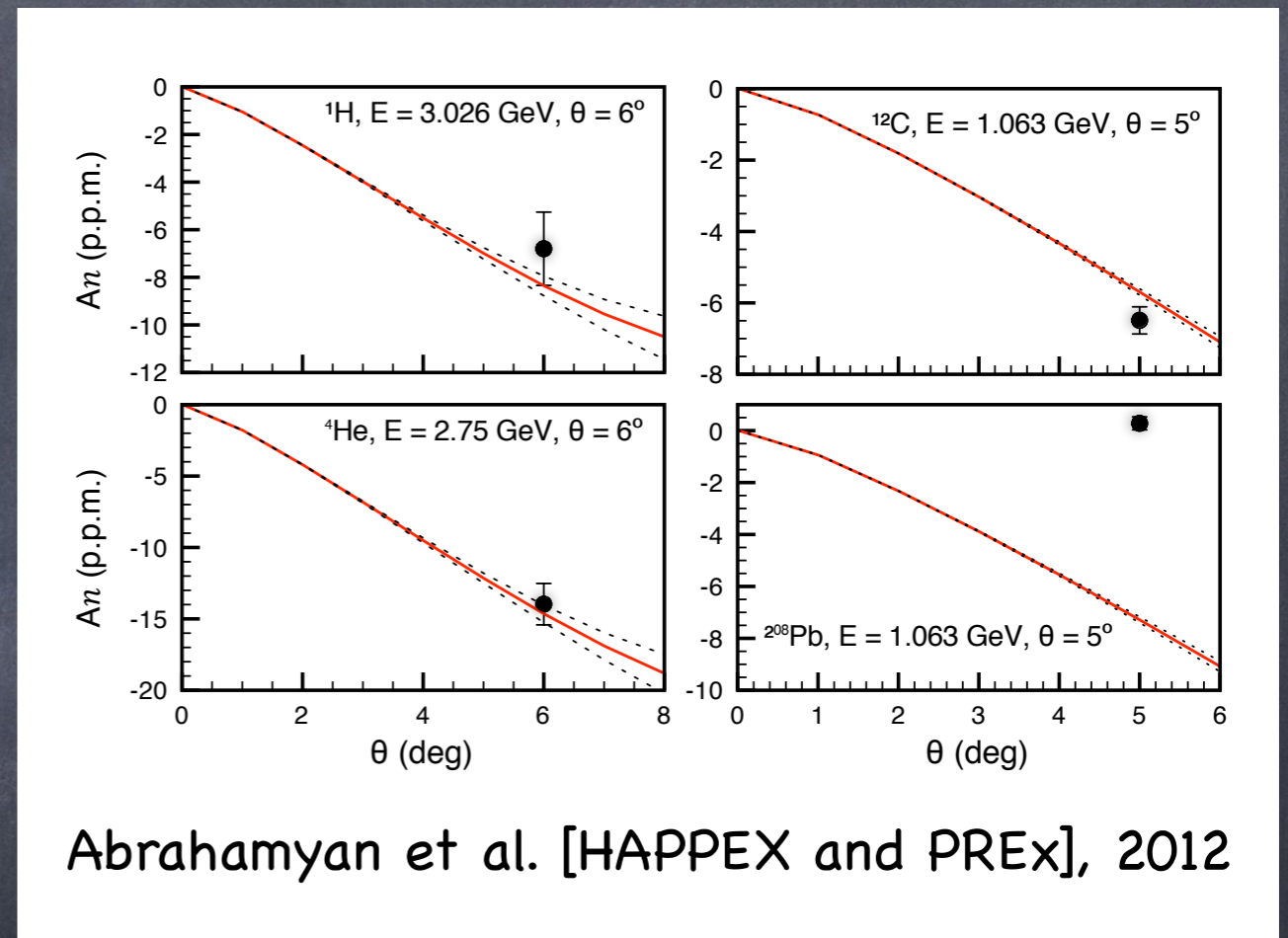
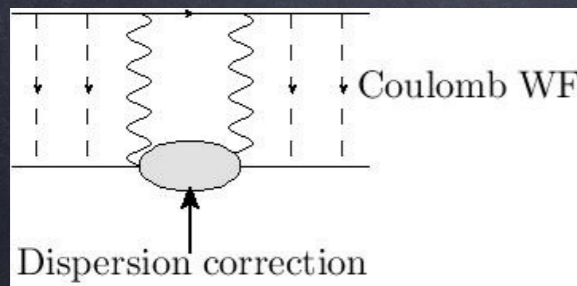
Bn features a large collinear log - $\ln(Q^2/m_e^2)$

$$B_n \approx -\frac{1}{4\pi^2} \frac{m_e \sqrt{Q^2}}{E^2} \ln\left(\frac{Q^2}{m_e^2}\right) \frac{e^{-BQ^2}}{F_C(Q^2)} \int_{\omega_\pi}^E d\omega \omega \sigma_{\gamma N}^{tot}(\omega)$$

Good quality data on selected nuclei - HAPPEX & PREx

Excellent description for light nuclei and very forward angles

Fails for lead - two photons is not enough



Work in progress with Xavi Roca Maza

Collinear logs are under control at forward angles for light nuclei

To summarize:

forward collinear logs are a well-established feature;
measured and confirmed for B_n

(where two-photon exchange dominates over h.o. effects);
modify the low- Q^2 asymptotics of observables;

Need to be assessed more accurately for PVES!

Calculate the coefficient $C_{2\gamma}^{PV}(E)$ in the forward regime

PV_{2γ} dispersive contribution to forward PVES

MG, H. Spieberger, arXiv: 1608.07484

$$\text{Im } T_{\gamma\gamma}^{PV} = e^4 \int \frac{d^4 k_1}{(2\pi)^4} 2\pi \delta(k_1^2 - m_e^2) \frac{2\pi \tilde{W}_{\gamma\gamma}^{\mu\nu} \ell_{\mu\nu}^{\gamma\gamma}}{Q^4}$$

Proton spin-independent case

$$\tilde{W}_{\gamma\gamma}^{\mu\nu} = \frac{i\varepsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(pq)} F_3^{\gamma\gamma}$$

Identify the sought for coefficient:

$$C_{2\gamma}^{PV}(E) = \frac{1}{M} \int_{E_\pi}^{\infty} \frac{d\omega}{\omega^2} F_3^{\gamma\gamma}(\omega) \left[\frac{\omega}{2E} \ln \left| \frac{E+\omega}{E-\omega} \right| + \frac{\omega^2}{4E^2} \ln \left| 1 - \frac{E^2}{\omega^2} \right| \right],$$

Does not vanish for E=0??? $C_{2\gamma}^{PV}(0) = \frac{3}{4M} \int_{\omega_\pi}^{\infty} \frac{d\omega}{\omega^2} F_3^{\gamma\gamma}(\omega)$

Compare to the PC case: $C_{2\gamma}(0) = 0$

Formal definition of the Q_W to remove $\square_{\gamma z}(E)$ - not viable??

$$- \lim_{E, Q^2 \rightarrow 0} \frac{4\pi\alpha\sqrt{2}}{G_F Q^2} A_{exp}^{PV}(E, Q^2) = Q_W^{p, 1-loop} - \frac{4\sqrt{2}\alpha^2}{G_F} C_{2\gamma}^{PV}(0) \ln \frac{4E^2}{Q^2}$$

General properties of the PV Compton amplitude

Low-energy expansion + high energy behavior \rightarrow
superconvergence relation (SCR)

Lukaszuk, arXiv: nucl-th/0207038;

Kurek, Lukaszuk, arXiv: hep-ph/0402297

$$\int_{E_\pi}^{\infty} \frac{d\omega}{\omega^2} F_3^{\gamma\gamma}(\omega) = 0$$

SM: shown to hold for $\gamma+e \rightarrow Z+e$, $\gamma+e \rightarrow W+\nu$

Altarelli, Cabibbo, Maiani 1972, ...

Check the SCR in ChPT

PV pion-nucleon coupling $\mathcal{L}_{\pi N}^{PV} = \frac{h_\pi^1}{\sqrt{2}} \bar{N} [\vec{\tau} \times \vec{\pi}]^3 N$

Donoghue, Desplanques, Holstein; Savage, Kaplan; ...

Heavy Baryon ChPT calculation of PV Compton amplitude

Cohen et al, arXiv: nucl-th/0009031

Result used by Kurek&Lukaszuk to check SCR: failed!

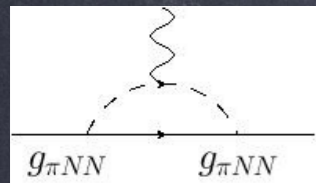
SCR important for the definition of Q_W^p - recheck!

Similar to the GDH sum rule proof to order $O(g_{\pi NN}^2)$

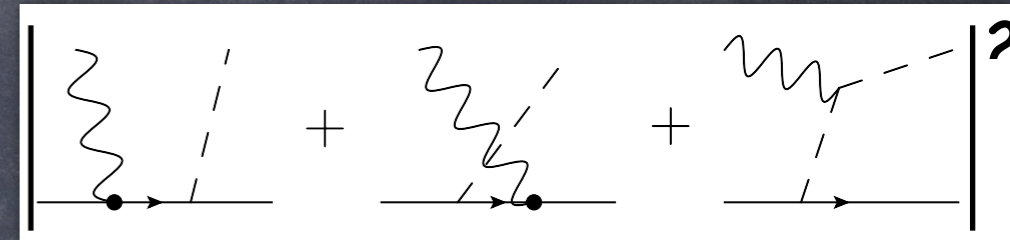
Holstein, Vanderhaeghen, Pascalutsa, 2005

Anomalous m.m. \longleftrightarrow Inelastic scattering of polarized photon on polarized proton w. helicities parallel (antiparallel)

1-loop level



$$\kappa_p^2 \sim \int_{E_\pi}^{\infty} d\omega \frac{\sigma_{\gamma p}^{3/2}(\omega) - \sigma_{\gamma p}^{1/2}(\omega)}{\omega}$$



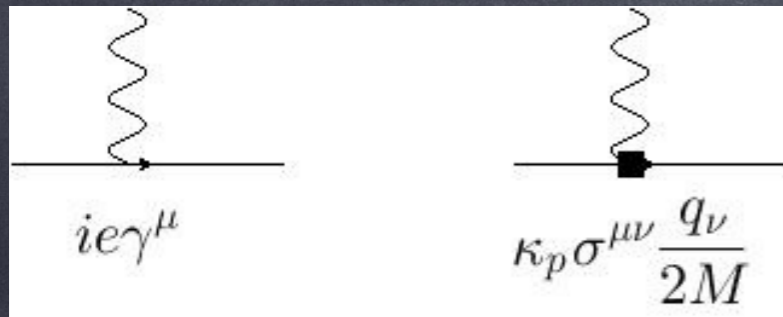
Scales as $g_{\pi NN}^2$

Scales as $g_{\pi NN}^4$

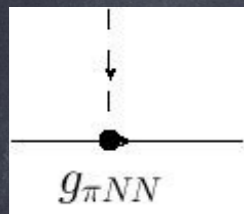
Holds in relativistic ChPT,
but not in heavy-baryon ChPT!

$$\int_{E_\pi}^{\infty} d\omega \frac{\sigma_{\gamma p}^{3/2}(\omega) - \sigma_{\gamma p}^{1/2}(\omega)}{\omega} \Bigg|_{\text{tree level}} = 0$$

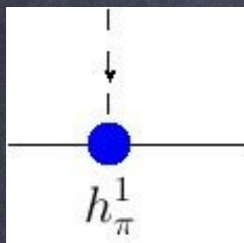
Prove SCR for PV Compton to order $O(g_{\pi NN} h_{\pi}^1)$



Electromagnetic vertex

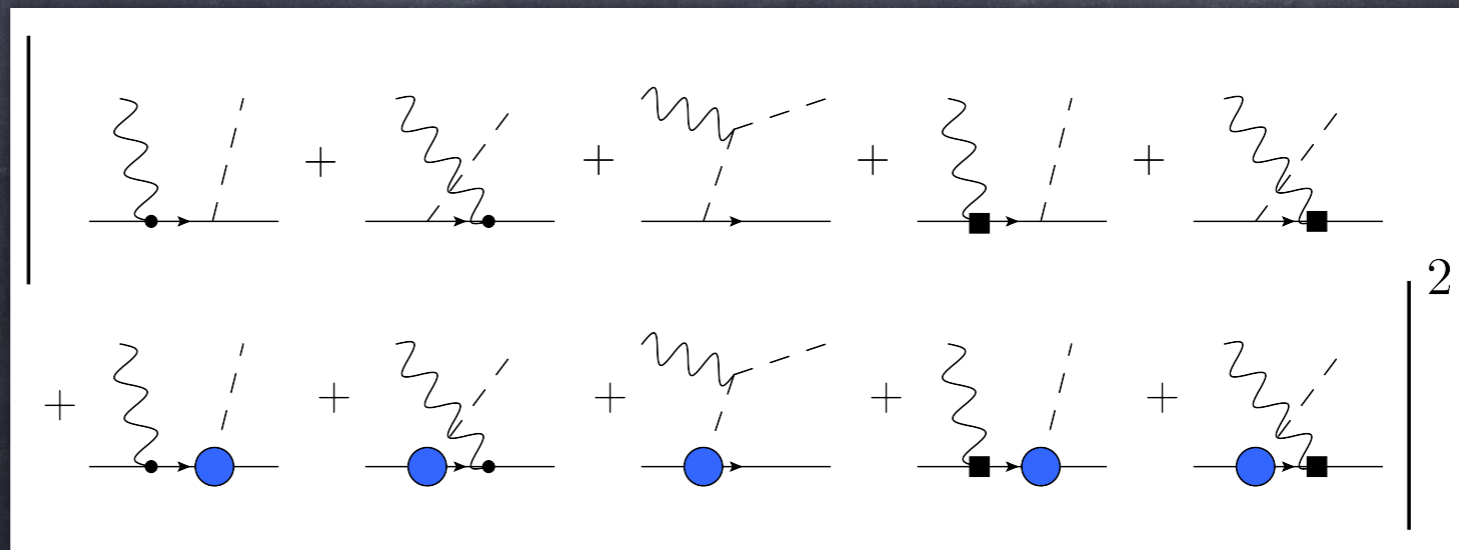


$$\mathcal{L}_{\pi N}^{PC} = \frac{g_A}{2f_\pi} \bar{N} \tau^a \not{\partial} \pi^a \gamma_5 N = -g_{\pi NN} \bar{N} \tau^a \gamma_5 N \pi^a$$



$$\mathcal{L}_{\pi N}^{PV} = \frac{h_\pi^1}{\sqrt{2}} \bar{N} [\vec{\tau} \times \vec{\pi}]^3 N = -ih_\pi^1 (\bar{n} \pi^+ p - \bar{p} \pi^- n)$$

$$F_3^{\gamma\gamma} \sim$$



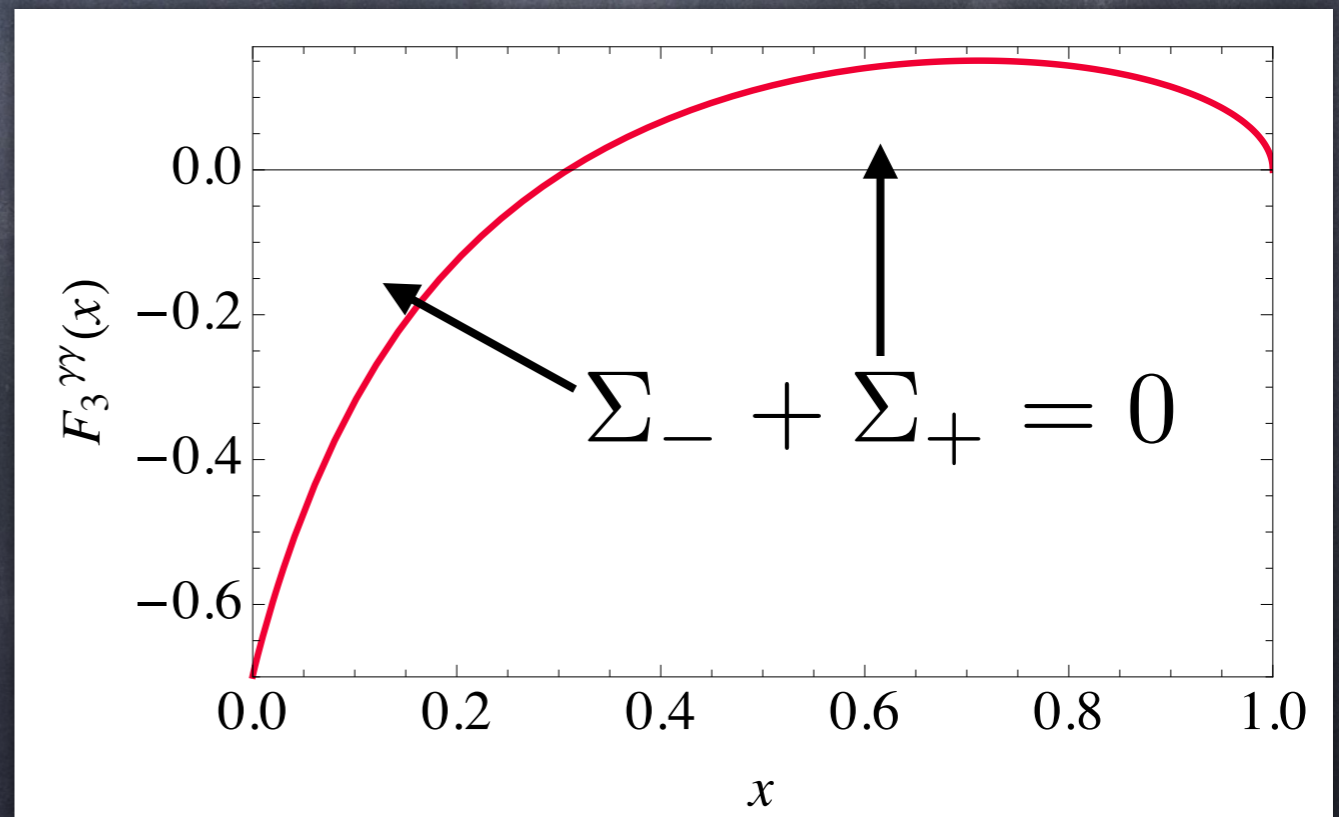
$$F_3^{\gamma\gamma}(\omega) = -\frac{g_{\pi NN} h_\pi^1 q_\pi}{2\sqrt{2}\pi^2 \sqrt{s}} \left\{ \mu^p \left[\frac{E'}{\sqrt{s}} - \frac{E_\pi}{q} + \frac{m_\pi^2}{2qq_\pi} \ln \frac{E_\pi + q_\pi}{E_\pi - q_\pi} \right] \right. \\ \left. - \mu^n \left[-\frac{E}{q} + \frac{m_\pi^2}{2qq_\pi} \ln \frac{E_\pi + q_\pi}{E_\pi - q_\pi} + \frac{M^2}{2qq_\pi} \ln \frac{E' + q_\pi}{E' - q_\pi} \right] \right. \\ \left. - \frac{qE'}{2M^2} \kappa^V \kappa^S + (\mu^n)^2 \frac{s - M^2}{4M^2} \left[-\frac{E'}{q} + \frac{M^2}{2qq_\pi} \ln \frac{E' + q_\pi}{E' - q_\pi} \right] \right\},$$

Terms $O(\kappa^2)$ - too many derivatives, SCR diverges;
not surprising: at tree level $O(g_{\pi NN}^5 h_\pi^1)$ incomplete

SCR - check numerically
up to terms linear in a.m.m.

Change variables $x = E_\pi/\omega$

$$\int_0^1 dx F_3^{\gamma\gamma}(E_\pi/x) = 0$$



Superconvergence relation for $F_3^{\gamma\gamma}$:

checked for the first time in relativistic field theory;

must be used as a basis for any reasonable estimate of the $PV_{2\gamma}$ correction to the weak charge

SCR ensures that the log term vanishes at $E=0$

$$C_{2\gamma}^{PV}(0) = \frac{3}{4M} \int_{\omega_\pi}^{\infty} \frac{d\omega}{\omega^2} F_3^{\gamma\gamma}(\omega)$$

The definition of the weak charge is still viable

$$Q_W^{p, 1\text{-loop}} = - \lim_{E, Q^2 \rightarrow 0} \frac{4\pi\alpha\sqrt{2}}{G_F Q^2} A_{exp}^{PV}(E, Q^2)$$

Numerical estimates: Input parameters

Origin: effective PV 4-quark operators



PV πNN coupling

$$h_{\pi}^1 = (1.1 \pm 1.0) 10^{-6}$$

De Vries et al, arXiv:1501.01832

$$h_{\pi}^1 = 3.8 \cdot 10^{-7}$$

DDH, 1979

PV $\gamma N \Delta$ coupling d_{Δ}

$$\mathcal{L}_{PV}^{\gamma N \Delta} = i \frac{e}{\Lambda_{\chi}} [d_{\Delta}^+ \bar{\Delta}_{\alpha}^+ \gamma_{\beta} p + d_{\Delta}^- \bar{\Delta}_{\alpha}^- \gamma_{\beta} n] F^{\alpha\beta}$$

Early claim: may be 10-100 x h_{π}^1

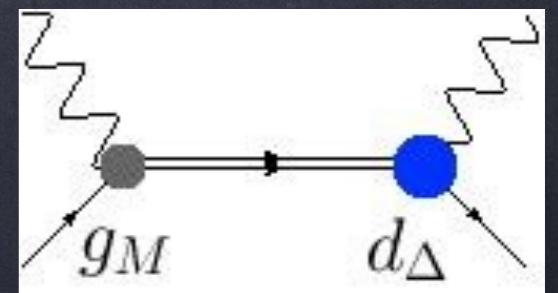
Zhu et al, arXiv:0106216

Not quite supported by exp. $|d_{\Delta}^-| = (0.31 \pm 0.91) 10^{-6}$

Androic et al [G0], arXiv:1112.1720

Q_{weak} has taken data that may further constrain d_{Δ}

$$F_{3\Delta}^{\gamma\gamma}(\omega)|_{\Gamma_{\Delta} \rightarrow 0} = \sqrt{\frac{2}{3}} \frac{4M g_M(0) d_{\Delta}^+}{\Lambda_{\chi}(M + M_{\Delta})} \omega_{\Delta}^2 \delta(\omega - \omega_{\Delta})$$



Δ contribution alone does not obey SCR

Supplement by a high energy
Regge-like background

$$F_{3\text{HE}}^{\gamma\gamma}(\omega) = C_\lambda(\Lambda) (\omega/\Lambda)^\lambda \Theta(\omega - \Lambda)$$

With $\Lambda \approx 1 \text{ GeV}$ and $\lambda < 1$ (SCR integral converges)

Fix HE contribution by imposing SCR $\int_{\omega_\pi}^{\infty} \frac{d\omega}{\omega^2} [F_{3\Delta}^{\gamma\gamma}(\omega) + F_{3\text{HE}}^{\gamma\gamma}(\omega)] = 0$

Normalization depends on λ $C_\lambda(\Lambda) = -\sqrt{\frac{2}{3}} \frac{4M g_M d_\Delta^+ \Lambda}{\Lambda_\chi (M + M_\Delta)} (1 - \lambda)$

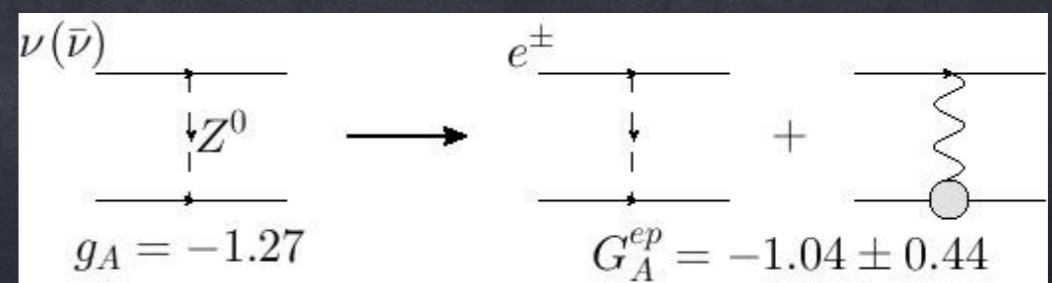
Explore $-1/2 < \lambda < 1/2$

Final ingredient (for completeness)

Anapole moment

$$\mathcal{L}_{PV} = ie a_0 \partial_\mu F^{\mu\nu} \bar{N} \gamma_\nu \gamma_5 N$$

Axial charge seen by
charged leptons is not g_A !



Results for the kinematics of relevant experiments

Object of interest $\delta Q_W^p(E, Q^2) = -\frac{4\sqrt{2}\alpha^2}{G_F} C_{2\gamma}^{PV}(E) \ln \frac{4E^2}{Q^2}$

The SM expectation: $Q_W^p = 0.0713(8)$

	Contribution	P2@MESA	Qweak	MOLLER
δQ_W^p	Elastic	$-(1.0 \pm 2.0) \cdot 10^{-4}$	$-(1.2 \pm 2.2) \cdot 10^{-5}$	$-(3 \pm 5) \cdot 10^{-7}$
	π	$-(2.0 \pm 2.0) \cdot 10^{-5}$	$-(5.5 \pm 5.5) \cdot 10^{-5}$	$-(2.8 \pm 2.8) \cdot 10^{-5}$
	$\Delta + \text{HE} (\lambda = 0.5)$	$-(0.67 \pm 2.0) \cdot 10^{-4}$	$-(1.3 \pm 3.8) \cdot 10^{-4}$	$-(1.1 \pm 3.3) \cdot 10^{-4}$
	$\Delta + \text{HE} (\lambda = 0)$	$-(0.4 \pm 1.2) \cdot 10^{-4}$	$-(1.1 \pm 3.3) \cdot 10^{-4}$	$-(0.5 \pm 1.4) \cdot 10^{-4}$
	$\Delta + \text{HE} (\lambda = -0.5)$	$-(0.32 \pm 0.93) \cdot 10^{-4}$	$-(1.1 \pm 3.3) \cdot 10^{-4}$	$-(0.2 \pm 0.6) \cdot 10^{-4}$
	Total	$-(1.7 \pm 0.3 \pm 2.5) \cdot 10^{-4}$	$-(1.9 \pm 0.1 \pm 3.6) \cdot 10^{-4}$	$-(0.9 \pm 0.5 \pm 1.8) \cdot 10^{-4}$

$$\delta Q_W^p \leq 0.3\%$$

$$\delta Q_W^p \leq 0.53\%$$

Cs-133 weak charge:

$$Q_W(^{113}\text{Cs}) = -72.58(29)_{exp}(32)_{th}$$

$$\delta Q_W(^{113}\text{Cs}) \sim 113\delta Q_W^p(0) = -(2.0 \pm 3.9) \cdot 10^{-2}$$

Summary

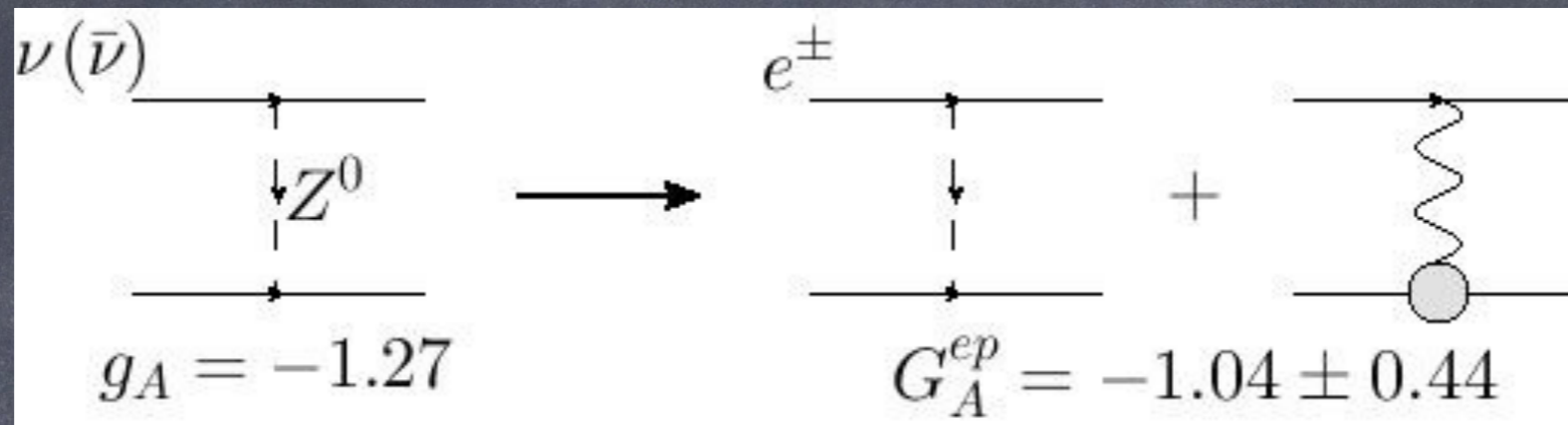
- 2γ -exchange induces a long-range interaction that modifies the extraction of charge radius and weak charge from electron scattering
- Formal definition of $Q_W(p)$ protected by a superconvergence relation;
- The superconvergence relation proved in relativistic ChPT;
- 0.5% uncertainties due to d_Δ - Q-Weak data may further reduce it!
- High energy part needed to obey SCR - unknown; Very mild sensitivity for Q-Weak, may matter for MOLLER e-p if $\lambda > 1/2$
- Sensitivity to anapole moment: non-negligible for MESA, but the uncertainty of G_A will be reduced w. MESA by a factor of 4
- Further hadronic PV couplings may be also included
- Atomic PV: hadronic 2γ -box purely short-range, small; nuclear resonances may change this behavior - more work needed

Backup slides

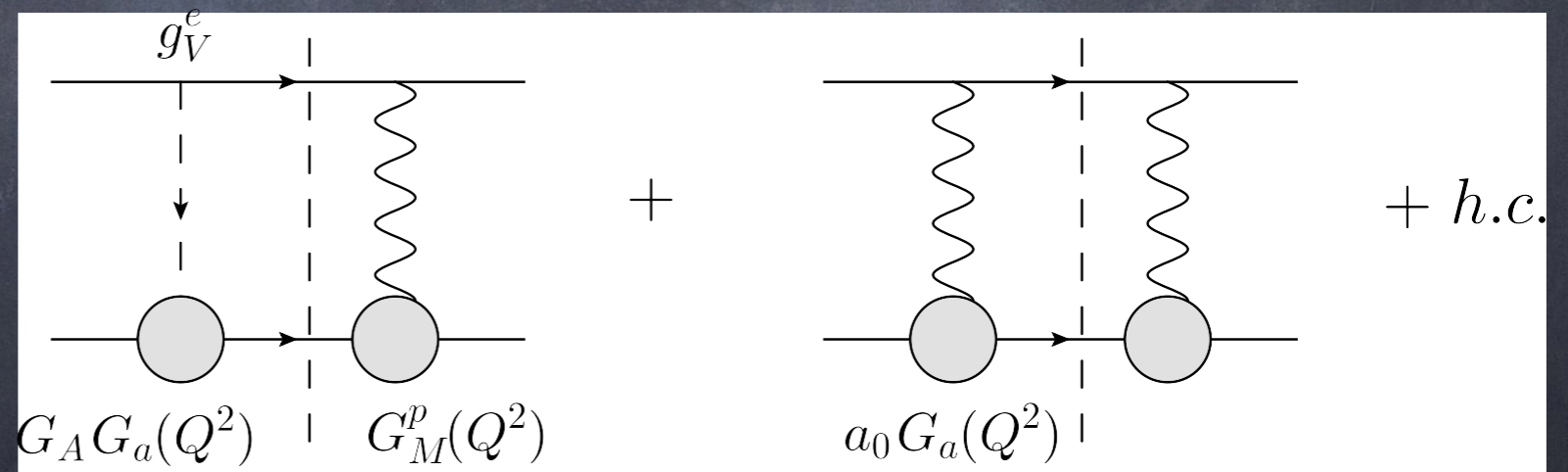
Anapole moment

$$\mathcal{L}_{PV} = ie a_0 \partial_\mu F^{\mu\nu} \bar{N} \gamma_\nu \gamma_5 N$$

Axial charge seen by charge leptons is not g_A !



Update the axial box: include uncertainty due to anapole



Attn: elastic contribution not enhanced by collinear log:
no anapole moment for real photons

Update the axial box: simply use G_A^{ep} instead of g_A

$$\delta(Q_W^p)^{el.} = \frac{\alpha g_V^e G_A^{ep}(0)}{ME} \int_0^\infty dQ^2 G_M(Q^2) G_a(Q^2) \left(\ln \left| \frac{E + E_Q}{E - E_Q} \right| + \frac{Q^2}{2ME} \ln \left| 1 - \frac{E^2}{E_Q^2} \right| \right)$$

Some caveats here!

Blunden et al. included running of

$\sin^2\theta_W \rightarrow g_V^e = 0.045$;

they used $g_A = -1.27$;

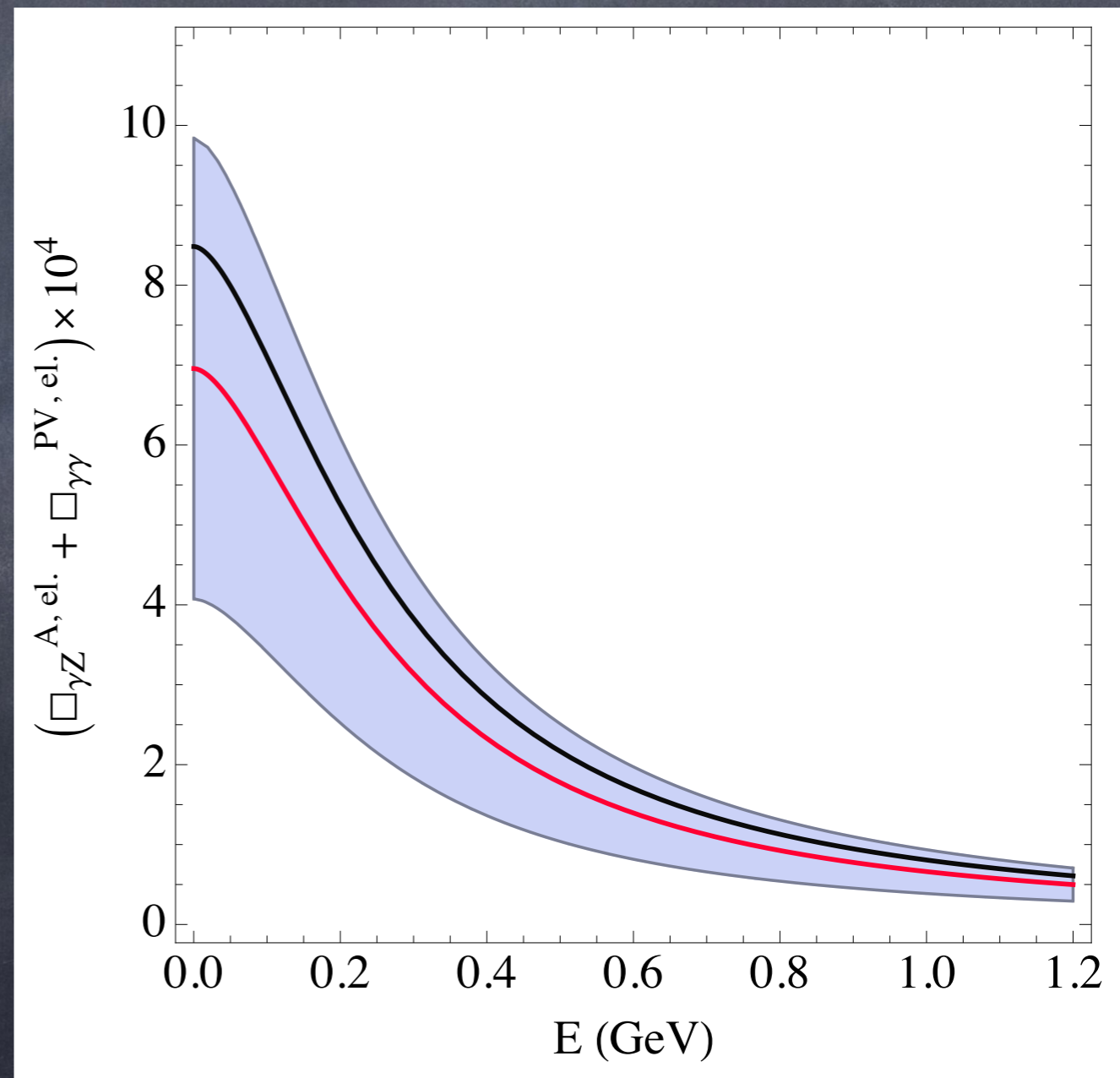
We use:

full one loop result $\rightarrow g_V^e = 0.07$,
and include RC in $G_A^{ep} = -1.04(43)$

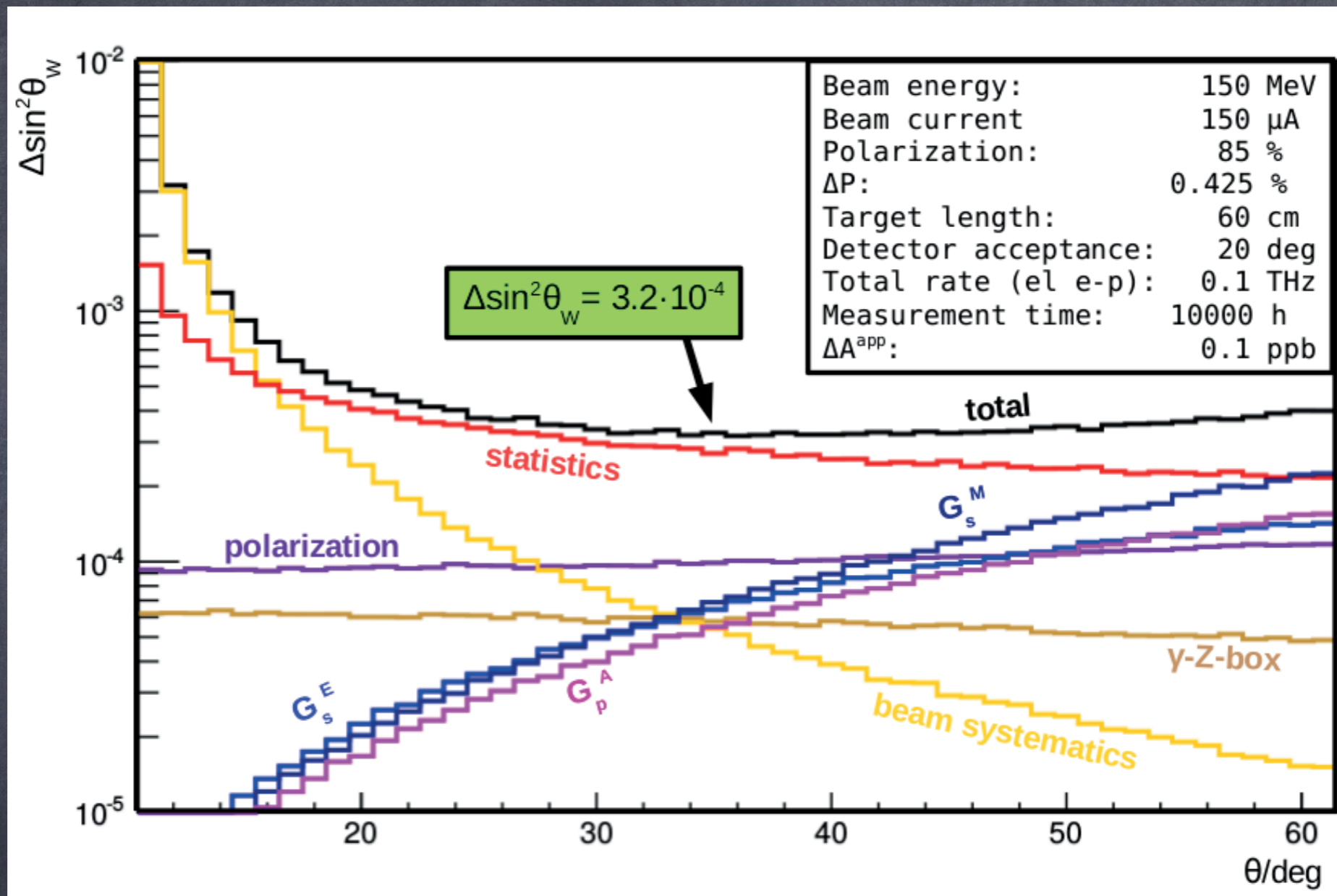
More natural from DR side

Central value almost identical;

Now can estimate an uncertainty!



WMA determination with MESA/P2

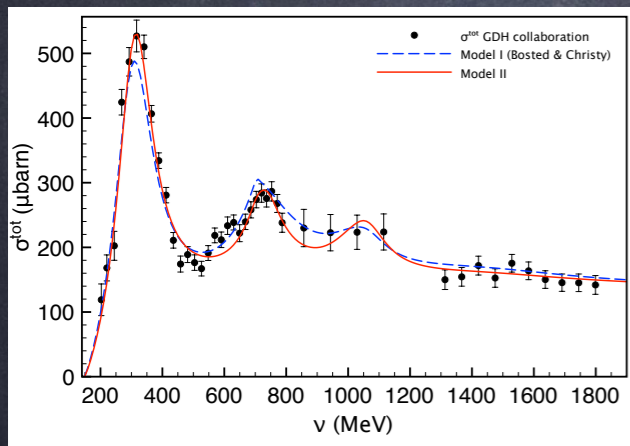


- Strange nucleon FFs: from the lattice
- Axial FF: from an auxiliary backward measurement
(will reduce the uncertainty on G_A by factor ~ 4)

Model dependence of the γZ box



$$\sigma \gamma^* p \rightarrow X$$



Model-dependent

Definite
spin, flavor
states

$$M_{\gamma^*} p \rightarrow H_{S,I}$$

Model-dependent

Weak Isospin
(flavor) rotation

$$M_{Z^*} p \rightarrow H_{S,I}$$