Long and short distances in non-leptonic K decays: ϵ'/ϵ

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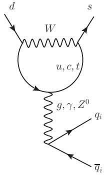
Outline

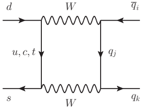
V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, J. Portolés, Rev. Mod. Phys. 84 (2012) 399

1. **Theoretical Framework**
2. **Octet Enhancement**
3. **CP Violation: ε'/ε**

Theoretical Framework

- **Sensitivity to Short-Distance Scales:**





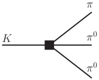
Charm mass prediction

Top quark

GIM cancellation

New Physics ?

- **Long-Distance Physics:**



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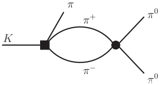


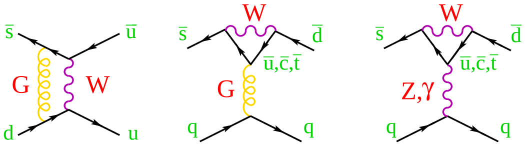
Chiral Dynamics

- **Multi-Scale Problem:**

$\log(M/\mu)$ **(OPE)** , $\log(\mu/m_\pi)$ **(χPT)**

Energy Scale	Fields	Effective Theory
M_W	W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model
	⬇ OPE	
$\lesssim m_c$	$\gamma, g\ ;\ \mu, e, \nu_i$ s, d, u	$\mathcal{L}_{\mathrm{QCD}}^{(n_f=3)}$, $\mathcal{L}_{\mathrm{eff}}^{\Delta S=1,2}$
	⬇ $N_C \to \infty$	
M_K	$\gamma\ ;\ \mu, e, \nu_i$ π, K, η	χPT

ΔS = 1 TRANSITIONS



$$\mathcal{L}_{\mathrm{eff}}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} \; V_{ud}\, V_{us}^* \; \sum_i C_i(\mu) \;\; Q_i(\mu)$$

$$Q_1 = (\bar{s}_\alpha u_\beta)_{\mathrm{V-A}} \, (\bar{u}_\beta d_\alpha)_{\mathrm{V-A}} \qquad Q_2 = (\bar{s}u)_{\mathrm{V-A}} (\bar{u}d)_{\mathrm{V-A}}$$

$$Q_{3,5} = (\bar{s}d)_{\mathrm{V-A}} \sum_q (\bar{q}q)_{\mathrm{V\mp A}} \qquad Q_4 = (\bar{s}_\alpha d_\beta)_{\mathrm{V-A}} \sum_q (\bar{q}_\beta q_\alpha)_{\mathrm{V-A}}$$

$$Q_{7,9} = \tfrac{3}{2}(\bar{s}d)_{\mathrm{V-A}} \sum_q e_q (\bar{q}q)_{\mathrm{V\pm A}} \qquad Q_{10} = \tfrac{3}{2}(\bar{s}_\alpha d_\beta)_{\mathrm{V-A}} \sum_q e_q \, (\bar{q}_\beta q_\alpha)_{\mathrm{V-A}}$$

$$Q_6 = -8 \sum_q (\bar{s}_L q_R)(\bar{q}_R d_L) \qquad Q_8 = -12 \sum_q e_q \, (\bar{s}_L q_R)(\bar{q}_R d_L)$$

$$Q_{11,12} = (\bar{s}d)_{\mathrm{V-A}} \sum_\ell (\bar{\ell}\ell)_{\mathrm{V,A}} \qquad Q_{13} = (\bar{s}d)_{\mathrm{V-A}} \sum_\nu (\bar{\nu}\nu)_{\mathrm{V-A}}$$

- $\boldsymbol{q > \mu}$ **:** $\qquad C_i(\mu) = z_i(\mu) - y_i(\mu) \, (V_{td} \, V_{ts}^* / V_{ud} \, V_{us}^*)$

  $O(\alpha_s^n t^n)$, $O(\alpha_s^{n+1} t^n)$ $\qquad [t \equiv \log (M/m)]$ $\qquad$ Munich / Rome

- $\boldsymbol{q < \mu}$ **:** $\quad \langle \pi\pi | Q_i(\mu) | K \rangle$? $\qquad$ **Physics does not depend on μ**

CHIRAL PERTURBATION THEORY (χPT)

- **Expansion in powers of p^2/Λ_χ^2 :** $\mathcal{A} = \sum_n \mathcal{A}^{(n)}$ $(\Lambda_\chi \sim 4\pi F_\pi \sim 1.2 \text{ GeV})$
- **Amplitude structure fixed by chiral symmetry**

$$\mathrm{SU(3)_L} \otimes \mathrm{SU(3)_R} \to \mathrm{SU(3)_V}$$

- **Short-distance dynamics encoded in Low-Energy Couplings**
- **$O(p^2)$ χPT: Goldstone interactions (π, K, η)** $\Phi \equiv \frac{1}{\sqrt{2}} \vec{\lambda}\,\vec{\varphi}$

$$\mathcal{L}_2^{\Delta S=1} = G_8\, F^4\, \mathrm{Tr}(\lambda L_\mu L^\mu) + G_{27}\, F^4 \left(L_{\mu 23} L^\mu_{11} + \frac{2}{3}\, L_{\mu 21} L^\mu_{13} \right)$$

$G_R \equiv -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^*\, g_R$; $L_\mu = -iU^\dagger D_\mu U$; $\lambda \equiv \frac{1}{2}\lambda_{6-i7}$; $U \equiv \exp\left\{ i\sqrt{2}\,\Phi/F \right\}$

- **Loop corrections (χPT logarithms) unambiguously predicted**
- **LECs can be determined at $N_C \to \infty$ (matching)**
- **$O(p^2)$ LECs (G_8, G_{27}) can be phenomenologically determined**

Nonleptonic Decays

- **Octet Enhancement:** $$\frac{A(K\to\pi\pi)_{I=0}}{A(K\to\pi\pi)_{I=2}} \approx 22$$
  - **Short-distance:** **gluonic corrections, penguins**
  - **Long-distance:** **large χPT corrections (FSI)**
  - **Ongoing Lattice efforts**

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- **Direct CP Violation:** $\eta_{ij} \equiv \frac{A(K_L \to \pi^i \pi^j)}{A(K_S \to \pi^i \pi^j)}$

$$\mathrm{Re}\,(\epsilon'/\epsilon) = \frac{1}{3}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|\right) = (16.8 \pm 1.4) \cdot 10^{-4}$$

NA31, E731, NA48, KTeV

$$\mathrm{Re}\,(\epsilon'/\epsilon)_{\mathrm{SM}} = \left(19 \pm 2\,^{+9}_{-6} \pm 6\right) \cdot 10^{-4}$$

Pallante-Pich-Scimemi

$K \to 2\pi$ Isospin Amplitudes

$$A[K^0 \to \pi^+\pi^-] \equiv A_0\, e^{i\chi_0} + \frac{1}{\sqrt{2}} A_2\, e^{i\chi_2}$$

$$A[K^0 \to \pi^0\pi^0] \equiv A_0\, e^{i\chi_0} - \sqrt{2}\, A_2\, e^{i\chi_2}$$

$$A[K^+ \to \pi^+\pi^0] \equiv \frac{3}{2} A_2^+\, e^{i\chi_2^+}$$

1) $\Delta I = 1/2$ Rule: $\qquad \omega \equiv \frac{\mathrm{Re}(A_2)}{\mathrm{Re}(A_0)} \approx \frac{1}{22}$

2) Strong Final State Interactions: $\qquad \chi_0 - \chi_2 \approx \delta_0 - \delta_2 \approx 45^\circ$

$$\varepsilon'_K = \frac{-i}{\sqrt{2}}\, e^{i(\chi_2 - \chi_0)}\, \omega \left\{ \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)} - \frac{\mathrm{Im}(A_2)}{\mathrm{Re}(A_2)} \right\}$$

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$$A[K^+ \to \pi^+\pi^0] \equiv \frac{3}{2} A_2^+\, e^{i\chi_2^+}$$

$$A_0\, e^{i\chi_0} = \mathcal{A}_{1/2}$$

$$A_2\, e^{i\chi_2} = \mathcal{A}_{3/2} + \mathcal{A}_{5/2}$$

$$A_2^+\, e^{i\chi_2^+} = \mathcal{A}_{3/2} - \frac{2}{3}\mathcal{A}_{5/2}$$

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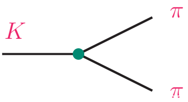
$\mathbf{O(p^2)} \quad \chi\mathbf{PT}$

$$\mathcal{L}_2^{\Delta S=1} = G_8\, F^4\, \langle \lambda L_\mu L^\mu \rangle \;+\; G_{27}\, F^4 \left(L_{\mu 23}\, L^\mu_{11} + \frac{2}{3}\, L_{\mu 21}\, L^\mu_{13} \right)$$

$$G_R \equiv -\frac{G_F}{\sqrt{2}}\, V_{ud} V^*_{us}\, g_R \quad ; \quad L_\mu = -i U^\dagger D_\mu U \quad ; \quad \lambda \equiv \tfrac{1}{2}\, \lambda_{6-i7} \quad ; \quad U \equiv \exp\left\{ i\sqrt{2}\,\Phi/F \right\}$$



$$\mathcal{A}_{1/2} = \sqrt{2}\, F_\pi \left(G_8 + \frac{1}{9}\, G_{27} \right) (M_K^2 - M_\pi^2)$$

$$\mathcal{A}_{3/2} = \frac{10}{9}\, F_\pi\, G_{27}\, (M_K^2 - M_\pi^2)$$

$$\mathcal{A}_{5/2} = 0 \qquad ; \qquad \delta_0 = \delta_2 = 0$$

$$[\Gamma(K \to 2\pi) + \delta_I]_{\mathrm{Exp}} \;\Longrightarrow\; |g_8| \approx 5.1 \quad ; \quad |g_{27}| \approx 0.29$$

$\mathbf{O(p^2, e^2p^0)}$ χPT

$\mathcal{Q} = \mathrm{diag}\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right)$

$$\mathcal{L}_2^{\Delta S=1} = G_8\, F^4\, \langle \lambda L_\mu L^\mu \rangle + G_{27}\, F^4 \left(L_{\mu 23}\, L^\mu_{11} + \frac{2}{3}\, L_{\mu 21}\, L^\mu_{13} \right) + e^2\, F^6\, G_8\, g_{ew}\, \langle \lambda U^\dagger \mathcal{Q} U \rangle$$

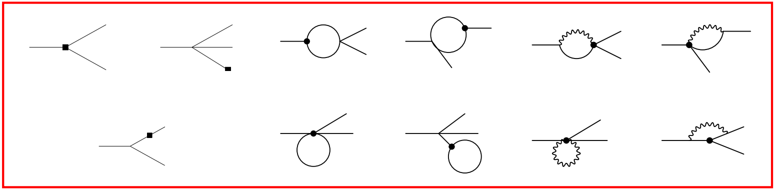
$$\mathcal{A}_{1/2} = \sqrt{2}\, F_\pi \left\{ G_8 \left[(M_K^2 - M_\pi^2) \left(1 - \frac{2}{3\sqrt{3}}\, \varepsilon^{(2)} \right) - \frac{2}{3}\, F_\pi^2\, e^2\, (g_{ew} + 2\, Z) \right] + \frac{1}{9}\, G_{27}\, (M_K^2 - M_\pi^2) \right\}$$

$$\mathcal{A}_{3/2} = \frac{2}{3}\, F_\pi \left\{ \left(\frac{5}{3}\, G_{27} + \frac{2}{\sqrt{3}}\, \varepsilon^{(2)}\, G_8 \right) (M_K^2 - M_\pi^2) - F_\pi^2\, e^2\, G_8\, (g_{ew} + 2\, Z) \right\}$$

$$\mathcal{A}_{5/2} = 0 \qquad ; \qquad \delta_0 = \delta_2 = 0$$

$\varepsilon^{(2)} = (\sqrt{3}/4)\, (m_d - m_u)/(m_s - \hat{m}) \approx 0.011$; $Z \approx (M^2_{\pi^\pm} - M^2_{\pi^0})/(2\, e^2 F_\pi^2) \approx 0.8$

$\mathbf{O}\left[\mathbf{p^4}, (\mathbf{m_u} - \mathbf{m_d})\,\mathbf{p^2}, \mathbf{e^2 p^0}, \mathbf{e^2 p^2}\right] \quad \chi\mathbf{PT}$



- **Nonleptonic weak Lagrangian:** $\mathcal{O}(G_F\, p^4)$

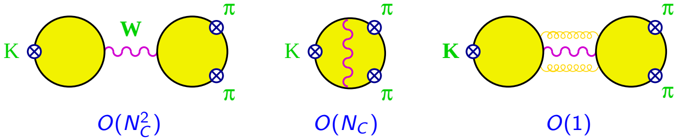
$$\mathcal{L}_{\rm weak}^{(4)} = \sum_i\; G_8\, N_i\, F^2\, O_i^8 \;+\; \sum_i\; G_{27}\, D_i\, F^2\, O_i^{27} \;+\; \mathrm{h.c.}$$

- **Electroweak Lagrangian:** $\mathcal{O}(G_F\, e^2 p^{0,2})$

$$\mathcal{L}_{\rm EW} = e^2 F^6 G_8\, g_{ew}\, \mathrm{Tr}(\lambda U^\dagger \mathcal{Q} U) + e^2 \sum_i\; G_8\, Z_i\, F^4\, O_i^{EW} \;+\; \mathrm{h.c.}$$

- $\mathcal{O}(e^2 p^{0,2})$ **Electromagnetic** $+$ $\mathcal{O}(p^4)$ **Strong:** $Z\,,\; K_i\,,\; L_i$

Weak Currents Factorize at Large N_C



$$A[K^0 \to \pi^0\pi^0] = 0 \quad \Longrightarrow \quad A_0 = \sqrt{2}\, A_2$$

No $\Delta I = \frac{1}{2}$ enhancement at leading order in $1/N_C$

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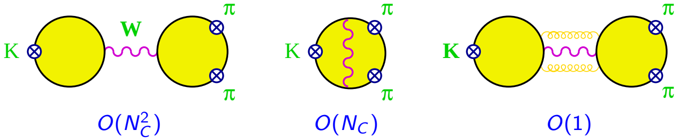

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- **Multiscale problem:** **OPE** $\quad \frac{1}{N_C} \log\left(\frac{M_W}{\mu}\right) \sim \frac{1}{3} \times 4$

Short-distance logarithms must be summed

- **Large χPT logarithms:** **FSI** $\quad \frac{1}{N_C} \log\left(\frac{\mu}{M_\pi}\right) \sim \frac{1}{3} \times 2$

Infrared logarithms must also be included $[\delta_I \sim O(1/N_C)\ ,\ \delta_0 - \delta_2 \approx 45^\circ]$

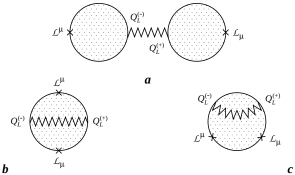
Dynamical understanding of the $\Delta I = 1/2$ rule

AP – E. de Rafael, PL B374 (1996) 186

$$\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} F^4 \left[a \; \mathrm{Tr}(Q_L^{(-)} L_\mu)\,\mathrm{Tr}(Q_L^{(+)} L^\mu) + b \; \mathrm{Tr}(Q_L^{(-)} L_\mu Q_L^{(+)} L^\mu) + c \; \mathrm{Tr}(Q_L^{(-)} Q_L^{(+)} L_\mu L^\mu) \right]$$

$\mathcal{O}(N_C^2)$

$\mathcal{O}(N_C)$



$$Q_L^{(+)} = \begin{pmatrix} 0 & V_{\rm ud} & V_{\rm us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad Q_L^{(-)} = Q_L^{(+)\dagger}$$

$$g_8 = \frac{3}{5}(a+b) - b + c$$

$$g_{27} = \frac{3}{5}(a+b)$$

$$a = 1 + \mathcal{O}\left(\frac{1}{N_c^2}\right) \quad ; \quad c = \mathrm{Re}C_4 - 16\, L_5\, \mathrm{Re}C_6(\mu^2) \left[\frac{<\bar\psi\psi>}{f_\pi^3}\right]^2 + \mathcal{O}\left(\frac{1}{N_c^2}\right) \simeq 0.3 \pm 0.2$$

$$|g_{27}| \simeq 0.29 \;\Longrightarrow\; b \simeq -0.52 + \mathcal{O}\left(\frac{1}{N_c^2}\right) \;\Longrightarrow\; g_8 \simeq 1.1 + \mathcal{O}\left(\frac{1}{N_c^2}\right)$$

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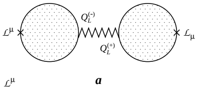

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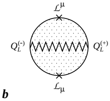

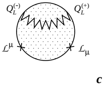

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$\mathbf{b < 0}$ predicted through explicit calculations

AP–E. de Rafael, NP B358 (1991) 311
Bardeen-Buras-Gerard, Bijnens-Prades, Bertolini et al

Confirmed through inclusive QCD analysis

M. Jamin–AP, NP B425 (1994) 15

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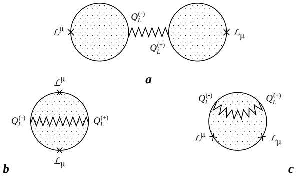

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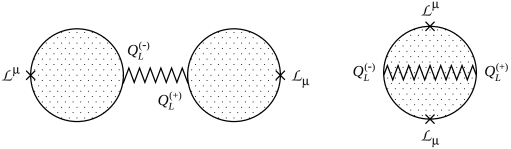
$\mathbf{b < 0}$ predicted through explicit calculations — AP–E. de Rafael, NP B358 (1991) 311; Bardeen-Buras-Gerard, Bijnens-Prades, Bertolini et al

Confirmed through inclusive QCD analysis — M. Jamin–AP, NP B425 (1994) 15

Confirmed recently by lattice calculations — RBC-UKQCD, PRL 110 (2013) 15, 152001; PRD 91 (2015) 7, 074502

"A qualitative picture towards the understanding of the underlying physics begins to emerge"

AP – E. de Rafael, PL B374 (1996) 186



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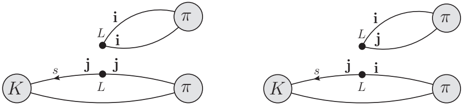
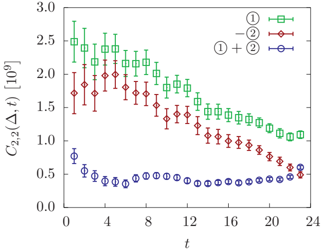
"Emerging understanding of the $\Delta I = 1/2$ Rule from Lattice QCD"

RBC-UKQCD, PRL 110 (2013) 15, 152001

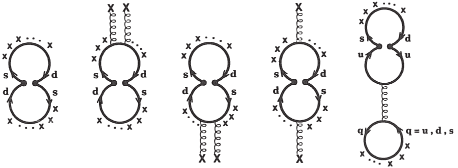


$$b \approx -0.7\, a$$



Effective Action Model: Bosonization in Gluonic Background

AP–E. de Rafael, NP B358 (1991) 311



$$\Delta \; = \; \frac{1}{N_C}\left[1 - \frac{N_C}{2}\,\frac{\langle\frac{\alpha_s}{\pi}\,G^2\rangle}{16\pi^2 f_\pi^4} + \mathcal{O}(\alpha_s^2 N_C^2)\right] \; < \; 0$$

$$g_{27} \;\approx\; \frac{3}{5}\, C_+(\mu^2)\;\left\{1 + \Delta + \mathcal{O}(1/N_C^2)\right\}$$

$$g_8 \;\approx\; \frac{1}{2}\, C_-(\mu^2)\;\left\{1 - \Delta + \mathcal{O}(1/N_C^2)\right\} + \frac{1}{10}\, C_+(\mu^2)\;\left\{1 + \Delta + \mathcal{O}(1/N_C^2)\right\} + c$$

$$c \;=\; C_4(\mu^2) - 16\, C_6(\mu^2)\, L_5\,\left[\frac{\langle\bar\psi\psi\rangle}{f_\pi^3}\right]^2 + \mathcal{O}(1/N_C^2)$$

$$\boxed{b \;=\; \frac{1}{2}\, C_+(\mu^2)\;\left\{1 + \Delta + \mathcal{O}(1/N_C^2)\right\} - \frac{1}{2}\, C_-(\mu^2)\;\left\{1 - \Delta + \mathcal{O}(1/N_C^2)\right\} \;<\; 0}$$

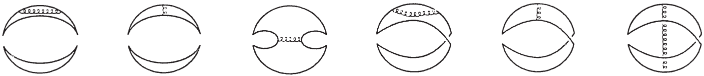
$\mu \sim m_c\,, \quad \langle\frac{\alpha_s}{\pi}\,G^2\rangle \sim 330\ \mathrm{MeV}^4$ $\quad\Longrightarrow\quad$ $b \sim -0.6 + \mathcal{O}(1/N_C^2)$

Two-point Functions

AP–E. de Rafael, NP B358 (1991) 311, PL B374 (1996) 186

M. Jamin–AP, NP B425 (1994) 15

$$\Psi^{\Delta S=1,2}(q^2) \equiv i\int d^4x\, e^{iq\cdot x}\,\langle 0|\, T\left(\mathcal{H}_{\mathrm{eff}}^{\Delta S=1,2}(x),\, \mathcal{H}_{\mathrm{eff}}^{\Delta S=1,2}(0)^\dagger\right)|0\rangle \;=\; \sum_{ij} C_i\, C_j^*\; \Psi_{ij}(q^2)$$



$$\frac{1}{\pi}\,\mathrm{Im}\Psi_{\pm\pm}(t) \;=\; \theta(t)\,\frac{2}{45}\,N_c^2\left(1\pm\frac{1}{N_c}\right)\frac{t^4}{(4\pi)^6}\,\alpha_s(t)^{-2a_\pm}\; C_\pm^2(M_W^2)\left[1+\frac{3}{4}\,\frac{\alpha_s(t)N_c}{\pi}\,\mathcal{K}_\pm\right]$$

$$a_\pm \;=\; \pm\tfrac{9}{11N_c}\;\tfrac{1\mp 1/N_c}{1-6/11N_c}$$

$$\mathcal{K}_+ \;=\; 1-\frac{30587}{3630}\,\frac{1}{N_c}+\frac{164936}{19965}\,\frac{1}{N_c^2}-\frac{51591}{14641}\,\frac{1}{N_c^3}+\frac{440193}{322102}\,\frac{1}{N_c^4}+\cdots = -\frac{3649}{3645}$$

$$\mathcal{K}_- \;=\; 1+\frac{30587}{3630}\,\frac{1}{N_c}+\frac{169706}{19965}\,\frac{1}{N_c^2}+\frac{70335}{14641}\,\frac{1}{N_c^3}+\frac{1810209}{322102}\,\frac{1}{N_c^4}+\cdots = +\frac{18278}{3645}$$

Multi-Scale Problem: Summation of logarithms needed

A large $\log(M_1/M_2)$ compensates a $1/N_C$ suppression

❶ **Short-distance:** $\frac{1}{N_C}\,\log\left(M_W/\mu\right)$ Bardeen-Buras-Gerard

$$\Rightarrow \begin{cases} g_8^\infty = 1.13 \pm 0.05_\mu \pm 0.08_{L_5} \pm 0.05_{m_s} \\ g_{27}^\infty = 0.46 \pm 0.01_\mu \end{cases}$$

Cirigliano et al, Pallante et al

❷ **Long-distance (χPT):** $\frac{1}{N_C}\,\log\left(\mu/m_\pi\right)$ Kambor et al, Pallante et al

$$g_8^{\mathrm{LO}} = 5.0 \;\Rightarrow\; g_8^{\mathrm{NLO}} = 3.6$$

$$g_{27}^{\mathrm{LO}} = 0.285 \;\Rightarrow\; g_{27}^{\mathrm{NLO}} = 0.286$$

Cirigliano et al

❸ **Isospin Violation:** $g_{27}^{\mathrm{NLO}} = 0.297$ Cirigliano et al

$N_C \to \infty$

$$g_8 = \left(\frac{3}{5}\, C_2 - \frac{2}{5}\, C_1 + C_4\right) - 16\; L_5 \;\left(\frac{\langle\bar q\, q\rangle(\mu)}{F_\pi^3}\right)^2 C_6(\mu)$$

$$g_{27} = \frac{3}{5}\,(C_2 + C_1)$$

$$e^2\, g_8\, g_{ew} = -3\;\left(\frac{\langle\bar q\, q\rangle(\mu)}{F_\pi^3}\right)^2 \left[C_8(\mu) \;+\; \frac{16}{9}\; C_6(\mu)\; e^2\;(K_9 - 2\, K_{10})\right]$$

$$\frac{\langle\bar q\, q\rangle(\mu)}{F_\pi^3} = \frac{M_{K^0}^2}{(m_s + m_d)(\mu)\, F_\pi}\;\left\{1 - \frac{8\, M_{K^0}^2}{F_\pi^2}\,(2L_8 - L_5) + \frac{4\, M_{\pi^0}^2}{F_\pi^2}\, L_5\right\}$$

- **Equivalent to standard calculations of $\mathbf{B_i}$**
- **μ dependence only captured for $\mathbf{Q_{6,8}}$**

Anomalous Dimension Matrix

$$\gamma_s^{(0)} = \begin{pmatrix}
-\frac{3}{N_c^2} & \frac{3}{N_c} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{3}{N_c} & -\frac{3}{N_c^2} & -\frac{1}{3N_c^2} & \frac{1}{3N_c} & -\frac{1}{3N_c^2} & \frac{1}{3N_c} & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{11}{3N_c^2} & \frac{11}{3N_c} & -\frac{2}{3N_c^2} & \frac{2}{3N_c} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{3}{N_c} - \frac{n_f}{3N_c^2} & \frac{n_f}{3N_c} - \frac{3}{N_c^2} & -\frac{n_f}{3N_c^2} & \frac{n_f}{3N_c} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{3}{N_c^2} & -\frac{3}{N_c} & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{n_f}{3N_c^2} & \frac{n_f}{3N_c} & -\frac{n_f}{3N_c^2} & -3 + \frac{n_f}{3N_c} + \frac{3}{N_c^2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{N_c^2} & -\frac{3}{N_c} & 0 & 0 \\
0 & 0 & \frac{-n_u+\frac{n_d}{2}}{3N_c^2} & \frac{n_u-\frac{n_d}{2}}{3N_c} & \frac{-n_u+\frac{n_d}{2}}{3N_c^2} & \frac{n_u-\frac{n_d}{2}}{3N_c} & 0 & -3 + \frac{3}{N_c^2} & 0 & 0 \\
0 & 0 & \frac{1}{3N_c^2} & -\frac{1}{3N_c} & \frac{1}{3N_c^2} & -\frac{1}{3N_c} & 0 & 0 & -\frac{3}{N_c^2} & 0 \\
0 & 0 & \frac{-n_u+\frac{n_d}{2}}{3N_c^2} & \frac{n_u-\frac{n_d}{2}}{3N_c} & \frac{-n_u+\frac{n_d}{2}}{3N_c^2} & \frac{n_u-\frac{n_d}{2}}{3N_c} & 0 & 0 & 0 & -\frac{3}{N_c^2}
\end{pmatrix}$$

Only γ_{66} and γ_{88} survive the large–N_C limit

Anatomy of ε'/ε calculation

$$\frac{\varepsilon'_K}{\varepsilon_K} \sim \left[\frac{105\ \mathrm{MeV}}{m_s(2\ \mathrm{GeV})}\right]^2 \left\{B_6^{(1/2)}\,(1-\Omega_{\mathrm{eff}}) - 0.4\ B_8^{(3/2)}\right\}$$

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$$\frac{\varepsilon'_K}{\varepsilon_K} \sim \left[\frac{105\ \text{MeV}}{m_s(2\ \text{GeV})}\right]^2 \left\{ B_6^{(1/2)} \left(1 - \Omega_{\text{eff}}\right) - 0.4\ B_8^{(3/2)} \right\}$$

❶ **$O(p^4)$ χPT Loops: Large correction** (NLO in $1/N_C$) FSI

$$\mathcal{A}_n^{(X)} = a_n^{(X)} \left[1 + \Delta_L \mathcal{A}_n^{(X)} + \Delta_C \mathcal{A}_n^{(X)} \right]$$ Pallante-Pich-Scimemi

$\Delta_L \mathcal{A}_{1/2}^{(8)} = 0.27 \pm 0.05 + 0.47\, i$;

$\Delta_L \mathcal{A}_{1/2}^{(27)} = 1.02 \pm 0.60 + 0.47\, i$; $\Delta_L \mathcal{A}_{3/2}^{(27)} = -0.04 \pm 0.05 - 0.21\, i$

$\Delta_L \mathcal{A}_{1/2}^{(g)} = 0.27 \pm 0.05 + 0.47\, i$; $\Delta_L \mathcal{A}_{3/2}^{(g)} = -0.50 \pm 0.20 - 0.21\, i$

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$$\frac{\varepsilon'_K}{\varepsilon_K} \sim \left[\frac{105\ \mathrm{MeV}}{m_s(2\ \mathrm{GeV})}\right]^2 \left\{ B_6^{(1/2)}\,(1-\Omega_{\mathrm{eff}}) - 0.4\ B_8^{(3/2)} \right\}$$

❶ $\mathbf{O(p^4)}$ χ**PT Loops: Large correction** (NLO in $1/N_C$) FSI

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Pallante-Pich-Scimemi

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❷ $\mathbf{O(p^4)}$ **LECs fixed at** $N_C \to \infty$: **Small correction** $\Delta_C\, \mathcal{A}_n^{(X)}$

❸ **Isospin Breaking** $O\left[(m_u - m_d)\, p^2, e^2 p^2\right]$: **Sizeable correction**

$$\Omega_{\mathrm{eff}} = 0.06 \pm 0.08$$

Cirigliano-Ecker-Neufeld-Pich

❹ **Re(g_8), Re(g_{27}), $\chi_0 - \chi_2$ fitted to data**

Isospin Breaking in ε'/ε

$$\epsilon'_K \sim \omega_+ \left\{ \frac{\mathrm{Im}\, A_0^{(0)}}{\mathrm{Re}\, A_0^{(0)}} \left(1+\Delta_0+f_{5/2}\right) - \frac{\mathrm{Im}\, A_2}{\mathrm{Re}\, A_2^{(0)}} \right\}$$

$$\sim \omega_+ \left\{ \frac{\mathrm{Im}\, A_0^{(0)}}{\mathrm{Re}\, A_0^{(0)}} \left(1-\Omega_{\mathrm{eff}}\right) - \frac{\mathrm{Im}\, A_2^{\mathrm{emp}}}{\mathrm{Re}\, A_2^{(0)}} \right\}$$

$$\omega \equiv \frac{\mathrm{Re}\, A_2}{\mathrm{Re}\, A_0} = \omega_+ \left(1+f_{5/2}\right) \quad ; \quad \omega_+ \equiv \frac{\mathrm{Re}\, A_2^+}{\mathrm{Re}\, A_0} \quad , \quad \Omega_{IB} = \frac{\mathrm{Re}\, A_0^{(0)}}{\mathrm{Re}\, A_2^{(0)}} \cdot \frac{\mathrm{Im}\, A_2^{\mathrm{non-emp}}}{\mathrm{Im}\, A_0^{(0)}}$$

Cirigliano-Ecker-Neufeld-Pich

$\times\ 10^{-2}$	$\alpha = 0$ LO	$\alpha = 0$ NLO	$\alpha \neq 0$ LO	$\alpha \neq 0$ NLO
Ω_{IB}	11.7	15.9 ± 4.5	18.0 ± 6.5	22.7 ± 7.6
Δ_0	-0.004	-0.41 ± 0.05	8.7 ± 3.0	8.4 ± 3.6
$f_{5/2}$	0	0	0	8.3 ± 2.4
Ω_{eff}	11.7	16.3 ± 4.5	9.3 ± 5.8	6.0 ± 7.7

$$\Omega_{\mathrm{eff}} = 0.06 \pm 0.08 \equiv \Omega_{IB} - \Delta_0 - f_{5/2}$$

$$\Omega_{\mathrm{IB}}^{\pi^0\eta} = 0.16 \pm 0.03$$

$$\frac{\varepsilon'_K}{\varepsilon_K} \sim \left[\frac{105\ \mathrm{MeV}}{m_s(2\ \mathrm{GeV})}\right]^2 \left\{B_6^{(1/2)}\,(1-\Omega_{\mathrm{eff}}) - 0.4\ B_8^{(3/2)}\right\}$$

Delicate Cancellation. Strong Sensitivity to:

- m_s (quark condensate) $m_s(2\ \mathrm{GeV}) = 110 \pm 20$ MeV
- Isospin Breaking $(m_u \neq m_d\ ,\ \alpha)$ $\Omega_{\mathrm{eff}} = 0.06 \pm 0.08$

  Cirigliano-Ecker-Neufeld-Pich
- Penguin Matrix Elements

χPT Loops (FSI): $B_{6,\infty}^{(1/2)} \times (1.35 \pm 0.05)$; $B_{8,\infty}^{(3/2)} \times (0.54 \pm 0.20)$

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Pallante–Pich–Scimemi '01: (updated '04)

$$\mathrm{Re}\left(\varepsilon'/\varepsilon\right) = \left(19 \pm 2_{\,\mu}\ {}^{+9}_{-6}{}_{\,m_s} \pm 6_{\,1/N_C}\right) \times 10^{-4}$$

Experimental world average: $\mathrm{Re}\left(\varepsilon'/\varepsilon\right) = (16.8 \pm 1.4) \times 10^{-4}$

Challenge: Control of subleading $1/N_C$ corrections to χPT couplings

Recent Lattice Results

Isospin limit: RBC-UKQCD 1505.07863, 1502.00263

$$\sqrt{\frac{3}{2}}\,\mathrm{Re}\,A_2 = (1.50 \pm 0.04 \pm 0.14)\cdot 10^{-8}\ \mathrm{GeV} \qquad \mathrm{exp}: 1.482\,(2)\cdot 10^{-8}\ \mathrm{GeV} \quad 0.1\,\sigma$$

$$\sqrt{\frac{3}{2}}\,\mathrm{Im}\,A_2 = -(6.99 \pm 0.20 \pm 0.84)\cdot 10^{-13}\ \mathrm{GeV}$$

$$\sqrt{\frac{3}{2}}\,\mathrm{Re}\,A_0 = (4.66 \pm 1.00 \pm 1.26)\cdot 10^{-7}\ \mathrm{GeV} \qquad \mathrm{exp}: 3.112\,(1)\cdot 10^{-7}\ \mathrm{GeV} \quad 1.0\,\sigma$$

$$\sqrt{\frac{3}{2}}\,\mathrm{Im}\,A_0 = -(1.90 \pm 1.23 \pm 1.08)\cdot 10^{-11}\ \mathrm{GeV}$$

$$\mathrm{Re}\,(\varepsilon'/\varepsilon) = (1.38 \pm 5.15 \pm 4.59)\cdot 10^{-4} \qquad \mathrm{exp}: (16.8 \pm 1.4)\cdot 10^{-4} \quad 2.2\,\sigma$$

$$\delta_0 = (23.8 \pm 4.9 \pm 1.2)^\circ \qquad \mathrm{exp}: (39.2 \pm 1.5)^\circ \quad 2.9\,\sigma$$

$$\delta_2 = -(11.6 \pm 2.5 \pm 1.2)^\circ \qquad \mathrm{exp}: -(8.5 \pm 1.5)^\circ \quad 1.0\,\sigma$$

Modelling (some) non-factorizable $1/N_C$ corrections

Buras-Gérard, 1507.06326

$$B_6^{(1/2)} = 1 - \frac{3}{2}\left[\frac{F_\pi}{F_K - F_\pi}\right]\frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln(1 + \frac{\Lambda^2}{\tilde{m}_6^2}) = 1 - 0.66\,\ln(1 + \frac{\Lambda^2}{\tilde{m}_6^2})$$

$$B_8^{(1/2)} = 1 + \frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln(1 + \frac{\Lambda^2}{\tilde{m}_8^2}) = 1 + 0.08\,\ln(1 + \frac{\Lambda^2}{\tilde{m}_8^2})$$

$$B_8^{(3/2)} = 1 - 2\frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln(1 + \frac{\Lambda^2}{\tilde{m}_8^2}) = 1 - 0.17\,\ln(1 + \frac{\Lambda^2}{\tilde{m}_8^2})$$

➡ $B_6^{(1/2)} \le B_8^{(3/2)} < 1$

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➡ $B_6^{(1/2)} \leq B_8^{(3/2)} < 1$

- FSI $(1/N_C)$ not included
- Part of 1-loop χPT corrections (?)
- Difficult to account in a matching calculation

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→ $B_6^{(1/2)} \leq B_8^{(3/2)} < 1$

Not true in QCD

- FSI $(1/N_C)$ not included
- Part of 1-loop χPT corrections (?)
- Difficult to account in a matching calculation

BBG Model

Bardeen-Buras-Gerard

$$\mathcal{L}_{\rm eff} = \frac{f_\pi^2}{4}\left\{\langle D_\mu U^\dagger D^\mu U\rangle + r\,\langle m\,(U+U^\dagger)\rangle - \frac{r}{\Lambda_\chi^2}\,\langle m\,(D^2U + D^2U^\dagger)\rangle\right\}$$

1. Equivalent to $\mathcal{O}(p^2)$ χPT + L_5 term $\qquad (L_i = 0\,,\ i\neq 5)$
   Most L_i are leading in N_C ➔ $\mathcal{L}_{\rm eff}$ **does not represent large-N_C QCD**

2. **Cut-off loop regularization:** **$M \sim (0.8 - 0.9)$ GeV**
   $f_\pi^2(M^2) = F_\pi^2 + 2\, I_2(m_\pi^2) + I_2(m_K^2)$, $\qquad I_2(m_i^2) = \frac{1}{16\pi^2}\left[M^2 - m_i^2\,\log\left(1+\frac{M^2}{m_i^2}\right)\right]$

3. **Large-N_C factorization assumed to hold in the IR ($\mu = 0$):** $\langle J\cdot J\rangle = \langle J\rangle\langle J\rangle$

4. **M identified with SD renormalization scale μ: $C_i(\mu)$ running**
   Meson evolution ⟷ Quark evolution

5. **Vector meson loops included through Hidden U(3) Gauge Symmetry**
   Could partially account for $L_{1,2,3,9,10}$
   L_8 still missing ➔ $\langle\bar{q}q\rangle$, $Q_{6,8}$ **not quite correct even at large-N_C**

6. **$\pi\pi$ re-scattering completely missing** ➔ $\delta_{0,2} = 0$, **FSI absent**

Summary

Kaons continue providing important physics information:

- **Interesting interplay of short and long-distances**
- **Sensitive to heavy mass scales. New Physics?**
- **Superb probe of flavour dynamics and $\not{\mathcal{C}}\not{\mathcal{P}}$**
- **Excellent testing ground of χPT dynamics**

Increased sensitivities at ongoing experiments ($K \to \pi\nu\bar{\nu}$)

Theoretical challenge: precise control of QCD effects

Successful SM prediction for ϵ'/ϵ

Pallante-Pich-Scimemi

$$\mathrm{Re}\,(\epsilon'/\epsilon)_{\mathrm{SM}} \;=\; \left(19 \pm 2\,{}^{+9}_{-6} \pm 6\right) \cdot 10^{-4}$$

Large uncertainty but no anomalies!

Phenomenological $K \to \pi\pi$ Fit

Cirigliano-Ecker-Neufeld-Pich

	LO-IC	LO-IB	NLO-IC	NLO-IB
Re g_8	4.96	4.99	3.62 ± 0.28	3.61 ± 0.28
Re g_{27}	0.285	0.253	0.286 ± 0.029	0.297 ± 0.029
$\chi_0 - \chi_2$	47.5°	47.8°	$(47.5 \pm 0.9)^\circ$	$(51.3 \pm 0.8)^\circ$

IC $\equiv [m_u - m_d = \alpha = 0]$; IB $\equiv [m_u - m_d \neq 0\, ,\ \alpha \neq 0]$

Isospin Limit: $[\delta_0 - \delta_2]_{K\to\pi\pi} = (52.5 \pm 0.8_{exp} \pm 2.8_{th})^\circ$

$\pi\pi \to \pi\pi$: $\delta_0 - \delta_2 = (47.7 \pm 1.5)^\circ$ Colangelo–Gasser–Leutwyler '01

Electroweak Penguins contribute at $\mathbf{O(p^0)}$ $\qquad (m_q, p \to 0)$

$$e^2 g_8 g_{ew} \, F^6 \; = \; 6 \, C_7(\mu) \, \langle \mathcal{O}_1(\mu) \rangle - 12 \, C_8(\mu) \, \langle \mathcal{O}_2(\mu) \rangle \; \stackrel{N_C \to \infty}{\longrightarrow} \; -\frac{1}{3} \, C_8(\mu) \, \langle \bar{q} q(\mu) \rangle^2$$

$$\langle \mathcal{O}_1(\mu) \rangle \; \equiv \; \langle 0 | (s_L \gamma^\mu d_L)(\bar{d}_R \gamma_\mu s_R) | 0 \rangle \qquad ; \qquad \langle \mathcal{O}_2(\mu) \rangle \; \equiv \; \langle 0 | (s_L s_R)(\bar{d}_R d_L) | 0 \rangle$$

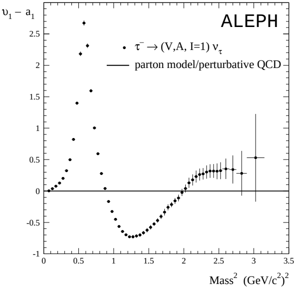
Electroweak Penguins contribute at $O(p^0)$ $\qquad (m_q, p \to 0)$

$$e^2 g_8 g_{ew} F^6 = 6\, C_7(\mu)\, \langle \mathcal{O}_1(\mu)\rangle - 12\, C_8(\mu)\, \langle \mathcal{O}_2(\mu)\rangle \xrightarrow{N_C \to \infty} -\frac{1}{3}\, C_8(\mu)\, \langle \bar{q} q(\mu)\rangle^2$$

$$\langle \mathcal{O}_1(\mu)\rangle \equiv \langle 0|(s_L \gamma^\mu d_L)(\bar{d}_R \gamma_\mu s_R)|0\rangle \quad ; \quad \langle \mathcal{O}_2(\mu)\rangle \equiv \langle 0|(s_L s_R)(\bar{d}_R d_L)|0\rangle$$

These D=6 vacuum condensates appear in the left-right correlator:

$$\Pi^{\mu\nu}_{LR}(q) \equiv 2i \int d^4x\, \mathrm{e}^{iqx} \langle 0|T(L^\mu(x), R^\nu(0)^\dagger)|0\rangle \equiv \left(-g^{\mu\nu} q^2 + q^\mu q^\nu\right)\, \Pi_{LR}(-q^2) = \tfrac{1}{2}\, \Pi^{\mu\nu}_{VV-AA}(q)$$



$$\Pi_{LR}(Q^2) = \underbrace{\frac{1}{\pi} \int_0^\infty dt\, \frac{\mathrm{Im}\Pi_{LR}(t)}{t + Q^2}}_{\text{Data}} = \underbrace{\frac{1}{2} \sum_{n=1}^{\infty} \frac{\langle \widetilde{\mathcal{O}}_{2n+4}\rangle}{(Q^2)^{n+2}}}_{\text{QCD OPE}}$$

$$\lim_{Q^2 \to \infty} -Q^6\, \Pi_{LR}(Q^2) = 4\pi\alpha_s \left[4\, \langle \mathcal{O}_2\rangle + \frac{2}{N_C}\, \langle \mathcal{O}_1\rangle\right] + O(\alpha_s^2)$$

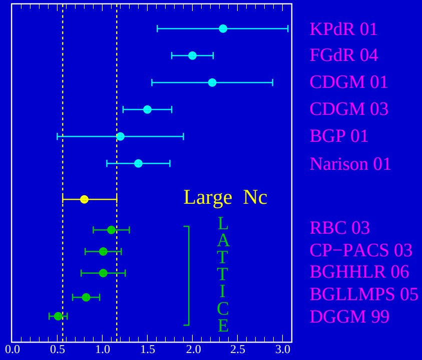
$$\langle \mathcal{O}_1\rangle = -\frac{i}{2}\, g_{\mu\nu} \int \frac{d^D q}{(2\pi)^D}\, \Pi^{\mu\nu}_{LR}(q) \sim \int_0^\infty dQ^2\, Q^D\, \Pi_{LR}(Q^2)$$

Electroweak Penguins contribute at $\mathbf{O(p^0)}$ $\quad (m_q, p \to 0)$

$$e^2 g_8 g_{ew}\, F^6 \;=\; 6\, C_7(\mu)\, \langle \mathcal{O}_1(\mu)\rangle - 12\, C_8(\mu)\, \langle \mathcal{O}_2(\mu)\rangle \;\xrightarrow{N_C\to\infty}\; -\frac{1}{3}\, C_8(\mu)\, \langle \bar{q} q(\mu)\rangle^2$$

$$\langle \mathcal{O}_1(\mu)\rangle \;\equiv\; \langle 0|(s_L\gamma^\mu d_L)(\bar{d}_R\gamma_\mu s_R)|0\rangle \qquad ; \qquad \langle \mathcal{O}_2(\mu)\rangle \;\equiv\; \langle 0|(s_L s_R)(\bar{d}_R d_L)|0\rangle$$

$\mathbf{M_8}$ (GeV^3)



$$\mathbf{M_8} \;\equiv\; \langle (2\pi)_{I=2}|Q_8(\mu_0)|K^0\rangle\Big|_{m_q=p=0} \;=\; \frac{8}{F^3}\,\langle \mathcal{O}_2(\mu_0)\rangle$$

$\mu_0 = 2$ GeV

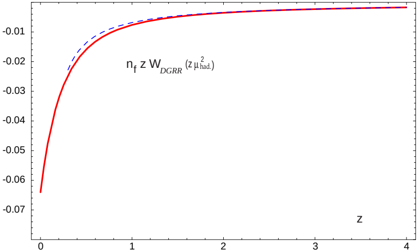
$$\mathbf{M_8} \;\xrightarrow{N_C\to\infty}\; \frac{2}{F^3}\,\langle \bar{q} q(\mu_0)\rangle^2 \;\approx\; \frac{2 M_K^4 F^3}{(m_s+m_q)^2(\mu_0)\, F_\pi^2}$$

n_f/N_C Correction to QCD PENGUIN $(m_q \to 0)$

Hambye-Peris-de Rafael '03

$$\mathrm{Im}(g_8) \doteq \mathrm{Im}[C_6(\mu)] \left\{ -16 L_5 \left(\frac{\langle \bar{q} q \rangle}{F^3} \right)^2 + \frac{8 n_f}{16\pi^2 F^4} \int_0^\infty dQ^2\, Q^{D-2}\, \mathcal{W}_{DGRR}(Q^2) \right\}$$

$$\left(\frac{q^\alpha q^\beta}{q^2} - g^{\alpha\beta} \right) \mathcal{W}_{DGRR}(-q^2) = \int d\Omega_q\, d^4x\, d^4y\, d^4z\;\; \mathrm{e}^{iqx}\, \langle T\, [(\bar{s}_L q_R)(x)\, (\bar{q}_R d_L)(0)\, (\bar{d}_R \gamma_\alpha u_R)(y)\, (\bar{u}_R \gamma^\alpha s_R)(z)] \rangle_{\mathrm{con}}$$



Available theoretical information: (poor)

$$\lim_{Q^2 \to \infty} Q^2\, \mathcal{W}_{DGRR}(Q^2) = -\frac{F^4 \pi \alpha_s}{6 Q^2} \left[1 - 16 L_5 \left(\frac{\langle \bar{q} q \rangle}{F^3} \right)^2 \right]$$

$$\lim_{Q^2 \to 0} Q^2\, \mathcal{W}_{DGRR}(Q^2) = \left(\frac{\langle \bar{q} q \rangle}{F^2} \right)^2 \left\{ \frac{F^2}{8 Q^2} - \left(L_5 - \frac{5}{2} L_3 \right) \right\}$$

Big enhancement (~ 3) claimed

Infrared unstability from pion pole: $\displaystyle \int_0^\infty dQ^2\, \frac{Q^{-\epsilon}}{Q^2 + m_\pi^2} \sim \frac{2}{\epsilon\, m_\pi^\epsilon}$

Large non-factorizable contribution claimed before — Bardeen et al, Bijnens-Prades