Wave-function renormalization and flavour mixing: the case for new physics

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EXCELENCIA MARÍA DE MAEZTU

Introduction

We have a 'composite Higgs' scenario in mind...

Motivation:
- Do we 'need' "anomalous" fermionic operators?
- What is the size of the EW radiative corrections to the CKM elements?
- In which renormalization scheme are the CKM given?
- Does it matter?
- What's the size of possible NP contributions (effective lagrangian)?
- Possible new manifestations of *CP*-violation and CKM non-unitarity.

I will review a consistent EW on-shell scheme for flavour changing transitions in presence of gauge-dependent branch cuts

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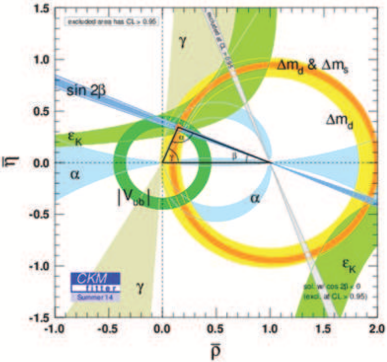
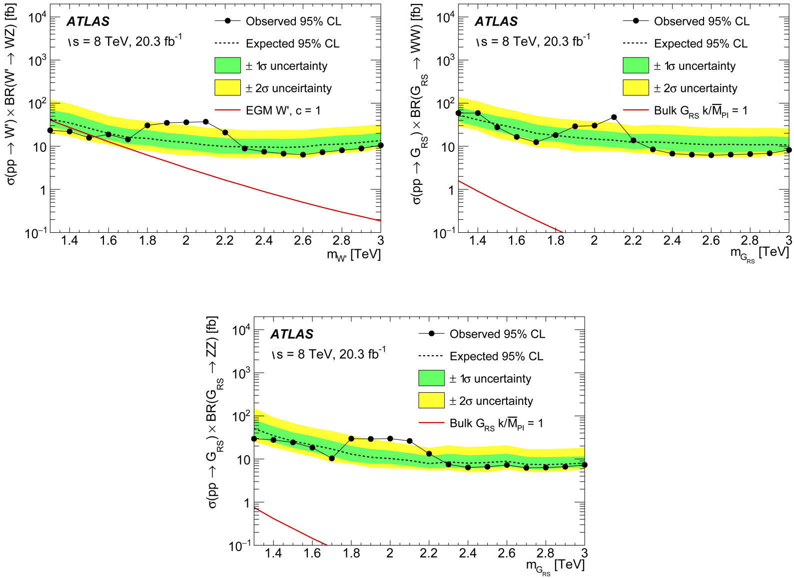
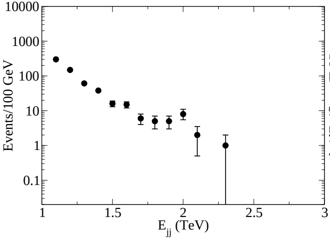


2002: $\sin 2\beta = 0.82 \pm 0.13$ (Belle)
2014: $\sin 2\beta = 0.691 \pm 0.017$ (WA)

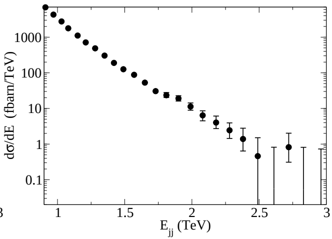
A resonance at 2 TeV?



Excess events at 2 TeV?

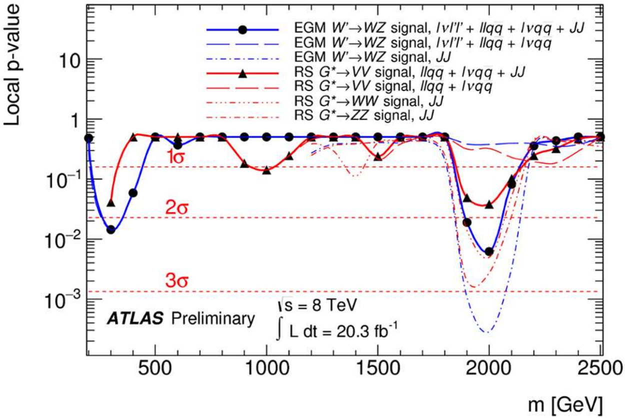


ATLAS WZ events



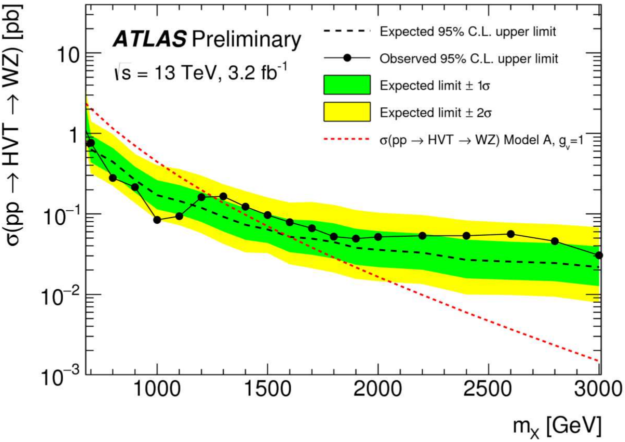
CMS WW events

Excess events at 2 TeV?



p-values for ATLAS WZ events

Excess events at 2 TeV??



However...no significant signal in early 13 TeV results

Parametrizing composite Higgs physics

A light "Higgs boson" with mass $M_H \sim 125$ GeV is coupled to the EW bosons according to (non-linear realization)

$$\mathcal{L}_{\rm eff} \supset -\frac{1}{2}{\rm Tr} W_{\mu\nu} W^{\mu\nu} - \frac{1}{4}{\rm Tr} B_{\mu\nu} B^{\mu\nu} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP} + \sum_i \mathcal{L}_{\rm i}$$
$$+ \left[1 + 2a\left(\frac{h}{v}\right) + b\left(\frac{h}{v}\right)^2\right] \frac{v^2}{4}{\rm Tr} D_\mu U^\dagger D^\mu U - V(h)$$

$$U = \exp(i\,\omega\cdot\tau/v)$$
$$D_\mu U = \partial_\mu U + \frac{1}{2} i g W_\mu^i \tau^i U - \frac{1}{2} i g' B_\mu^i U \tau^3$$

and additional gauge-invariant operators are encoded in $\mathcal{L}_{\rm i}$.
Setting $a = b = 1$ (and $\mathcal{L}_{\rm i}=0$) reproduces the SM interactions

$\mathcal{O}(p^4)$ operators

The $\mathcal{L}_i$ are a full set of C, P, and $SU(2)_L \times U(1)_Y$ gauge invariant, $d = 4$ operators that parameterize the *low-energy effects* of the *model-dependent high-energy EWSB sector* along with *a,b*.
The two relevant *custodial-symmetry preserving* operators are

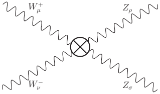
$$\mathcal{L}_4 = a_4 \left(\mathrm{Tr}\left[V_\mu V_\nu\right]\right)^2 \qquad \mathcal{L}_5 = a_5 \left(\mathrm{Tr}\left[V_\mu V^\mu\right]\right)^2 \qquad V_\mu = \left(D_\mu U\right) U^\dagger$$

The a_i could be *functions of* $\frac{h}{v}$

- For example:

		Heavy Higgs	QCD-like technicolor
a_4	$=$	0	$-2a_5$
a_5	$=$	$\frac{v^2}{8M_H^2}$	$\frac{N_{TC}}{96\pi^2}$

(up to logarithmic corrections)



$$ig^4\left[a_4\left(g^{\mu\sigma}g^{\nu\rho} + g^{\mu\rho}g^{\nu\sigma}\right) + 2a_5 g^{\mu\nu}g^{\rho\sigma}\right]$$

Unitarization: Inverse Amplitude Method

Partial wave unitarity **requires**

$$\mathrm{Im}\ t_{I\,J}(s) = \underbrace{\sigma(s)|t_{I\,J}(s)|^2}_{\substack{\text{Elastic} \\ WW \to WW}} + \underbrace{\sigma_H(s)|t_{H,I\,J}(s)|^2}_{\substack{\text{Inelastic} \\ WW \to hh}}$$

where σ and σ_H are phase space factors.

Given a perturbative expansion

$$t_{I\,J} \approx \underbrace{t_{I\,J}^{(2)}}_{\text{tree}} + \underbrace{t_{I\,J}^{(4)}}_{\substack{\text{one-loop} \\ +\ a_i \text{ terms}}} + \cdots$$

we can require unitarity to hold *exactly* if *(Note: non-coupled channels)*

$$t_{I\,J} \approx \frac{t_{I\,J}^{(2)}}{1 - t_{I\,J}^{(4)}/t_{I\,J}^{(2)}}$$

Several mild analyticity assumptions are implied.

Is this unitarization method unique?

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No, it is not. Many methods exist: IAM, K-matrix approach, N/D expansions, Roy equations,... qualitatively they all agree.

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Several mild analyticity assumptions are implied.

For a detailed comparative analysis see e.g. Delgado, Dobado and Llanes (2015)

New resonances

The unitarization of the amplitudes may result in the appearance of *new heavy resonances* associated with the high-energy theory

$$t_{00} \rightarrow \text{Scalar isoscalar}$$
$$t_{11} \rightarrow \text{Vector isovector}$$
$$t_{20} \rightarrow \text{Scalar isotensor}$$

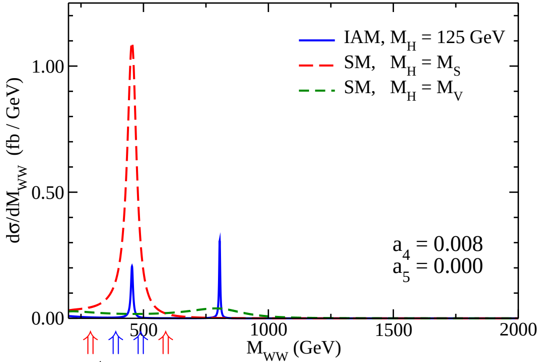
Will search for poles in $t_{I\,J}(s)$ up to $(4\pi v) \sim 3$ TeV (domain of applicability)

True resonances will have the phase shift pass through $+\pi/2$

$$\delta_{IJ} = \tan^{-1}\left(\frac{\mathrm{Im}\, t_{IJ}}{\mathrm{Re}\, t_{IJ}}\right)$$

This method is known to work remarkably well in strong interactions: $\pi\pi$ scattering $\Rightarrow$ σ and ρ meson masses and widths

Are these resonances detectable?



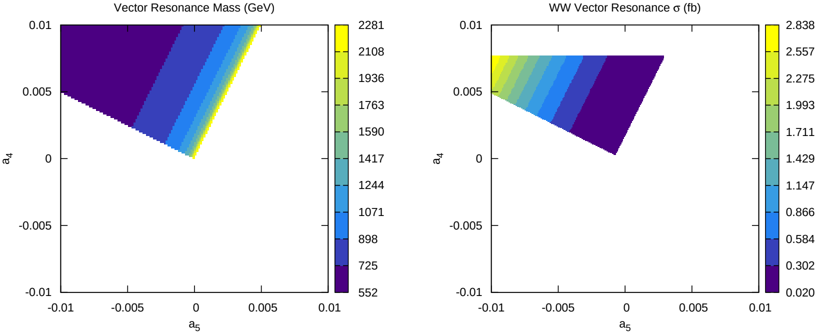
Signal of IAM scalar/vector vs. SM Higgs of *same mass*

The large contribution that the SM Higgs represents leaves little room for additional resonances.

Note: only in $WW \to WW$ or $WW \to ZZ$ channels!

D.E., Mescia, Yencho (2012)

Masses and cross-sections



- $M_V \sim 550 - 2300$ GeV, $\Gamma_V \sim 2 - 24$ GeV (narrow resonances)

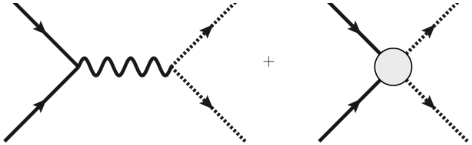
There are constraints on vector masses from S, T, U parameter constraints in some models. *e.g. Pich, Rosell and Sanz-Cillero (2014)*.

A resonance in the 2 TeV region requires a_4, a_5 in the $10^{-3} - 10^{-4}$ range. Natural from an EFT point of view?

Cross-sections off by a factor 10^{-2}

DY: anomalous $\bar{q}_L q_L$ coupling

Drell-Yan + rescattering



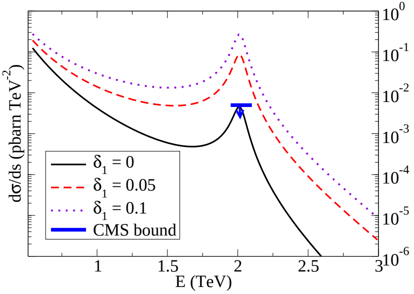
Production of a pair of Goldstone bosons by $\bar{q}q'$ annihilation through a W-meson and anomalous BSM vertex enhancing it

Gauge invariant operator: $\delta_1 \bar{\psi}_L U \not{D} U^\dagger \psi_L$
(one of several $d = 4$ Longhitano's operators)

Changes the relation between G_F and the TGB vertex

Dobado, D.E., Llanes-Estrada (2015)

Effect of an anomalous $\bar{q}_L q_L$ coupling



$$\left[\frac{d\hat{\sigma}}{d\Omega}\right]_{\rm cm} = \frac{1}{64\pi^2 s}\frac{g^4}{32}\sin^2\theta\left(1+\frac{\delta_1 s}{v^2}\right)^2 |F_V(s)|^2 \ ,$$

F_V : Form factor (Watson's theorem)

$a = 0.90$, $a_4 = 7\times 10^{-4}$, $a_5 = 0$ (computed via ET).

Difficult to set precise bounds on δ_1 but at least $\delta_1 < 0.01$.

$d = 4$ EFT operators

Longhitano (1980) —extended to the flavour sector:

$$
\begin{aligned}
\mathcal{L}_L^1 &= i\bar{f} M_L^1 \gamma^\mu U \left(D_\mu U\right)^\dagger Lf + h.c.,\\
\mathcal{L}_L^2 &= i\bar{f} M_L^2 \gamma^\mu \left(D_\mu U\right) \tau^3 U^\dagger Lf + h.c.,\\
\mathcal{L}_L^3 &= i\bar{f} M_L^3 \gamma^\mu U \tau^3 U^\dagger \left(D_\mu U\right) \tau^3 U^\dagger Lf + h.c.,\\
\mathcal{L}_L^4 &= i\bar{f} M_L^4 \gamma^\mu U \tau^3 U^\dagger D_\mu^L Lf + h.c.,\\
\mathcal{L}_R^1 &= i\bar{f} M_R^1 \gamma^\mu U^\dagger \left(D_\mu U\right) Rf + h.c.,\\
\mathcal{L}_R^2 &= i\bar{f} M_R^2 \gamma^\mu \tau^3 U^\dagger \left(D_\mu U\right) Rf + h.c.,\\
\mathcal{L}_R^3 &= i\bar{f} M_R^3 \gamma^\mu \tau^3 U^\dagger \left(D_\mu U\right) \tau^3 Rf + h.c..
\end{aligned}
$$

M_L^1, M_R^1, M_L^3 and M_R^3 hermitian
M_L^2, M_R^2 and M_L^4 are completely general.
If we require the above operators to be CP conserving, the matrices $M_{L,R}^i$ must be real.

The 'SM' after integrating out heavy d.o.f.

$$
\begin{aligned}
\mathcal{L}^L_{kin} &= i\bar{\mathrm{f}} X_L \gamma^\mu D^L_\mu L \mathrm{f}, \\
\mathcal{L}^R_{kin} &= i\bar{\mathrm{f}} \left(\tau^u X_{Ru} + \tau^d X_{Rd}\right) \gamma^\mu D^R_\mu R \mathrm{f}, \\
\mathcal{L}_m &= -\bar{\mathrm{f}} \left(U \left(\tau^u \tilde{y}^f_u + \tau^d \tilde{y}^f_d\right) R + \left(\tau^u \tilde{y}^{f\dagger}_u + \tau^d \tilde{y}^{f\dagger}_d\right) U^\dagger L \right) \mathrm{f}.
\end{aligned}
$$

X_L, X_{Ru} and X_{Rd} are non-singular Hermitian matrices with family indices

$\tilde{y}^f_u$ and $\tilde{y}^f_d$ are arbitrary matrices and have only family indices too.

In the Standard Model, the $X_{L,R}$ can always be reabsorbed so one does not even contemplate the possibility that left and right 'kinetic' terms are differently normalized, but this is perfectly possible in an EFT.

In addition we have the Longhitano operators:

$$
\begin{aligned}
\mathcal{L}_L &= \bar{\mathrm{f}} \gamma_\mu M_L O^\mu_L L \mathrm{f} + h.c., \\
\mathcal{L}_R &= \bar{\mathrm{f}} \gamma_\mu M_R O^\mu_R R \mathrm{f} + h.c.,
\end{aligned}
$$

Longhitano's operators

How do all these possible $d = 4$ terms in an EFT contribute to observables such as effective couplings and so on?

My prejudice: forget about large effects.

They have to be treated on the same footing as radiative corrections (effects at the 1% to 0.1% level)

Some notation...

WFR:

$$\Psi_0 = Z^{\frac{1}{2}}\Psi\,, \qquad \bar{\Psi}_0 = \bar{\Psi}\bar{Z}^{\frac{1}{2}}\,.$$

For reasons that will become clear along the discussion, we shall allow Z and $\bar{Z}$ to be independent renormalisation constants (hermiticity?)
These renormalisation constants contain flavour, family and Dirac indices. We can decompose them into

$$Z^{\frac{1}{2}} = Z^{u\frac{1}{2}}\tau^u + Z^{d\frac{1}{2}}\tau^d\,, \qquad \bar{Z}^{\frac{1}{2}} = \bar{Z}^{u\frac{1}{2}}\tau^u + \bar{Z}^{d\frac{1}{2}}\tau^d\,, \tag{1}$$

with τ^u and τ^d the up and down flavour projectors and furthermore each piece in left and right chiral projectors, L and R respectively,

$$Z^{u\frac{1}{2}} = Z^{uL\frac{1}{2}}L + Z^{uR\frac{1}{2}}R\,, \qquad \bar{Z}^{u\frac{1}{2}} = \bar{Z}^{uL\frac{1}{2}}R + \bar{Z}^{uR\frac{1}{2}}L\,. \tag{2}$$

Analogous decompositions hold for $Z^{d\frac{1}{2}}$ and $\bar{Z}^{d\frac{1}{2}}$.

Example: top decay

Tree level:

$$f_i(p_1) \to W^+(q)\, f_j(p_2)$$

There are two different Lorentz structures (at the one loop level)

$$\begin{aligned} M_L^{(1)} &= \bar{u}_j(p_2)\, \not{\varepsilon}^*(q)\, L u_i(p_1)\,, \qquad (L \leftrightarrow R)\,, \\ M_L^{(2)} &= \bar{u}_j(p_2)\, L u_i(p_1)\, p_1 \cdot \varepsilon^*(q)\,, \qquad (L \leftrightarrow R)\,. \end{aligned}$$

At tree level only $M_L^{(1)}$ appears.

The transition amplitude at tree level is

$$\mathcal{M}_0 = -\frac{eK_{ij}}{2s_W} M_L^{(1)}\,,$$

t-decay: one loop

$$\begin{aligned}\mathcal{M}_1 &= -\frac{e}{2s_W} M_L^{(1)} [K_{ij}(1 + \frac{\delta e}{e} - \frac{\delta s_W}{s_W} + \frac{1}{2}\delta Z_W) + \delta K_{ij} \\ &+ \frac{1}{2}\sum_r (\delta \bar{Z}_{ir}^{Lu} K_{rj} + K_{ir}\delta Z_{rj}^{Ld})] \\ &- \frac{e}{2s_W}\left(\delta F_L^{(1)} M_L^{(1)} + M_L^{(2)}\delta F_L^{(2)} + M_R^{(1)}\delta F_R^{(1)} + M_R^{(2)}\delta F_R^{(2)}\right)\end{aligned}$$

$\delta F_{L,R}^{(1,2)}$: electroweak form factors coming from one-loop vertex diagrams.

$$\begin{aligned}\frac{\delta e}{e} &= -\frac{1}{2}\left[\left(\delta Z_2^A - \delta Z_1^A\right) + \delta Z_2^A\right] = -\frac{s_W}{c_W M_Z^2}\Pi^{ZA}(0) + \frac{1}{2}\frac{\partial \Pi^{AA}}{\partial k^2}(0) , \\ \frac{\delta s_W}{s_W} &= -\frac{c_W^2}{2s_W^2}\left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2}\right) = -\frac{c_W^2}{2s_W^2} Re\left(\frac{\Pi^{WW}(M_W^2)}{M_W^2} - \frac{\Pi^{ZZ}(M_Z^2)}{M_Z^2}\right) , \\ \delta Z_W &= -\frac{\partial \Pi^{WW}}{\partial k^2}(M_W^2) ,\end{aligned}$$

CKM renormalization versus WFR

Weak basis

$$\begin{pmatrix} u_0 \\ d_0 \end{pmatrix} = Z_L \begin{pmatrix} u \\ d \end{pmatrix}$$

Mass diagonal basis

$$\begin{pmatrix} \tilde{u}_0 \\ \tilde{d}_0 \end{pmatrix} = \begin{pmatrix} Z_u^{\frac{1}{2}} u \\ Z_d^{\frac{1}{2}} d \end{pmatrix}$$

$$\tilde{u} = V_u^\dagger u, \quad \tilde{u}_0 = (V_u^0)^\dagger u_0, \quad \tilde{d} = V_d^\dagger d, \quad \tilde{d}_0 = (V_d^0)^\dagger d_0.$$

$$Z_L^{\frac{1}{2}} = V_u^0 Z_u^{\frac{1}{2}} V_u^\dagger, \qquad Z_L^{\frac{1}{2}} = V_d^0 Z_d^{\frac{1}{2}} V_d^\dagger$$

$$Z_u^{\frac{1}{2}} K = K^0 Z_d^{\frac{1}{2}} \quad \Rightarrow \quad \delta K = \frac{1}{2}\delta Z_u K - \frac{1}{2} K \delta Z_d$$

Ward identity (unitarity of K and K^0): $(Z_u^\dagger)^{\frac{1}{2}} Z_u^{\frac{1}{2}} K = K (Z_d^\dagger)^{\frac{1}{2}} Z_d^{\frac{1}{2}}$ $\Rightarrow$

$$\delta K_{jk} = \frac{1}{4}\left[\left(\delta\hat{Z}^{uL} - \delta\hat{Z}^{uL\dagger}\right) K - K \left(\delta\hat{Z}^{dL} - \delta\hat{Z}^{dL\dagger}\right)\right]_{jk}$$

$\hat{Z}$ means that the wfr. constants here are not necessarily the same ones used for the LSZ factors.

What about WFR?

Due to radiative corrections the propagator mixes fermion of different family indices

$$iS^{-1}(p) = \bar{Z}^{\frac{1}{2}} \left(\not{p} - m - \delta m - \Sigma(p) \right) Z^{\frac{1}{2}} \, ,$$

Introducing the family indices explicitly we have

$$iS^{-1}_{ij}(p) = (\not{p} - m_i)\, \delta_{ij} - \hat{\Sigma}_{ij}(p) \; .$$

The one-loop renormalised self-energy is given by

$$\hat{\Sigma}_{ij}(p) = \Sigma_{ij}(p) - \frac{1}{2} \delta \bar{Z}_{ij} (\not{p} - m_j) - \frac{1}{2} (\not{p} - m_i)\, \delta Z_{ij} + \delta m_i \delta_{ij} \, .$$

$$\hat{\Sigma}_{ij}(p) = \not{p} \left(\hat{\Sigma}^{\gamma R}_{ij}(p^2)\, R + \hat{\Sigma}^{\gamma L}_{ij}(p^2)\, L \right) + \hat{\Sigma}^{R}_{ij}(p^2)\, R + \hat{\Sigma}^{L}_{ij}(p^2)\, L$$

Note that we acount for Z and $\bar{Z}$ separately.

On-shell conditions

Off-diagonal conditions:

The conditions will be

$$\begin{aligned} \hat{\Sigma}_{ij}(p)\, u_j^{(s)}(p) &= 0\,, \qquad (p^2 \to m_j^2)\,, \quad \text{(incoming particle)} \\ \bar{v}_i^{(s)}(-p)\, \hat{\Sigma}_{ij}(p) &= 0\,, \qquad (p^2 \to m_i^2)\,, \quad \text{(incoming anti−particle)} \\ \bar{u}_i^{(s)}(p)\, \hat{\Sigma}_{ij}(p) &= 0\,, \qquad (p^2 \to m_i^2)\,, \quad \text{(outgoing particle)} \\ \hat{\Sigma}_{ij}(p)\, v_j^{(s)}(-p) &= 0\,, \qquad (p^2 \to m_j^2)\,, \quad \text{(outgoing anti−particle)} \end{aligned}$$

where no summation over repeated indices is assumed and $i \neq j$.

(Plus the unit-residue condition)

Solving the on-shell conditions

$$\delta Z_{ij}^{L} = \frac{2}{m_j^2 - m_i^2}\left[\Sigma_{ij}^{\gamma R}\left(m_j^2\right) m_i m_j + \Sigma_{ij}^{\gamma L}\left(m_j^2\right) m_j^2 + m_i \Sigma_{ij}^{L}\left(m_j^2\right) + \Sigma_{ij}^{R}\left(m_j^2\right) m_j\right]$$

$$\delta Z_{ij}^{R} = \frac{2}{m_j^2 - m_i^2}\left[\Sigma_{ij}^{\gamma L}\left(m_j^2\right) m_i m_j + \Sigma_{ij}^{\gamma R}\left(m_j^2\right) m_j^2 + m_i \Sigma_{ij}^{R}\left(m_j^2\right) + \Sigma_{ij}^{L}\left(m_j^2\right) m_j\right]$$

$$\delta \bar{Z}_{ij}^{L} = \frac{2}{m_i^2 - m_j^2}\left[\Sigma_{ij}^{\gamma R}\left(m_i^2\right) m_i m_j + \Sigma_{ij}^{\gamma L}\left(m_i^2\right) m_i^2 + m_i \Sigma_{ij}^{L}\left(m_i^2\right) + \Sigma_{ij}^{R}\left(m_i^2\right) m_j\right]$$

$$\delta \bar{Z}_{ij}^{R} = \frac{2}{m_i^2 - m_j^2}\left[\Sigma_{ij}^{\gamma L}\left(m_i^2\right) m_i m_j + \Sigma_{ij}^{\gamma R}\left(m_i^2\right) m_i^2 + m_i \Sigma_{ij}^{R}\left(m_i^2\right) + \Sigma_{ij}^{L}\left(m_i^2\right) m_j\right]$$

Solving the on-shell condition

Diagonal conditions:

$$\delta m_i = -\frac{1}{2} Re \left\{ m_i \Sigma_{ii}^{\gamma L}\left(m_i^2\right) + m_i \Sigma_{ii}^{\gamma R} + \Sigma_{ii}^{L}\left(m_i^2\right) + \Sigma_{ii}^{R}\left(m_i^2\right) \right\} .$$

$$\delta \bar{Z}_{ii}^{L} = \delta Z_{ii}^{L}, \qquad \delta \bar{Z}_{ii}^{R} = \delta Z_{ii}^{R}$$

$$Z_{ii}^{L} = \Sigma_{ii}^{\gamma L}\left(m_i^2\right) + m_i^2 \left(\Sigma_{ii}^{\gamma L\prime}\left(m_i^2\right) + \Sigma_{ii}^{\gamma R\prime}\left(m_i^2\right) \right) + m_i \left(\Sigma_{ii}^{L\prime}\left(m_i^2\right) + \Sigma_{ii}^{R\prime}\left(m_i^2\right) \right)$$

Note that $\Sigma_{ii}^{R}\left(m_i^2\right) = \Sigma_{ii}^{L}\left(m_i^2\right)$ at one loop in the SM.

Hermiticity?

$$\delta \bar{Z}^{L}_{ij} - \delta Z^{L\dagger}_{ij} = \frac{2}{m_i^2 - m_j^2} \Big\{ \left(\Sigma^{\gamma R}_{ij}\left(m_i^2\right) - \Sigma^{\gamma R*}_{ji}\left(m_i^2\right) \right) m_i m_j$$
$$+ \left(\Sigma^{\gamma L}_{ij}\left(m_i^2\right) - \Sigma^{\gamma L*}_{ji}\left(m_i^2\right) \right) m_i^2$$
$$+ \left(m_i^2 + m_j^2\right) \left(\Sigma^{S}_{ij}\left(m_i^2\right) - \Sigma^{S*}_{ji}\left(m_i^2\right) \right) \Big\} \neq 0 \,,$$

$$\Sigma^{R}_{ij}\left(p^2\right) \equiv \Sigma^{S}_{ij}\left(p^2\right) m_j \,, \qquad \Sigma^{L}_{ij}\left(p^2\right) \equiv m_i \Sigma^{S}_{ij}\left(p^2\right)$$

A similar relation holds for $\delta \bar{Z}^{R}_{ij} - \delta Z^{R\dagger}_{ij}$

i.e.

$$\delta \bar{Z} \neq \delta Z^{\dagger}$$

D.E., Manzano, Talavera (2002)

Hermiticity?

The non-vanishing difference $\delta\bar{Z} \neq \delta Z^\dagger$ is due to the presence of branch cuts in the self-energies that invalidate the pseudo-hermiticity relation

$$\Sigma_{ij}(p) \neq \gamma^0 \Sigma^\dagger_{ij}(p)\gamma^0 \,.$$

In the SM these branch cuts are generically gauge dependent!

Some popular prescriptions existing in the literature do not consider the contribution from absorptive cuts in the WFR constants. Then $\delta\bar{Z} = \delta Z^\dagger$ but the amplitudes are *gauge dependent*.

There is no issue of loss of hermiticity in the bare lagrangian. These WFR are to be used only for the LSZ reduction formulae

These constants need not be the same that appear in δK ($\delta\hat{Z}$). In fact they are not.

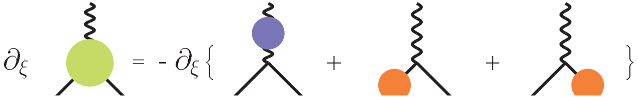
Aoki et al (1982), Denner and Sack (1990),Gambino, Grassi and Madricardo (1999), Diener, Kniehl, Madricardo, Sirlin, Steinhauser (2000 - 2006),...

Gauge invariance

Gauge invariance is an issue

$$\partial_\xi \delta F_L^{(2)} = \partial_\xi \delta F_R^{(1)} = \partial_\xi \delta F_R^{(2)} = 0\,.$$

Nielsen identities:



$$\partial_\xi \left(\bar{u}_u \epsilon^\mu \Gamma^{(1)}_{W^+_\mu \bar{u}_i d_j} v_d \right) = \frac{e}{2 s_W} M_L^{(1)} \partial_\xi \left(\delta \bar{Z}^{uL}_{ir} K_{rj} + K_{ir} \delta Z^{dL}_{rj} + \delta Z_W K_{ij} \right)$$

Absorptive parts are absolutely necessary to fulfill the Nielsen identities. Then

$$0 = \partial_\xi \mathcal{M}_1 = -\frac{e}{2 s_W} M_L^{(1)} \partial_\xi \left[K_{ij} \left(\frac{\delta e}{e} - \frac{\delta s_W}{s_W} \right) + \delta K_{ij} \right]$$

The combination $\frac{\delta e}{e} - \frac{\delta s_W}{s_W}$ is gauge independent, so

$$\partial_\xi \delta K_{ij} = 0$$

Absorptive parts

$$
\begin{aligned}
i\widetilde{Im}_{\xi}\left(\delta Z^{uL}_{ij}\right) &= \sum_h \frac{iK_{ih}K^{\dagger}_{hj}}{8\pi v^2 m^{u2}_j}\theta\left(m^u_j - m^d_h - \sqrt{\xi}M_W\right)\left(m^{u2}_j - m^{d2}_h - \xi M^2_W\right) \\
&\times\sqrt{\left(\left(m^u_j - m^d_h\right)^2 - \xi M^2_W\right)\left(\left(m^u_j + m^d_h\right)^2 - \xi M^2_W\right)}\,, \\
i\widetilde{Im}_{\xi}\left(\delta \bar{Z}^{uL}_{ij}\right) &= \sum_h \frac{iK_{ih}K^{\dagger}_{hj}}{8\pi v^2 m^{u2}_i}\theta\left(m^u_i - m^d_h - \sqrt{\xi}M_W\right)\left(m^{u2}_i - m^{d2}_h - \xi M^2_W\right) \\
&\times\sqrt{\left(\left(m^u_i - m^d_h\right)^2 - \xi M^2_W\right)\left(\left(m^u_i + m^d_h\right)^2 - \xi M^2_W\right)}\,, \\
\widetilde{Im}_{\xi}\left(\delta Z^{uR}_{ij}\right) &= \widetilde{Im}_{\xi}\left(\delta \bar{Z}^{uR}_{ij}\right) = 0\,,
\end{aligned}
$$

For d-type quarks δZ's we have the same formulae replacing $u \leftrightarrow d$ and $K \leftrightarrow K^{\dagger}$.

For t-decay the correction is at the 1 per mille level.

To take home

The scheme proposed is LSZ-compliant (at the one-loop level)

Is gauge invariant. Other prescriptions found in the literature are not.

Even if $\bar{Z} \neq \gamma^0 Z^\dagger \gamma^0$ one can check easily that no poblems with CP or CPT arise:

- $\Gamma = \bar{\Gamma}$,
- $B(i \to f) = B(\bar{i} \to \bar{f})$ if CP holds,
- etc.

EW radiative corrections in this sector are small

BSM and mass-matrix diagonalization

The $D = 4$ fermionic operators in the weak basis need to be transformed to the physical basis to be of any use

$$\mathrm{f} = \left[\tilde{V}_L L + \left(\tilde{V}_{Ru} \tau^u + \tilde{V}_{Rd} \tau^d \right) R \right] \mathrm{f},$$

with the help of the unitary matrices $\tilde{V}_L$,$\tilde{V}_{Ru}$ and $\tilde{V}_{Rd}$

$$\left(\tilde{y}_u^f \tau^u + \tilde{y}_d^f \tau^d \right) \to \left(\tilde{V}_L^\dagger \tilde{y}_u^f \tilde{V}_{Ru} \tau^u + \tilde{V}_L^\dagger \tilde{y}_d^f \tilde{V}_{Rd} \tau^d \right)$$

Then

$$\begin{aligned} X_L &\to \tilde{V}_L^\dagger X_L \tilde{V}_L = D_L, \\ X_{Ru} &\to \tilde{V}_{Ru}^\dagger X_{Ru} \tilde{V}_{Ru} = D_{Ru}, \\ X_{Rd} &\to \tilde{V}_{Rd}^\dagger X_{Rd} \tilde{V}_{Rd} = D_{Rd}, \end{aligned}$$

D_L, D_{Ru} and D_{Rd} are diagonal matrices with eigenvalues $\neq 0$.

BSM and mass-matrix diagonalization

Then, with the help of the non-unitary transformation

$$\mathrm{f} \to \left[D_L^{-\frac{1}{2}} L + \left(D_{Ru}^{-\frac{1}{2}} \tau^u + D_{Rd}^{-\frac{1}{2}} \tau^d \right) R \right] \mathrm{f},$$

The matrix $\tilde{y}_u^f \tau^u + \tilde{y}_d^f \tau^d$ transforms to

$$\left(D_L^{-\frac{1}{2}} \right)^* \tilde{V}_L^\dagger \tilde{y}_u^f \tilde{V}_{Ru} D_{Ru}^{-\frac{1}{2}} \tau^u + \left(D_L^{-\frac{1}{2}} \right)^* \tilde{V}_L^\dagger \tilde{y}_d^f \tilde{V}_{Rd} D_{Rd}^{-\frac{1}{2}} \tau^d \equiv y_u^f \tau^u + y_d^f \tau^d$$

y_u^f and y_d^f are the Yukawa couplings.

The left and right kinetic terms can be brought to the canonical form at the sole expense of redefining the Yukawa couplings. Since this is all there is in the Standard Model, we see that the effect of considering the more general coefficients for the kinetic terms is irrelevant

Effect on BSM terms

These transformations leave some traces

$$
\begin{aligned}
\mathrm{f} \quad \rightarrow \quad & \tilde{V}_L \left(D^L\right)^{\frac{-1}{2}} \left(V_{Lu}\tau^u + V_{Ld}\tau^d\right) L\mathrm{f} \\
& + \left(\tilde{V}_{Ru} \left(D^R_u\right)^{\frac{-1}{2}} V_{Ru}\tau^u + \tilde{V}_{Rd} \left(D^R_d\right)^{\frac{-1}{2}} V_{Rd}\tau^d \right) R\mathrm{f} \\
\equiv \quad & \left(C^u_L\tau^u + C^d_L\tau^d\right) L\mathrm{f} + \left(C^u_R\tau^u + C^d_R\tau^d\right) R\mathrm{f}.
\end{aligned}
$$

Note that because of the presence of the matrices D, the matrices C are in general non-unitary

For R operators: they just redefine the matrices M^i_R $(i = 1, 2, 3)$

For L operators new structures appear

$$\mathcal{L}_L \rightarrow \bar{\mathrm{f}}\gamma_\mu \mathcal{O}^\mu_L L\mathrm{f} + h.c.$$

$$\mathcal{O}^\mu_L = N\tau^u O^\mu_L \tau^u + NK\tau^u O^\mu_L \tau^d + K^\dagger NK\tau^d O^\mu_L \tau^d + K^\dagger N\tau^d O^\mu_L \tau^u$$

$$N \equiv C^{u\dagger}_L M_L C^u_L$$

Why WFR is relevant

Consider

$$\mathcal{L}_L^4 = -\bar{\mathrm{f}}\gamma^\mu \left\{ \left(N^4\tau^u - K^\dagger N^4 K\tau^d\right)\left[-i\partial_\mu + eQA_\mu + \frac{e}{c_W s_W}\left(\frac{\tau^3}{2} - Qs_W^2\right)Z_\mu + g_s\frac{\lambda}{2}\cdot G_\mu\right] + \frac{e}{s_W}\left(N^4 K\frac{\tau^-}{2}W_\mu^+ - K^\dagger N^4\frac{\tau^+}{2}W_\mu^-\right)\right\} L\mathrm{f} + h.c.$$

$\mathcal{L}_L^4$ is the only operator potentially contributing to the gluon and photon effective couplings.

The photon and the gluon are associated to currents which are exactly conserved and radiative corrections (including those from NP) are prohibited at zero momentum transfer.
However one must take into account the WFR.

In fact $\mathcal{L}_L^4$ is the only operator that can possibly contribute to such renormalization at the order we are working.

Eventually $\mathcal{L}_L^4$ drops from observables involving neutral couplings.

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Eventually $\mathcal{L}_L^4$ drops from observables involving neutral couplings.

Effective couplings

Z effective couplings

$$
\begin{aligned}
g_L^u &= -N^1 - N^{1\dagger} + N^{2\dagger} + N^2 + N^3 + N^{3\dagger}, \\
g_L^d &= K^\dagger \left(N^1 + N^{1\dagger} + N^{2\dagger} + N^2 - N^3 - N^{3\dagger}\right) K, \\
g_R^u &= \tilde{M}_R^1 + \tilde{M}_R^{1\dagger} + \tilde{M}_R^2 + \tilde{M}_R^{2\dagger} + \tilde{M}_R^3 + \tilde{M}_R^{3\dagger}, \\
g_R^d &= \tilde{M}_R^2 + \tilde{M}_R^{2\dagger} - \tilde{M}_R^1 - \tilde{M}_R^{1\dagger} - \tilde{M}_R^3 - \tilde{M}_R^{3\dagger}.
\end{aligned}
$$

W couplings (everything included: WFR, CKM etc.)

$$
\begin{aligned}
h_L &= \left(-N^1 - N^{1\dagger} + N^2 - N^{2\dagger} - N^3 - N^{3\dagger} + N^4 - N^{4\dagger}\right) K, \\
h_R &= \left(\tilde{M}_R^1 + \tilde{M}_R^{1\dagger} + \tilde{M}_R^2 - \tilde{M}_R^{2\dagger} - \tilde{M}_R^3 - \tilde{M}_R^{3\dagger}\right).
\end{aligned}
$$

$$\mathcal{L}_L = \bar{\mathrm{f}}\gamma_\mu S^\mu L \mathrm{f} + h.c.$$

$$S^\mu \equiv N\tau^u O^\mu \tau^u + NK\tau^u O^\mu \tau^d + K^\dagger NK\tau^d O^\mu \tau^d + K^\dagger N\tau^d O^\mu \tau^u$$

Under *CP* $S^\mu \to S'^\mu$

$$S'^\mu \equiv N^t\tau^u O^\mu \tau^u + K^t N^t \tau^d O^\mu \tau^u + K^t N^t K^* \tau^d O^\mu \tau^d + N^t K^* \tau^u O^\mu \tau^d$$

For *CP* invariance we require

$$\begin{aligned} N &= N^*, \\ NK &= NK^*, \\ K^t N K^* &= K^\dagger N K \end{aligned}$$

Sufficient (but not necessary) condition:

$$N = N^*, \qquad K = K^*$$

More about CP-violation

Even if the matrices $M_{L,R}$ were real phases do appear after the diagonalization:

- due to the appearance of the usual CKM matrix in operators involving L fields
- diagonalization matrices appear explicitly, both for left and right-handed operators
- the effective operators couplings are redefined by matrices which are not unitary in general.
- large custodially breaking contributions in the NP could give different values for X_{Ru} and X_{Rd}, yielding eigenvalues < 1 possibly enhancing CP violation in R sector.

In the Standard Model there is a link between the existence of three families and the presence of CP violation. This disappears completely, both in the left and right-handed sectors, once additional operators are included.

Unitarity triangle

How can we check for the presence of all this wealth of new phases?

In the left-handed sector the analysis is usually done in terms of the unitarity triangle. Clearly the unitarity triangle as such is gone once the additional $d = 4$ operators are included.

$$\mathcal{U} = K + GK$$

where G is a combination of the N matrices. Since G is not antihermitian, $\mathcal{U}$ is not unitary in a perturbative sense.

However, these deviations of unitarity due to radiative corrections shall be small.

Conclusions

- Unitarity is a powerful constraint on scattering amplitudes. Its validity is well tested. Even in the presence of a light Higgs, it helps constrain "anomalous" couplings by helping predict heavier resonances.
- An extended EWSBS would typically have such resonances even in the presence of a light 'Higgs'. However their properties are radically different from the 'naive expectations'
- Current LHC searches do not yet probe the IAM resonances: at least $10\times$ statistics is required. X-sections are too smalls to explain 'resonances'.
- Direct coupling of the resonances to quarks (Drell-Yan) probe the "anomalous" fermionin operators. They can be extended to the flavour sector non-trivially
- NP (if present at all) seems to be hidden in small LEC (of order 10^{-3} or perhaps less).
- An extended fermionic sector would lead to a wealth of new phenomena in flavour physics: new *CP* violating phases, non-unitarity of (measured) CKM.

THANK YOU!