B physics with lattice QCD: status and prospects

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Outline

- Motivation and Introduction
  - lattice QCD
- Results
  - leptonic decays
  - semileptonic decays
  - neutral meson mixing
  - summary of B,D,K results
- Phenomenology
  - CKM determinations
  - UT analysis
  - BSM phenomenology
- Summary and Outlook

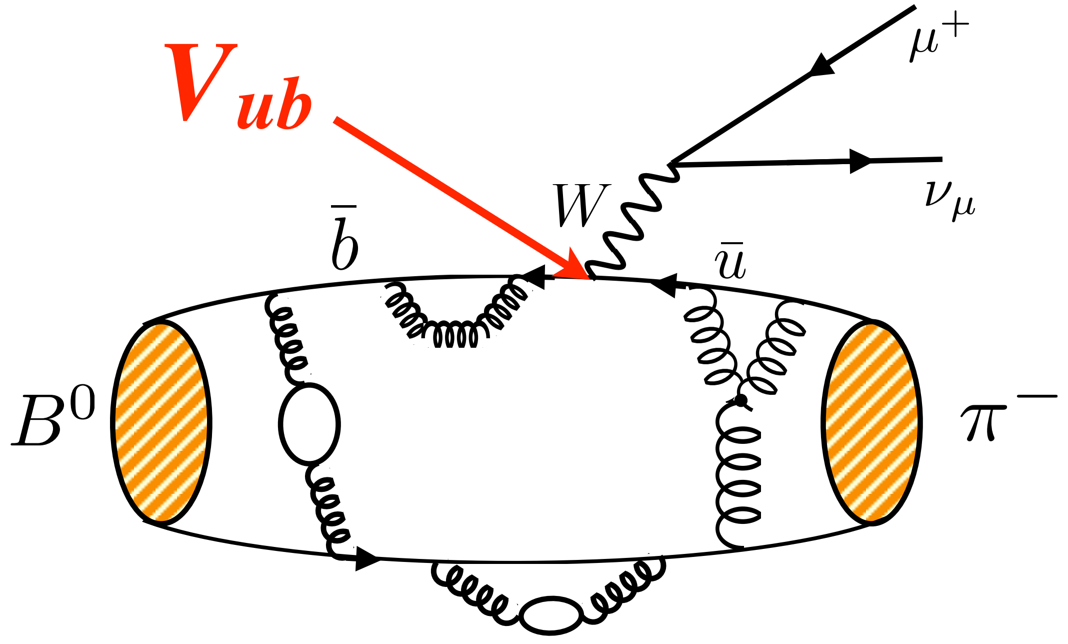
Outline

- Motivation and Introduction
  - lattice QCD
- Results

The focus of this talk is on “**simple**” quantities:
hadronic matrix elements of local operators between single (stable) meson states for which lattice results exist with complete systematic error budgets.

- Phenomenology
  - CKM determinations
  - UT analysis
  - BSM phenomenology
- Summary and Outlook

Introduction



example: $B^0 \to \pi^- \ell^+ \nu_\ell$

Experiment vs. SM theory:

(experiment) = (known) x (**CKM factor**) x (had. matrix element)

$$\frac{d\Gamma(B\to\pi\ell\nu)}{dq^2}, \frac{d\Gamma(B\to K\ell^+\ell^-)}{dq^2}, \ldots$$

$$\frac{d\Gamma(B\to D\ell\nu)}{d\omega}, \frac{d\Gamma(B\to D\tau\nu)}{d\omega}, \ldots$$

$$\Delta m_{d(s)}$$

$$\vdots$$

Lattice QCD

parameterize the MEs in terms of form factors, decay constants, bag parameters, ...

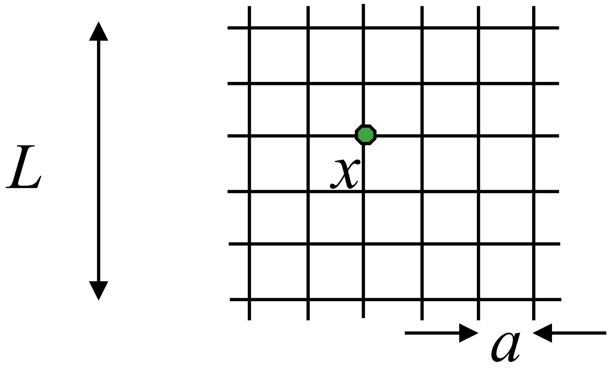
simple processes for CKM determinations

$$\begin{pmatrix}
\begin{array}{c} V_{ud} \\ \pi \to \mu\nu \end{array} & \begin{array}{c} V_{us} \\ K \to \pi\ell\nu \\ K \to \mu\nu \end{array} & \begin{array}{c} V_{ub} \\ B \to \pi\ell\nu,\ B_s \to K\ell\nu \\ \Lambda_b \to p\ell\nu \end{array} \\
\begin{array}{c} V_{cd} \\ D \to \pi\ell\nu \\ D \to \ell\nu \end{array} & \begin{array}{c} V_{cs} \\ D \to K\ell\nu \\ D_s \to \ell\nu \end{array} & \begin{array}{c} V_{cb} \\ B_{(s)} \to D_{(s)}, D^*_{(s)}\ell\nu \end{array} \\
\begin{array}{c} V_{td} \\ B^0 - \overline{B^0} \\ B \to \pi\ell\ell \end{array} & \begin{array}{c} V_{ts} \\ B^0_s - \overline{B^0_s} \\ B \to K\ell\ell \end{array} & \begin{array}{c} V_{tb} \end{array}
\end{pmatrix}$$

$$(\rho, \eta) \quad K^0 - \overline{K^0}$$

Lattice QCD Introduction

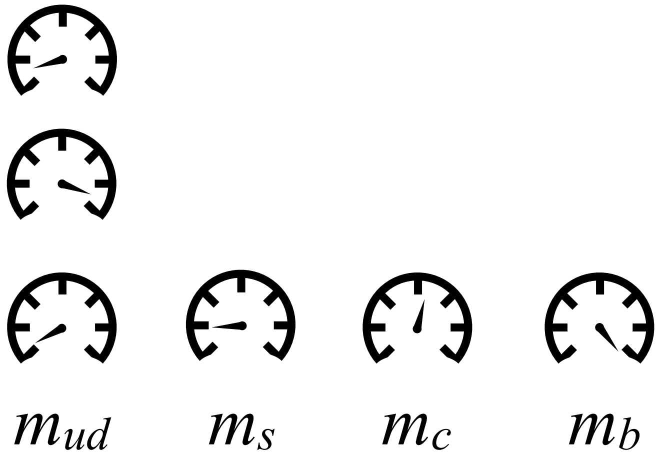
$$\mathcal{L}_{\text{QCD}} = \sum_f \bar\psi_f (\not{D} + m_f)\psi_f + \frac{1}{4}\text{tr}F_{\mu\nu}F^{\mu\nu}$$



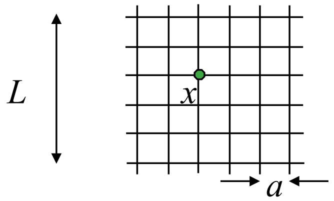
- ✦ discrete Euclidean space-time (spacing a)
  derivatives ➙ difference operators, etc…
- ✦ finite spatial volume (L)
- ✦ finite time extent (T)

adjustable parameters

- ❖ lattice spacing: $a \to 0$
- ❖ finite volume, time: $L \to \infty,\ T > L$
- ❖ quark masses (m_f): $M_{H,\text{lat}} = M_{H,\text{exp}}$
  tune using hadron masses
  extrapolations/interpolations $m_f \to m_{f,\text{phys}}$



- ❖ also: n_f = number of sea quarks: 3 (2+1), 4 (2+1+1)



Lattice QCD Introduction

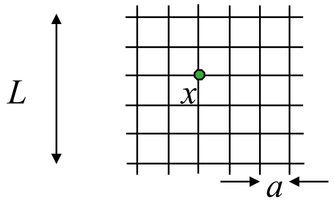
$$\langle \mathcal{O} \rangle \sim \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \, \mathcal{O}(\psi, \bar{\psi}, A) \, e^{-S} \qquad S = \int d^4x \left[\bar{\psi}(\not{D} + m)\psi + \frac{1}{4}(F^a_{\mu\nu})^2 \right]$$

use monte carlo methods (importance sampling) to evaluate the integral.

Note: Integrating over the fermion fields leaves $\det(\not{D} + m)$ in the integrand. The correlation functions, $\mathcal{O}$, are then written in terms of $(\not{D}+m)^{-1}$ and gluon fields.

steps of a lattice QCD calculation:

1. generate gluon field configurations according to $\det(\not{D}+m)\, e^{-S}$
2. calculate quark propagators, $(\not{D}+m_q)^{-1}$, for each valence quark flavor and source point
3. tie together quark propagators into hadronic correlation functions (usually 2 or 3-pt functions)
4. statistical analysis to extract hadron masses, energies, hadronic matrix elements, …. from correlation functions
5. systematic error analysis



Lattice QCD Introduction

systematic error analysis

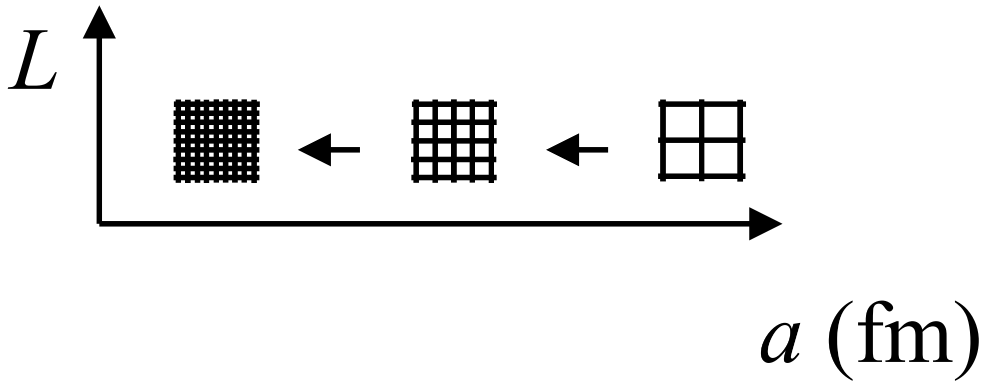
...of lattice spacing, chiral, heavy quark, and finite volume effects is based on EFT (Effective Field Theory) descriptions of QCD
→ ab initio

The EFT description:

- provides functional form for extrapolation (or interpolation)
- can be used to build improved lattice actions/methods
- can be used to anticipate the size of systematic effects

To control and reliably estimate the systematic errors

- repeat the calculation on several lattice spacings, light quark masses, spatial volumes, ...



Heavy Quark Treatment

- For light quarks ($m_\ell < \Lambda_{\mathrm{QCD}}$), leading discretization errors ~ $\alpha_s^k (a\Lambda_{\mathrm{QCD}})^n$
- For heavy quarks, leading discretization errors ~ $\alpha_s^k (am_h)^n$
  with currently available lattice spacings

  for b quarks $am_b > 1$

  for charm $am_c \sim 0.15\text{-}0.6$

⟹ need effective field theory methods for *b* quarks
for charm can use light quark methods, if action is sufficiently improved

- avoid errors of $(am_b)^n$ in the action by using EFT:
  - ✦ relativistic HQ actions (Fermilab, Columbia, Tsukuba)
  - ✦ HQET
  - ✦ NRQCD

or

- use improved light quark actions for charm (HISQ, tmWilson, NP imp. Wilson,...)
  and for *b*:
  - ✦ use same LQ action as for charm but keep $am_h < 1$,
  - ✦ use HQET and/or static limit to extrapolate/interpolate to *b* quark mass

chiral-continuum extrapolation

Some ensembles still have

$$m_{\text{light}} > 1/2\ (m_u + m_d)_{\text{phys}}$$

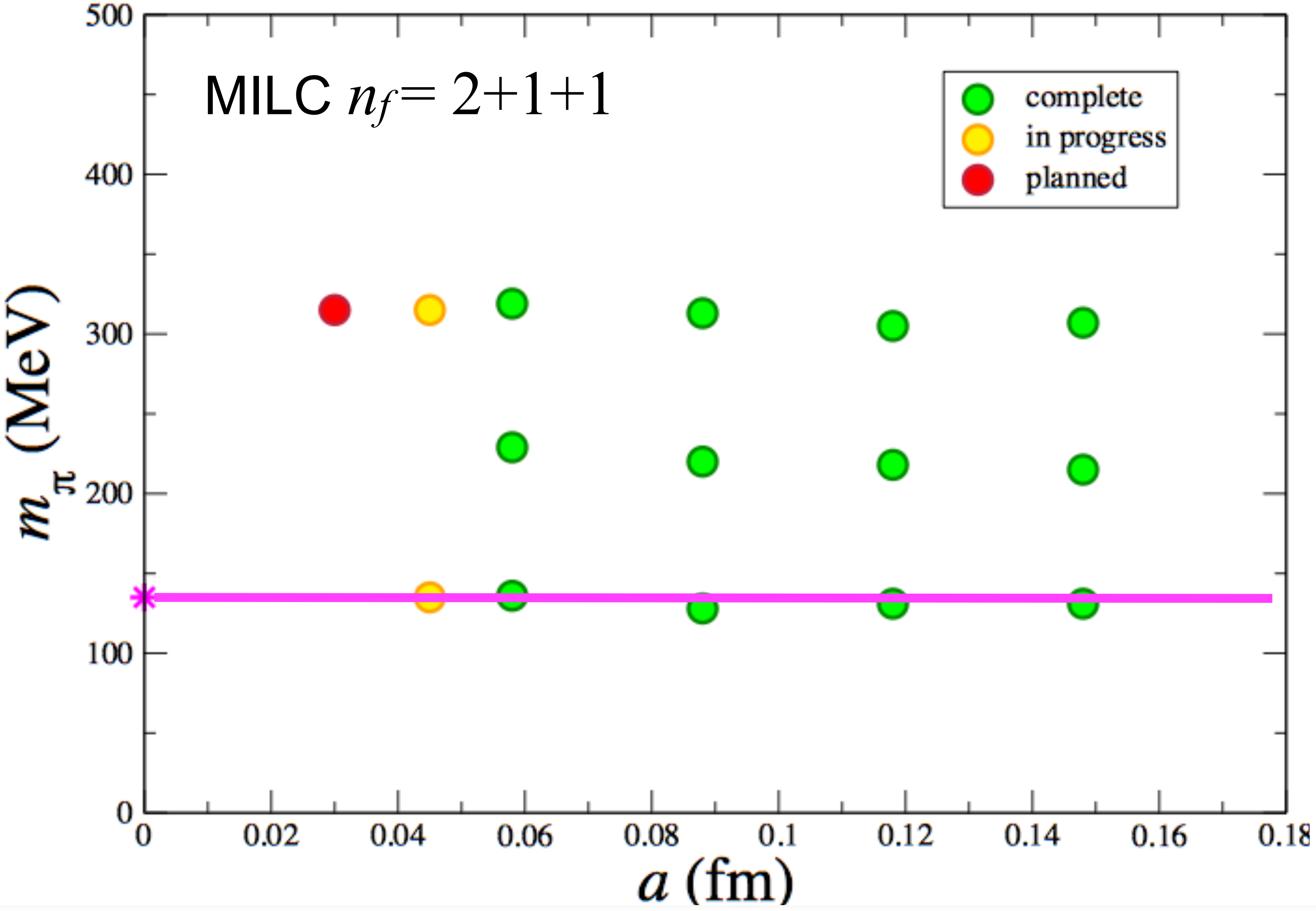
χPT guides the extrapolation/interpolation to the physical point.

- include (light quark) discretization effects (for example, staggered χPT)
- can also add HQ discretization terms to chiral-continuum fits
- combined chiral-continuum extrapolation/interpolation

- for B,D meson processes use Heavy Meson χPT: χPT + $1/M$ expansion

chiral-continuum extrapolation

Example: Set of ensembles by MILC collaboration



Five collaborations have now generated sets of ensembles that include sea quarks with physical light-quark masses:

PACS-CS, BMW, MILC, RBC/UKQCD, ETM

finite volume effects

One stable hadron (meson) in initial/final state:

If L is large enough, FV error $\sim e^{-m_\pi L}$

- keep $m_\pi L \gtrsim 4$

To quantify residual error:

- include FV effects in χPT
- compare results at several Ls (with other parameters fixed)

The story changes completely with two or more hadrons in initial/final state!
(or if there are two or more intermediate state hadrons)

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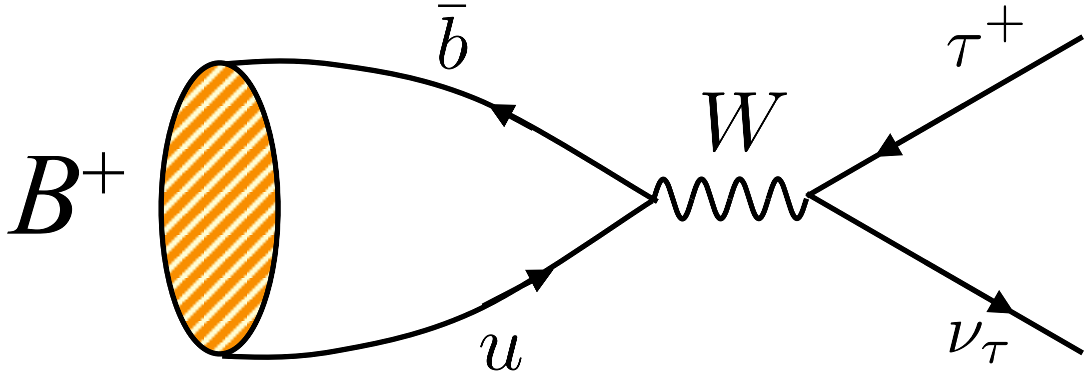
Leptonic B-meson decay

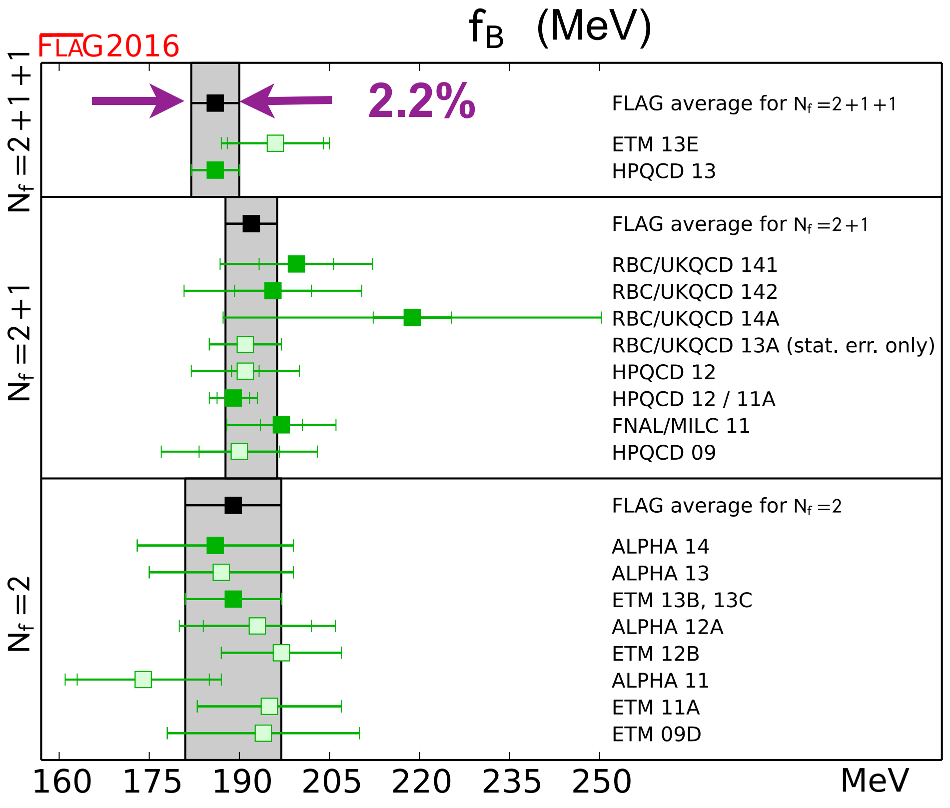
Example: $B^+ \to \tau^+ \nu_\tau$

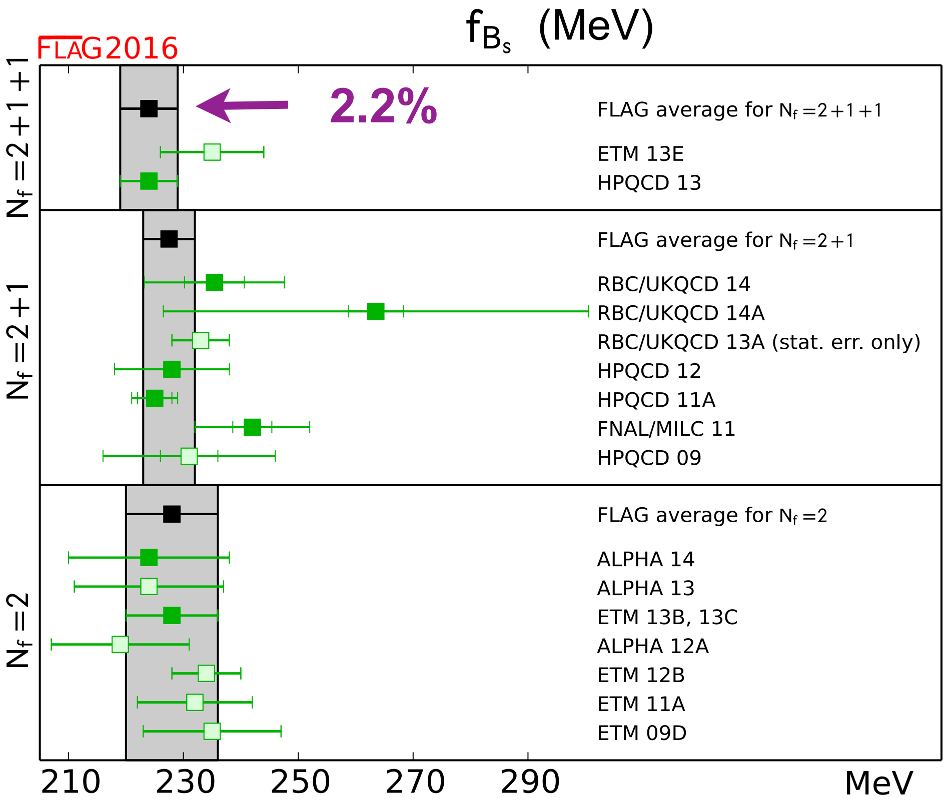


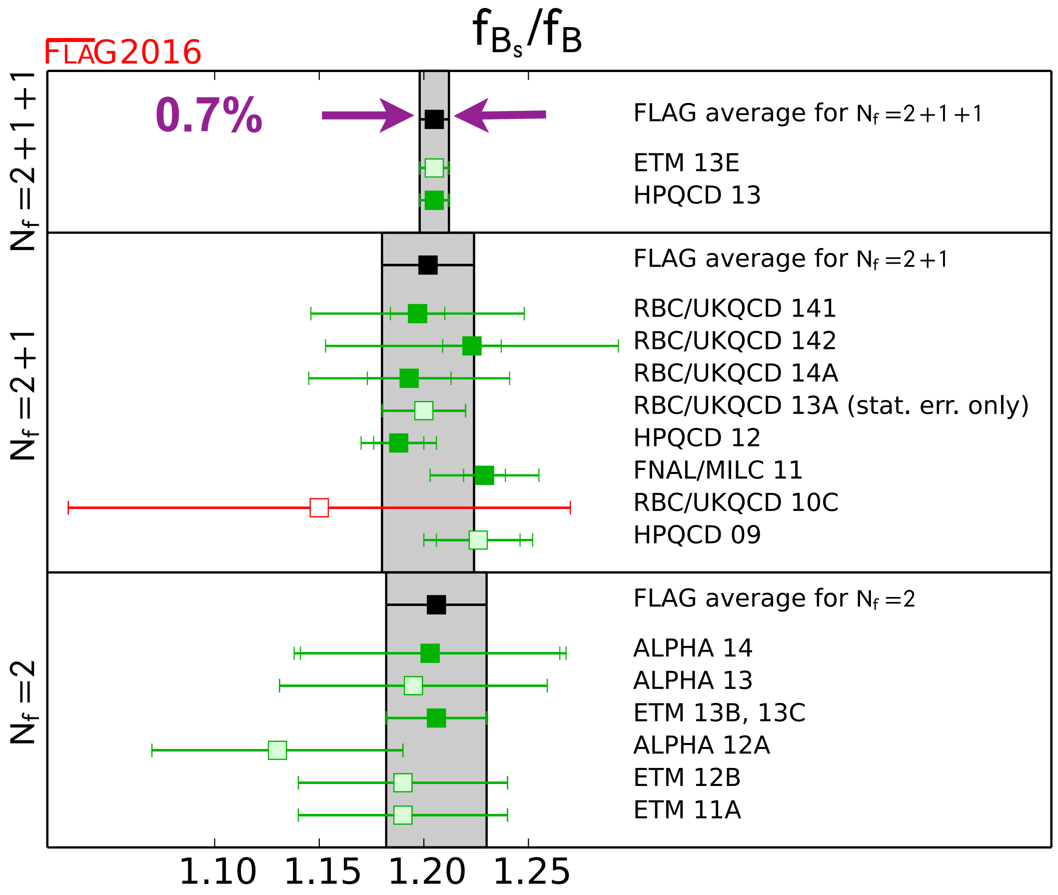
$$\Gamma(B^+ \to \tau^+ \nu_\tau) = (\text{known}) \times |V_{ub}|^2 f_B^2$$

- use experiment + LQCD input for determination of CKM element.
- SU(3) **ratio** f_{B_s}/f_{B_d}: statistical and systematic errors tend to cancel.
- Decay constants are also needed for rare leptonic decay, $B_{s(d)} \to \mu\mu$.

B decay constant summary



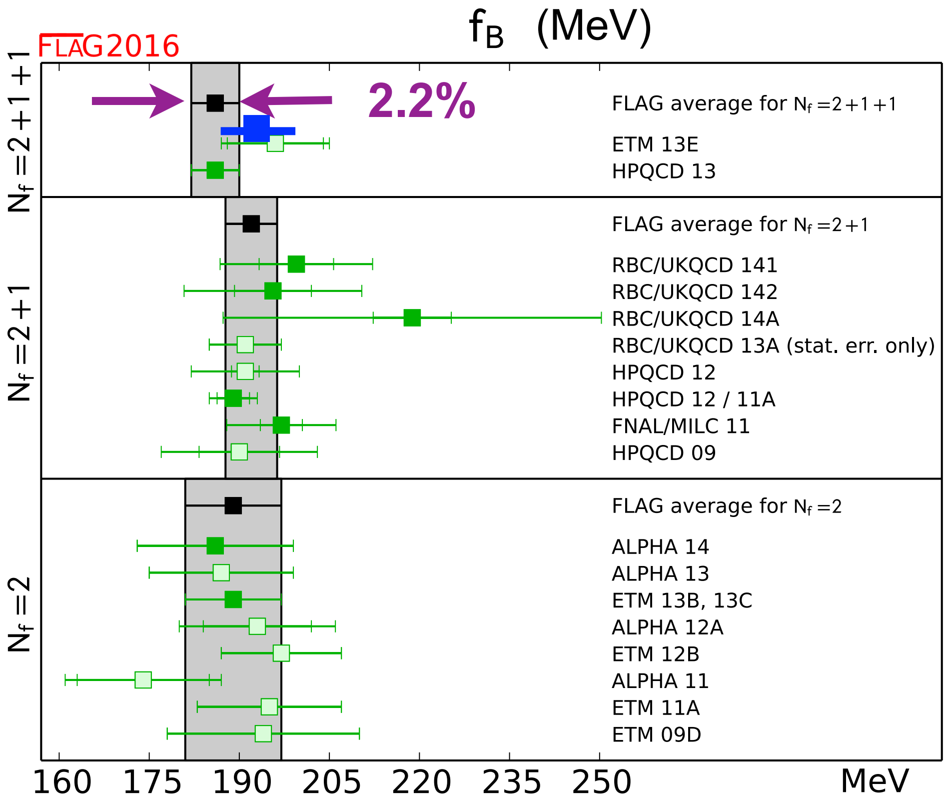


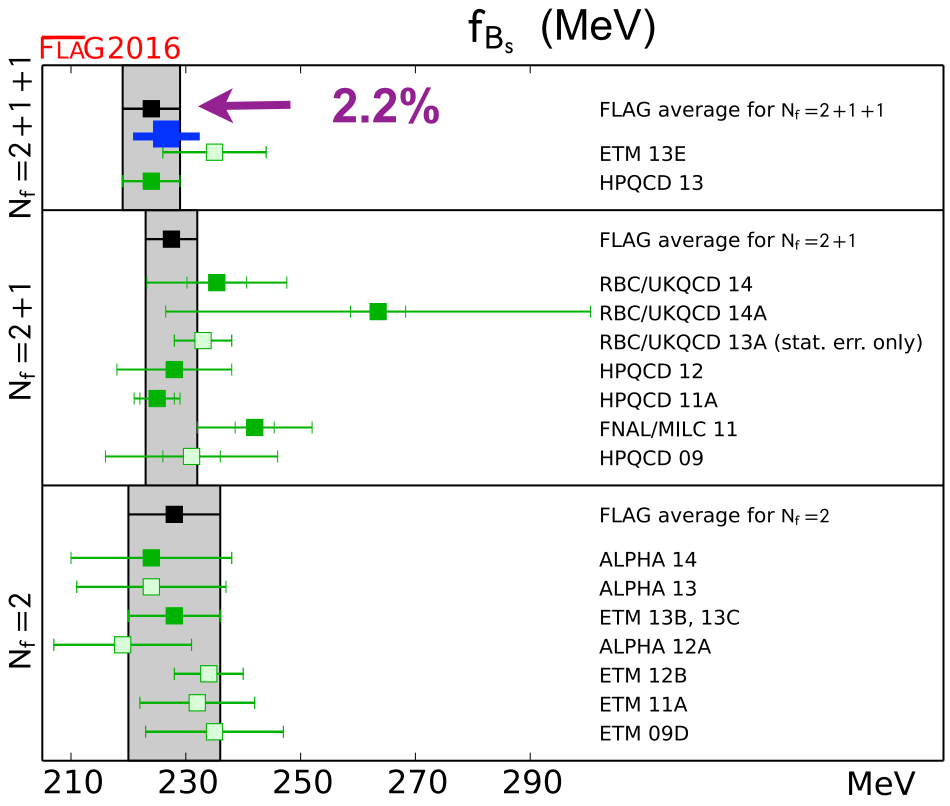


S. Aoki et al
(FLAG-3 review,
arXiv:1607.00299)

status
end 2015

B decay constant summary





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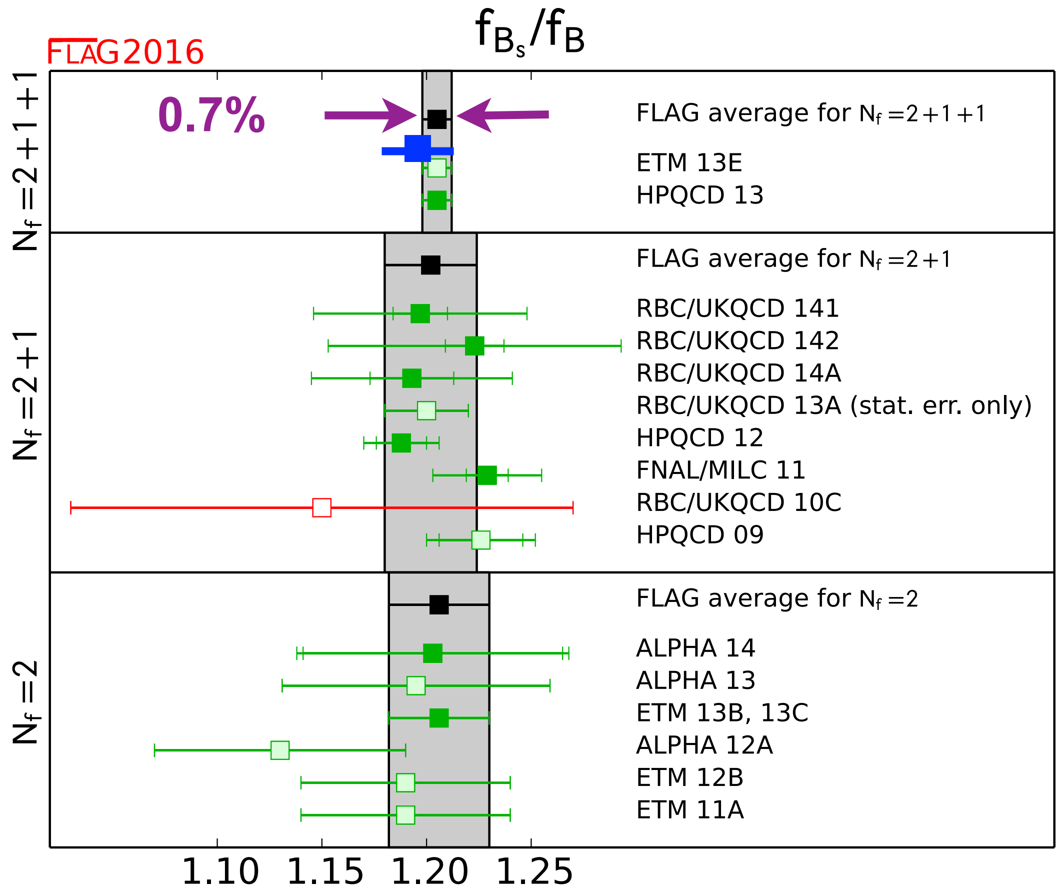
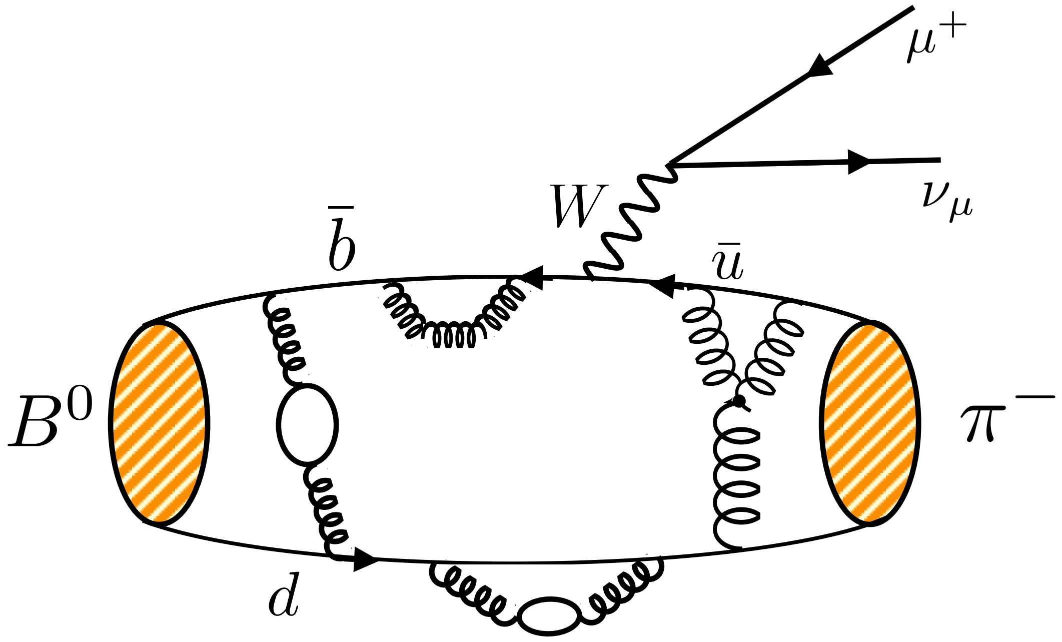


- ✦ new results by ETM (arXiv:1603.04306, 2016 PRD)
- ✦ ongoing work by FNAL/MILC (Komijani @ Lattice 2016), RBC/UKQCD, …

  ➠ expect to reduce errors on f_B, f_{B_s} to ≲ 1%

Semileptonic B decay to light hadrons

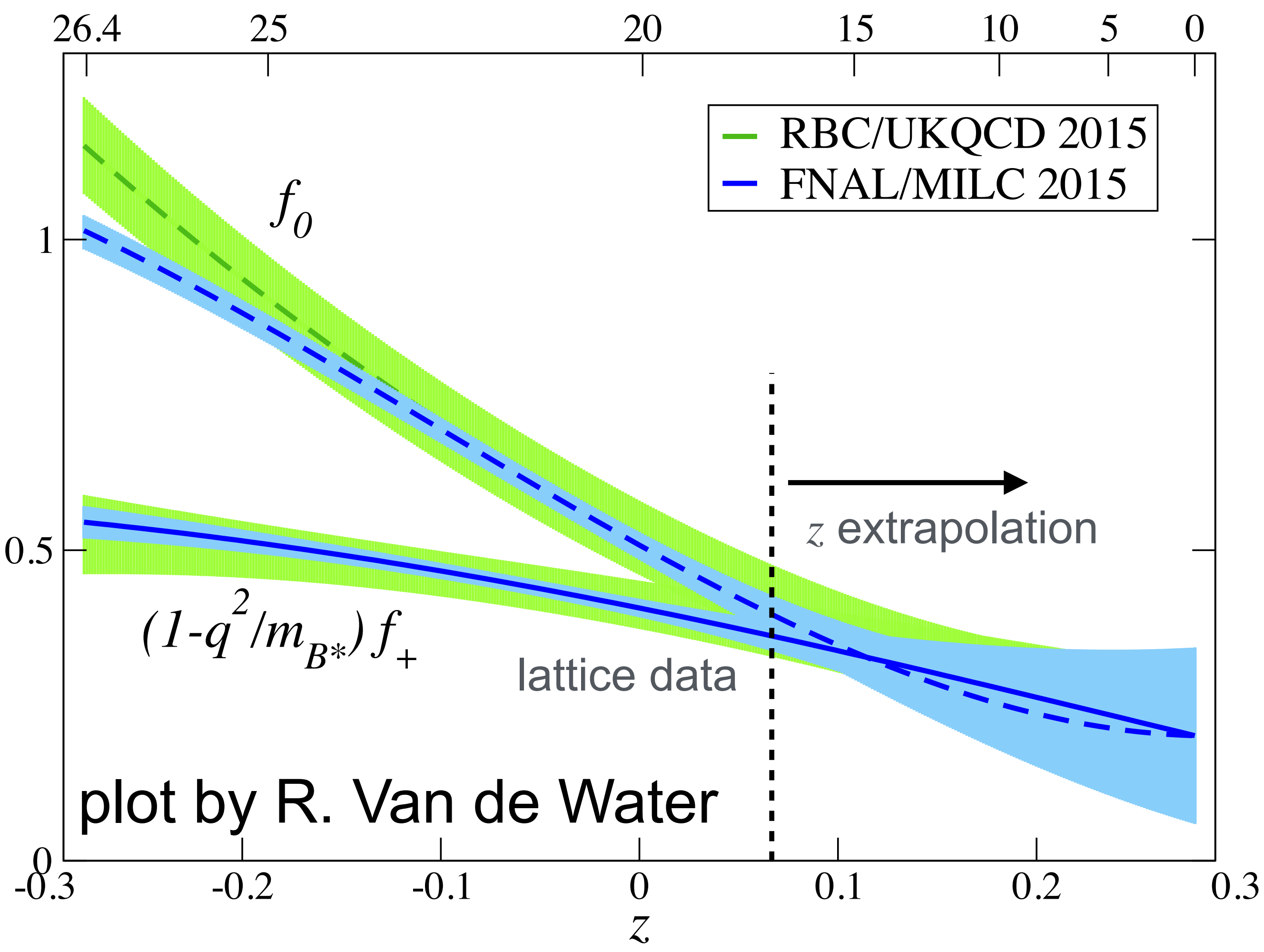
Example: $B \to \pi \ell \nu$



$$\frac{d\Gamma(B\to\pi\ell\nu)}{dq^2} = (\text{known}) \times \left|V_{ub}\right|^2 \times \left|f_+(q^2)\right|^2$$

- ⋆ calculate the form factors in the low recoil energy (high q^2) range.
- ⋆ use z-expansion for model-independent parameterization of q^2 dependence.
- ⋆ calculate the complete set of form factors, $f_+(q^2), f_0(q^2)$ and $f_T(q^2)$.
- ⋆ for $f_+(q^2)$ compare shape between experiment and lattice.

form factors for $B \to \pi \ell \nu$ & V_{ub}

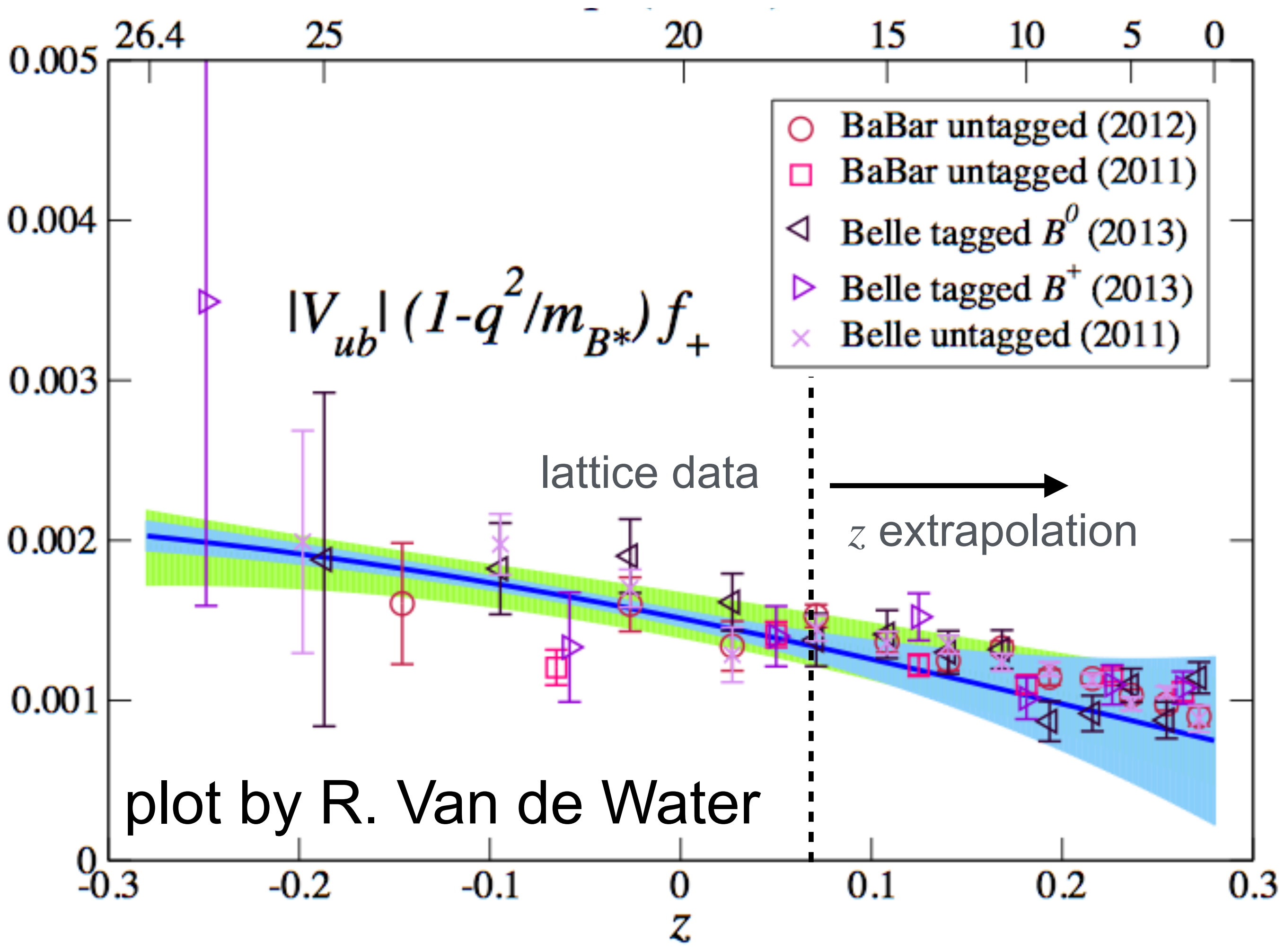


RBC (arXiv:1501.05373, PRD 2015)

FNAL/MILC (arXiv:1503.07839, PRD 2015)

- FNAL/MILC & RBC form factors are in good agreement
- HPQCD (arXiv:1510.07446, PRD 2016): f_0 with physical light quarks at zero recoil satisfies soft-pion theorem
- Note: two independent LQCD **predictions** for $B_s \to K\ell\nu$ form factors (HPQCD, arXiv:1406.2279, PRD 2014; RBC, arXiv:1501.05373, PRD 2015)
  + ongoing work by ALPHA (Banerjee, Koren @ Lattice 2016), FNAL/MILC, …

form factors for $B \to \pi \ell \nu$ & V_{ub}



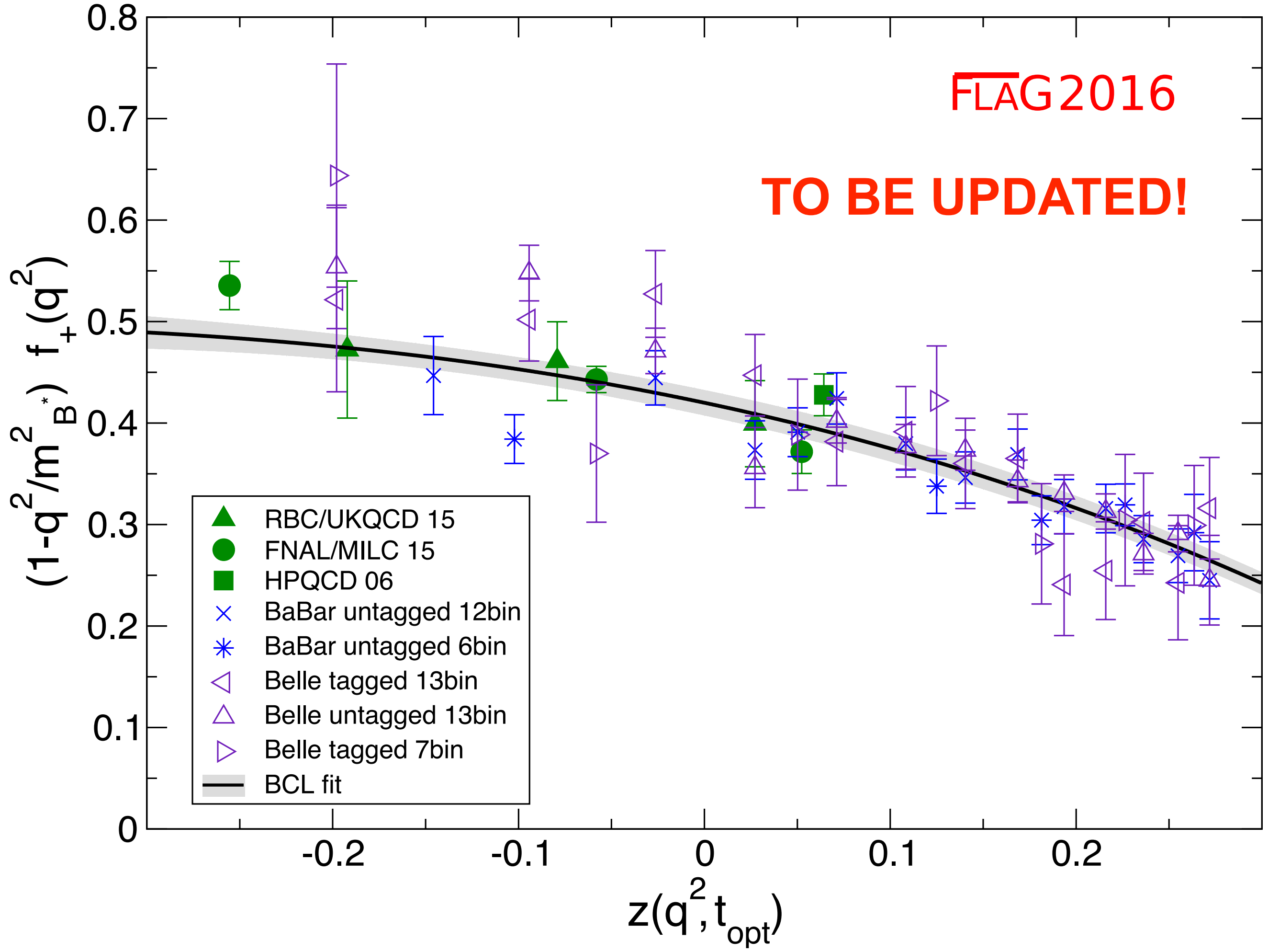
RBC (arXiv:1501.05373, PRD 2015)

FNAL/MILC (arXiv:1503.07839, PRD 2015)

$|V_{ub}| = 3.72\,(16)\,10^{-3}$

- shape of f_+ agrees with experiment and uncertainties are commensurate
- fit lattice form factors together with experimental data to determine $|V_{ub}|$ **and** obtain form factors (f_+, f_0) with improved precision…

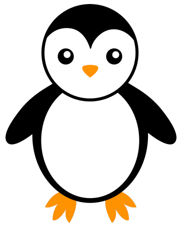
form factors for $B \to \pi \ell \nu$ & V_{ub}



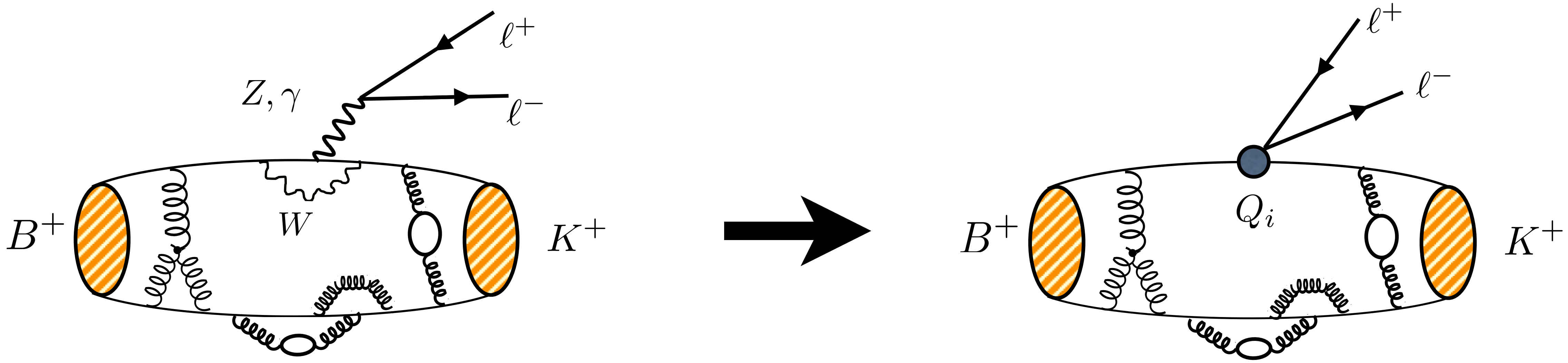
S. Aoki et al (FLAG-3 review, arXiv:1607.00299)

- shape of f_+ agrees with experiment and uncertainties are commensurate
- fit lattice form factors together with experimental data to determine $|V_{ub}|$ **and** obtain form factors (f_+, f_0) with improved precision…

- **Note:** plot is for illustration only. FLAG-3 will update this combined fit soon!



Rare semileptonic B decay



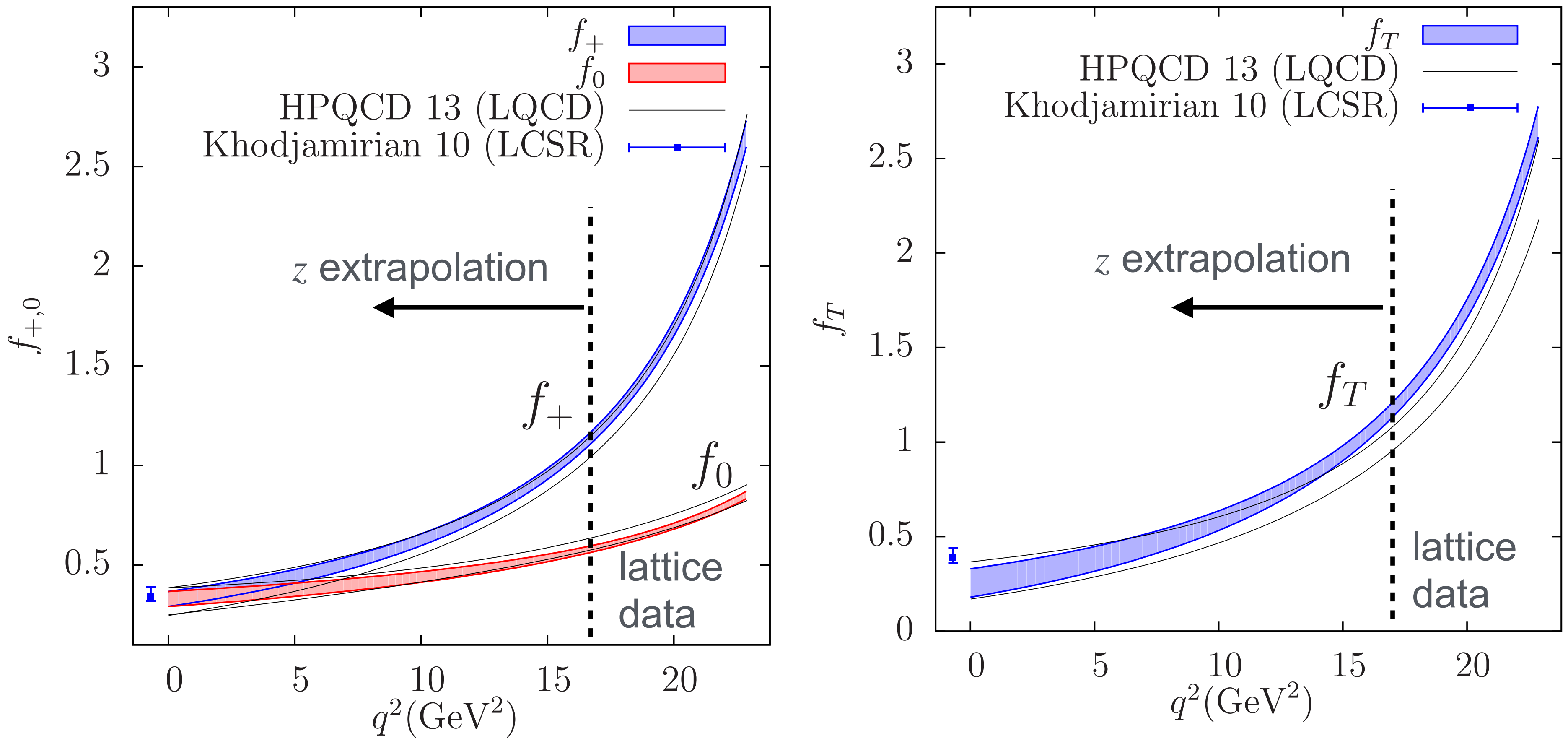
$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tq}^* V_{tb} \sum_i C_i(\mu) Q_i + \dots$$

Parameterize the amplitude in terms of the three form factors $f_{+,0,T}(q^2)$:

$$A(B \to P\,\ell\ell) \sim C_7^{\text{eff}} f_T + (C_9^{\text{eff}} + C_{10}) f_+ + \text{nonfactorizable terms}$$

see Hurth talk

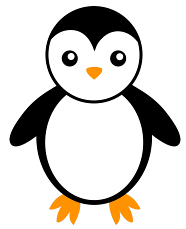
form factors for $B \to K\,\ell\ell$



HPQCD (arXiv:1306.0434, 1306.2384, PRL 2013)

FNAL/MILC
(arXiv:1509.06235, PRD 2016)

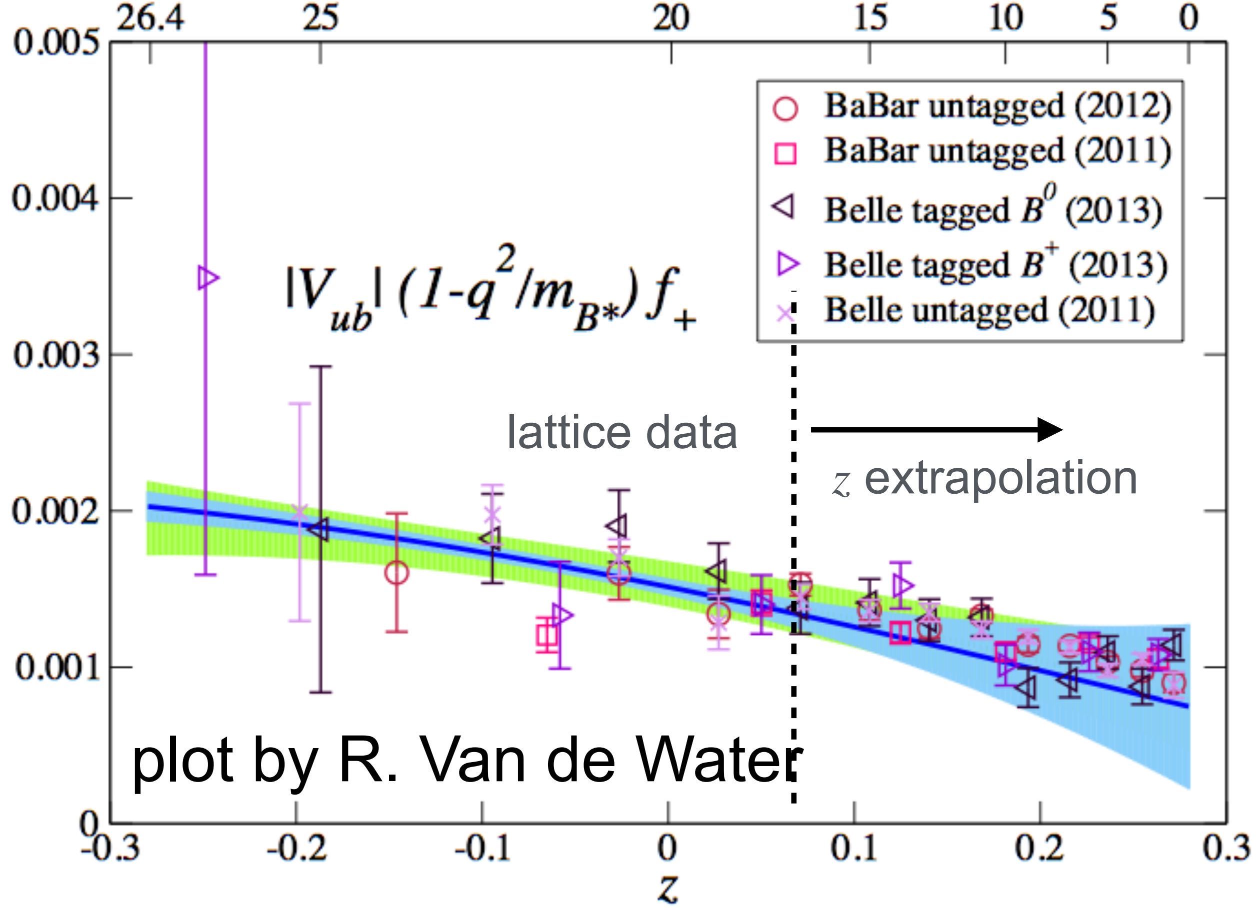
- ☆ Two LQCD calculations (on overlapping ensemble sets, different valence actions):
  HPQCD (NRQCD b + HISQ), FNAL/MILC (Fermilab b + asqtad)
- ☆ consistent results for all three form factors
- ☆ also consistent with LCSR (Khodjamarian et al, arXiv:1006.4945, JHEP 2010)
- ☆ Note: First LQCD calculation of $\Lambda_b \to \Lambda\,\ell^+\ell^-$ form factors (10 total)
  (see Meinel talk)



form factors for $B \to \pi \, \ell\ell$

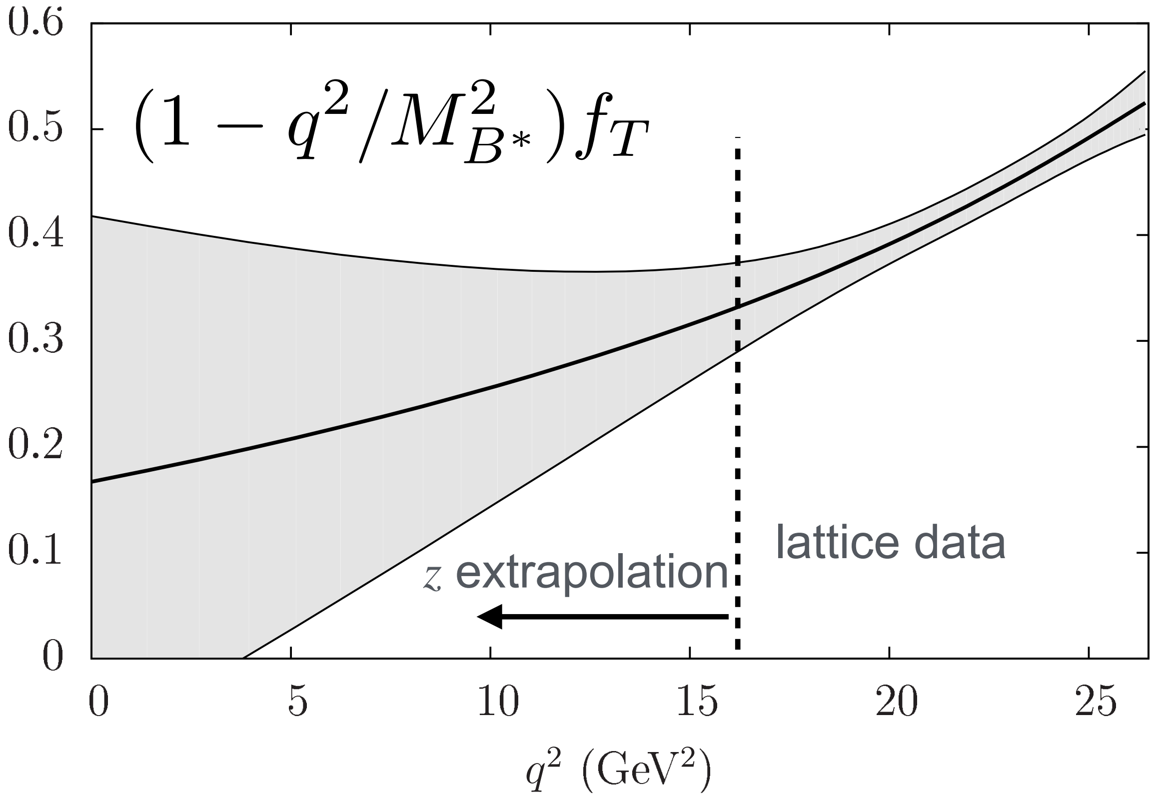
RBC (arXiv:1501.05373, PRD 2015)

FNAL/MILC (arXiv:1503.07839, PRD 2015)



plot by R. Van de Water

FNAL/MILC (arXiv:1507.01618, PRL 2015)

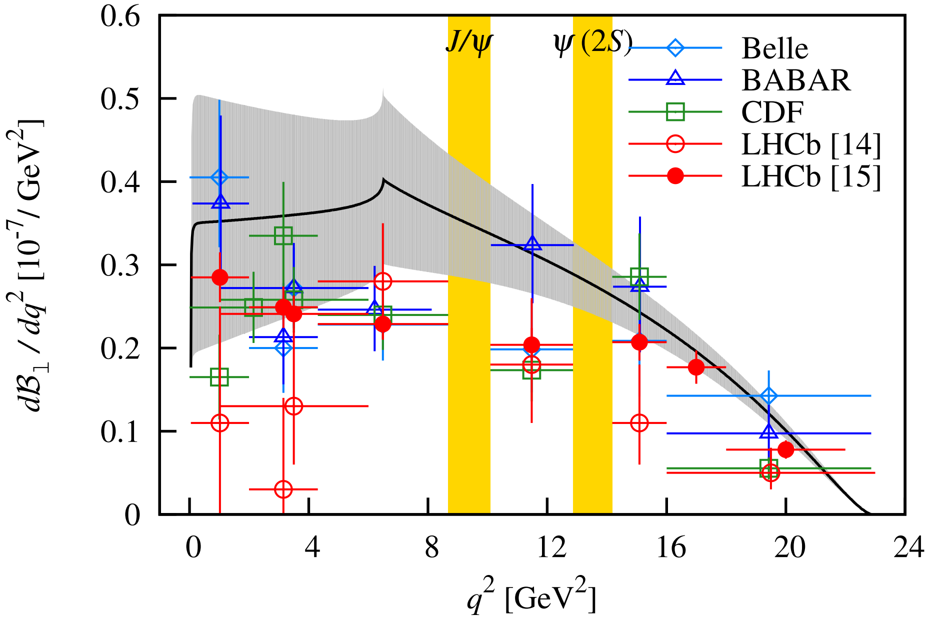


- **First** LQCD calculation of f_T by FNAL/MILC
- Take f_+, f_0 from combined fit of lattice form factors + experimental data for $d\mathcal{B}(B \to \pi \ell \nu)/dq^2$

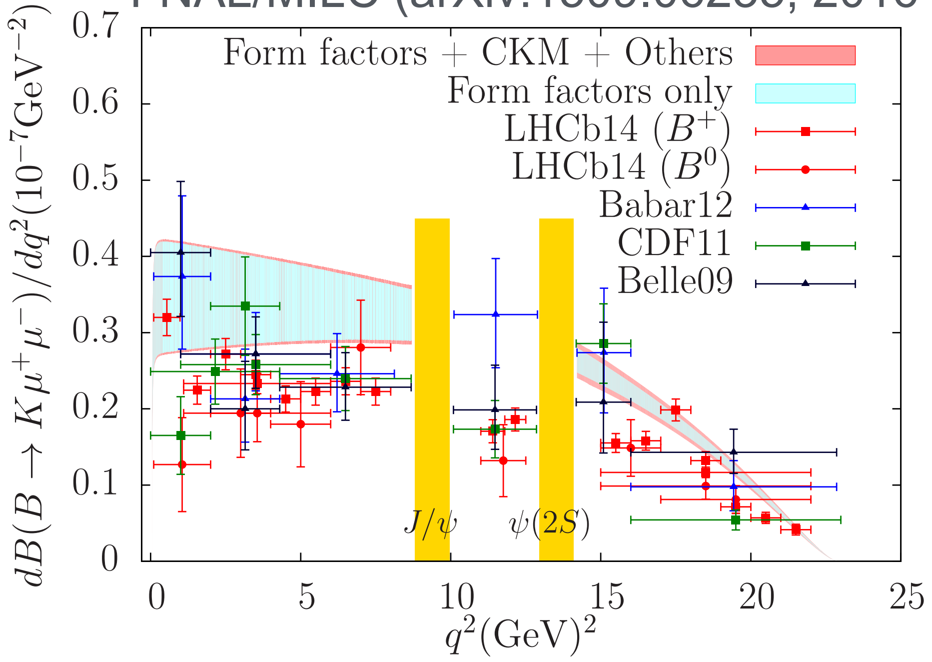
Phenomenology for $B \to K, \pi\, \ell^+ \ell^-$

Experiment vs. Theory

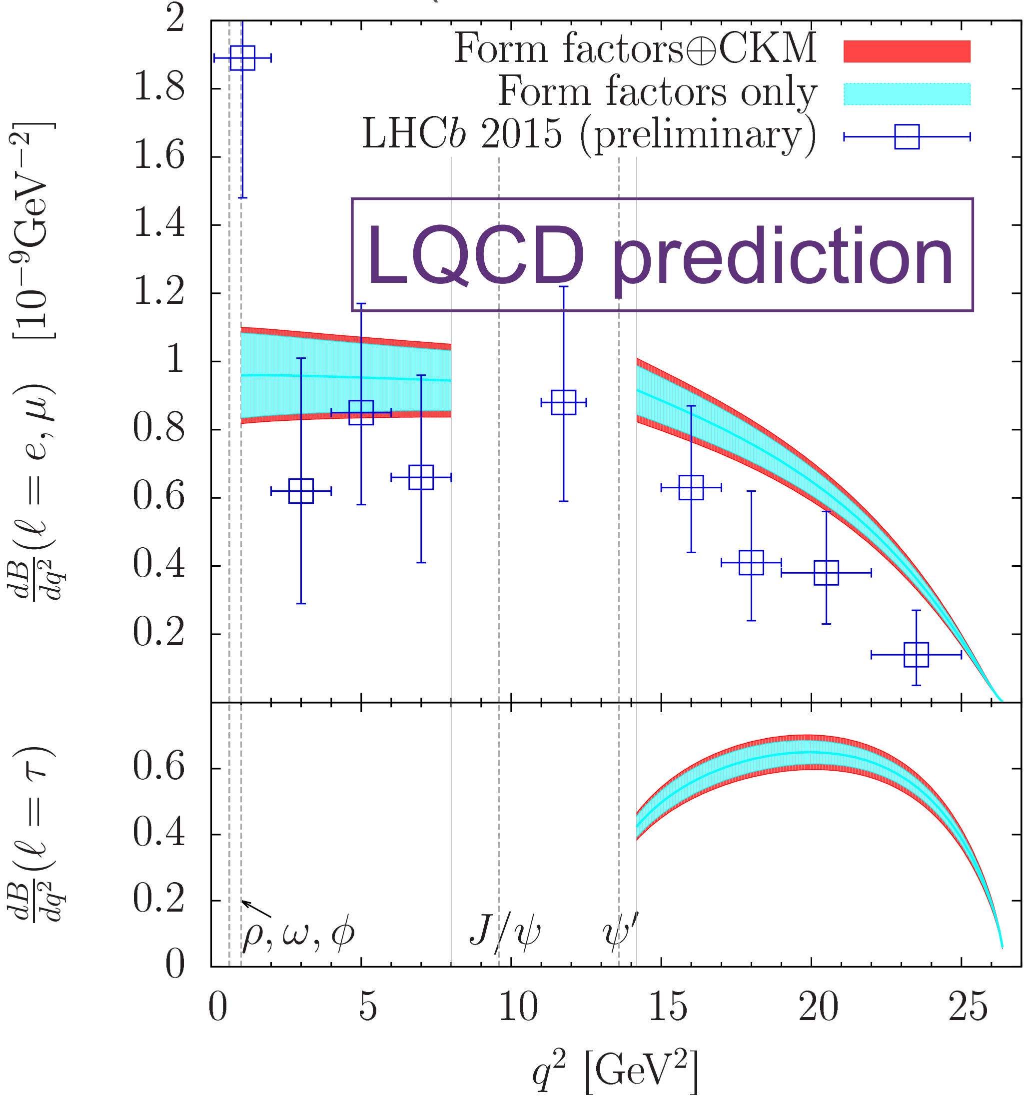
HPQCD (arXiv:1306.0434, 2013 PRL)

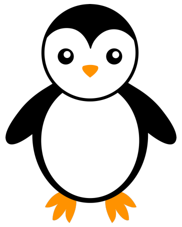


FNAL/MILC (arXiv:1509.06235, 2016 PRD)



FNAL/MILC (arXiv:1507.01618, 2015 PRL)





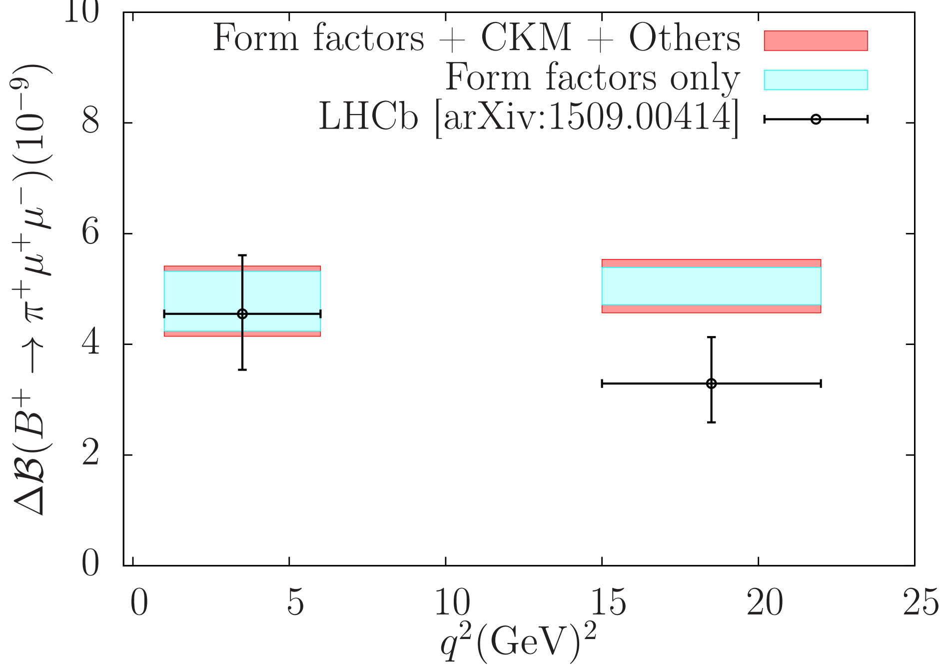
Phenomenology for $B \to K, \pi \, \ell^+ \ell^-$

Experiment vs. theory

- LHCb data + FNAL/MILC form factors (arXiv:1509.00414, JHEP 2015;1403.8044, JHEP 2014)
- focus on large bins above and below charmonium resonances
- theory errors commensurate with experiment
- yields $\sim$1-2σ tensions
- ⇒ determine $|V_{td}/V_{ts},|V_{td}|,|V_{ts}|$

  or constrain Wilson coefficients

D. Du et al (arXiv:1510.02349, PRD 2016)



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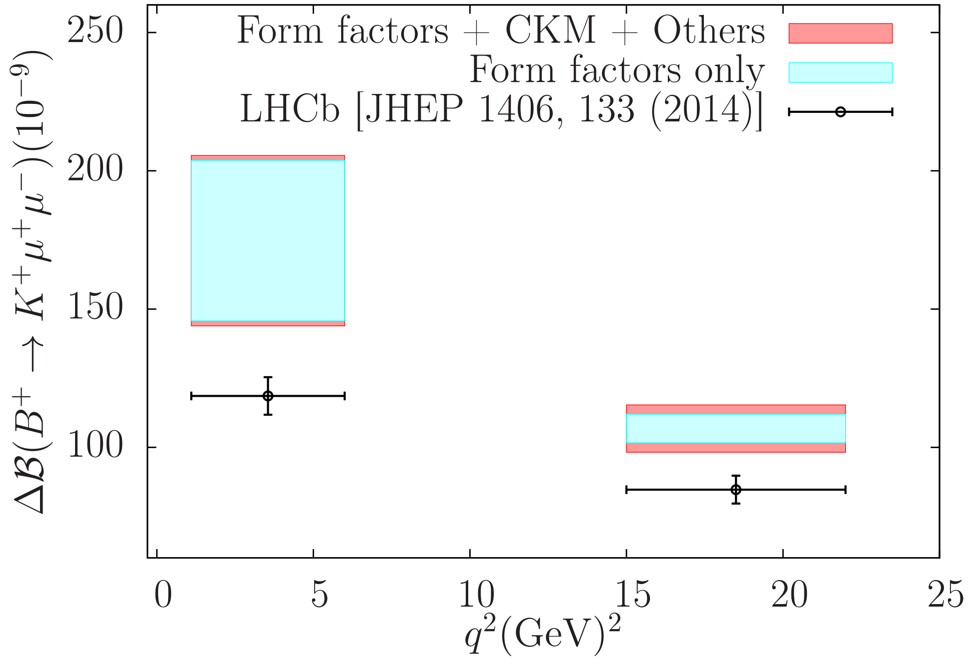
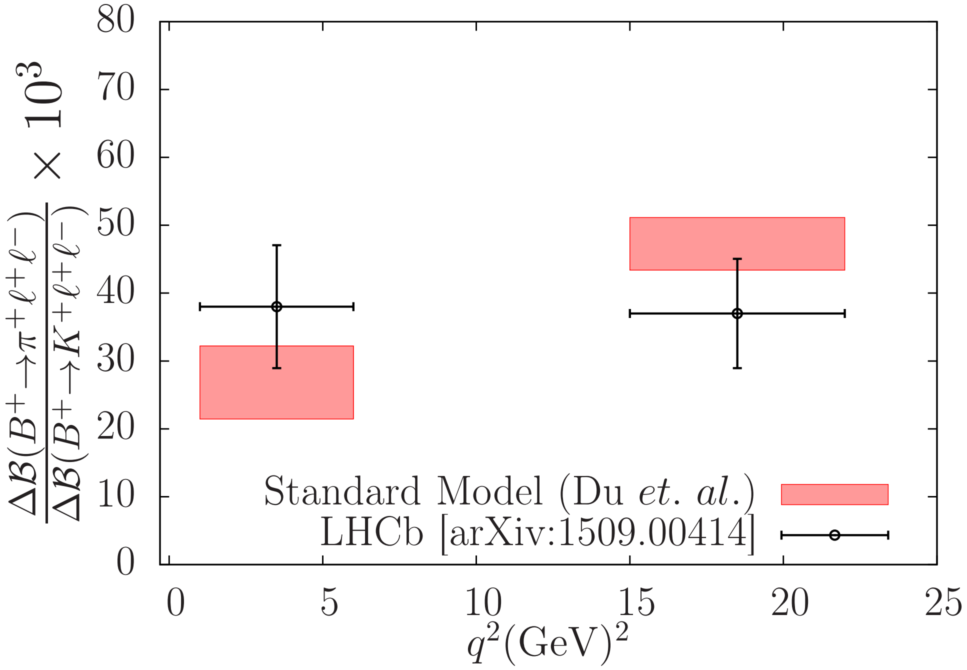
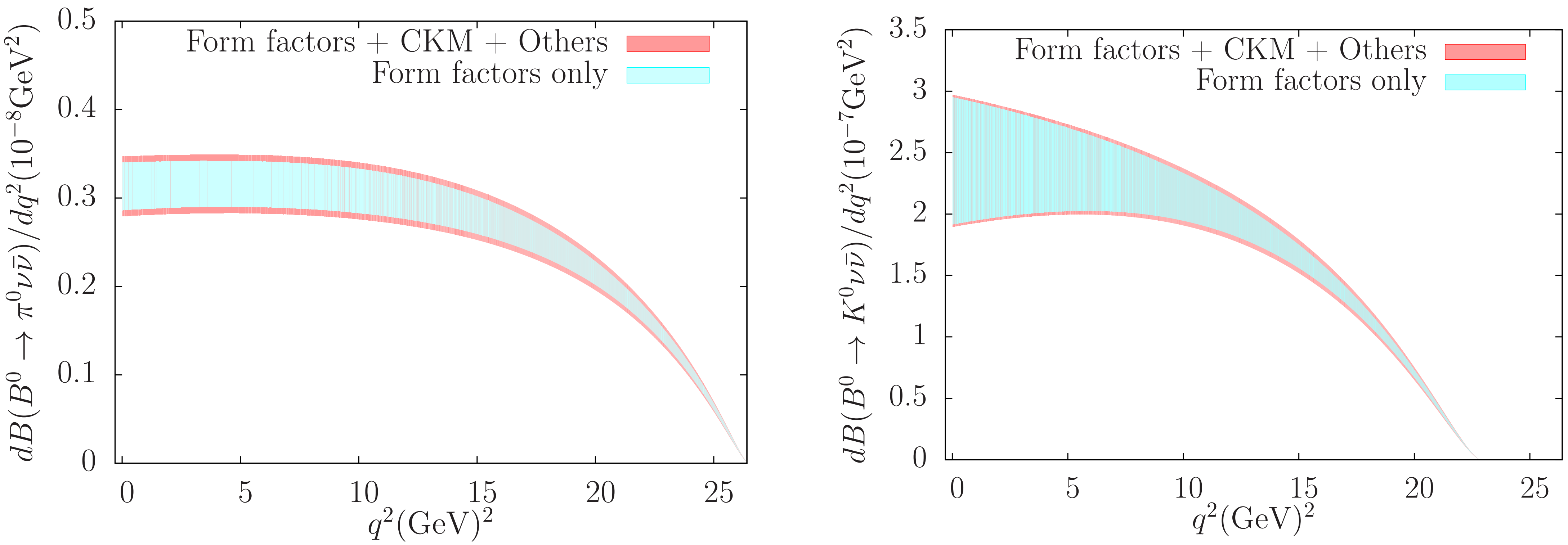




Phenomenology for $B \to K,\pi\,\nu\,\bar{\nu}$

theoretically clean

D. Du et al (arXiv:1510.02349, PRD 2016)



form factors for $B \to D^{(*)}\ell\nu$ & V_{cb}

$$\frac{d\Gamma(B\to D^*\ell\nu)}{d\omega} = (\text{known}) \times |V_{cb}|^2 \times (\omega^2-1)^{1/2}|\mathcal{F}(\omega)|^2$$

$$\frac{d\Gamma(B\to D\ell\nu)}{d\omega} = (\text{known}) \times |V_{cb}|^2 \times (\omega^2-1)^{3/2}|\mathcal{G}(\omega)|^2$$

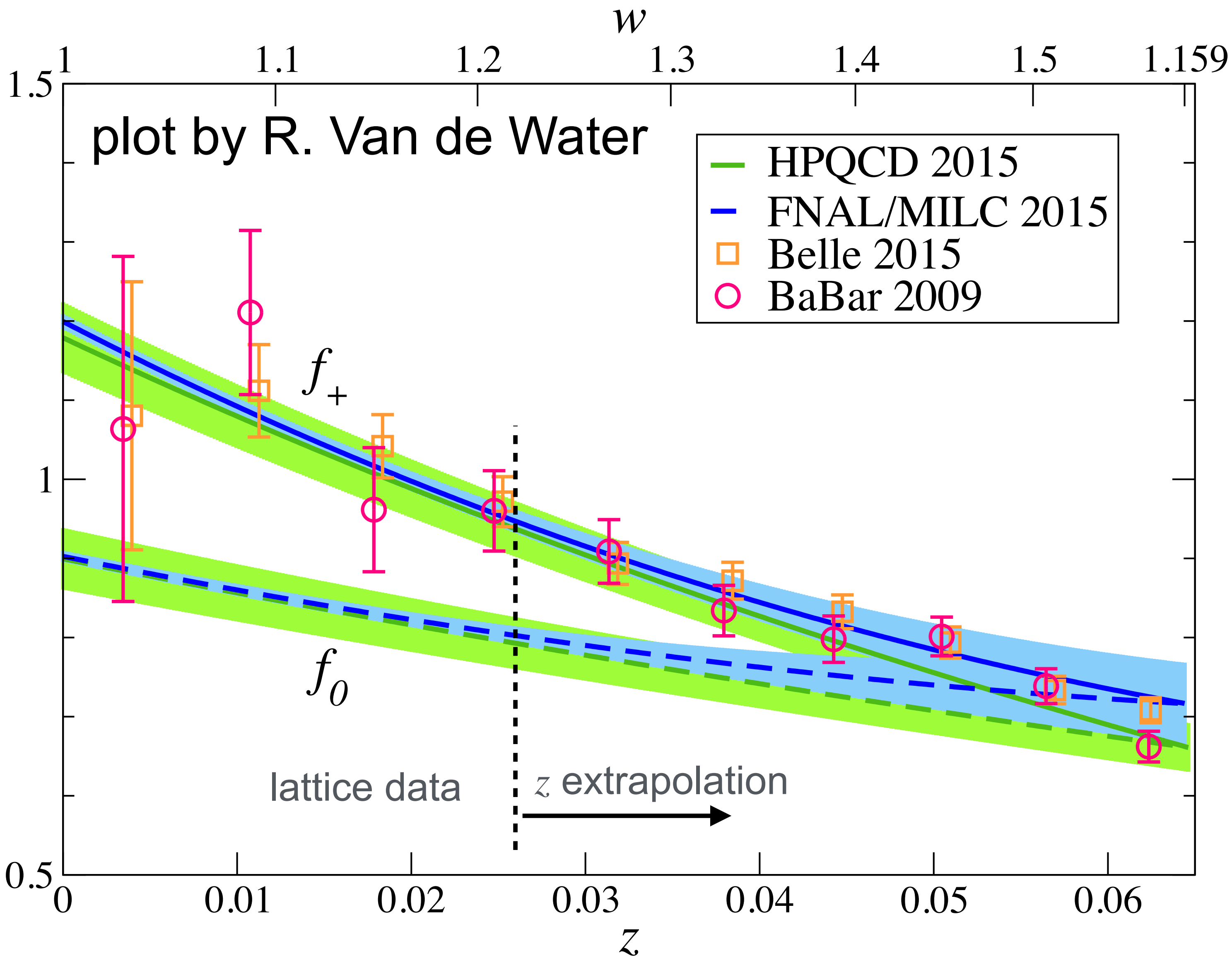
at zero recoil (HFAG 2014):

$$B \to D^*\ell\nu: \quad \eta_{\mathrm{EW}}|V_{cb}|\mathcal{F}(1) = (35.81 \pm 0.11 \pm 0.44)\ 10^{-3}$$

$$B \to D\ell\nu: \quad \eta_{\mathrm{EW}}|V_{cb}|\mathcal{G}(1) = (42.65 \pm 0.71 \pm 1.35)\ 10^{-3}$$

- need form-factors at non-zero recoil for shape comparison, $R(D^{(*)})$
- new LQCD results for $B \to D$ form factors at non-zero recoil
- ongoing LQCD calculations for $B \to D^*$ form factors at non-zero recoil by HPQCD, FNAL/MILC, RBC/UKQCD, LANL using different methods.

form factors for $B \to D\,\ell\nu, \quad (\ell = e, \mu, \tau)$



HPQCD (arXiv:1505.03925, PRD 2015)
FNAL/MILC (arXiv:1503.07237, PRD 2015)

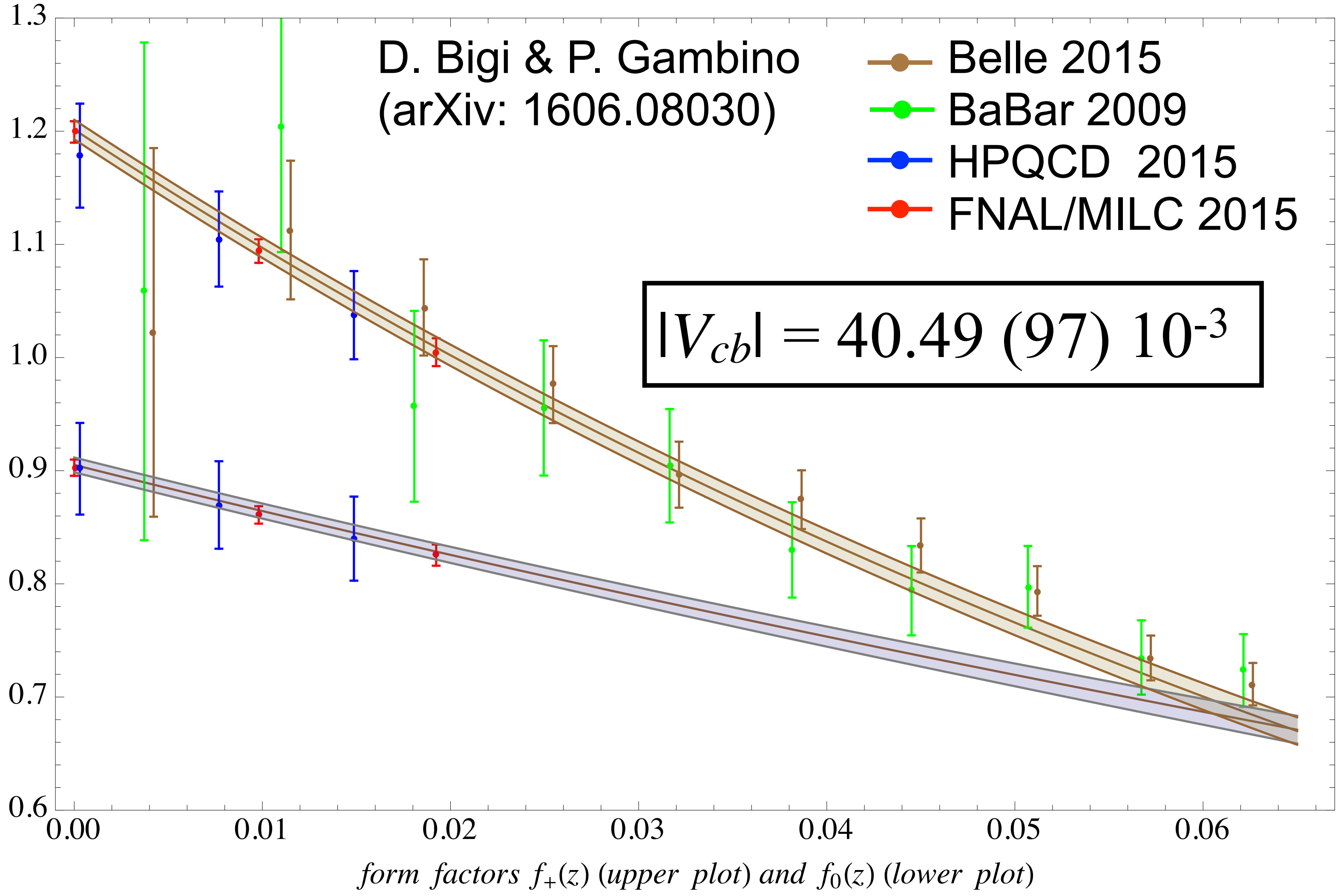
- ☆ Two LQCD calculations (FNAL/MILC, HPQCD)
- ☆ LQCD form factor uncertainties (~1.2%) smaller than experiment.

- ☆ LQCD form factors can be used to calculate the CKM free ratio:

$$R(D) \equiv \frac{\mathcal{B}(B \to D\tau\nu_\tau)}{\mathcal{B}(B \to D\ell\nu)}$$

form factors for $B \to D\,\ell\nu, \quad (\ell = e, \mu, \tau)$

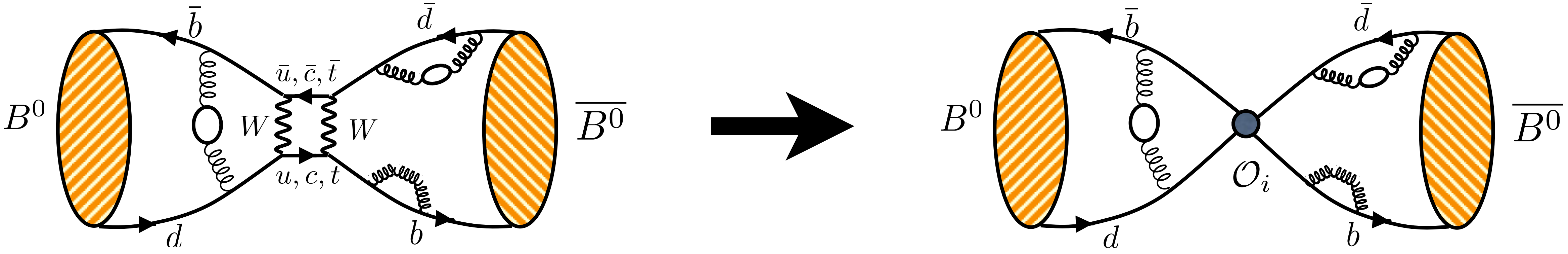
- combine LQCD form factors with experiment:



- FLAG-3 combined fit is currently being updated.

Neutral B meson mixing

Standard Model



SM: $\Delta M_q = (\text{known}) \times |V^*_{tq} V_{tb}|^2 \times \langle \bar{B}^0_q | \mathcal{O}_1 | B^0_q \rangle$

also:

$$\frac{\Delta M_s}{\Delta M_d} = \frac{m_{B_s}}{m_{B\,d}} \times \left| \frac{V_{ts}}{V_{td}} \right|^2 \times \xi^2 \quad \text{with} \quad \xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

$$\Delta\Gamma_q = \left[G_1 \, \langle \bar{B}^0_q | \mathcal{O}_1 | B^0_q \rangle + G_3 \, \langle \bar{B}^0_q | \mathcal{O}_3 | B^0_q \rangle \right] \cos\phi_q + O(1/m_b)$$

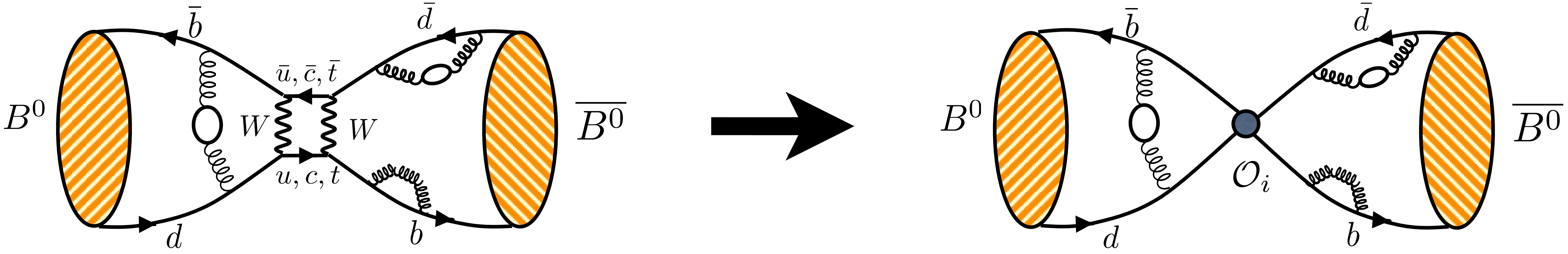
HFAG, PDG 2016 averages:

$$\Delta M_d = (0.5055 \pm 0.0020)\ \text{ps}^{-1}\ (0.4\%)$$
$$\Delta M_s = (17.575 \pm 0.021)\ \text{ps}^{-1}\ \ (0.1\%)$$

$$\Delta\Gamma_d/\Gamma_d = 0.001 \pm 0.010$$
$$\Delta\Gamma_s/\Gamma_s = 0.124 \pm 0.009\ \ (7.3\%)$$

Neutral B meson mixing

Standard Model



In general :

$$\mathcal{H}_{\text{eff}} = \sum_{i=1}^{5} c_i(\mu)\mathcal{O}_i(\mu)$$

SM:

$$\mathcal{O}_1 = (\bar{b}^\alpha \gamma_\mu L q^\alpha)\ (\bar{b}^\beta \gamma_\mu L q^\beta)$$

$$\mathcal{O}_2 = (\bar{b}^\alpha L q^\alpha)\ (\bar{b}^\beta L q^\beta)$$

$$\mathcal{O}_3 = (\bar{b}^\alpha L q^\beta)\ (\bar{b}^\beta L q^\alpha)$$

BSM:

$$\mathcal{O}_4 = (\bar{b}^\alpha L q^\alpha)\ (\bar{b}^\beta R q^\beta)$$

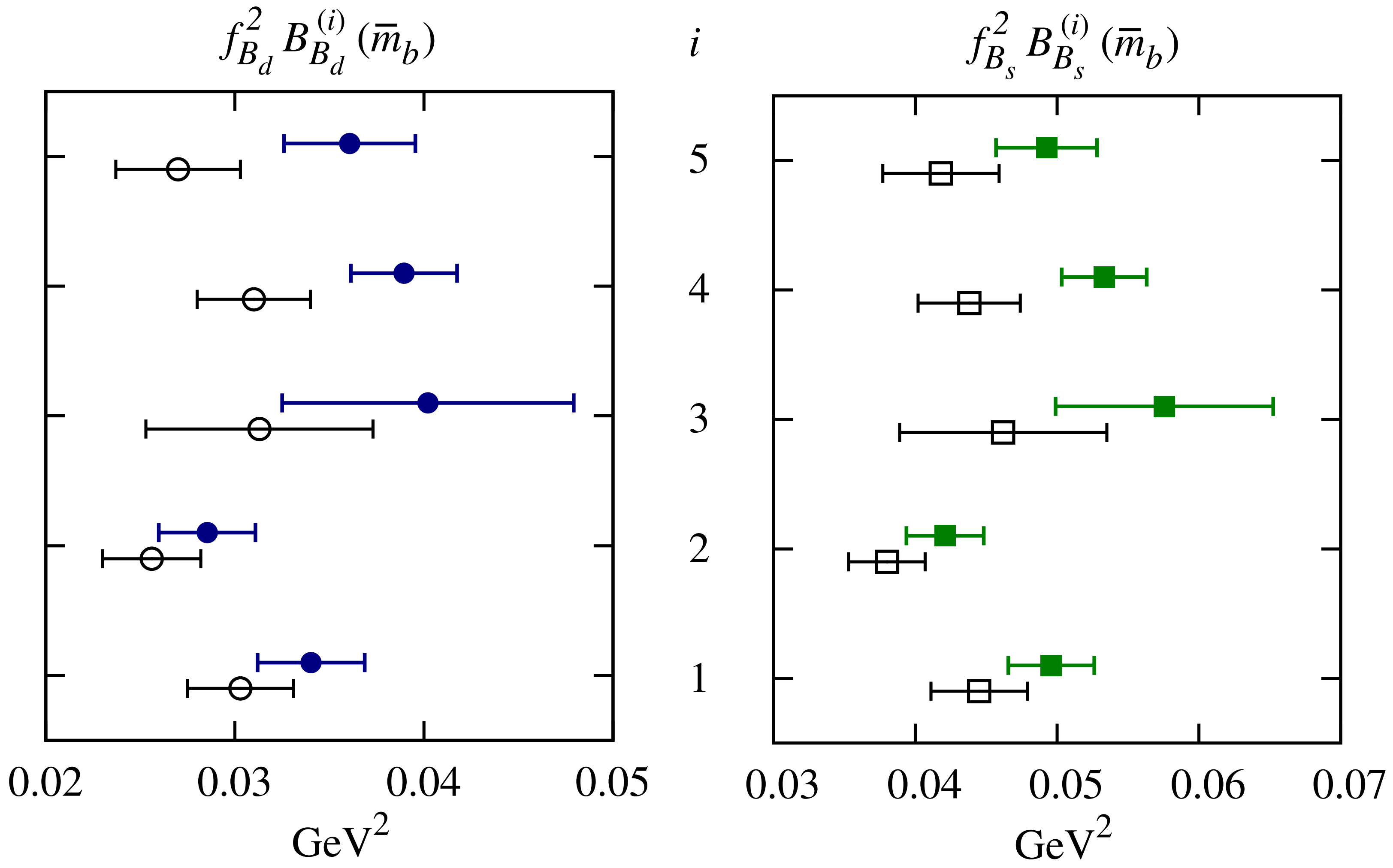
$$\mathcal{O}_5 = (\bar{b}^\alpha L q^\beta)\ (\bar{b}^\beta R q^\alpha)$$

$$\langle \mathcal{O}_i \rangle \equiv \langle \bar{B}_q^0 | \mathcal{O}_i | B_q^0 \rangle(\mu) = e_i\ m_{B_q}^2\ f_{B_q}^2\ B_{B_q}^{(i)}(\mu)$$

The matrix elements of all five operators can be calculated in LQCD.

B mixing results in comparison

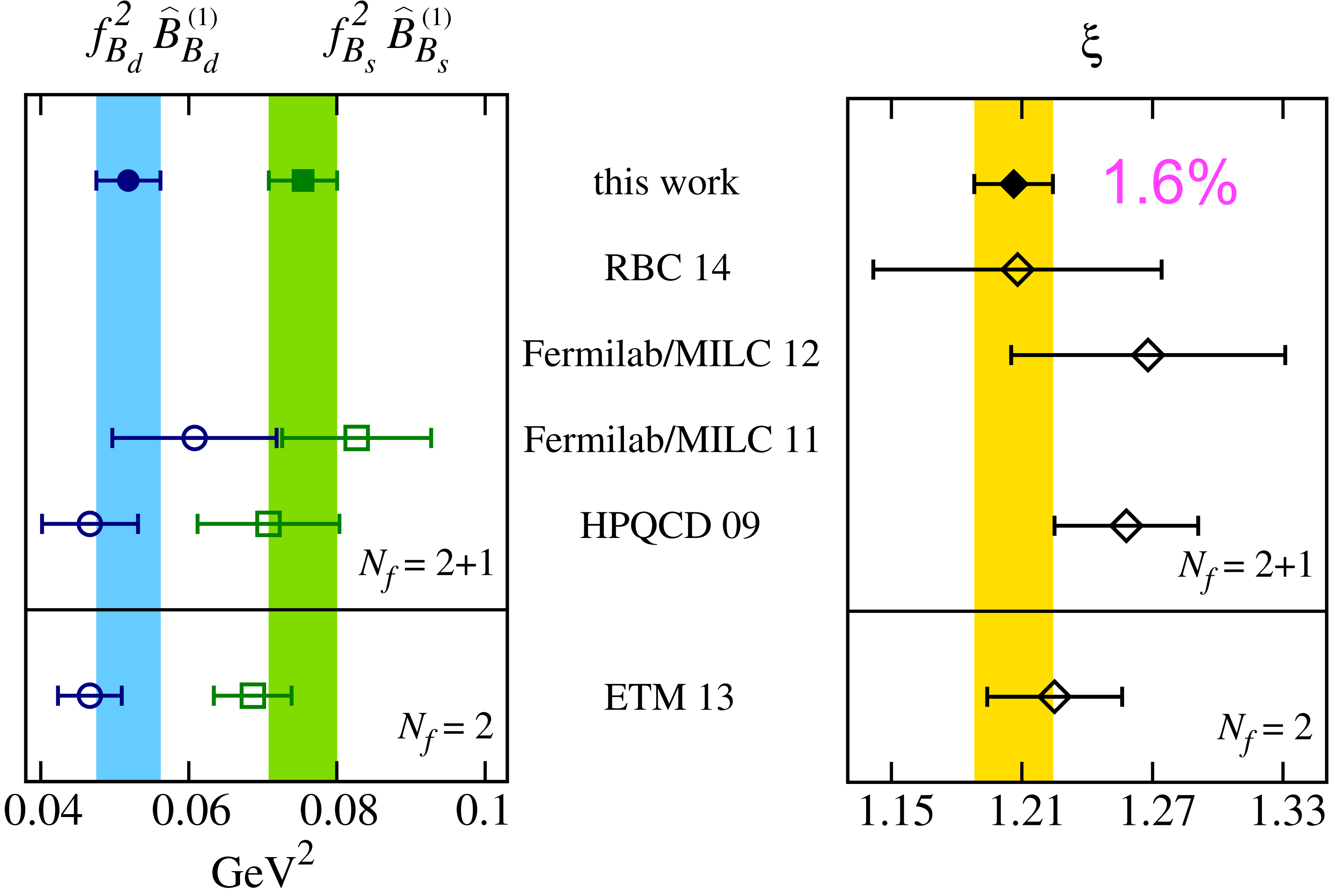
ETM (n_f=2, arXiv:1308.1851, JHEP 2014) vs. FNAL/MILC (n_f=3, arXiv:1602.03560, PRD 2016)



- ⭐ **First** three flavor LQCD results for all five matrix elements including the correlations between all 10 MEs.

B mixing results in comparison

FNAL/MILC (arXiv:1602.03560, PRD 2016)

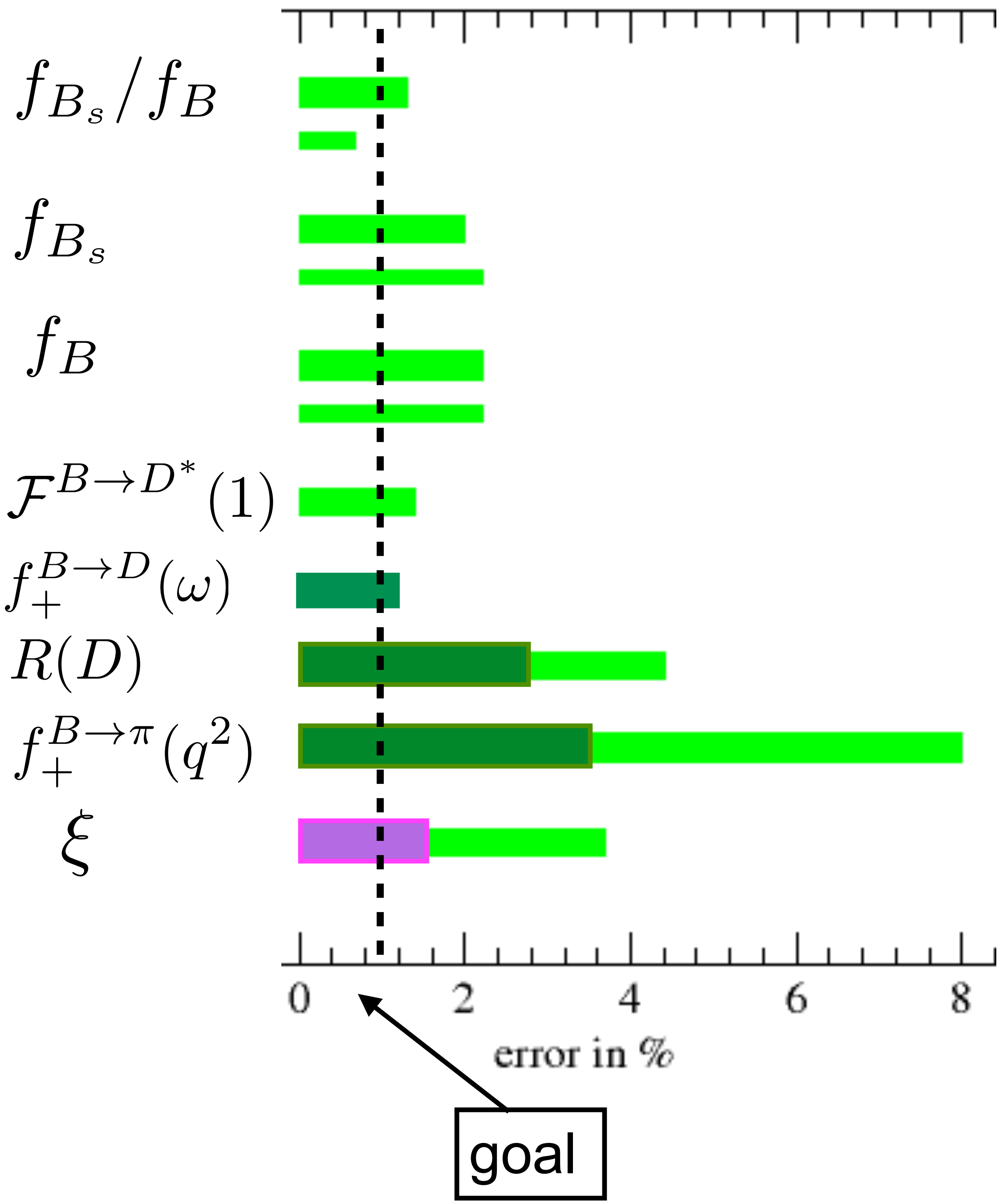


Significant reduction of errors compared to previous three flavor results, especially for ξ

- Note: FLAG-3 is currently updating their averages for B mixing quantities to include the new FNAL/MILC results.
- ongoing LQCD calculations by HPQCD, ETM, RBC/UKQCD, …

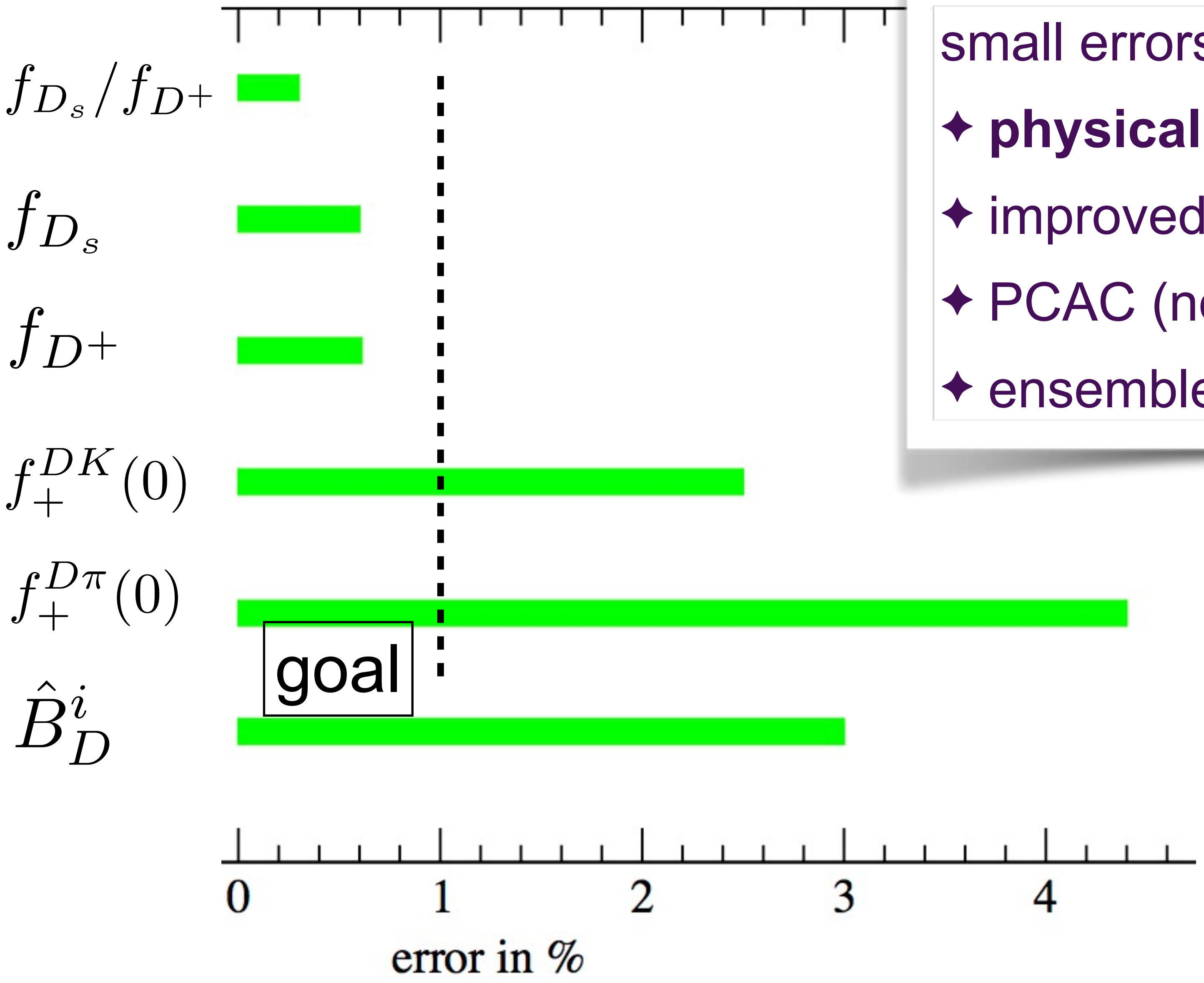
B meson Summary

errors (in %) **FLAG-2/3 averages** + **new results**



D meson summary

errors (in %) comparison:



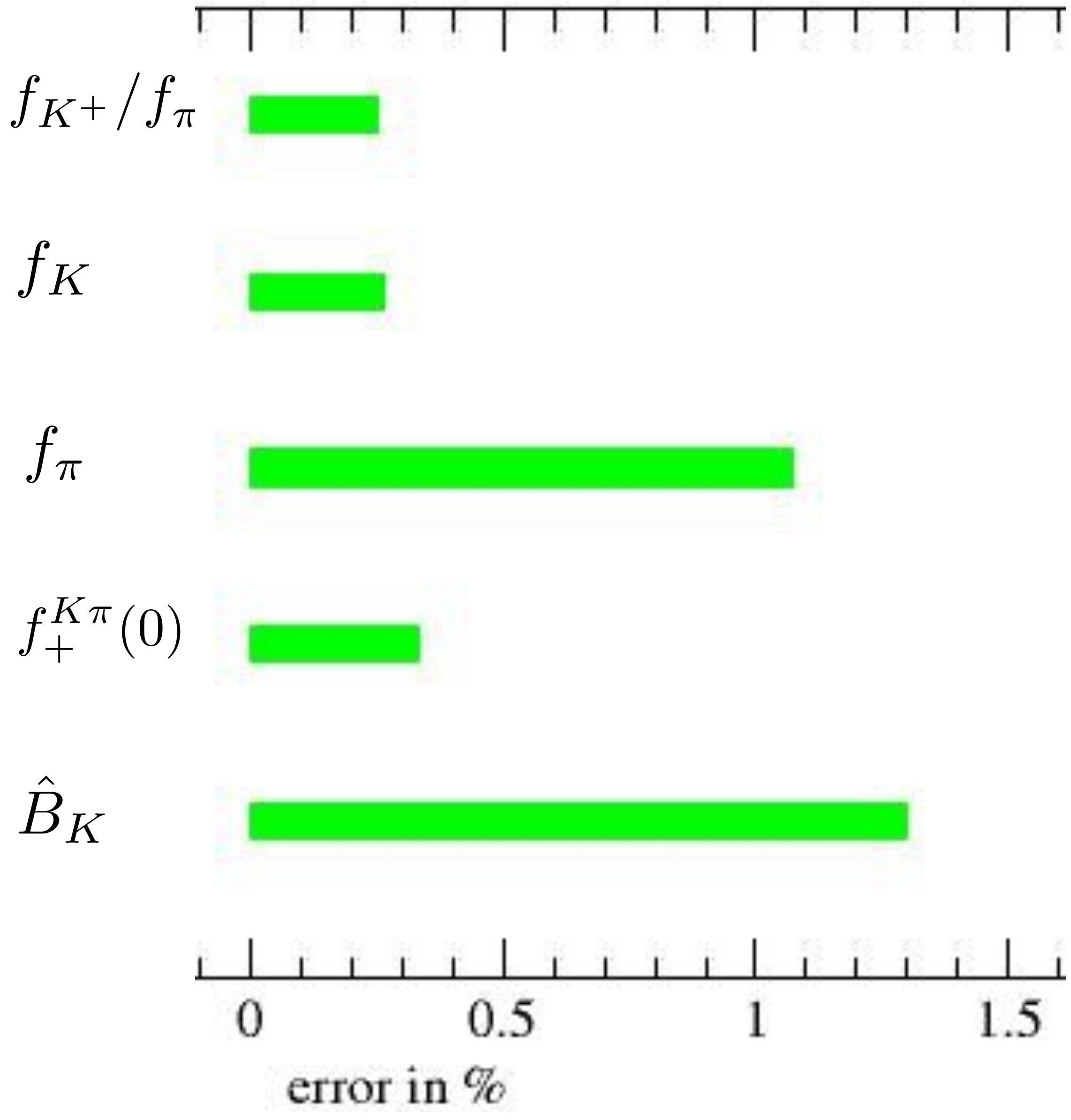
small errors due to

- ✦ **physical light quark masses**
- ✦ improved charm-quark action (HISQ)
- ✦ PCAC (no renormalization)
- ✦ ensembles with small lattice spacings

Kaon summary

For all quantities there are results that use **physical mass** ensembles

errors (in %) **FLAG-3 averages**



independent results (different methods)

small errors due to

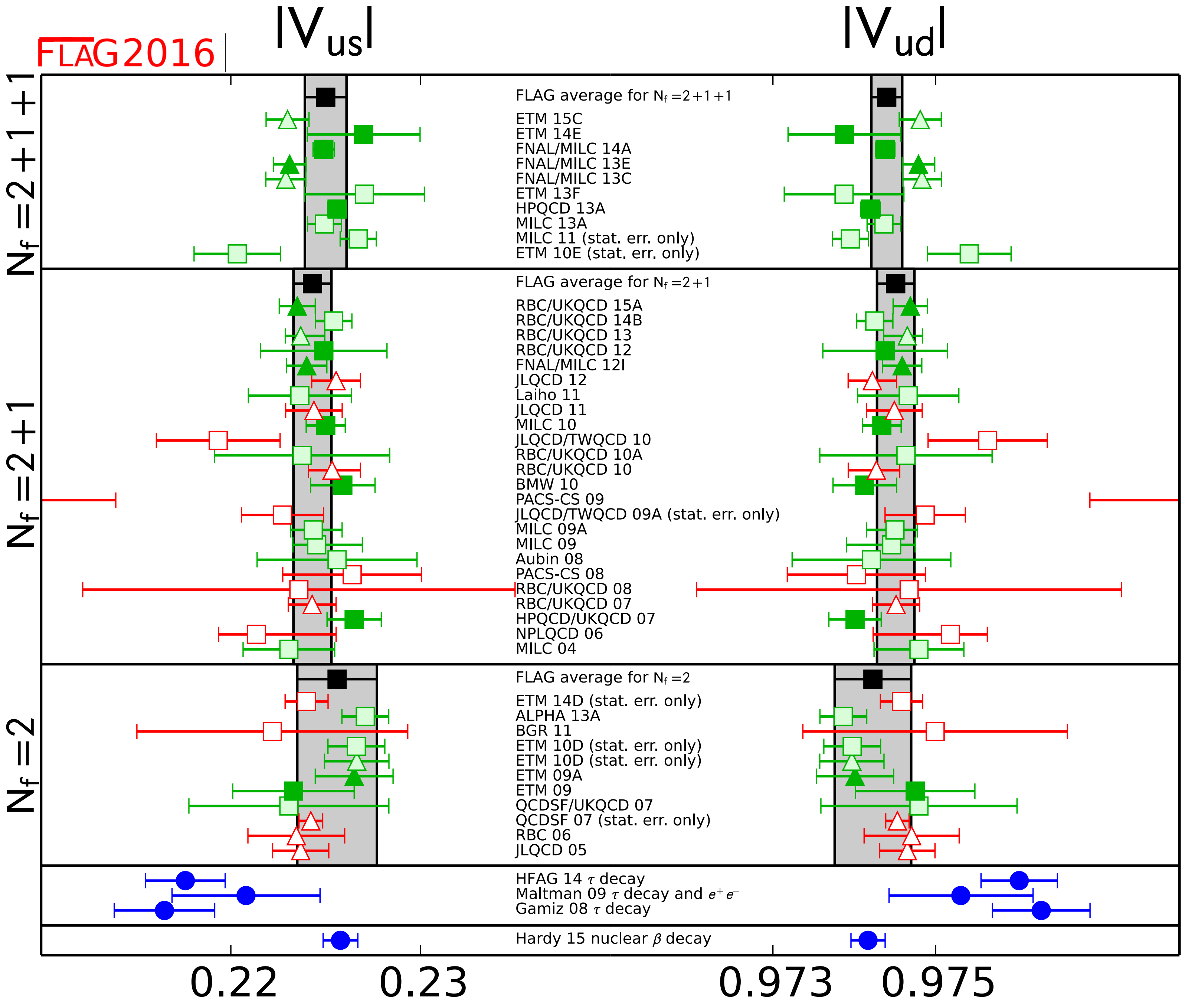
- ✦ **physical light quark masses**
- ✦ improved light-quark actions
- ✦ ensembles with small lattice spacings
- ✦ NPR or no renormalization

Outline

- Motivation and Introduction
  - lattice QCD
- Results
  - leptonic decays
  - semileptonic decays
  - neutral meson mixing
  - summary of B,D,K results
- Phenomenology
  - CKM determinations
  - UT analysis
  - BSM phenomenology
- Summary and Outlook

Implications for $|V_{us}|$, $V_{ud}|$

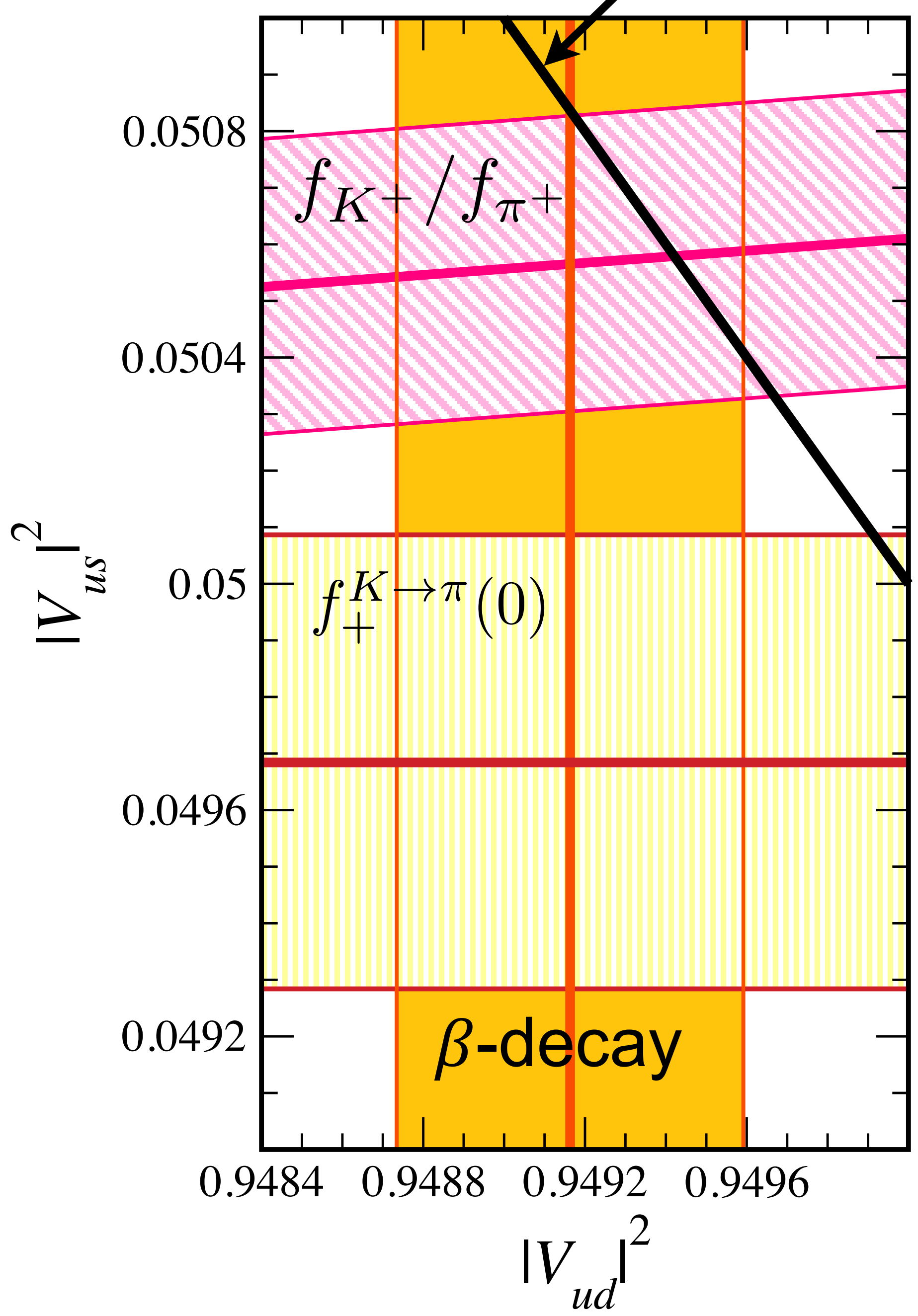
S. Aoki et al (FLAG-3 review, arXiv:1607.00299)



1^{st} row CKM unitarity test

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$|V_{ub}| \approx 4 \times 10^{-3} \approx 0$



Constraining $|V_{us}|$ using FLAG-3 averages for K_{l3} form factor or for f_{K^+}/f_{π^+}.

CKM unitarity test with K_{l3}:

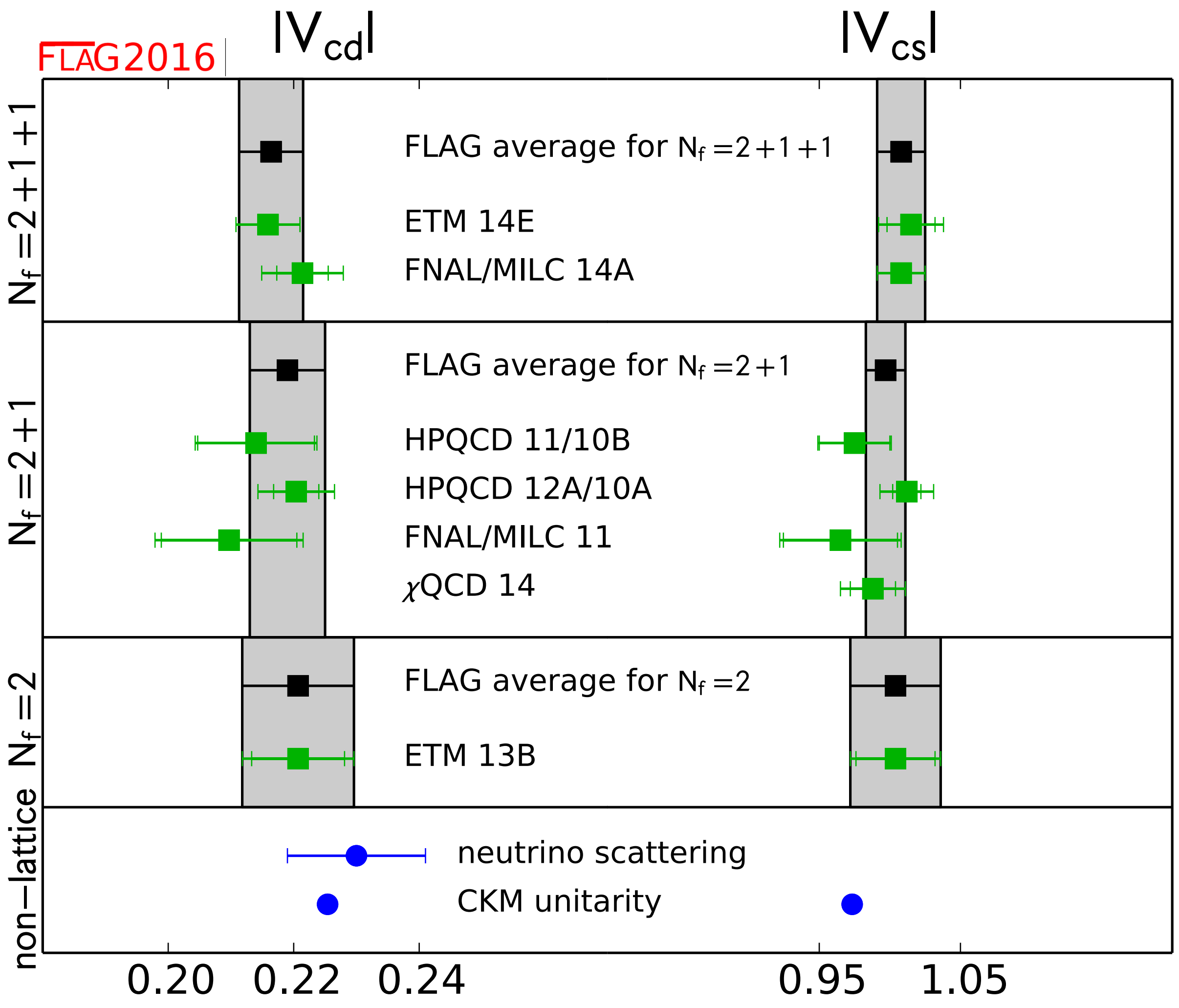
$$\boxed{1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2 = 0.0012(6)}$$

commensurate $|V_{us}|$ and $|V_{ud}|$ uncertainty contributions.

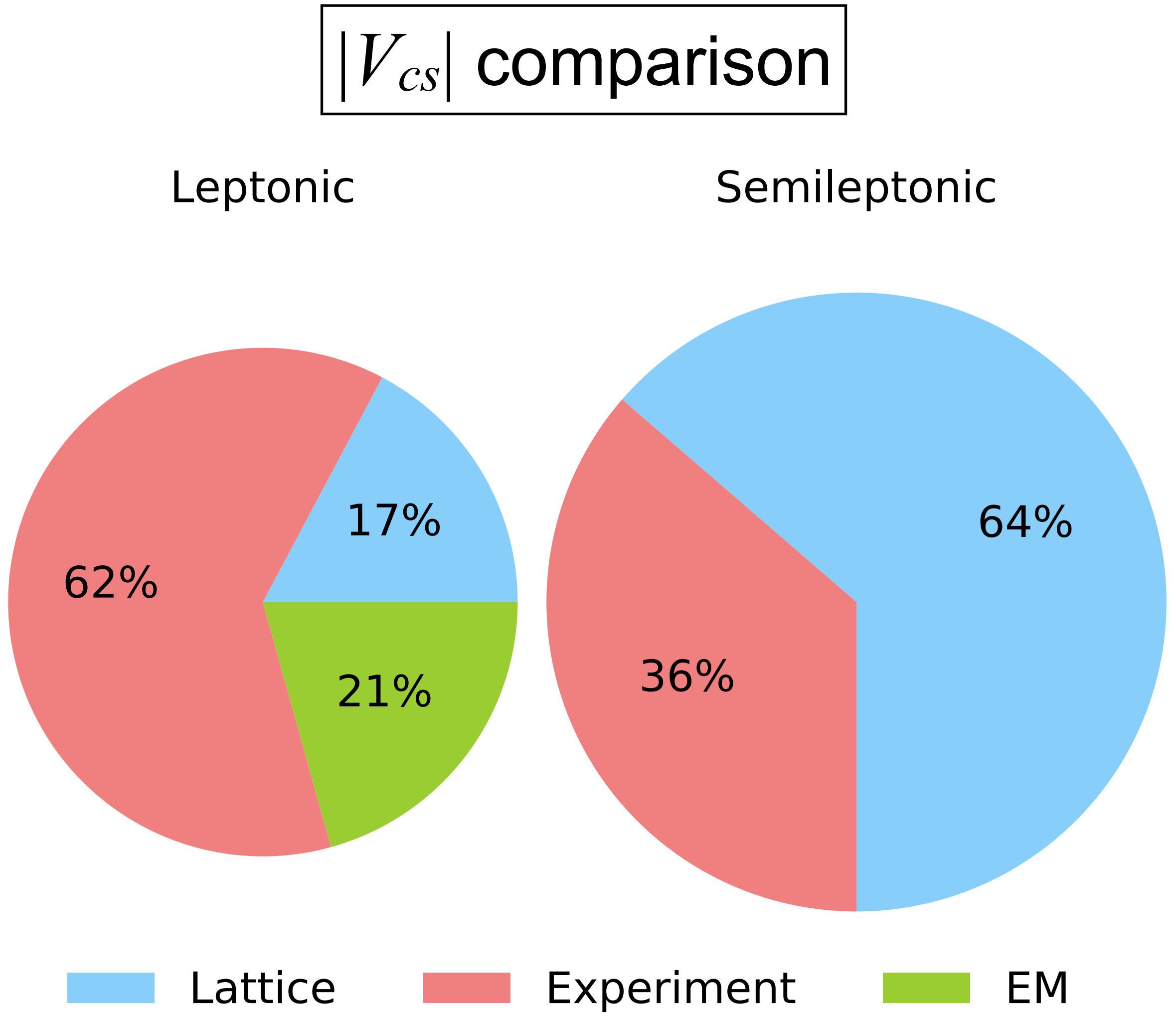
Slight (2σ) tension.

Implications for $|V_{cs}|$, $|V_{cd}|$

S. Aoki et al (FLAG review, arXiv:1607.00299)

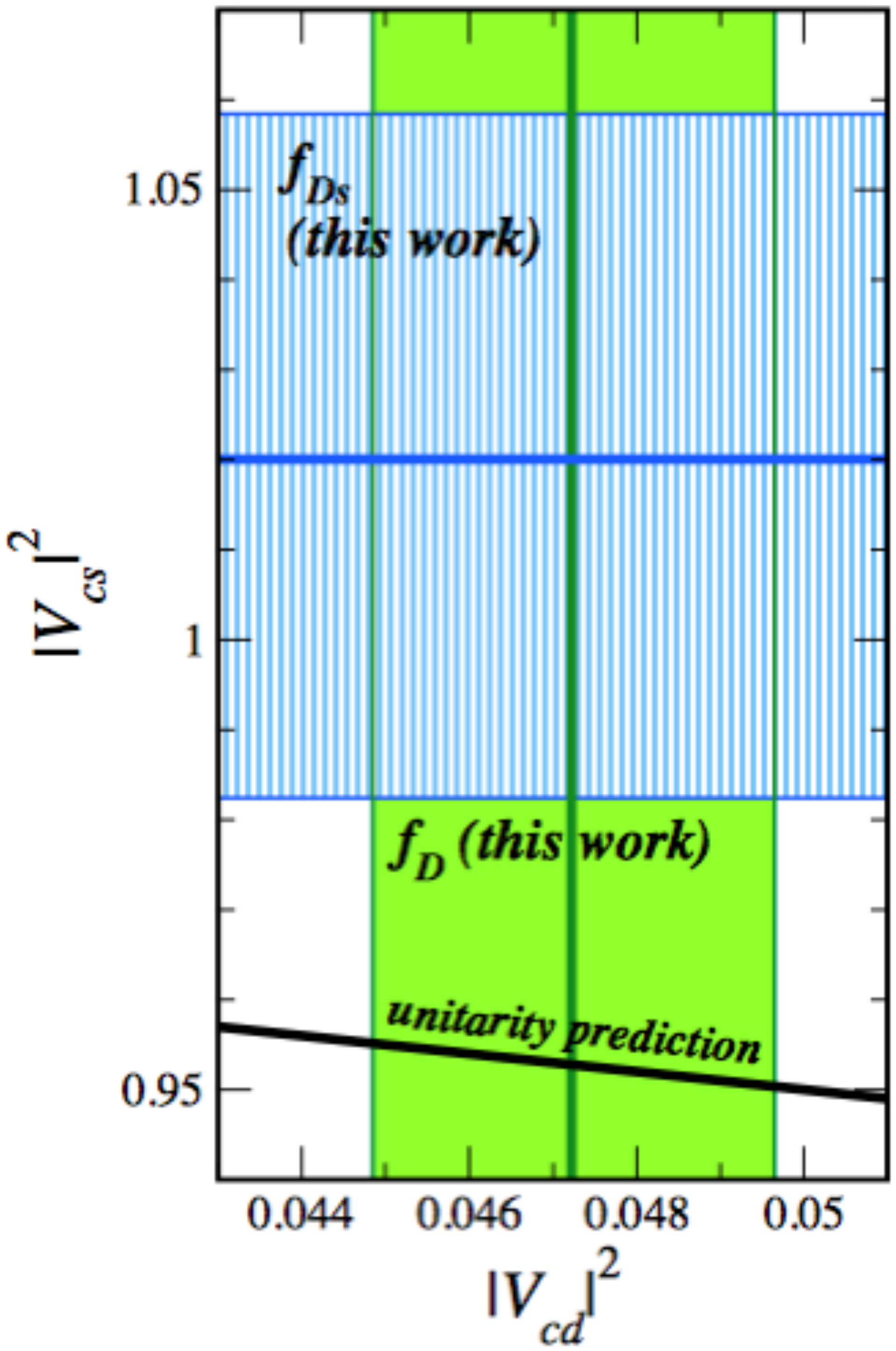


S. Gottlieb, T. Primer (FNAL/MILC) @ Lattice 2016



Implications for the 2nd row of the CKM Matrix

FNAL/MILC (arXiv:1407.3772, 2014 PRD)



errors on $|V_{cs}|$ and $|V_{cd}|$ are dominated by experiment (PDG 2015, arXiv:509.02220):

$$\boxed{\begin{array}{l}|V_{cd}| = 0.217\ (1)_{\mathrm{LQCD}}\ (5)_{\mathrm{exp}} \\ |V_{cs}| = 1.007\ (4)_{\mathrm{LQCD}}\ (16)_{\mathrm{exp}}\end{array}}$$

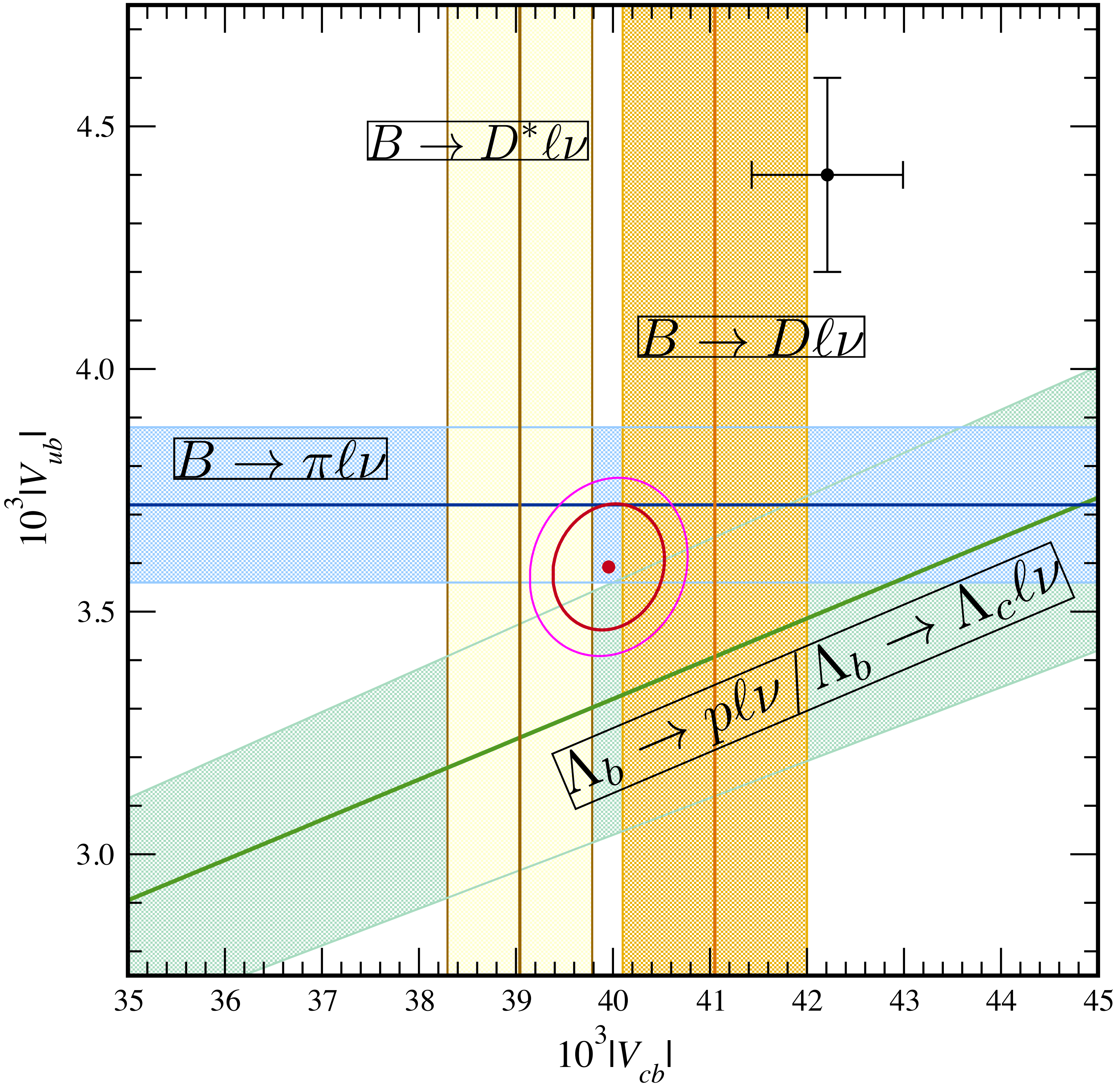
(based on the PDG average of 2+1 & 2+1+1 flavor LQCD results; average is dominated by FNAL/MILC)

2σ tension with unitarity:

$$\boxed{|V_{cs}|^2 + |V_{cd}|^2 + |V_{cb}|^2 - 1 = 0.064(32)}$$

Exclusive vs. inclusive $|V_{cb}|$ and $|V_{ub}|$

A. Kronfeld (priv. communication)



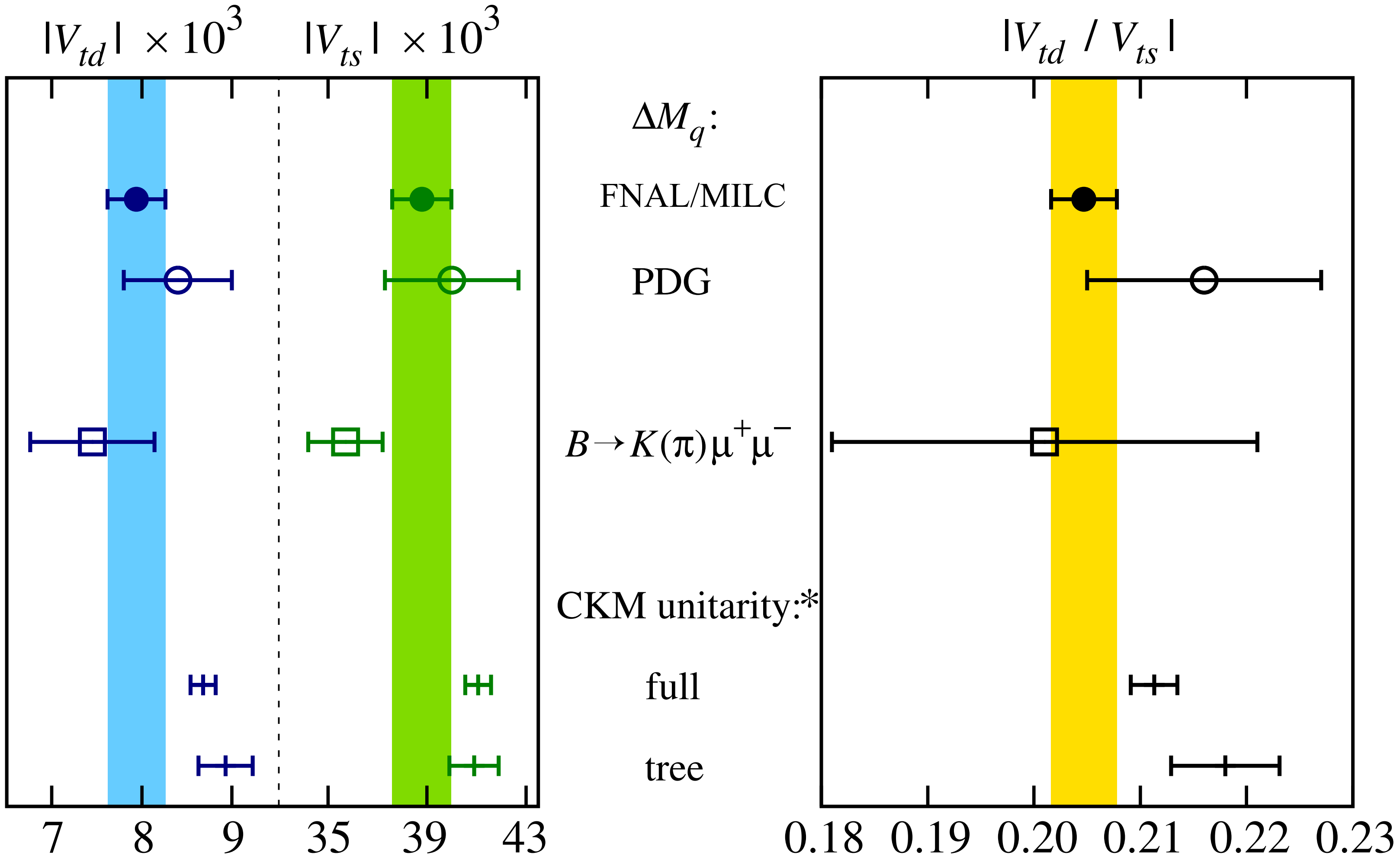
$\sim 3\sigma$ tension between inclusive and exclusive $|V_{cb}|$ and $|V_{ub}|$

New in 2015:

- $|V_{cb}|$ from $B \to D\ell\nu$
- $|V_{ub}|$ from $B \to \pi\ell\nu$
- $|V_{ub}/V_{cb}|$ from $\Lambda_b \to p\ell\nu / \Lambda_b \to \Lambda_c\ell\nu$

Implications for $|V_{ts}|$, $|V_{td}|$, $|V_{td}/V_{ts}|$

D. Du et al (arXiv:1510.02349, PRD 2016)



Blanke & Buras:
(arXiv:1602.04020, EPJC 2016)

tension between $\Delta M_{s,d}$ & ϵ_K inconsistent with CMFV (Constrained Minimal Flavor Violation)

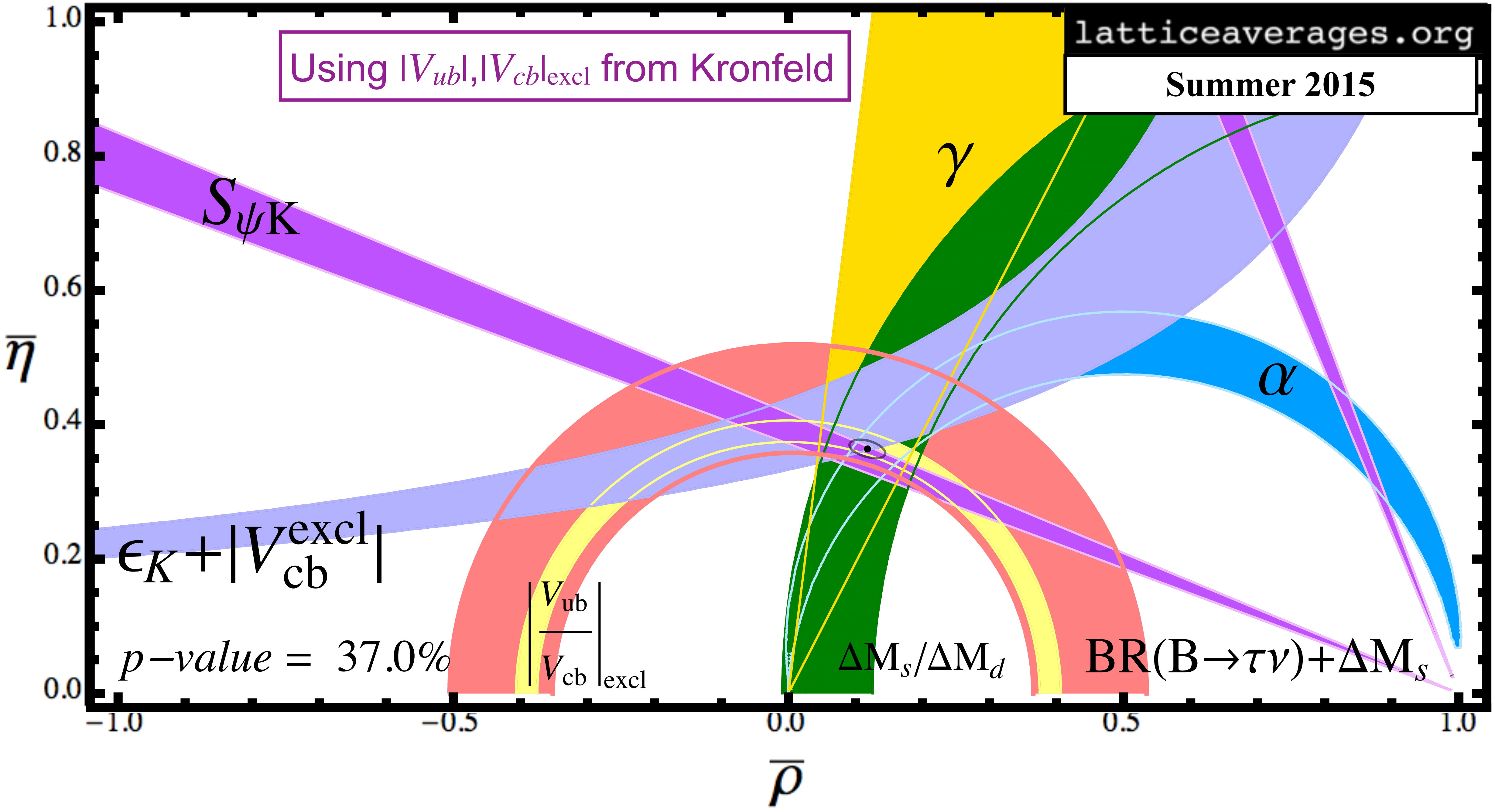
Buras & De Fazio:
(arXiv:1604.02334)
implications for “331” models

$\sim 2\sigma$ tensions between loop processes and CKM unitarity.

*from CKMfitter 2015
(hep-ph/0406186,
http://ckmfitter.in2p3.fr)

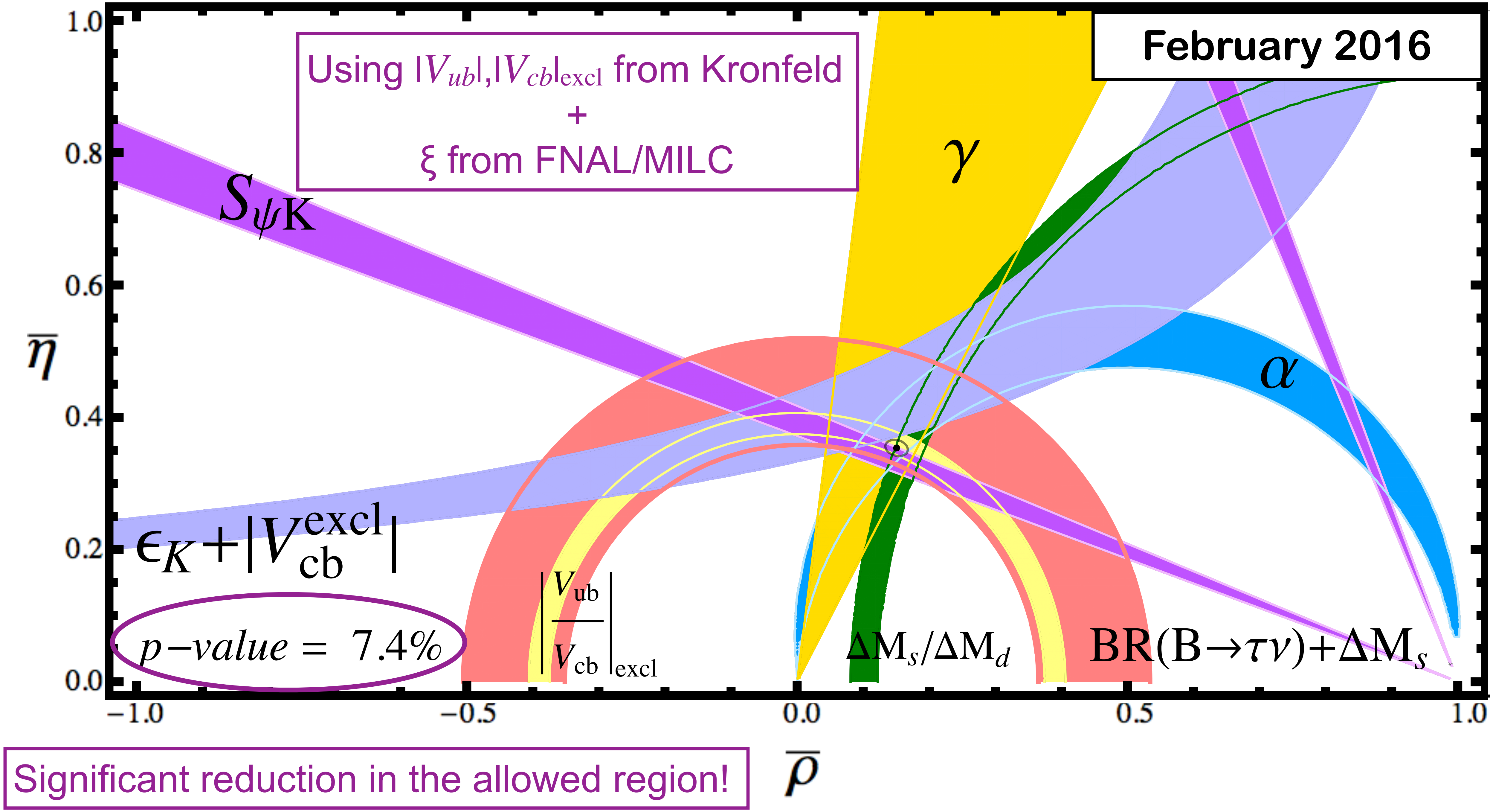
UT analysis

Laiho, Lunghi & Van de Water (Phys.Rev.D81:034503,2010), E. Lunghi, private comm.



UT analysis

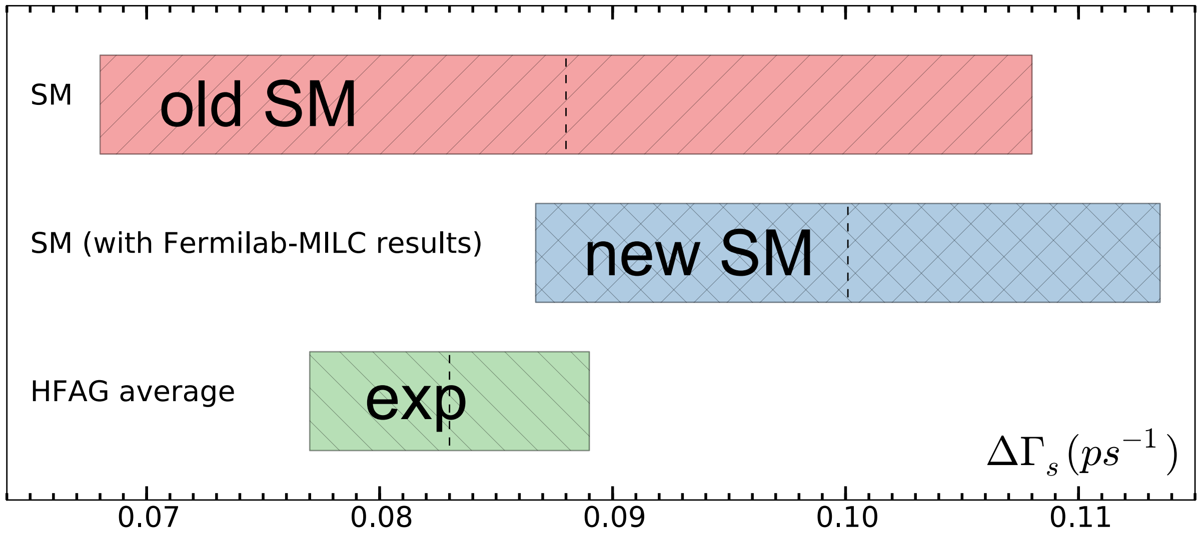
Laiho, Lunghi & Van de Water (Phys.Rev.D81:034503,2010), E. Lunghi, private comm.



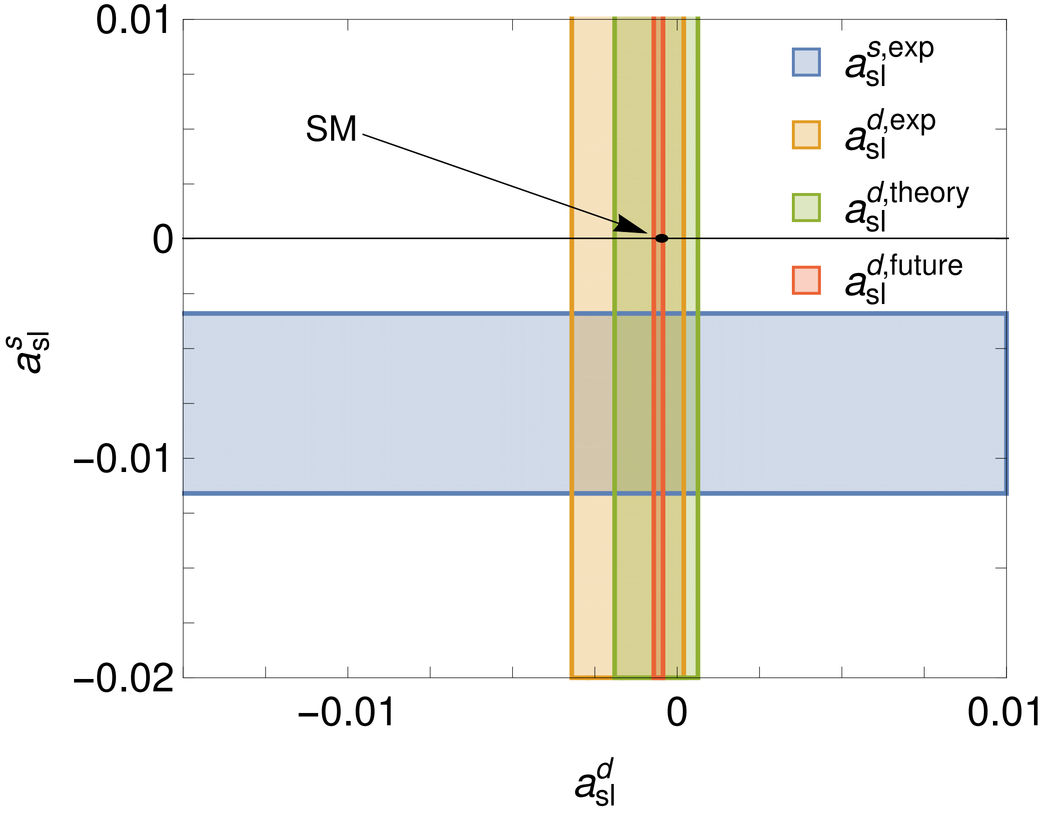
Significant reduction in the allowed region!

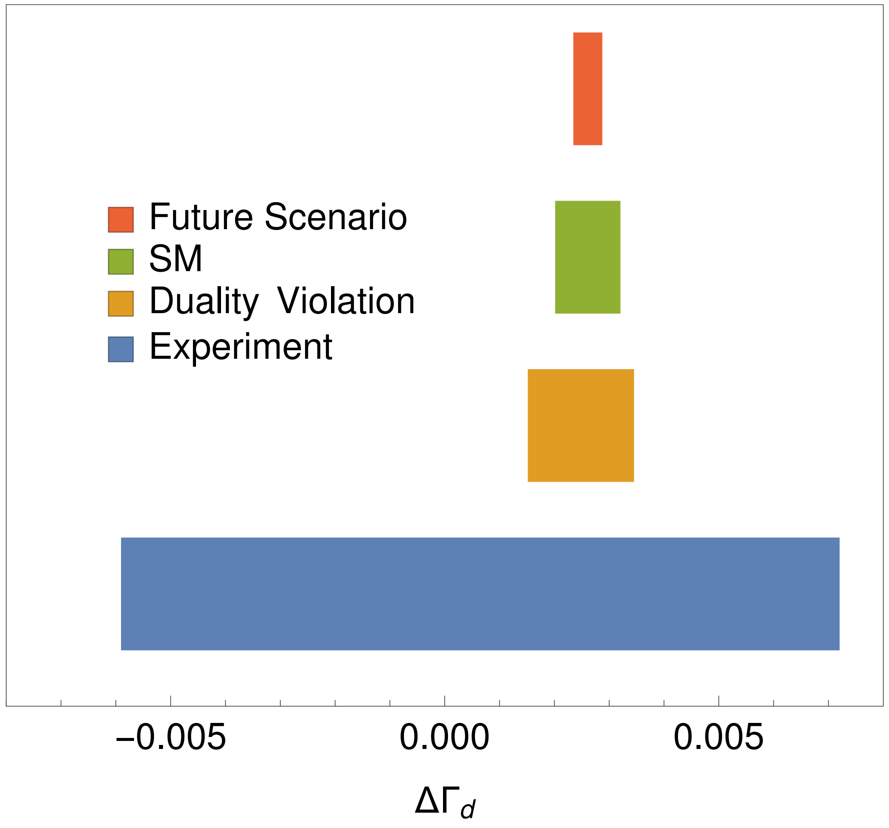
implicatons for $\Delta\Gamma_{s(d)}$ & a_{SL}

Standard Model theory from Jubb et al (arXiv:1603.07770) and M. Kirk @ Lattice 2016:

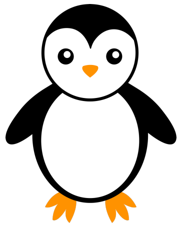


using mixing matrix elements from FNAL/MILC

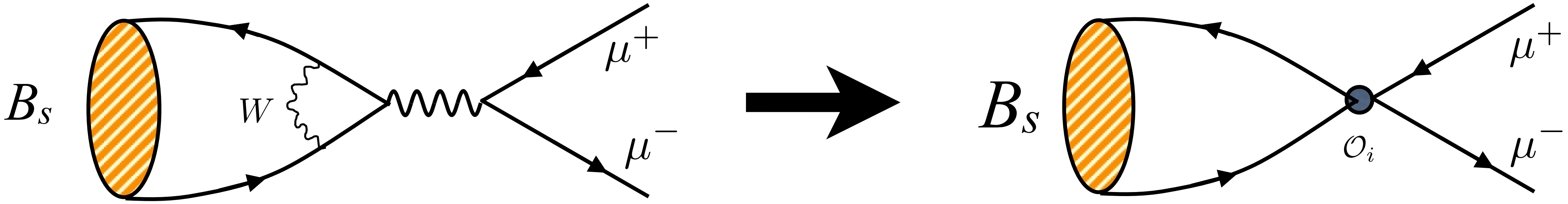




- HPQCD (Wingate @ Lattice 2016): first look at dimension 7 $\Delta B = 2$ matrix elements

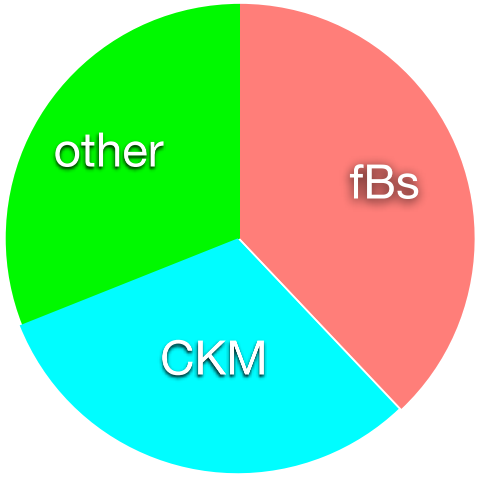


Rare leptonic decay $B_s \to \mu^+\mu^-$

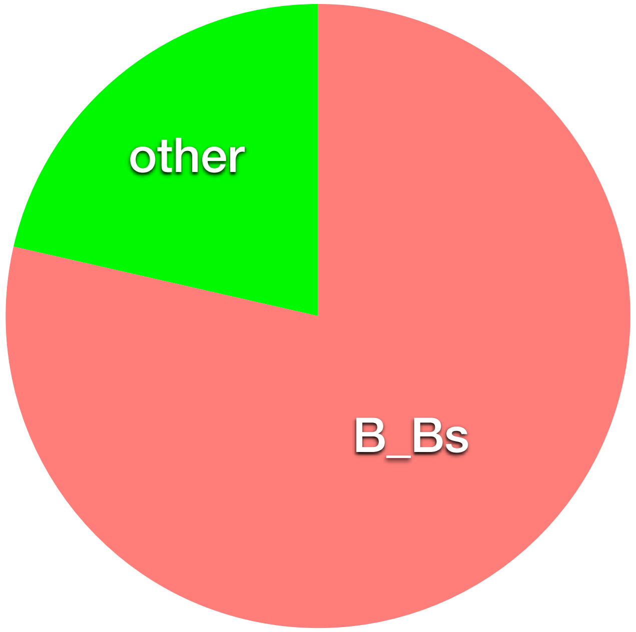


Standard Model prediction: Buras, et al (arXiv:1303.3820, JHEP 2013), Bobeth, et al (arXiv:1311.0903, PRL 2014)

$$\bar{\mathcal{B}}(B_s \to \mu^+\mu^-) = 3.53(11)(9)(9) \times 10^{-9}$$



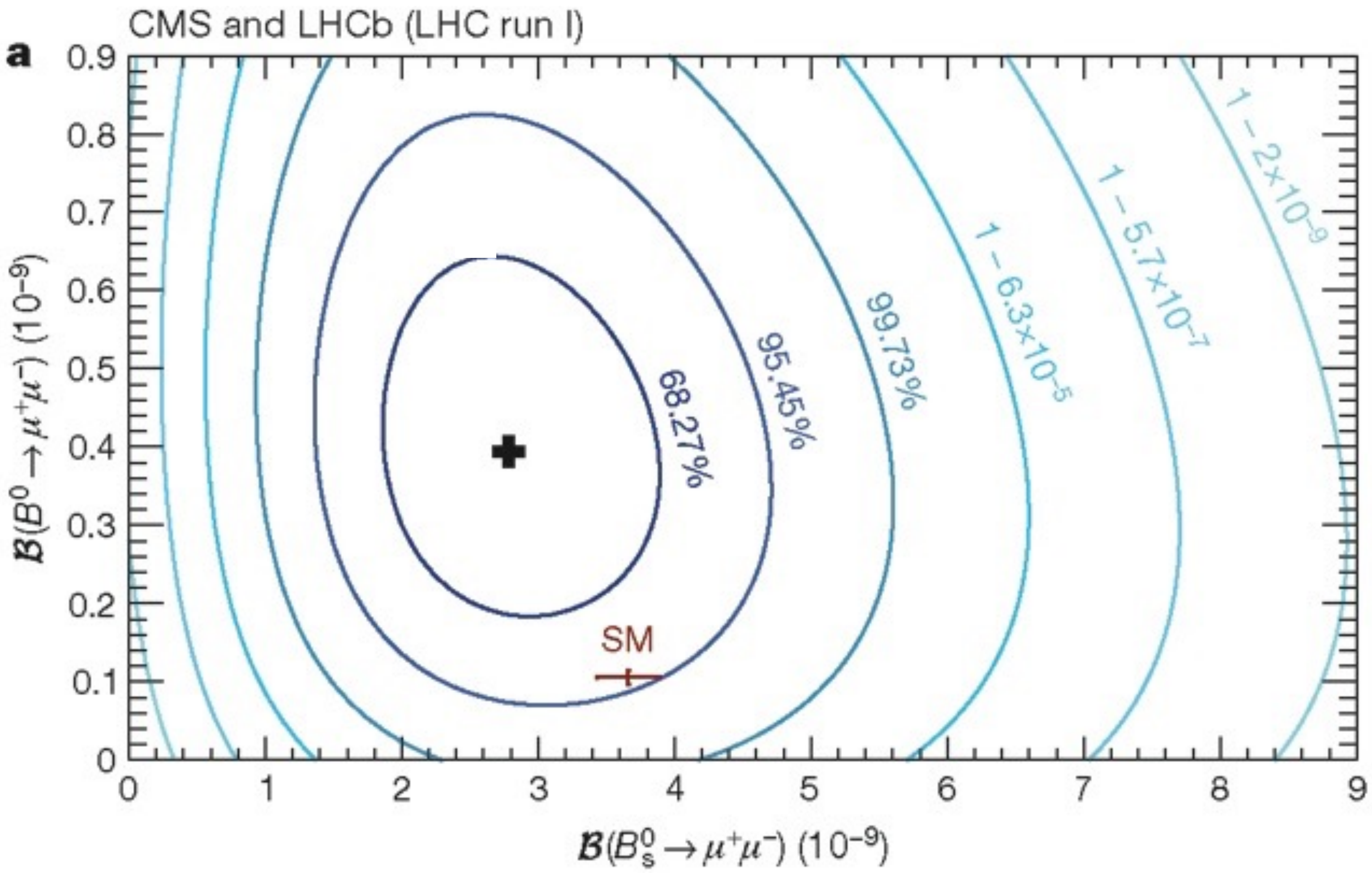
$$\bar{\mathcal{B}}(B_s \to \mu^+\mu^-) = 3.22(22)(6) \times 10^{-9}$$



FNAL/MILC (arXiv:1602.03560)

BSM phenomenology $B_{s(d)} \to \mu^+\mu^-$

CMS+LHCb combined (arXiv:1411.4413, Nature 2015)

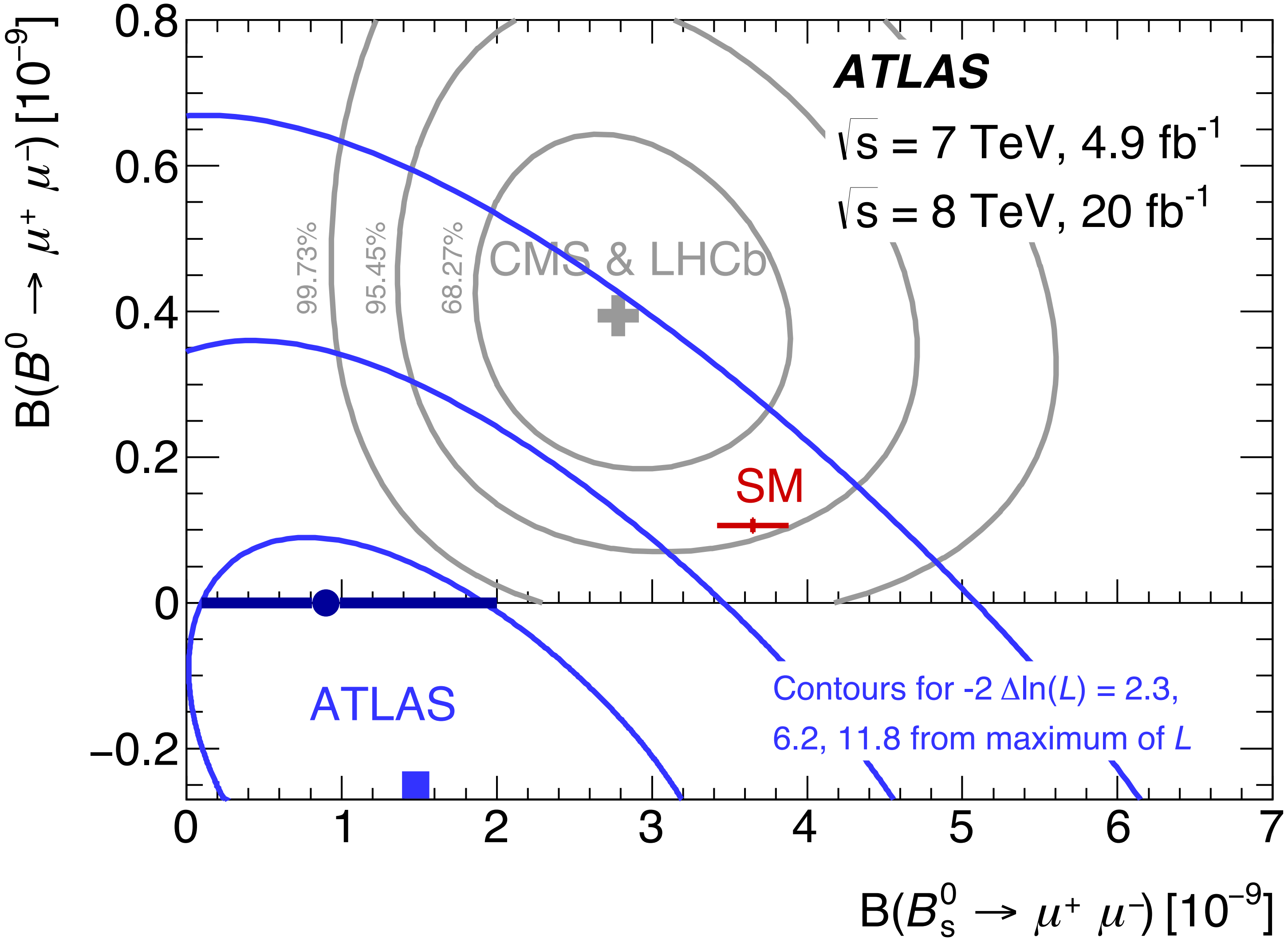


exp. measurements consistent with SM expectations, but with ample room for NP.

SM predictions depend on $f_{B(s)}$ or $\hat{B}_{B_s}$

BSM phenomenology $B_{s(d)} \to \mu^+\mu^-$

CMS+LHCb combined (arXiv:1411.4413, Nature 2015) and ATLAS (arXiv:1604.04263)



exp. measurements consistent with SM expectations, but with ample room for NP.

BSM Phenomenology for $B \to K, \pi\, \ell^+ \ell^-$

Constraints on Wilson coefficients (C_9, C_{10})

- New physics contributions modify the Wilson coefficients:

  $$C_i \to C_i + C_i^{\mathrm{NP}}$$

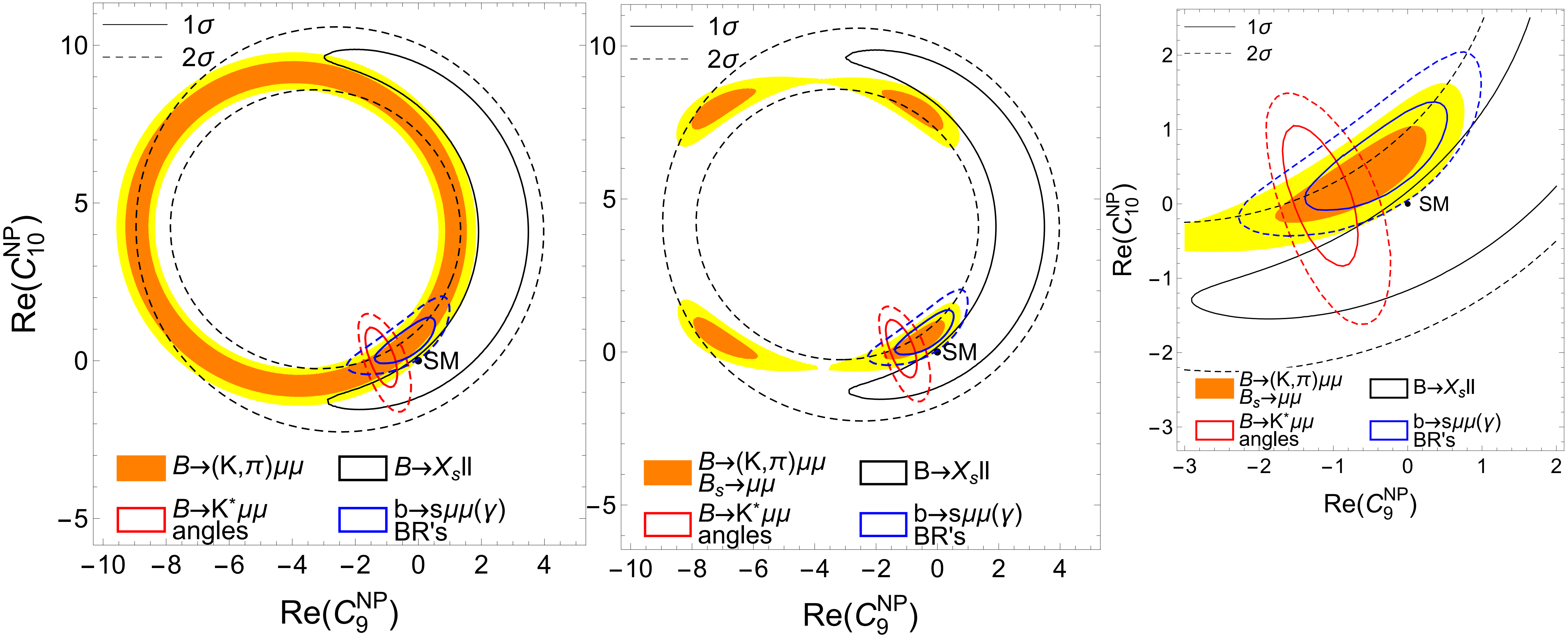
  at the high scale, $\mu_0 = 120\ \mathrm{GeV}$
- take $C_{7,8}^{\mathrm{NP}} = 0$ using constraints from $B \to X_s \gamma$
- assume MFV so that $C_i(b \to s\, \ell\ell) = C_i(b \to d\, \ell\ell)$
- assume $C_{9,10}^{\mathrm{NP}}$ are real (no new CP violating phases)
- take measured $\Delta\mathcal{B}(B \to K, \pi\, \mu^+\mu^-)$ in $\Delta q^2 = 1-6, 15-22\ \mathrm{GeV}^2$
- and FNAL/MILC form factors

- add $B_s \to \mu^+\mu^-$ constraint with lattice f_{Bs}

BSM Phenomenology for $B \to K, \pi\, \ell^+ \ell^-$

Constraints on Wilson coefficients (C_9, C_{10})

D. Du et al (arXiv:1510.02349, PRD 2016)

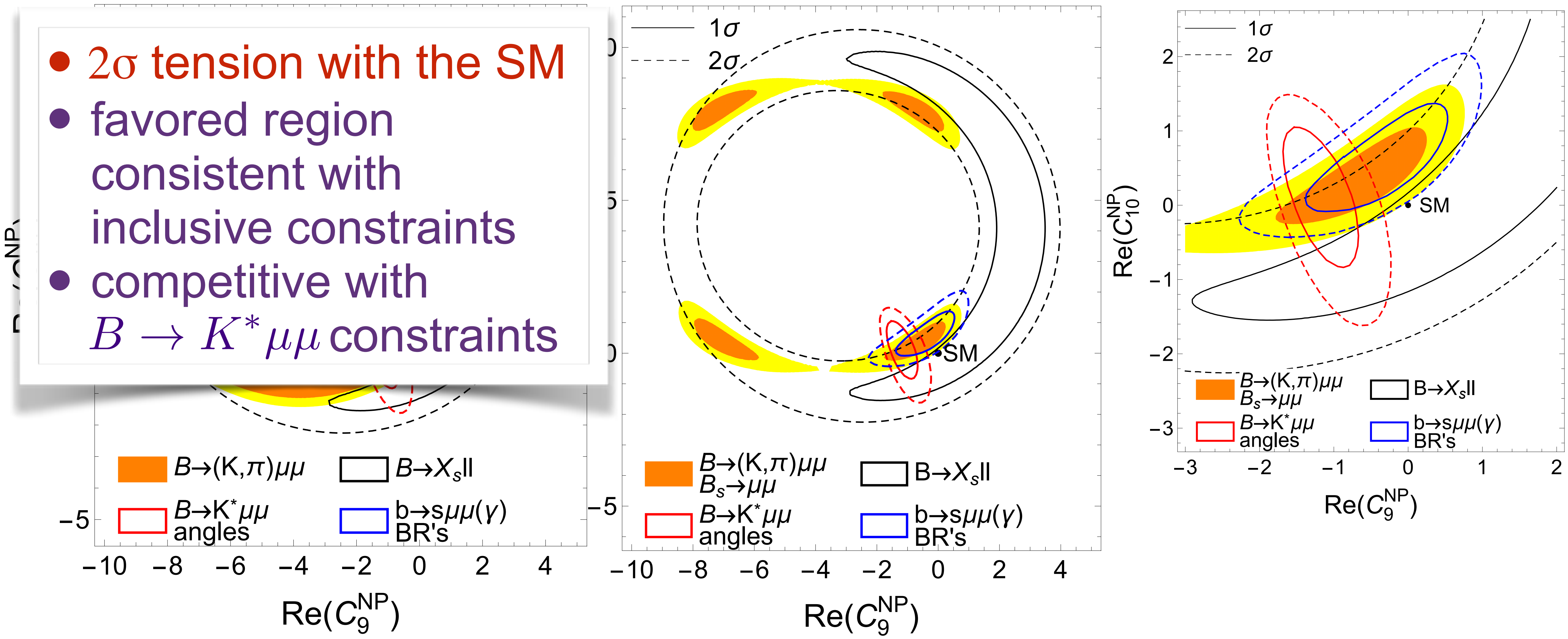


$B \to K\,\mu\mu$, high q^2 bin dominates constraint

BSM Phenomenology for $B \to K, \pi\, \ell^+ \ell^-$

Constraints on Wilson coefficients (C_9, C_{10})

D. Du et al (arXiv:1510.02349, PRD 2016)



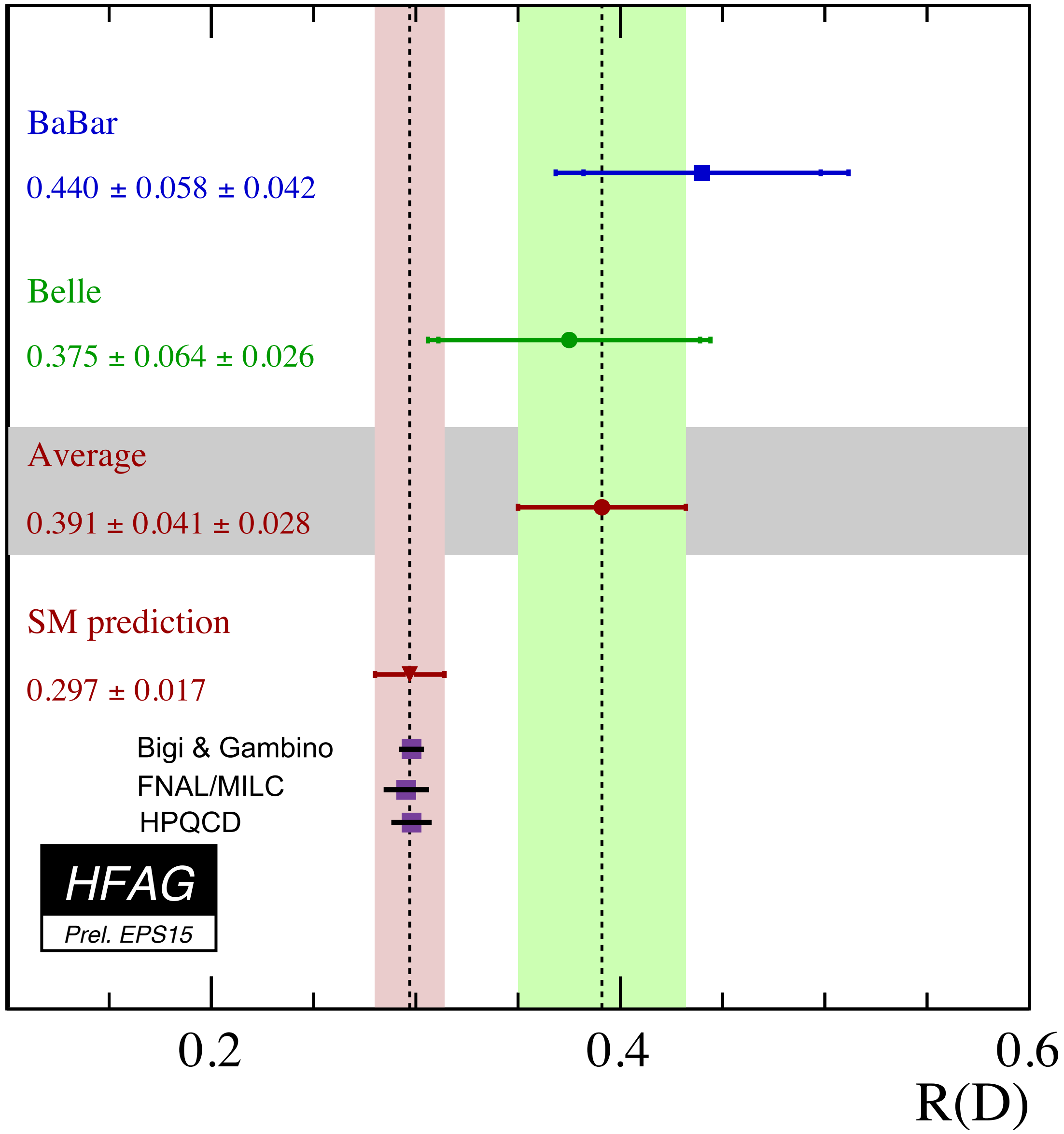
- 2σ tension with the SM
- favored region consistent with inclusive constraints
- competitive with $B \to K^* \mu\mu$ constraints

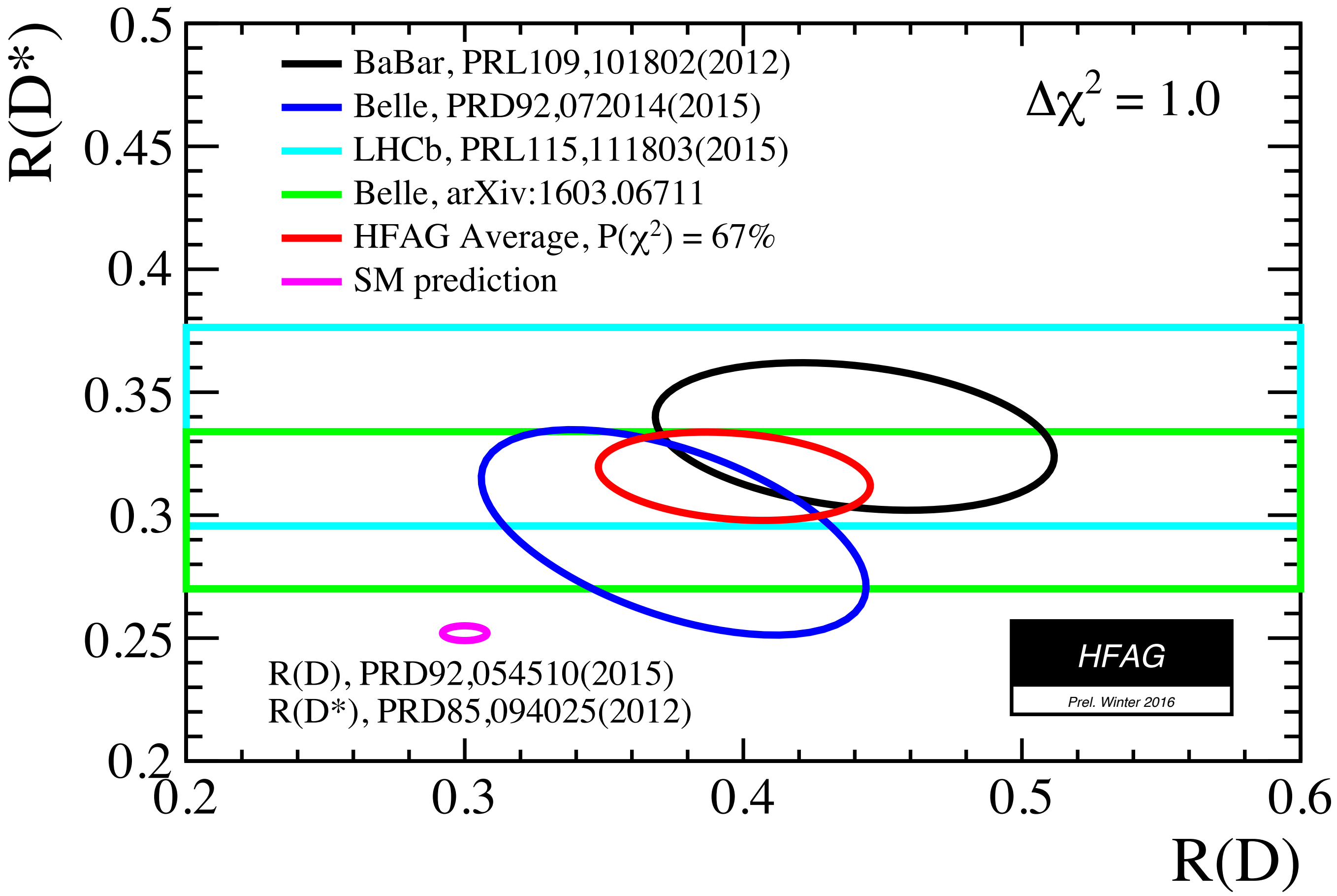
$B \to K\, \mu\mu$, high q^2 bin dominates constraint

BSM phenomenology: LFU τ/ℓ

$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu)}$$

HFAG average for EPS 2015



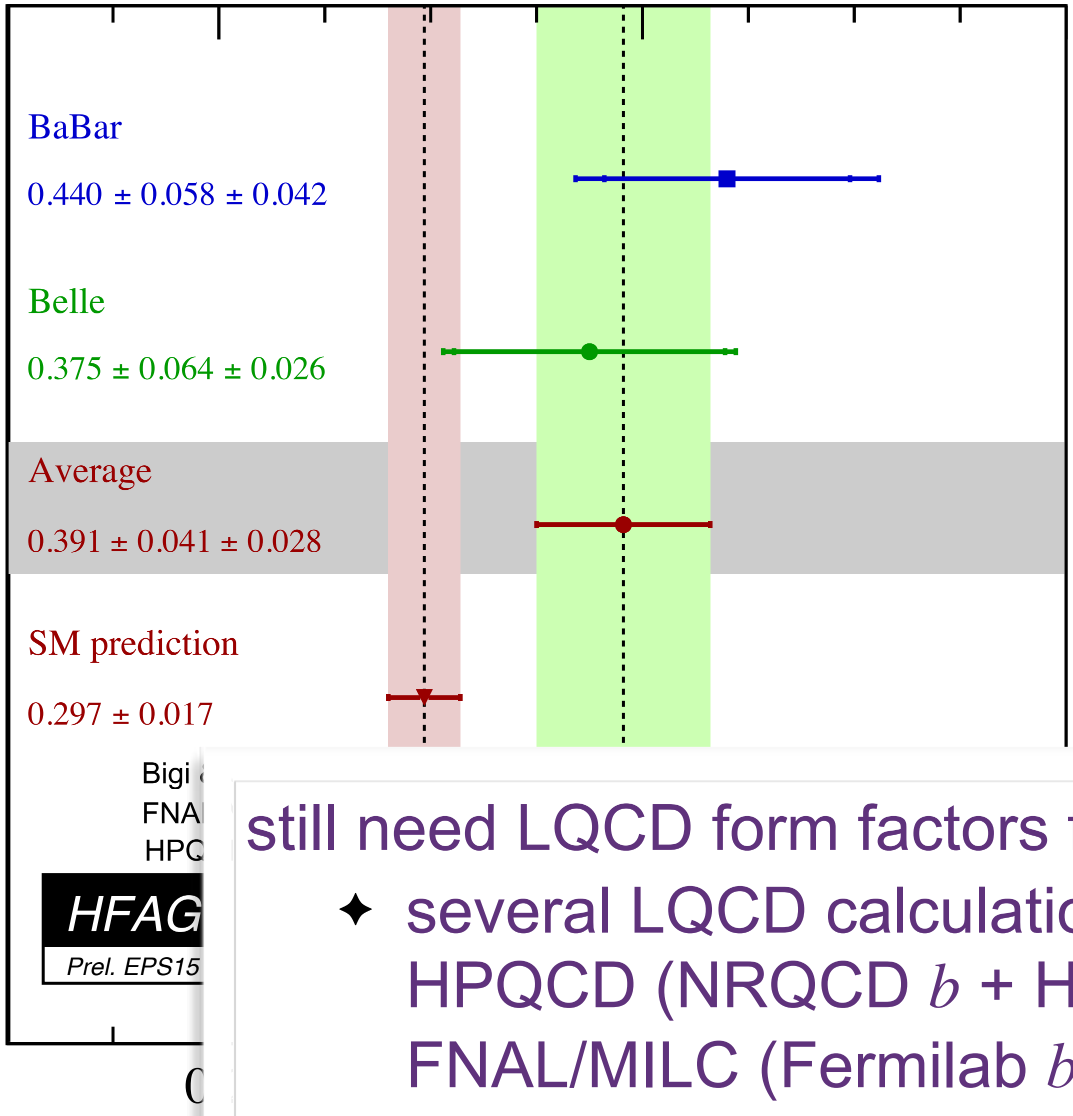


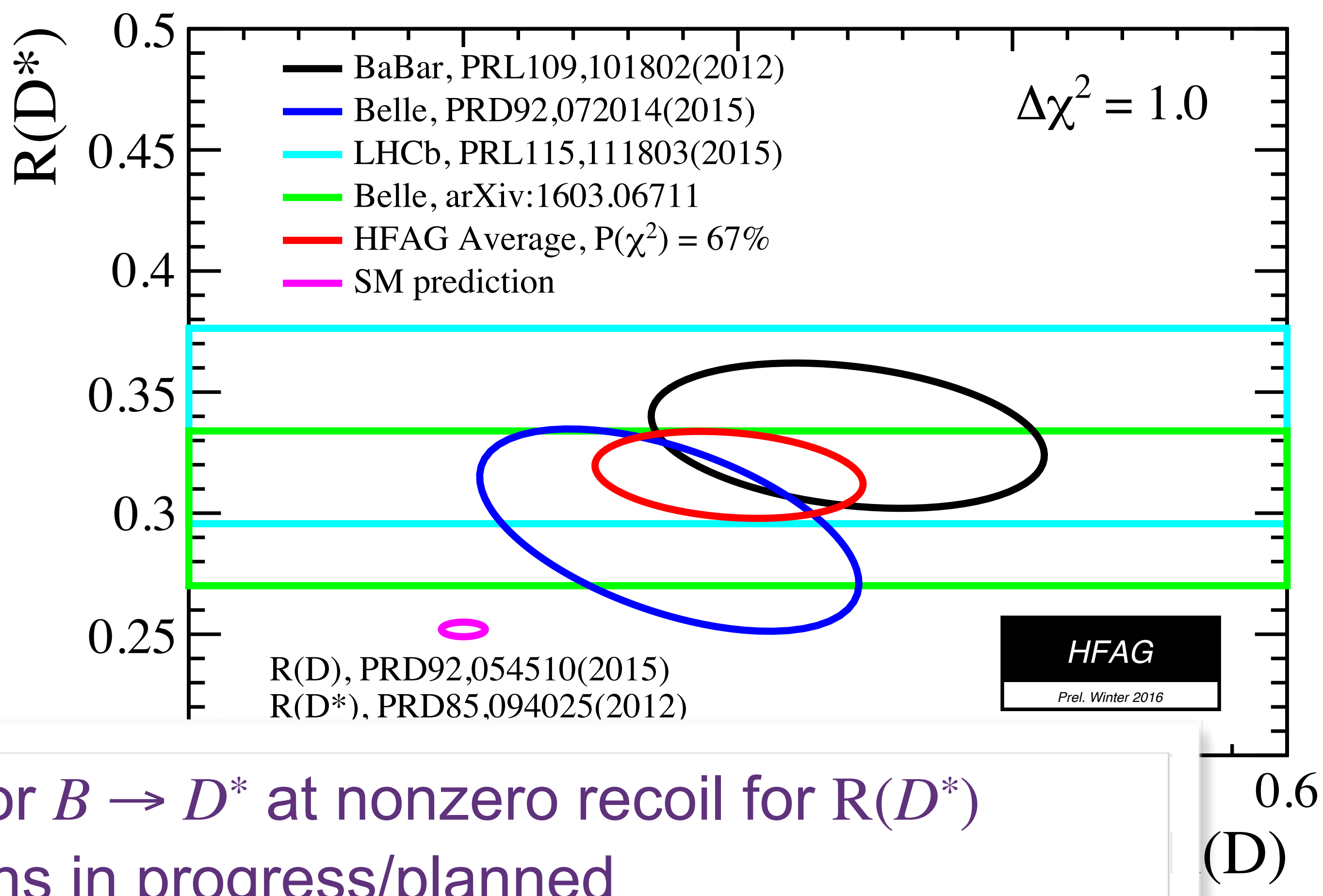
HFAG average: combined 4σ excess

BSM phenomenology: LFU τ/ℓ

$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu)}$$

HFAG average for EPS 2015





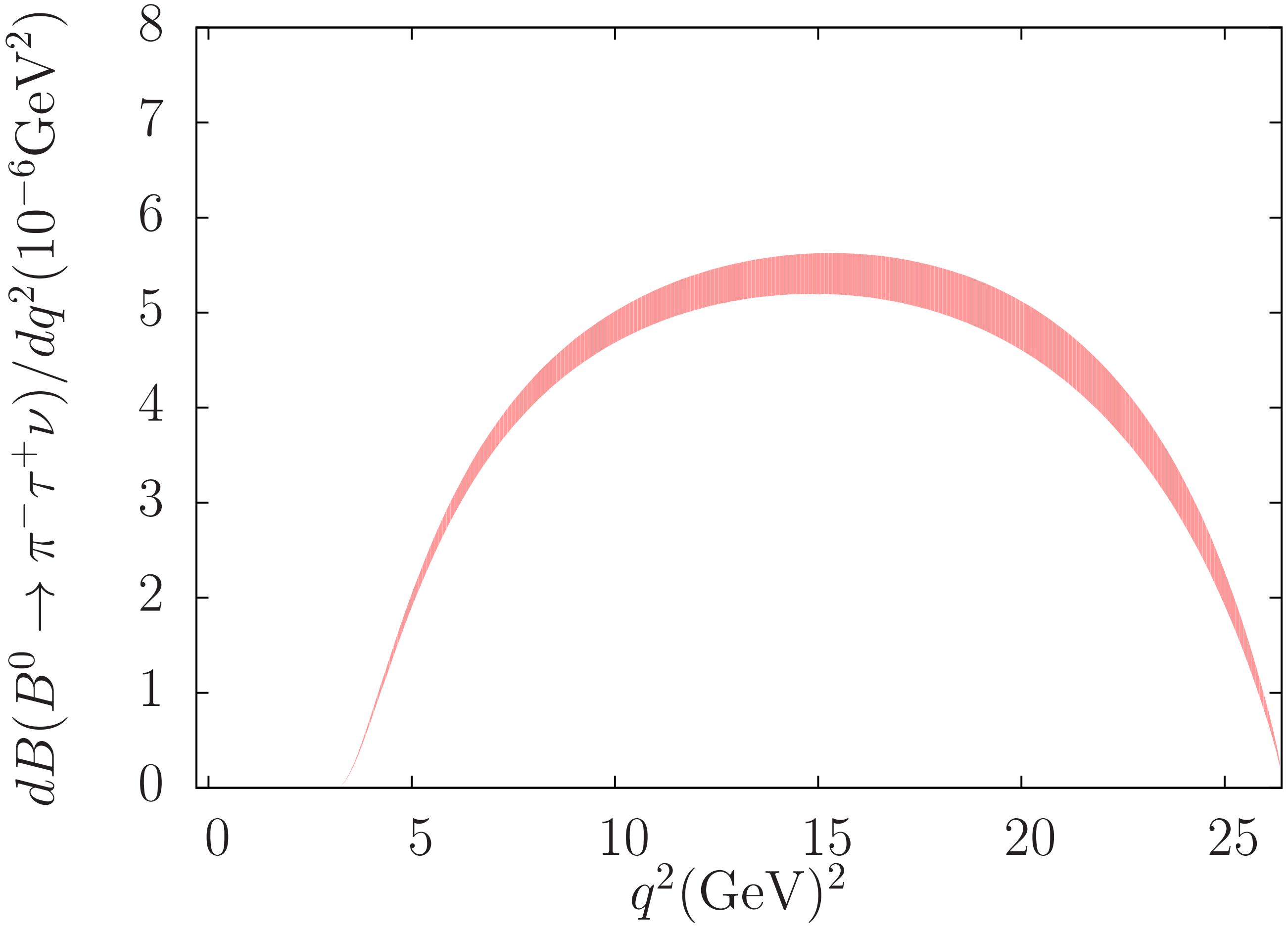
still need LQCD form factors for $B \to D^*$ at nonzero recoil for $\mathrm{R}(D^*)$

- ✦ several LQCD calculations in progress/planned
  HPQCD (NRQCD b + HISQ): Harrison, Monahan @ Lattice 2016
  FNAL/MILC (Fermilab b + asqtad, Fermilab b + HISQ)
  RBC (RHQ b + DWF): Witzel @ Lattice 2016
  LANL-SNU (OK b + HISQ): Bailey, Leem @ Lattice 2016

BSM phenomenology: LFU τ/ℓ

D. Du et al (arXiv:1510.02349, PRD 2016)

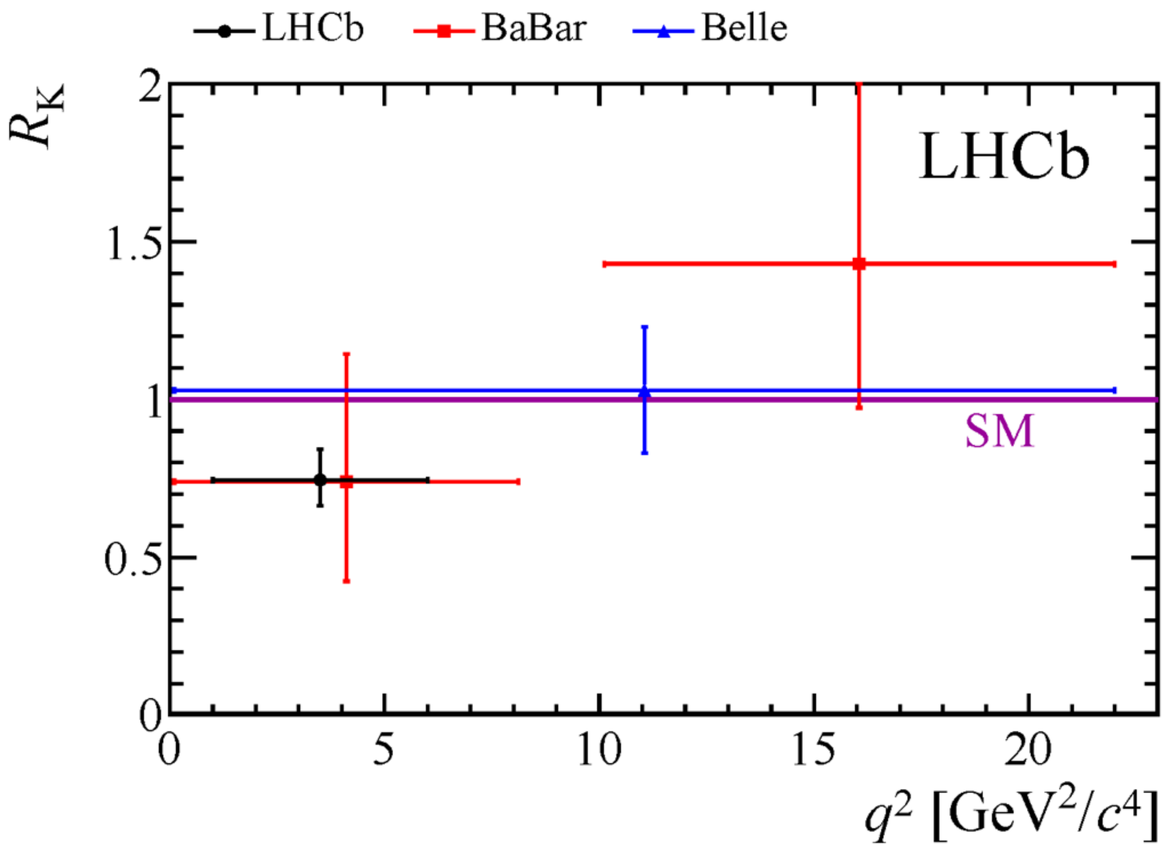
SM prediction for $R(\pi) = \dfrac{\mathcal{B}(B \to \pi\,\tau\nu_\tau)}{\mathcal{B}(B \to \pi\,\ell\nu)} = 0.641(17)$



Uses the form factors from the combined LQCD + exp. fit to $d\mathcal{B}(B \to \pi\ell\nu)/dq^2$

BSM phenomenology: LFU μ/e

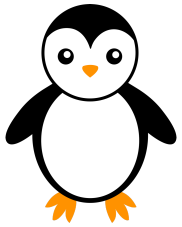
Lepton universality test: $B \to K\mu^+\mu^- / B \to K e^+ e^-$



LHCb (arXiv:1406.6482, PRL 2014):

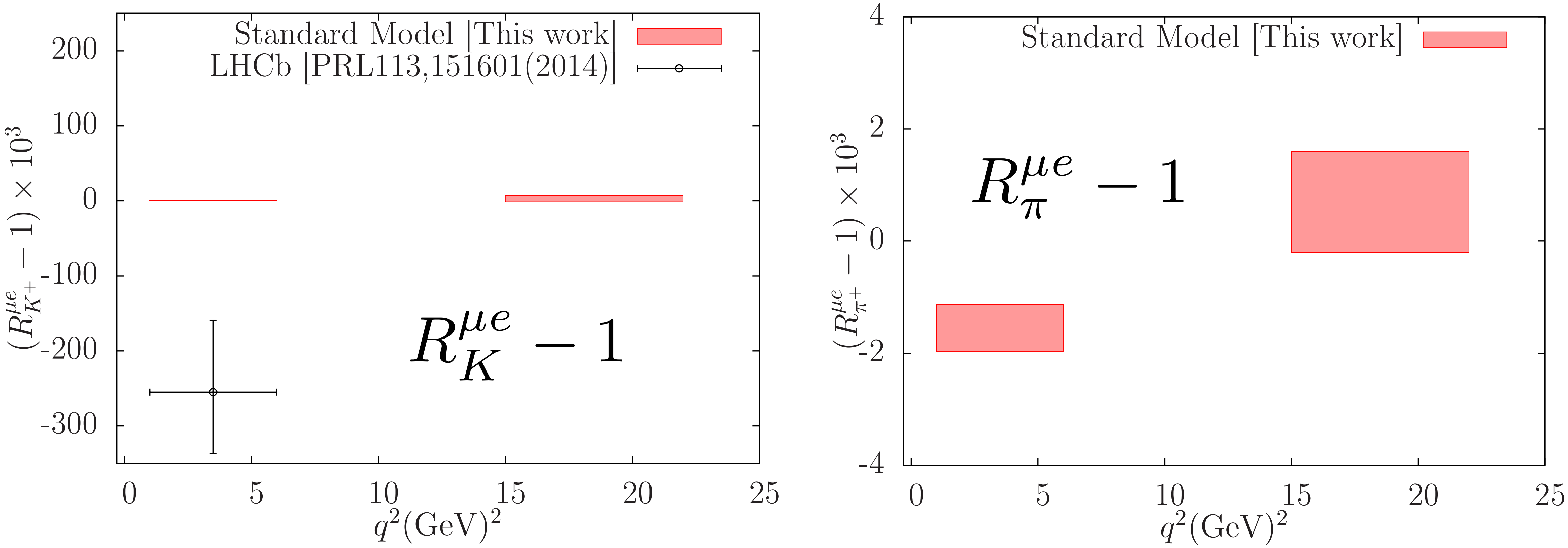
$$\boxed{R_K = 0.745\,\left({}^{90}_{74}\right)(36)}$$

~$2.6\,\sigma$ tension between LHCb measurement and SM theory



BSM phenomenology: LFU μ/e

D. Du et al (arXiv:1510.02349, PRD 2016)



~2.6 σ tension between LHCb measurement and SM theory

In the SM these ratios are insensitive to the form factors

(see also C. Bouchard et al, arXiv:1303.0434, PRL 2013)

Summary

- Gauge field ensembles with light sea quarks at their **physical masses** are being used in a growing number of LQCD calculations.
  ➠ removes chiral extrapolation errors ➠ better precision

- LQCD results for K, π, $D_{(s)}$ decay constants and $K_{\ell 3}$ form factor are very precise (0.25~0.5% errors), B decay constants still at 2% level
  ➠ slight (2σ) tensions with 1st and 2nd row unitarity

- Precise LQCD results for semileptonic form factors for $B \to \pi, K, D$ transitions
  ➢ SM pre/postdictions with theory errors that are commensurate with experimental uncertainties
  ➢ tension for $|V_{cb}|$ and $|V_{ub}|$ between exclusive and inclusive determinations remains, but $|V_{cb}|$ from new $B \to D$ analysis with LQCD form factors at nonzero recoil is consistent with inclusive result.
  ➠ need LQCD form factors for $B \to D^*$ at nonzero recoil
  ➢ 2σ tensions in LFU observables
- new LQCD results for neutral B meson mixing matrix elements with significantly smaller theory uncertainties than before … but still larger than experimental errors …
  ➠ emerging $\sim 2\sigma$ tensions between loop processes and CKM unitarity

Outlook

Amala Willenbrock

Outlook

How do/did we get to 1% total errors (or below)?

- physical mass ensembles are essential
- small lattice spacings
- calculate renormalizations nonperturbatively
- small statistical errors (straightforward, but expensive)

- will need to include
  - strong isospin breaking ($m_u \neq m_d$) effects ✓
  - QED effects

  ➢ program being developed for kaon quantities, muon g-2

Extend LQCD calculations to include "hard(er)" quantities

- theoretical framework for semileptonic B decays to vector meson final states under development (Briceño et al, arXiv:1406.5965, 2015 PRD; Agadjanov et al, arXiv:1605.03386).

  ➢ LQCD calculations of form factors for $B_s \to K^* \ell\nu,\ B \to K^* \ell\ell, \ldots$

  pilot studies are underway
- Ongoing work for kaons (RBC/UKQCD, JLQCD): $K \to \pi\pi, \epsilon', \Delta M_K, \ldots$

Amala Willenbrock

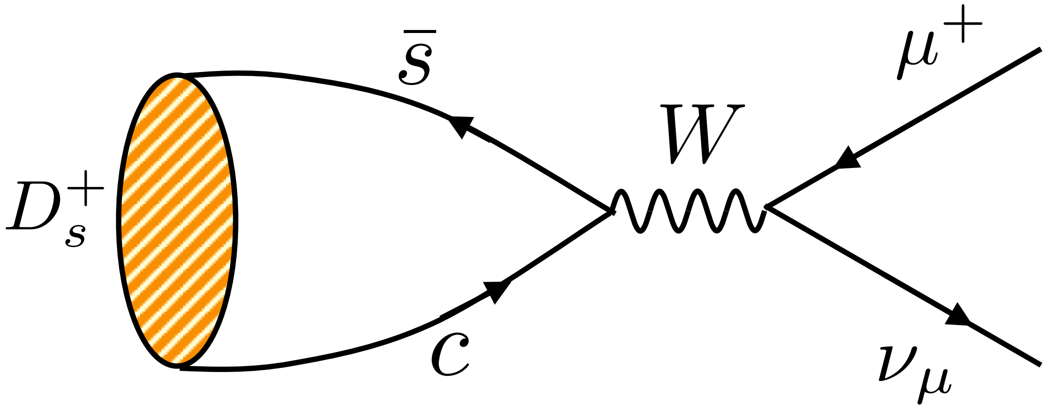
Thank you!

Farah Willenbrock

Backup slides

Leptonic D, K decay

example: $D_s^+ \to \mu^+ \nu_\mu$



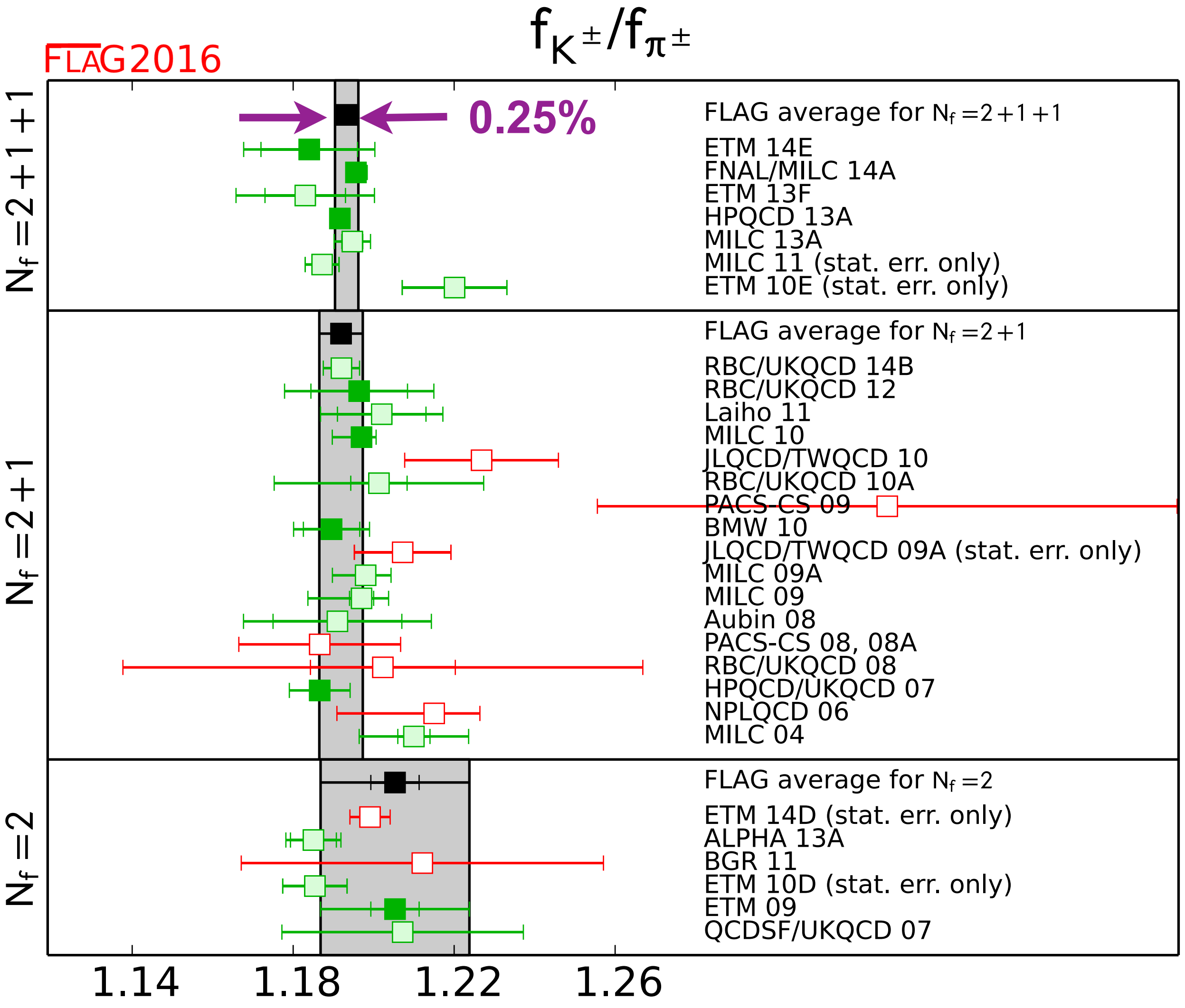
$$\Gamma(D_s^+ \to \ell^+ \nu_\ell(\gamma)) = (\text{known}) \times (1 + \delta_{\text{EM}}^{\ell}) \times |V_{cs}|^2 f_{D_s}^2$$

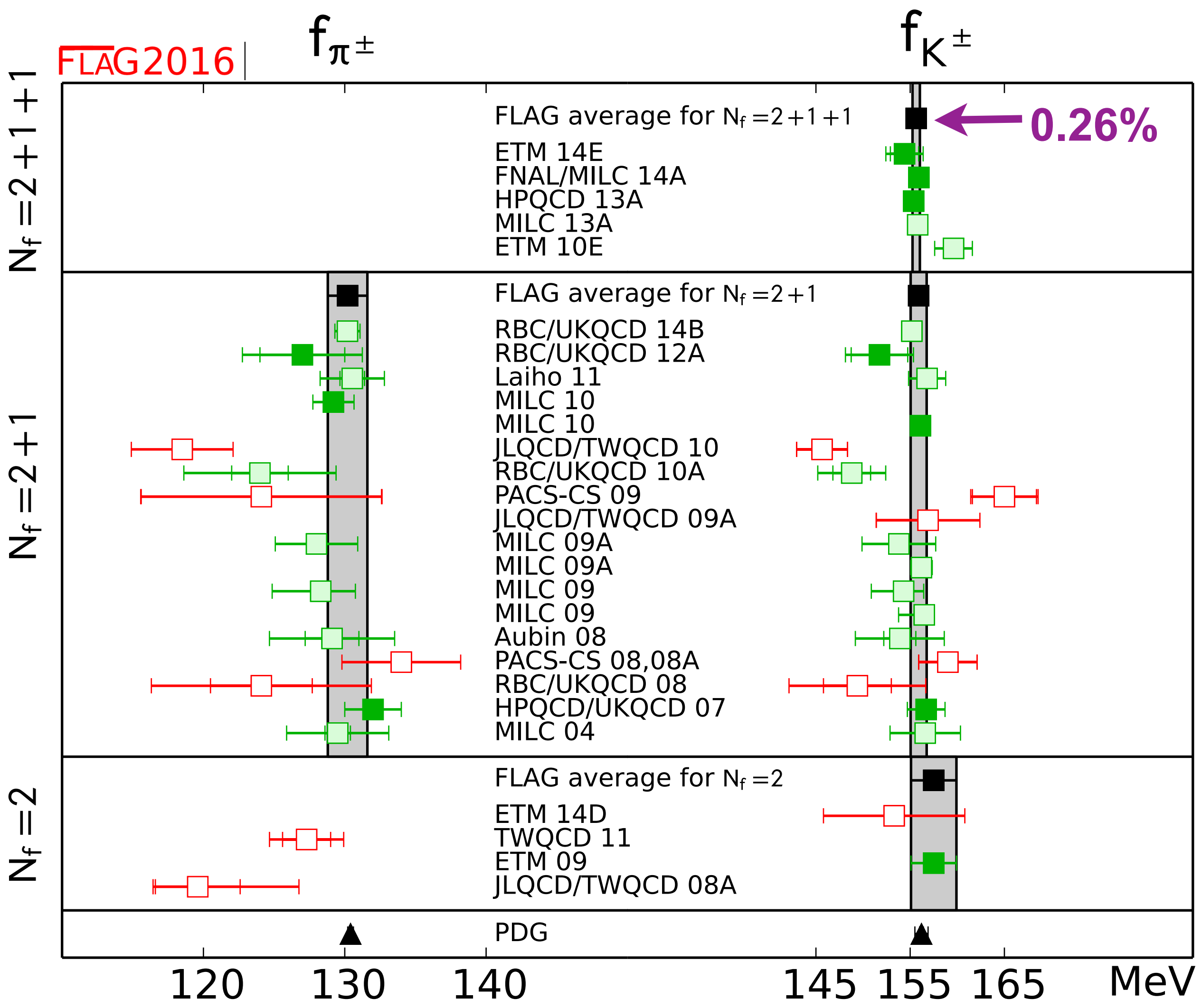
- $\delta_{\text{EM}}^{\ell}$ includes structure dependent EM corrections. It is needed to relate the “pure QCD” decay constant to experiment and is currently estimated phenomenologically.

Kaon decay constant summary

S. Aoki et al (FLAG-3 review, arXiv:1607.00299)

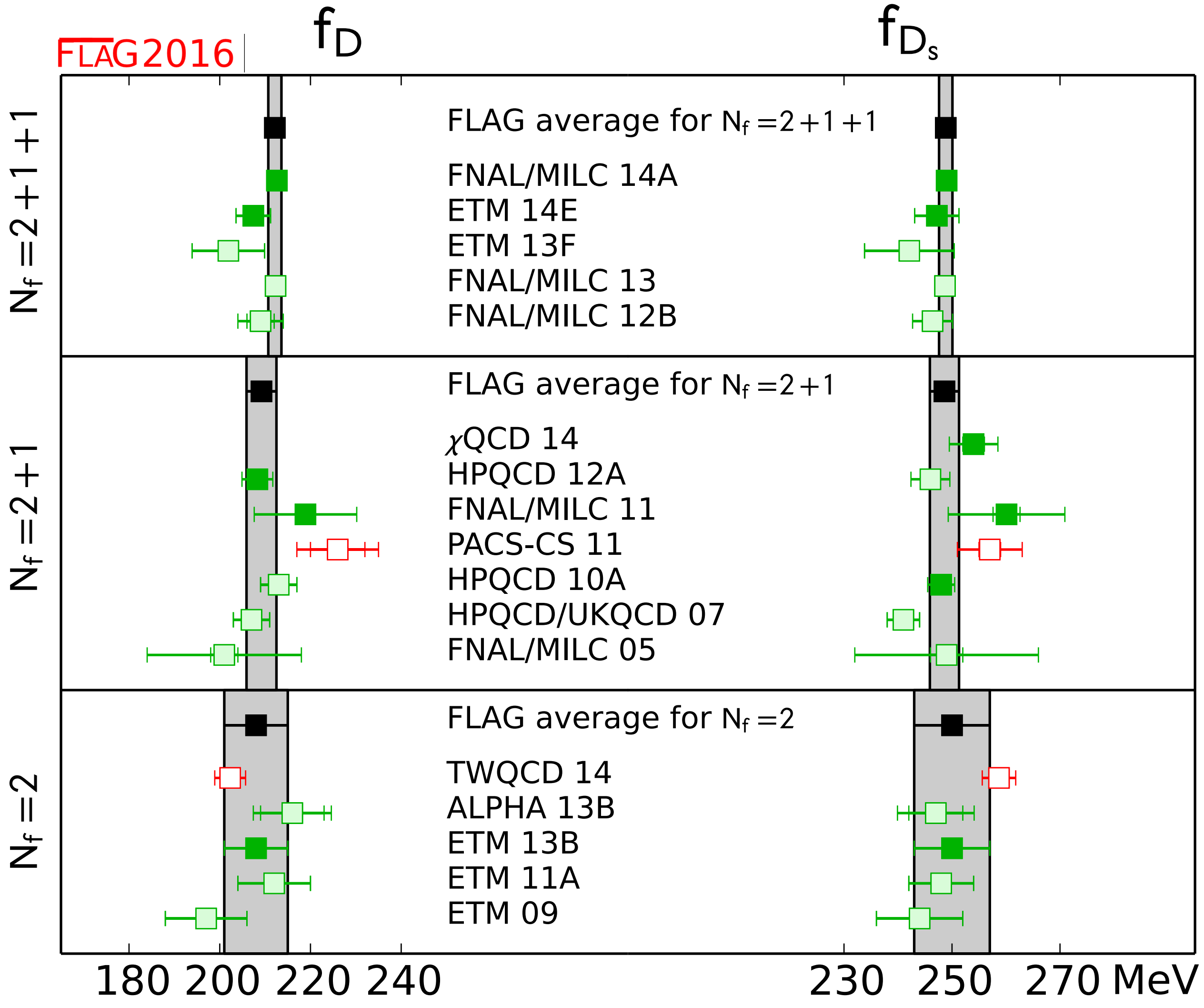
status
end 2015

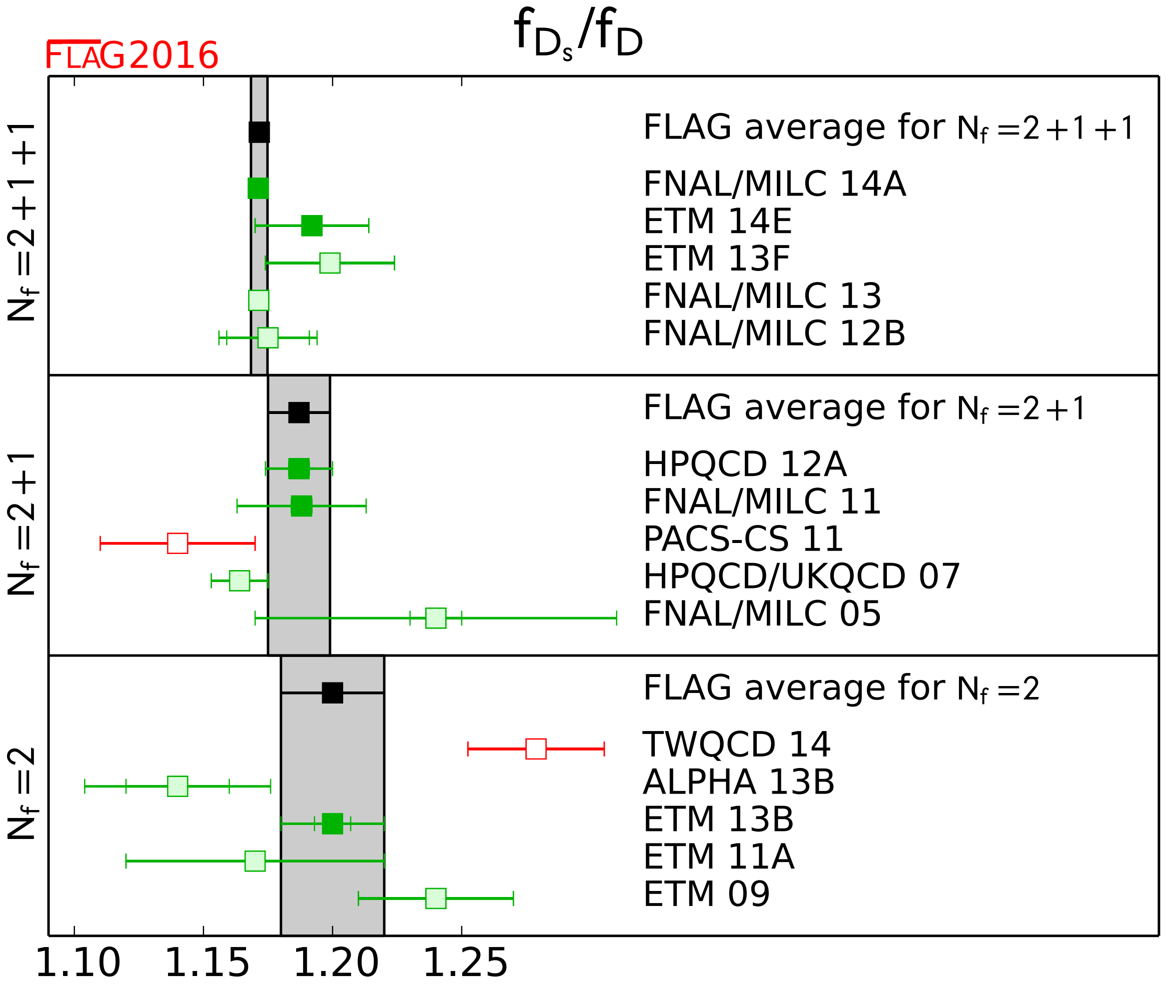




$D_{(s)}$ decay constant summary

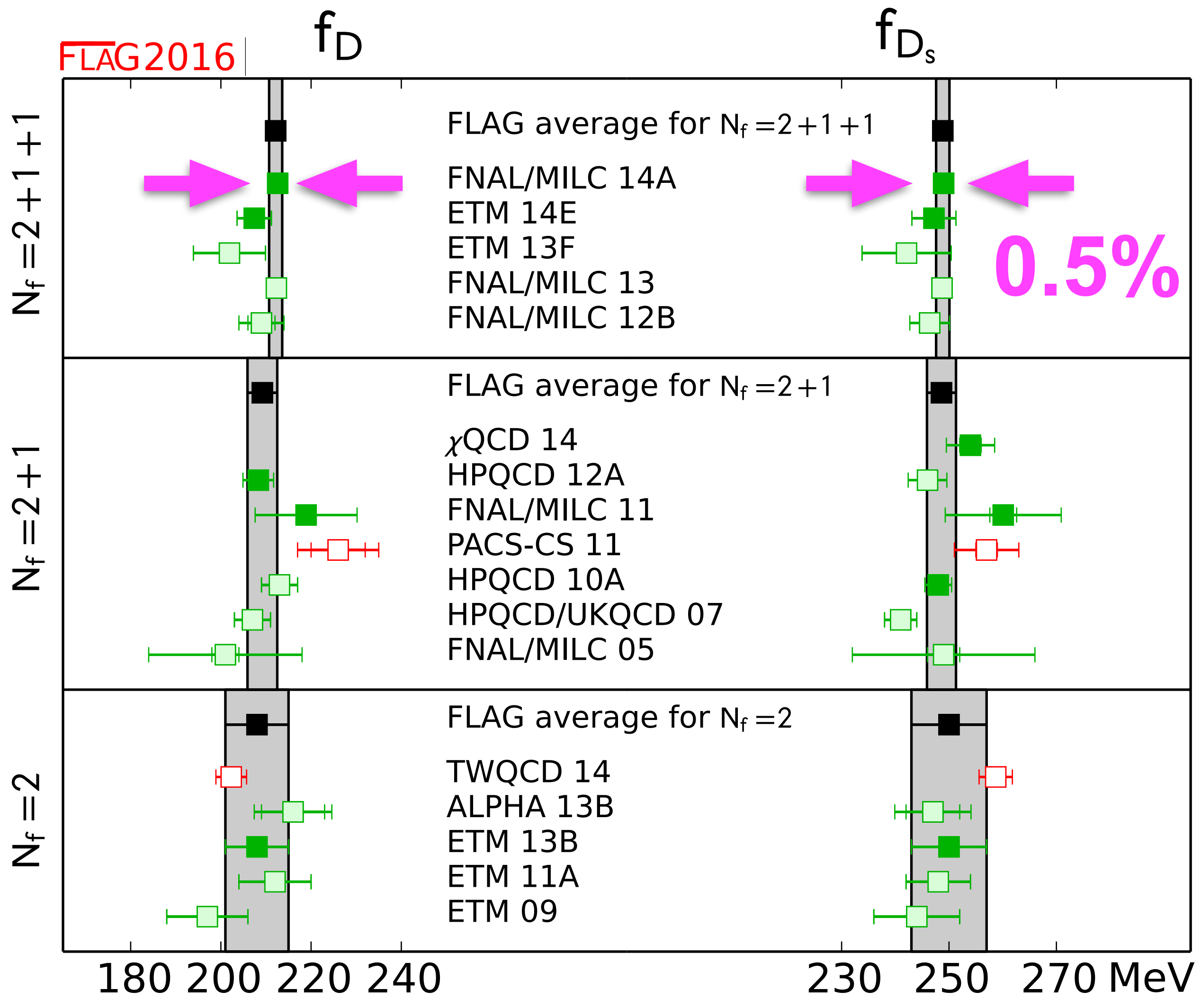
S. Aoki et al (FLAG-3 review, arXiv:1607.00299)



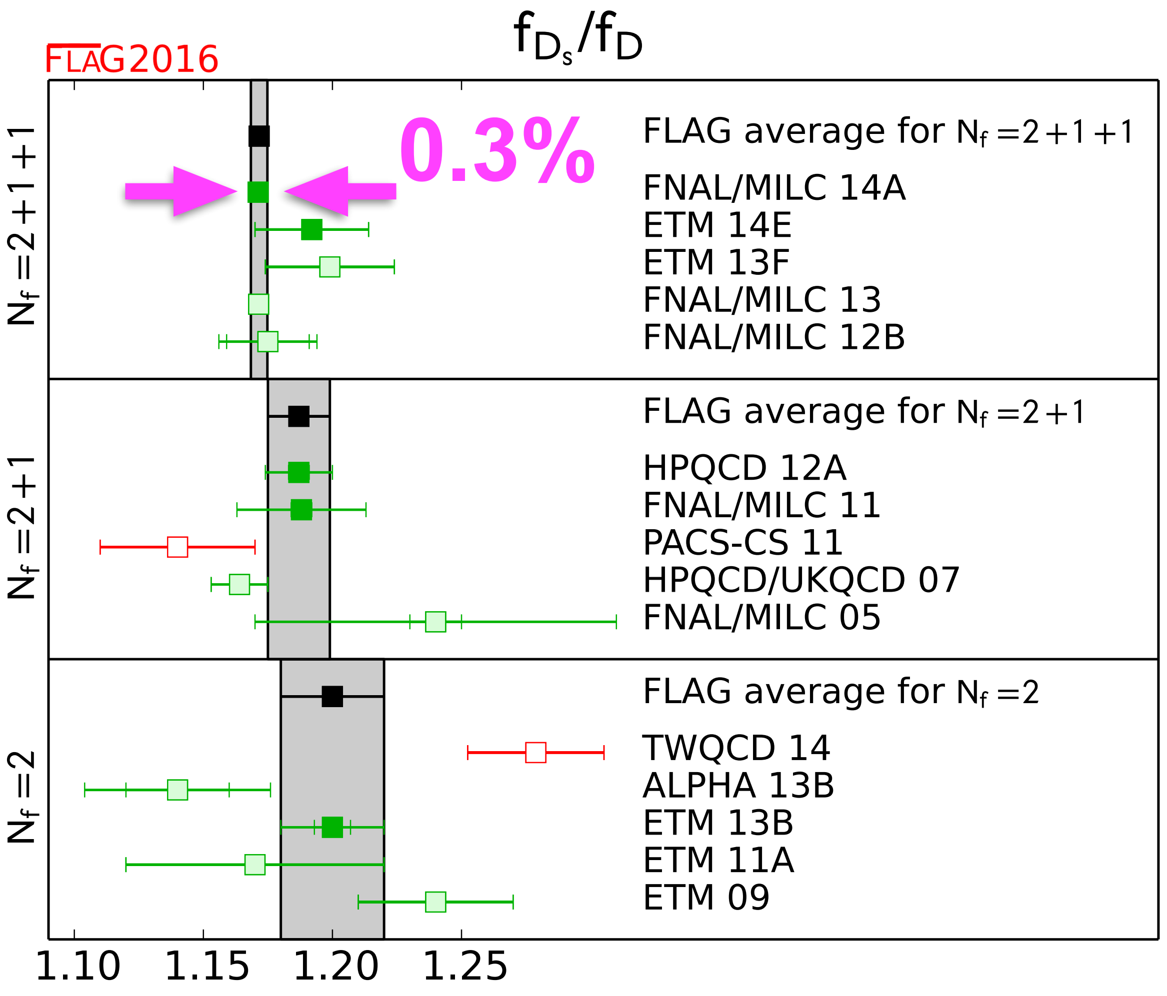


$D_{(s)}$ decay constant summary

S. Aoki et al (FLAG-3 review, arXiv:1607.00299)



0.5%



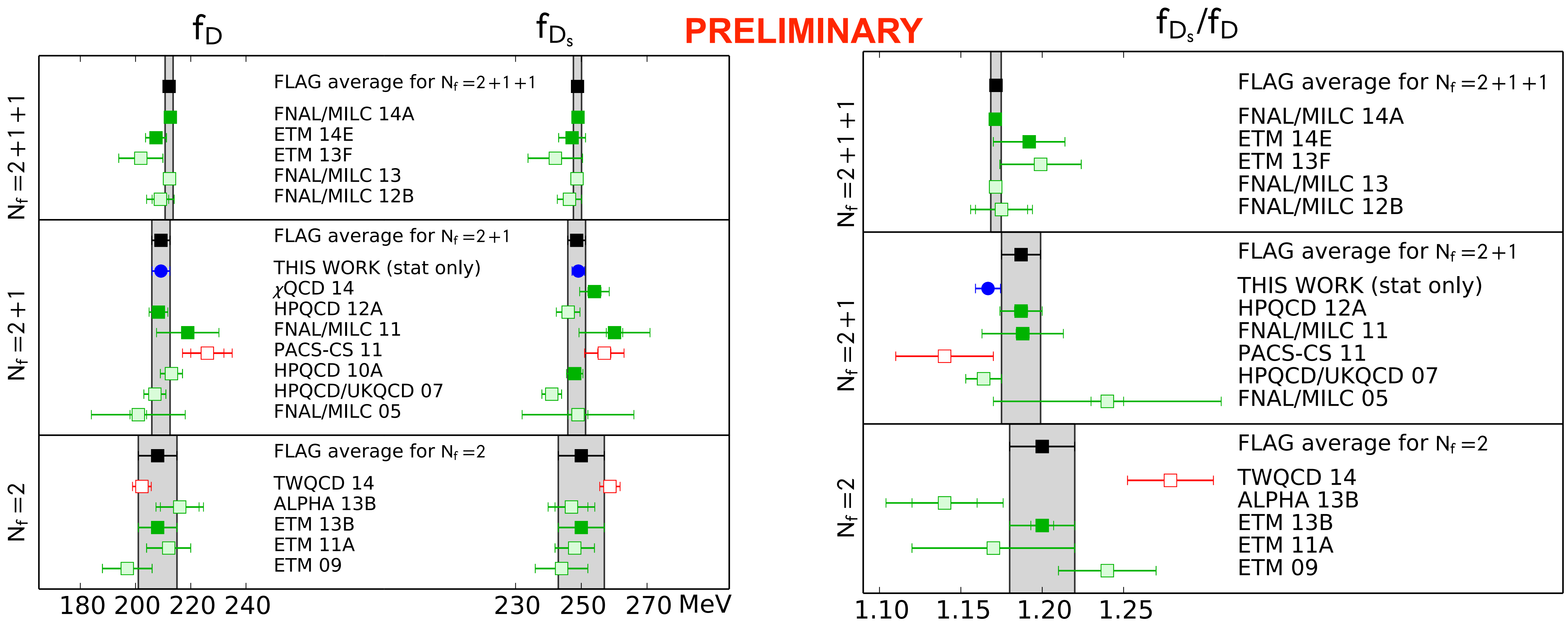
0.3%

small errors in FNAL/MILC 14A (arXiv:1407.3772, 2014 PRD) due to

- physical mass ensembles
- improved action (small discretization errors)
- small lattice spacings
- PCAC (no renormalization)

$D_{(s)}$ decay constant summary

J. T. Tsang (RBC/UKQCD) @ Lattice 2016:



RBC/UKQCD (J.T. Tsang @ Lattice 2016):

- 2+1 flavors of DW fermions
- physical mass ensembles
- PCAC (no renormalization)

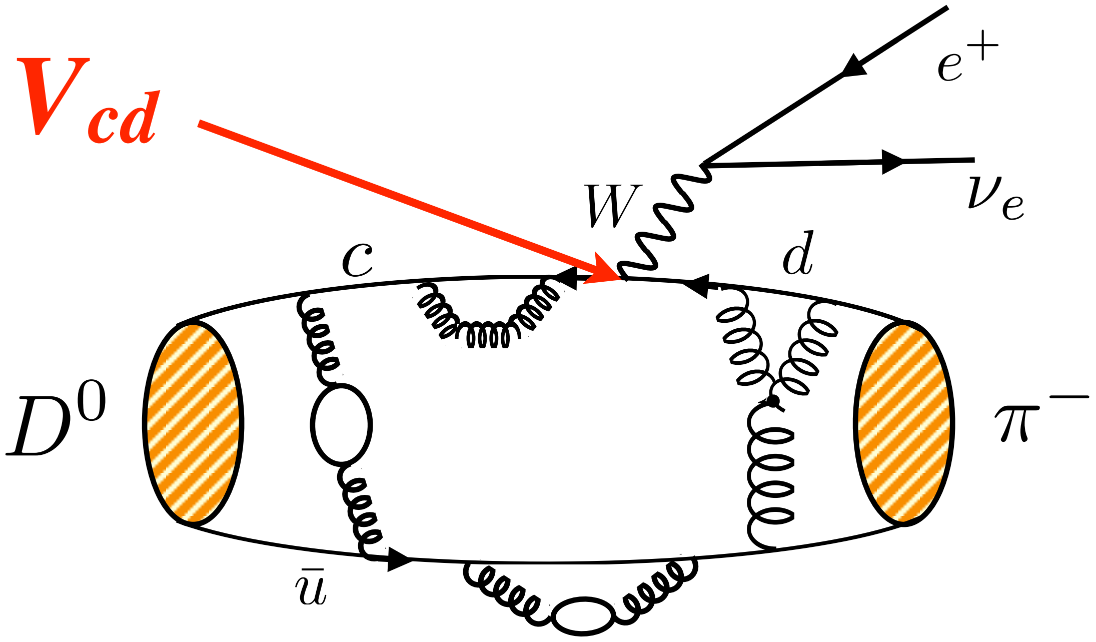
also ongoing work by:

- ALPHA/RQCD (imp. Wilson)
- FNAL/MILC (with Fermilab charm)

+ new results from ETM on $f_{D^*(s)}$ (Melis @ Lattice 2016)

Semileptonic D-meson decay

Example: $D \to \pi \ell \nu$



$$\frac{d\Gamma(D \to \pi \ell \nu)}{dq^2} = (\text{known}) \times |V_{cd}|^2 \, f_+^2(q^2)$$

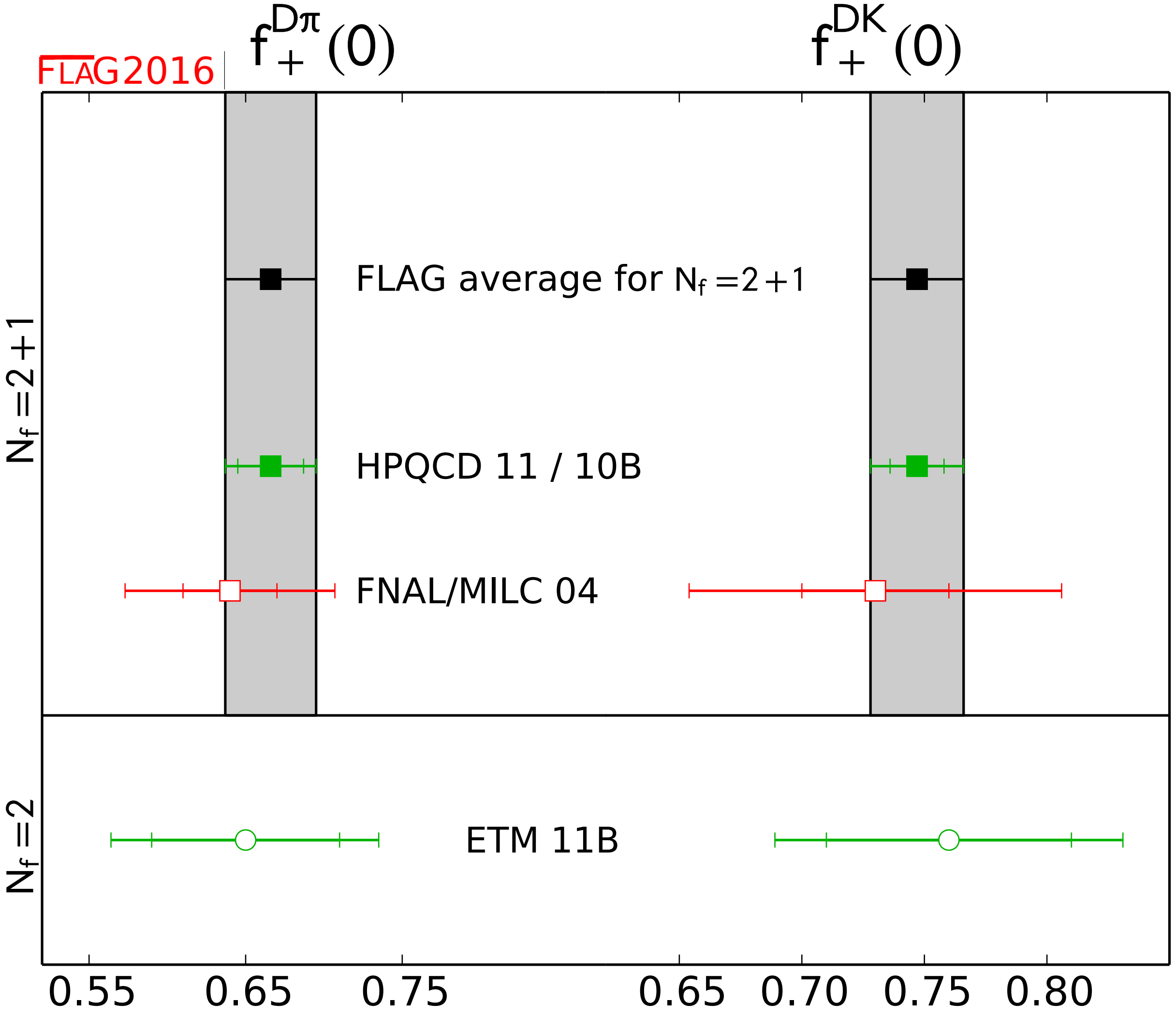
$$\boxed{\ell = e, \mu}$$

- ★ can calculate the form factors for the entire recoil energy range
- ★ can use z-expansion* for model-independent parameterization of q^2 dependence
- ★ calculate both form factors $f_+(q^2), f_0(q^2)$
- ★ can compare shape between experiment and lattice
- ★ extension to rare SL decay form factors (f_T) straightforward

*see backup slides

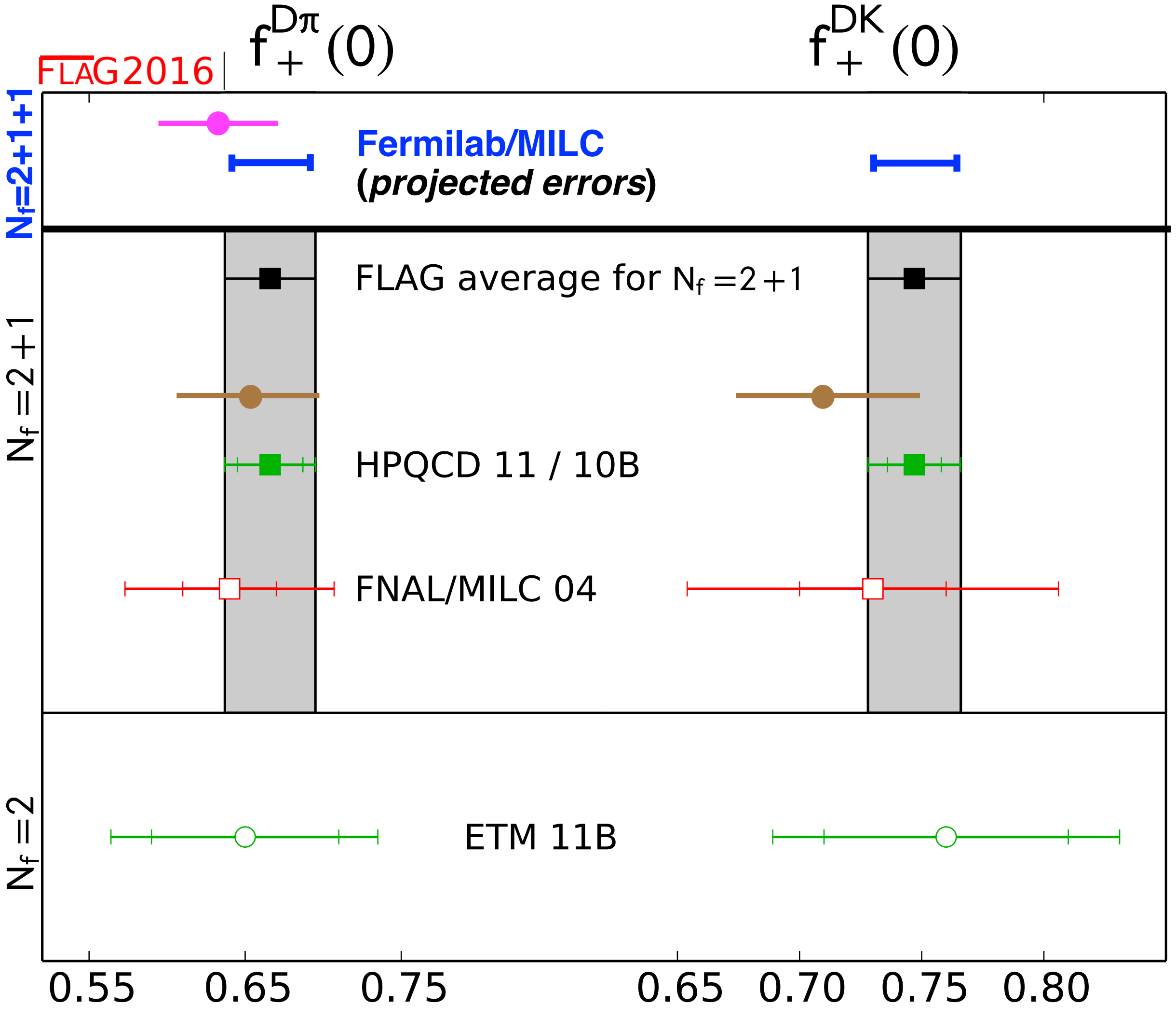
D SL form factor results

S. Aoki et al (FLAG review, arXiv:1607.00299)



D SL form factor results

adapted from S. Aoki et al (arXiv:1607.00299)



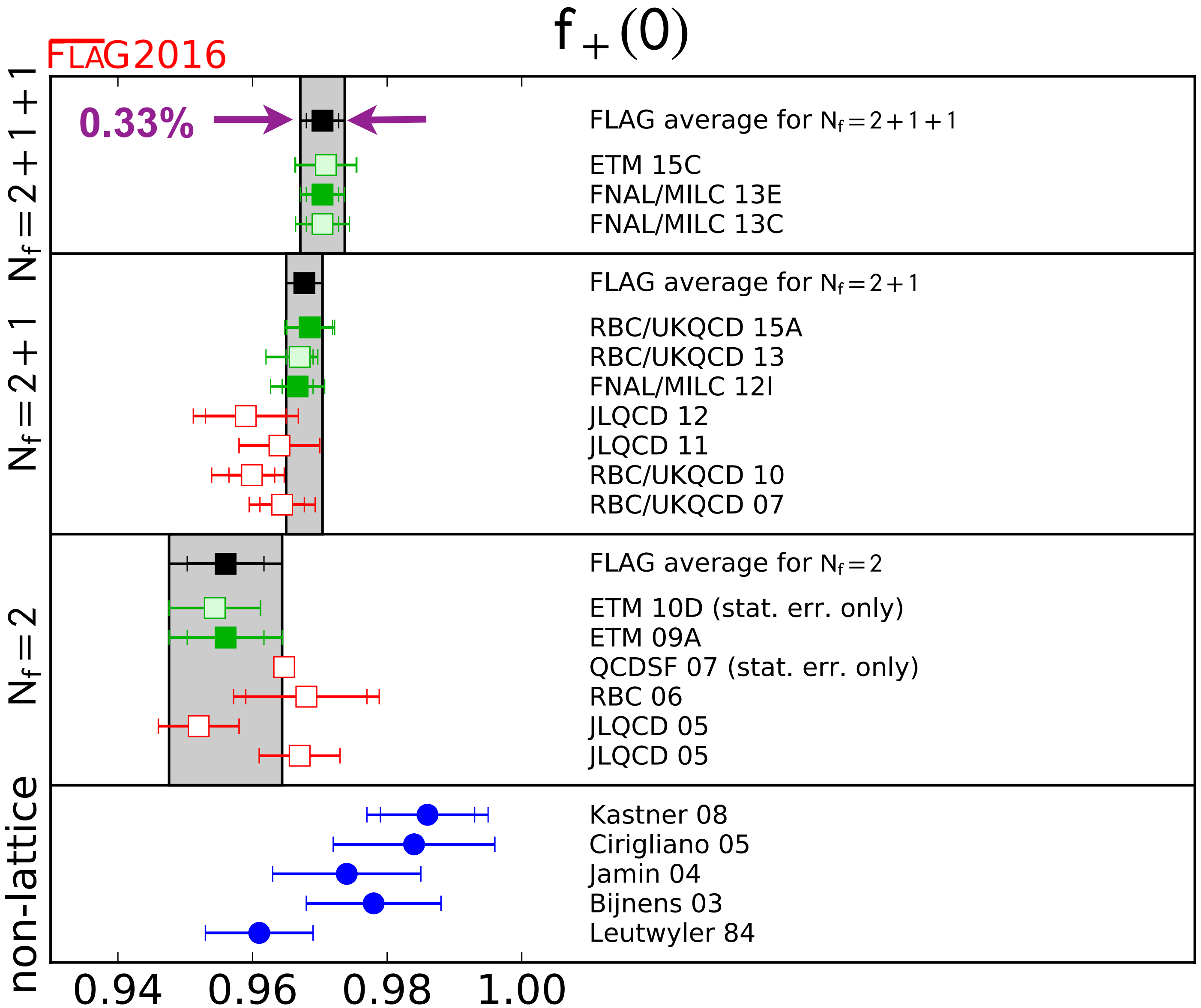
new preliminary results @ Lattice 2016:

- ETM (G. Salerno)
  2+1+1 flavors of tmWilson
  calculate all form factors over whole q^2 range
  modified z-expansion
  preliminary sys. errors
- FNAL/MILC (S. Gottlieb, T. Primer)
  no central values (yet)
  2+1+1 flavors of HISQ
  physical mass ensembles
  calculate directly at zero q^2
- JLQCD (T. Kaneko)
  2+1 flavors of DW fermions
  extrapolate to zero q^2 with z-expansion
  chiral-continuum extrapolaton
  still adding ensembles to analysis

summary for $K_{\ell 3}$ form factor

S. Aoki et al (FLAG-3 review, arXiv:1607.00299)

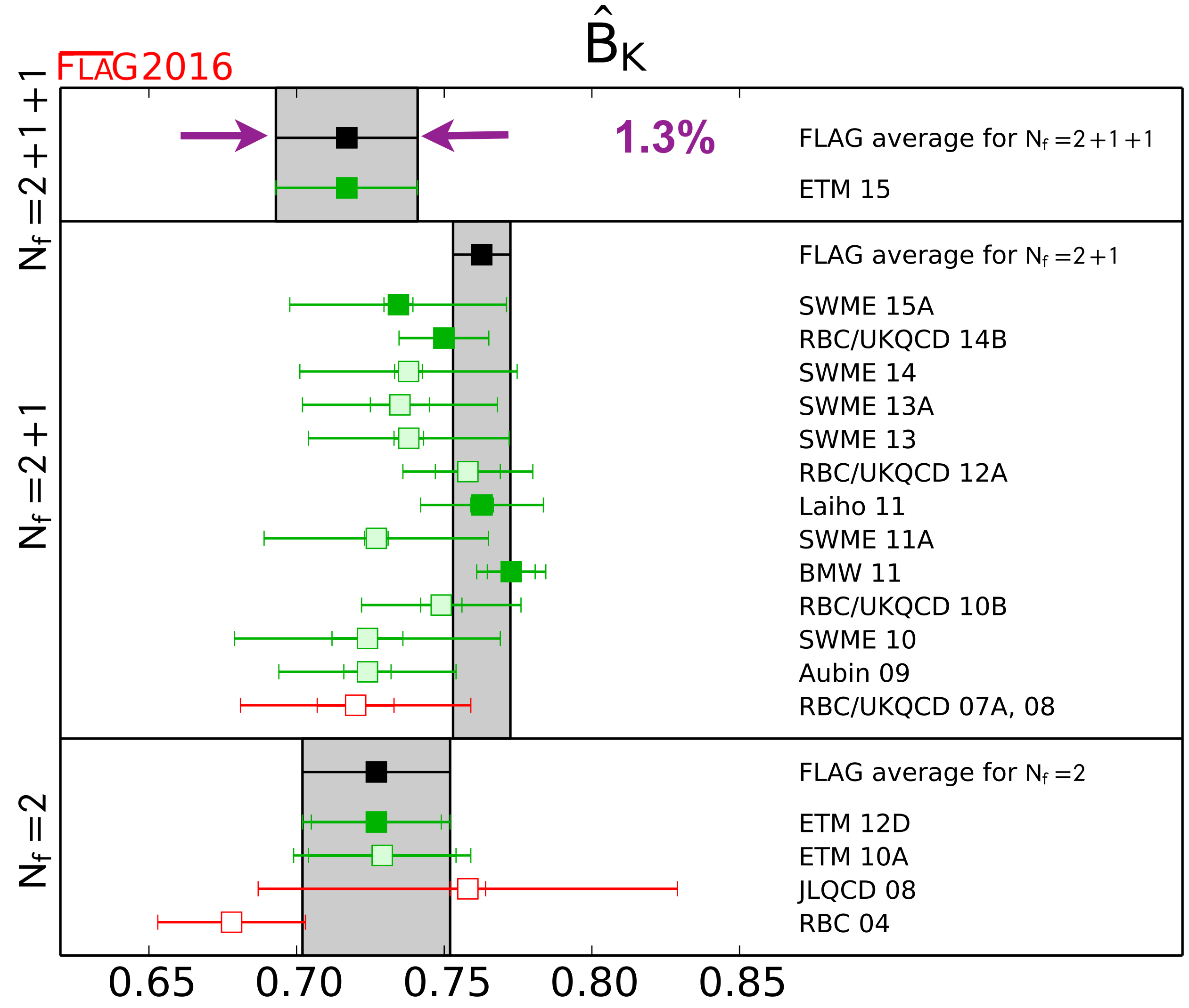
status
end 2015

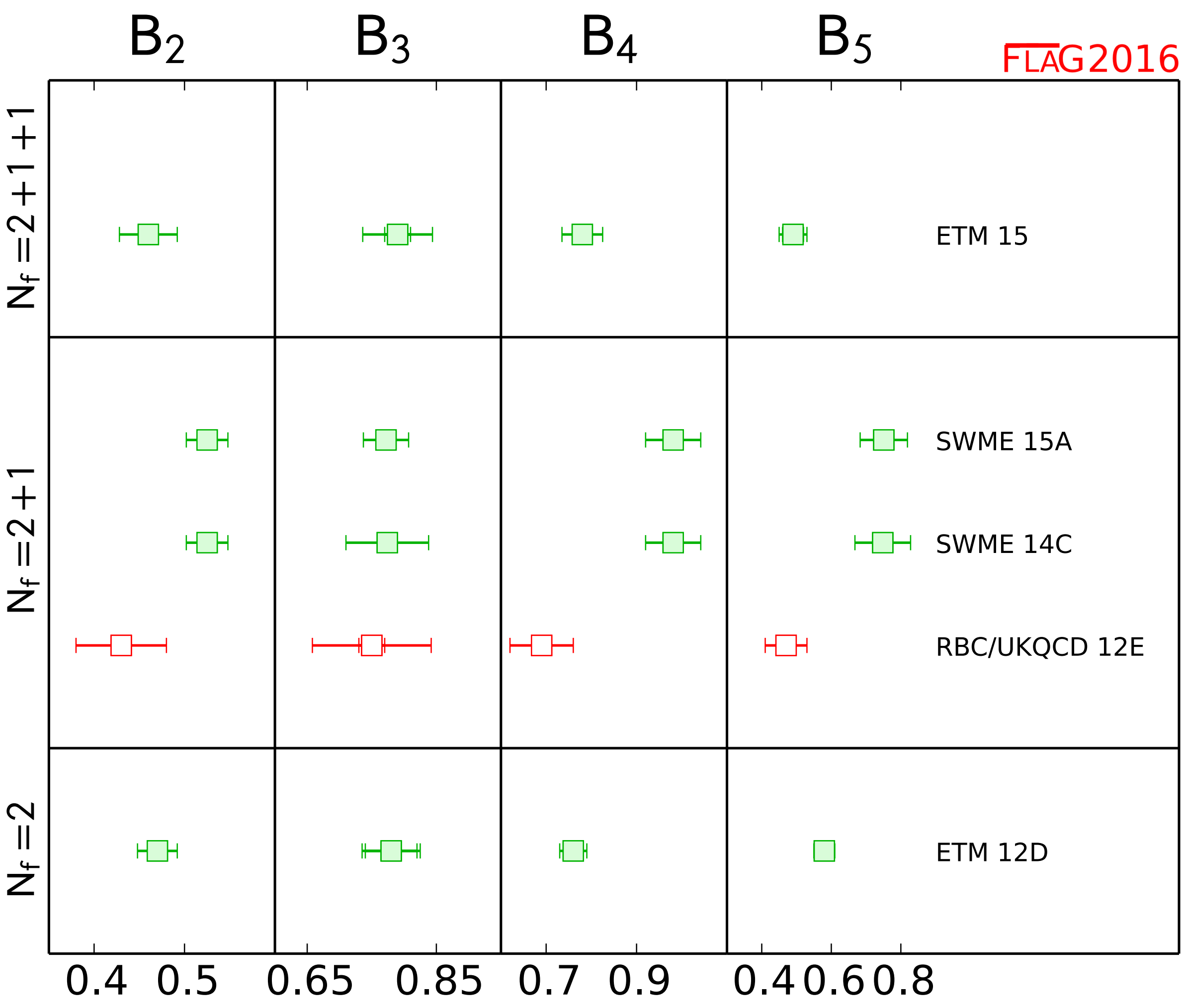


summary for B_K

S. Aoki et al (FLAG-3 review, arXiv:1607.00299)

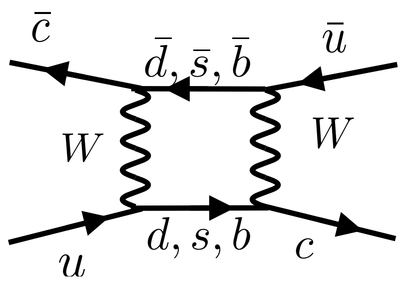
status
end 2015

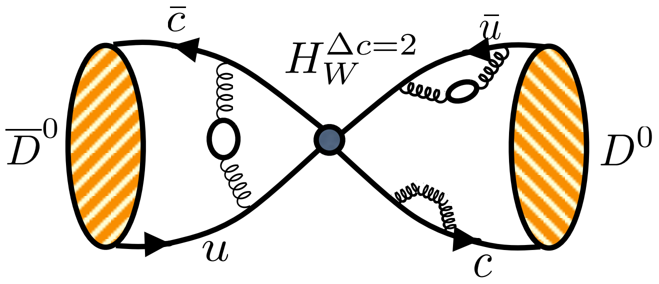


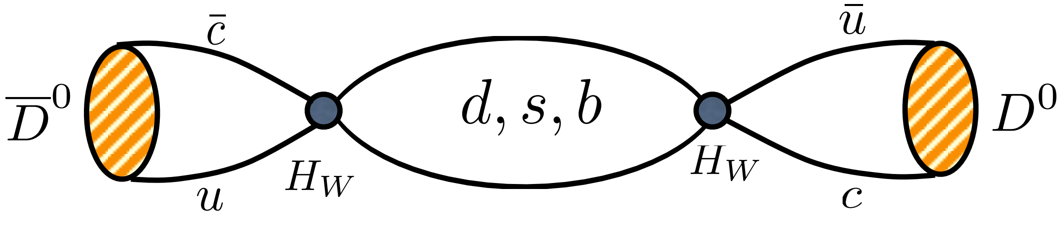


Neutral D meson mixing

$$M_{12} - \frac{i}{2}\Gamma_{12} \propto \langle D^0 | H_W^{\Delta c=2} | \overline{D}^0 \rangle + \sum_n \frac{\langle D^0 | H_W^{\Delta c=1} | n \rangle \langle n | H_W^{\Delta c=1} | \overline{D}^0 \rangle}{M_D - E_n + i\epsilon}$$







short distance

long distance

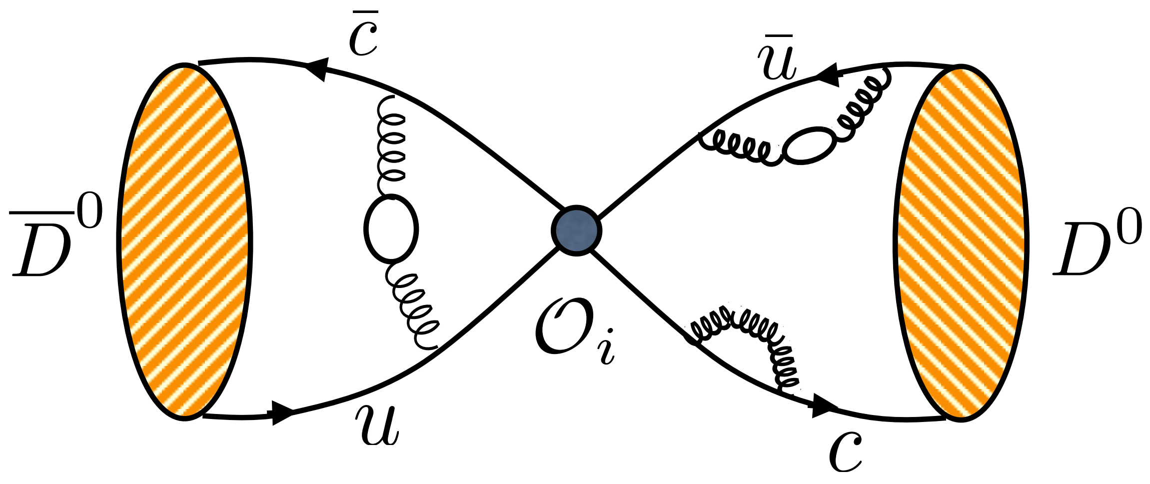
“Simple”

- can use the same methods as for B mixing (and decay constants, form factors)
- BSMs with heavy new particles can contribute here

“Hard”

- large contribution
- intermediate state can include multiple (>2) hadrons:
  - ✦ formalism for multi-hadron states still under development (Hansen & Sharpe, arXiv:1602.00324, 2016 PRD)
  - ✦ not a problem for Kaon mixing
    ➟ first calculation of long-distance contribution already exists (RBC/UKCD, arXiv:1406.0916, 2014 PRL)

Neutral D meson mixing



In the SM and beyond:

$$\mathcal{H}_{\text{eff}} = \sum_{i=1}^{5} c_i(\mu)\mathcal{O}_i(\mu)$$

$$\mathcal{O}_1 = \bar{c}\gamma^\mu L u\, \bar{c}\gamma^\mu L u$$

$$\mathcal{O}_2 = \bar{c} L u\, \bar{c} L u$$

$$\mathcal{O}_3 = \bar{c}^\alpha L u^\beta\, \bar{c}^\beta L u^\alpha$$

$$\mathcal{O}_4 = \bar{c} L u\, \bar{c} R u$$

$$\mathcal{O}_5 = \bar{c}^\alpha L u^\beta\, \bar{c}^\beta R u^\alpha c$$

$$\langle \mathcal{O}_i \rangle \equiv \langle D^0 | \mathcal{O}_i | \overline{D}^0 \rangle(\mu) = e_i M_D^2\, f_D^2 B_D^{(i)}(\mu)$$

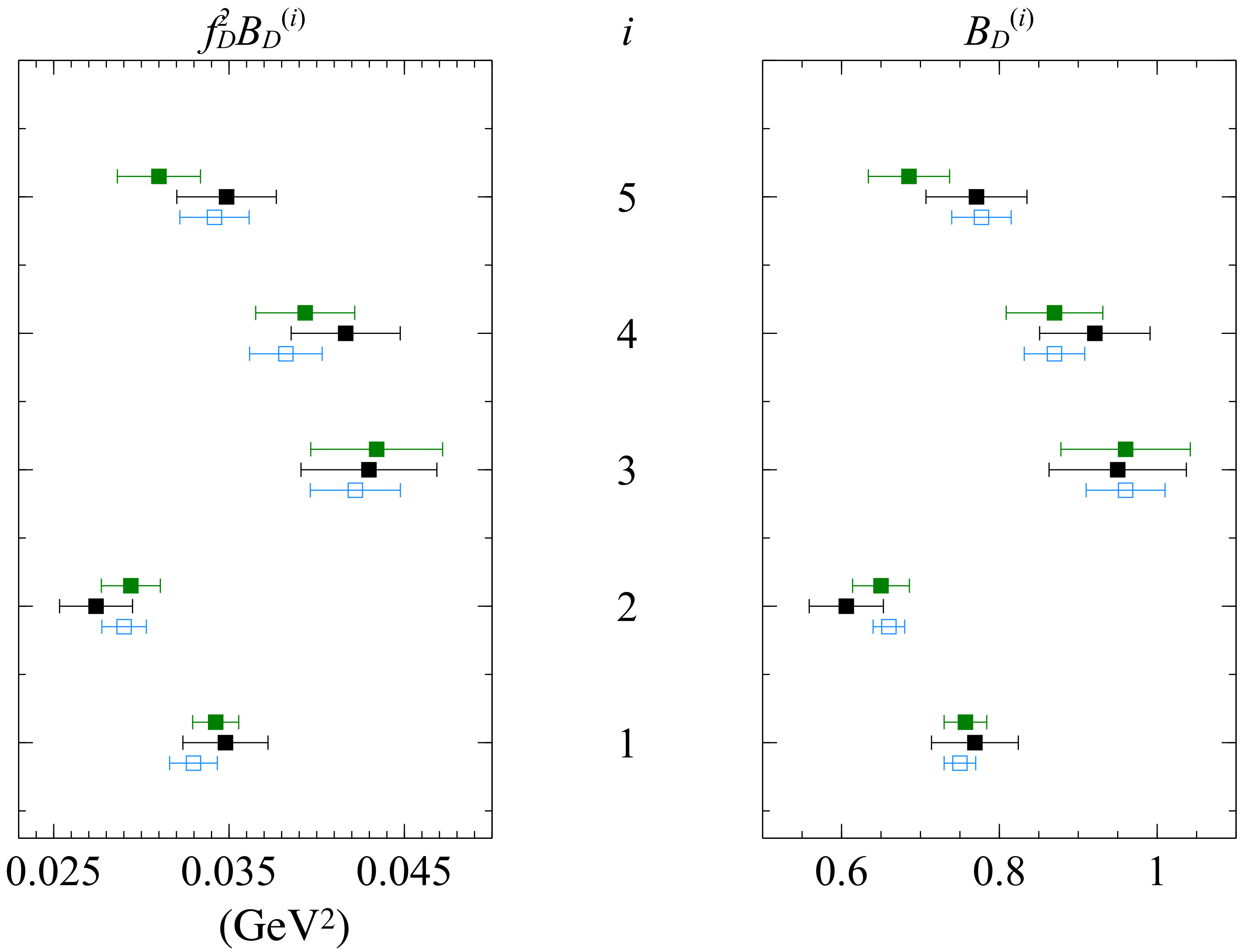
choose $\mu = 3$ GeV

- calculate the matrix elements of all five local operators.

D mixing results in comparison

$\boxed{\mu = 3\ \mathrm{GeV}}$

A. Kronfeld @ Lattice 2016 (plot by C.C. Chang)



- ETM:
  $n_f = 2{+}1{+}1$
  arXiv:1505.06639
- Fermilab/MILC:
  $n_f = 2{+}1$
- ETM:
  $n_f = 2$
  arXiv:1403.7302