What is the size of the DM nucleus cross section?

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Outline

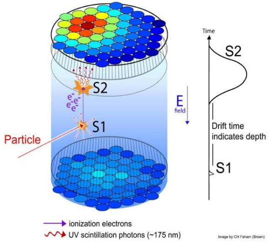
- We would like to answer the following question:
  - "Given any interaction in the UV, what is the DM nucleus cross section?"
- Identify all relevant scales (EFTs)
- Take leading term as estimate
  - Operator mixing can lead to deviation from "naive" estimate

Direct Detection Basics

- Direct detection – scattering on nuclei
  - Complementary information, proves cosmological lifetime
  - Assume velocity distribution (Maxwell); $v \sim 10^{-3}$
  - Maximal momentum transfer is $q \lesssim 200\,\text{MeV}$

$$\frac{dR}{dE_R} \propto \int_{v_{min}} dv\, v\, f_1(v)\ \frac{d\sigma}{dE_R}(v, E_R)\,.$$

[Lewin & Smith, Astropart.Phys.6 (1996)]



LUX

Estimating the cross section

- Many scales:
  - Heavy mediators – Λ
  - Electroweak symmetry breaking – v_{EW}
  - Quark thresholds – m_b, m_c
  - Chiral symmetry breaking of QCD – Λ_χ, m_N
  - Momentum transfer – $\vec{q}$
  - Mass of the atomic nucleus – A
  - DM mass – m_χ
- Power counting scheme ("expansion in small ratios")
- Use appropriate effective theories

Nonrelativistic limit – cartoon

- *DM currents:*
  - Vector: $\bar{\chi}\gamma^\mu\chi \quad \rightsquigarrow \quad \Psi_\chi^\dagger\Big(1, \quad \vec{v}^\perp + \frac{\vec{q}}{2m_N} + i\frac{\vec{q}\times\vec{S}_\chi}{m_\chi}\Big)\Psi_\chi$
  - Axial vector: $\bar{\chi}\gamma^\mu\gamma_5\chi \quad \rightsquigarrow \quad \Psi_\chi^\dagger\Big(\vec{v}^\perp\cdot\vec{S}_\chi + \frac{\vec{q}\cdot\vec{S}_\chi}{2m_N}, \ \vec{S}_\chi\Big)\Psi_\chi$
- *SM currents:*
  - Vector: $\bar{p}\gamma^\mu p \quad \rightsquigarrow \quad \Psi_p^\dagger\Big(1, \ \frac{\vec{q}}{2m_N} - i\frac{\vec{q}\times\vec{S}_p}{m_N}\Big)\Psi_p$
  - Axial vector: $\bar{p}\gamma^\mu\gamma_5 p \quad \rightsquigarrow \quad \Psi_p^\dagger\Big(\frac{\vec{q}\cdot\vec{S}_p}{m_N}, 2\vec{S}_p\Big)\Psi_p$
- "Spin independent" vs. "spin dependent" scattering
- Momentum / velocity - suppressed interactions

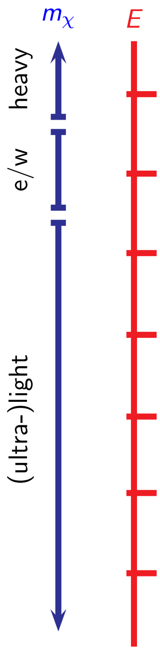
EFT and operator mixing

- A consistent EFT framework is needed:
  - Connect all scales from the UV to the atomic nuclei
  - Keep dependence on UV physics explicit
  - Consistent power counting (identify all leading effects)
- Operator mixing is important:
  - Momentum-dependent interactions are leading in many UV models
  - Electroweak loops can mix suppressed and unsuppressed operators
    [Freytsis & Ligeti, 1012.5317; Kopp et al. 0907.3159; see also Haisch et al. 1302.4454; Crivellin et al. 1402.1173, 1408.5046; D'Eramo et al. 1411.3342]

The setup

- Assume DM is an electroweak multiplet χ, with $m_\chi \sim v_{\text{ew}}$
  - Here, DM is a fermion
- Several examples:
  - Neutralinos in the MSSM (bino, higgsino, wino)
  - Minimal Dark Matter [Cirelli et al. hep-ph/0512090, ...]
  - "Technibaryons" [Nussinov, Phys.Lett. B165 (1985) 55, ...]
- Potential mediators ϕ, integrated out at $\Lambda \sim m_\phi \gg m_\chi$
  - Dim.4 gauge interactions
  - Higher-dimensional effective operators

Relevant Scales



Λ – heavy mediators

“SM-EFT” + DM

$v_{\mathbf{EW}}$ – electroweak symmetry breaking

five-flavor QCD + DM

m_b – b-quark threshold

four-flavor QCD + DM

m_c – c-quark threshold

three-flavor QCD + DM

$\Lambda_{\mathbf{QCD}}$ – chiral symmetry breaking

(Heavy Baryon) Chiral Effective Theory

m_π – pion threshold

pionless effective theory

$v, \vec{q}$ – DM-nucleus scattering kinematics

NR theory of nuclear response

Above v_{EW}: Mixing

Effective Lagrangian above v_{EW}

- Construct operators in unbroken e/w phase

$$\mathcal{L}^{\text{eff}} = \mathcal{L}^{\text{SM}} + \mathcal{L}^{\text{DM}} + \sum \frac{C_j^{(5)}}{\Lambda} Q_j^{(5)} + \sum \frac{C_j^{(6)}}{\Lambda^2} Q_j^{(6)} + \dots$$

- Expansion in inverse mediator mass Λ
- Generalizes "SM-EFT"

  [Buchmüller et al. Nucl.Phys. B268 (1986) 621, Grzadkowski et al. 1008.4884]

  - Dim.5: $Q_1^{(5)} = \frac{g_1}{8\pi^2}(\bar{\chi}\sigma^{\mu\nu}\chi)B_{\mu\nu}$
  - Dim.5: $Q_3^{(5)} = (\bar{\chi}\chi)(H^\dagger H)$
  - Dim.6: $Q_{2,i}^{(6)} = (\bar{\chi}\gamma_\mu\chi)(\bar{Q}_L^i\gamma^\mu Q_L^i)$
  - Dim.6: $Q_{16}^{(6)} = (\bar{\chi}\gamma^\mu\chi)(H^\dagger i \overleftrightarrow{D}_\mu H)$

Mixing – General Structure

- The RGE (Renormalization Group Equations) tell us:

$$C_i(M_W) = C_i(\Lambda) + C_j(\Lambda)\gamma_{ji}\frac{\alpha}{4\pi}\log\frac{M_W}{\Lambda} + \dots$$

- Both weak and Yukawa interaction mix left- and right-handed structures
- Have huge hierachy in matrix elements
- Do we need to sum $\alpha \log \frac{M_W}{\Lambda}$ to all orders?
  - $\alpha_1(\mu_{EW}) \approx 0.01$, $\alpha_2(\mu_{EW}) \approx 0.03$, $\alpha_\lambda(\mu_{EW}) \approx 0.04$, $\alpha_t(\mu_{EW}) \approx 0.08$
  - Only if $\Lambda \gtrsim 10^4$ TeV (strongly coupled models?)
  - However, anomalous dimensions can be large
  - Mixing via several steps can be important $\Rightarrow$ full resummation
- We calculated all mixing up to dimension-six UV operators

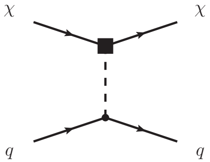
Below v_{EW}:

Matching and Running

Effective Lagrangian below v_{EW}

$$\mathcal{L}^{\text{eff}} = \mathcal{L}^{(4)}|_{n_f} + \mathcal{L}^{\text{DM}}|_{n_f} + \sum \hat{\mathcal{C}}_j^{(5)}|_{n_f} Q_j^{(5)} + \sum \hat{\mathcal{C}}_j^{(6)}|_{n_f} Q_j^{(6)} + \sum \hat{\mathcal{C}}_j^{(7)}|_{n_f} Q_j^{(7)} + \dots$$

- Dim.5: $\mathcal{Q}_1^{(5)} = \frac{e}{8\pi^2}(\bar{\chi}\sigma^{\mu\nu}\chi)F_{\mu\nu}$
- Dim.6: $\mathcal{Q}_{1,f}^{(6)} = (\bar{\chi}\gamma_\mu\chi)(\bar{f}\gamma^\mu f)$
- Dim.7: $\mathcal{Q}_{5,f}^{(7)} = m_f(\bar{\chi}\chi)(\bar{f}f)$

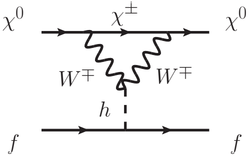


- Now have expansion in $1/\Lambda$, $1/v_{\text{EW}}$, and $1/m_\chi$
- E.g. $\hat{\mathcal{C}}_{5,f}^{(7,2)} = \mathcal{C}_{5,f}^{(7)\{2,0\}}/(\Lambda v_{\text{EW}}^2) + \dots$

Matching and HDMET

- Recall $m_\chi \sim v_{\text{EW}}$ – need "HQET" version of dark matter
  [Hill, Solon 1111.0016; 1409.8290]

- E.g. "Higgs penguin" contribution
  - Match onto $\mathcal{Q}_{5,f}^{(7)} = m_f(\bar{\chi}_v \chi_v)(\bar{f} f)$
  - $w = M_W^2/m_\chi^2$, $z = M_Z^2/m_\chi^2$



$$\mathcal{C}_{5,f}^{(7)\{3,0\}} = \frac{e^3}{8\pi^2 s_w^3 \lambda}\left[\left(I_\chi(I_\chi+1) - \frac{Y_\chi^2}{4}\right) f(w) + \frac{Y_\chi^2}{4c_w^3} f(z)\right]$$

$$f(x) = \frac{2(x^2-2x-2)}{\sqrt{x-4}} \log\left(\frac{\sqrt{x}}{2} + \sqrt{\frac{x}{4}-1}\right) + \sqrt{x}(2 - x\log x)\,.$$

Transition to the nucleon picture

Transition to the nucleon picture

- Recall maximum momentum transfer in DM scattering is $q_{\text{max}} \approx 200\,\text{MeV}$
- Expansion in $q/(4\pi f_\pi)$ is good to $\mathcal{O}(20\%)$
- Can use (Heavy Baryon) Chiral Perturbation Theory (HBChPT)
  [Jenkins et al. Phys.Lett. B255 (1991) 558, see also Hoferichter et al. 1503.04811]
- Treat DM currents as $SU(3)_L \times SU(3)_R$ flavor-symmetric spurions
- Can write hadronization of quark currents explicitly, e.g.:
  - Pseudo-scalar meson current: $\bar{q} i \gamma_5 q \to -B_0 f_\pi m_u \big(\pi^0 + \eta/\sqrt{3}\big) + \ldots$
  - Nuclear vector current: $\bar{u} \gamma^\mu u \to v^\mu (2\bar{p}_v p_v + \bar{n}_v n_v) + \ldots$
- Describe hadronic physics in terms of few parameters $(f_\pi, g_A, \mu_N, \sigma_{\pi N} \ldots)$

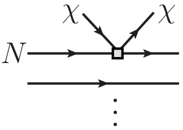
Chiral power counting

- Power counting scheme: $M_{A,\chi} \sim p^{\nu}$

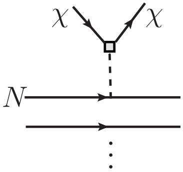
  [Weinberg NPB363,3 (1991); Kaplan, Savage, Wise, nucl-th/9605002; Cirigliano, Graesser, Ovanesyan 1205.2695]

  - $\nu = 4 - A - 2C + 2L + \sum_i V_i(d_i - n_i/2 - 2) + \epsilon_W$
  - Resonances, shallow bound states etc. can upset power counting

    [Bedaque et al. nucl-th/0203055, Epelbaum et al. 0811.1338, Epelbaum 1001.3229]



- Only leading diagram for most DM-SM interactions
- Leading diagram for $A \cdot A$ interaction



- Only leading diagram for $S \cdot P$ and $P \cdot P$
- Leading diagram for $A \cdot A$ interaction

HBChPT beyond LO

- Beyond LO effects can be important!
  - Recall NR scaling: $\bar{\psi}\gamma^\mu\psi \sim (1, q)$, $\bar{\psi}\gamma^\mu\gamma_5\psi \sim (q, 1)$
- Need to retain (many, partially new) NLO terms in HBChPT Lagrangian
  - E.g. $g_4' \epsilon^{\alpha\beta\lambda\sigma} v_\alpha \,\mathrm{Tr}\left(\bar{B}_v S_{N\beta} B_v\right) \partial_\lambda \,\mathrm{Tr}(V_\sigma)$
  - Vector current at NLO: μ_N, μ_N^s + 1 unknown constant (lattice?)
  - Axial-vector current at NLO: 3 unknown constants (lattice?)
- "Reparameterization invariance" fixes some coefficients...
  - ...such that "magically" the resulting NR theory is Galilean invariant!

Nonrelativistic EFT

- Match to nonrelativistic, Galilean-invariant EFT, constructed from
  - momentum transfer $i\vec{q}$
  - relative transversal incoming DM velocity $v_T^\perp$
  - nucleon spin $\vec{S}_N$ (DM spin $\vec{S}_\chi$)
- To LO, need only single-nucleon operators, e.g.
  - Spin-independent (“M”): e.g. $\mathcal{O}_1^p = 1_\chi \, 1_N$
  - Spin-dependent (“Σ', Σ”): e.g. $\mathcal{O}_4^p = \vec{S}_\chi \cdot \vec{S}_N$
  - Nuclear angular momentum (“Δ”): e.g. $\mathcal{O}_9^p = \vec{S}_\chi \cdot \left(\vec{S}_p \times \frac{i\vec{q}}{m_N}\right)$

Nuclear matrix elements

- Calculation of nuclear response functions for all NR operators (available for F, Na, Ge, I, Xe)
  [Fitzpatrick et al. 1203.3542]
- Rough scaling:
  - $W_M \sim \mathcal{O}(A^2)$
  - $W_{\Sigma'}\,, W_{\Sigma''}\,, W_{\Delta}\,, W_{\Delta\Sigma'} \sim \mathcal{O}(1)$
- Finally, convolution with velocity distribution and experimental efficiencies allows to calculate scattering rate for different experiments

Illustrative Example

DM triplet ($I_\chi = 1$) with hypercharge $Y_\chi = 0$ (no coupling to Z)

Vector DM – Axial 1st gen. (No Mixing)

- Start with

  $-Q_{2,1}^{(6)} + Q_{3,1}^{(6)} + Q_{4,1}^{(6)} = (\bar{\chi}\gamma_\mu\chi)(\bar{u}\gamma^\mu\gamma_5 u + \bar{d}\gamma^\mu\gamma_5 d)$

- "Vector – axial-vector" has response $v^2\, W_{\Sigma',\Sigma''}$, so the cross section scales roughly as

$$\sigma \propto v^2 A^0 \left(\frac{1}{\Lambda^2}\right)^2$$

Vector DM – Axial 1rst gen. (Mixing)

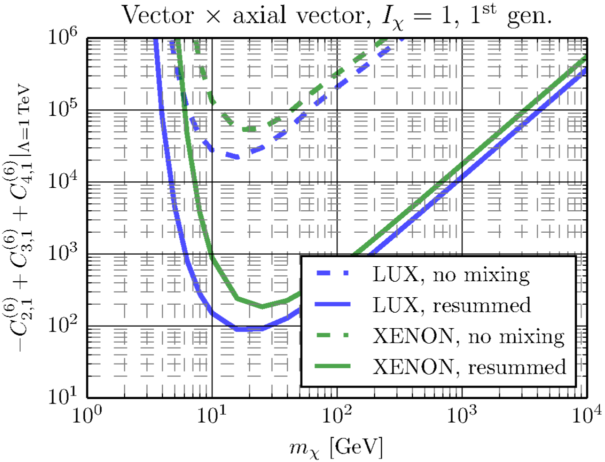
- Start with $-Q_{2,1}^{(6)} + Q_{3,1}^{(6)} + Q_{4,1}^{(6)} = (\bar\chi\gamma_\mu\chi)(\bar u\gamma^\mu\gamma_5 u + \bar d\gamma^\mu\gamma_5 d)$
- No mixing into unsuppressed operators at one loop!
- However, have two-step mixing $Q_{2,1}^{(6)} \to Q_{5,1}^{(6)} \to Q_{2,1}^{(6)}$
  with large anomalous dimension
- Breaks original alignment, generates "vector – vector" component

$$\frac{1}{\Lambda^2} \xrightarrow{\mu\sim\Lambda} \frac{\alpha_2}{\Lambda^2} \xrightarrow{\mu\sim\Lambda} \frac{\alpha_2^2}{\Lambda^2}$$

- "Vector – vector" has response $A^2\ W_M$, so the cross section *really* scales as

$$\sigma \propto v^0 A^2 \left(\frac{\alpha_2^2}{\Lambda^2}\right)^2$$

Vector DM – Axial 1st gen.



PRELIMINARY

Summary

- Established explicit connection between UV and nuclear physics
  - General setup that covers many models
  - Radiative corrections can have significant impact on interpretation of data
- What's new?
  - Full connection between $\Lambda \gg M_W$ and nuclear scale
  - Operator mixing at dim.5 and dim.6 with e/w charges
  - One-loop matching to HDMET
  - NLO terms in HBChPT Lagrangian

Outlook

- Provide public code for automatic running from UV to nuclear scale
- Several multiplets and Higgs interactions
- Scalar and vector DM
- Dimension-seven operators in the UV
- Heavy DM