Determinations of baryon sigma terms from chiral extrapolations of lattice QCD baryon masses

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Contents

- Motivation: Dark Matter Direct Detection, quark-flavor structure of the nucleon
- A systematic study of octet baryon sigma terms from chiral extrapolation of lattice QCD baryon masses
- Matching SU(3) to SU(2)—check on the validity of SU(3) BChPT
- Summary

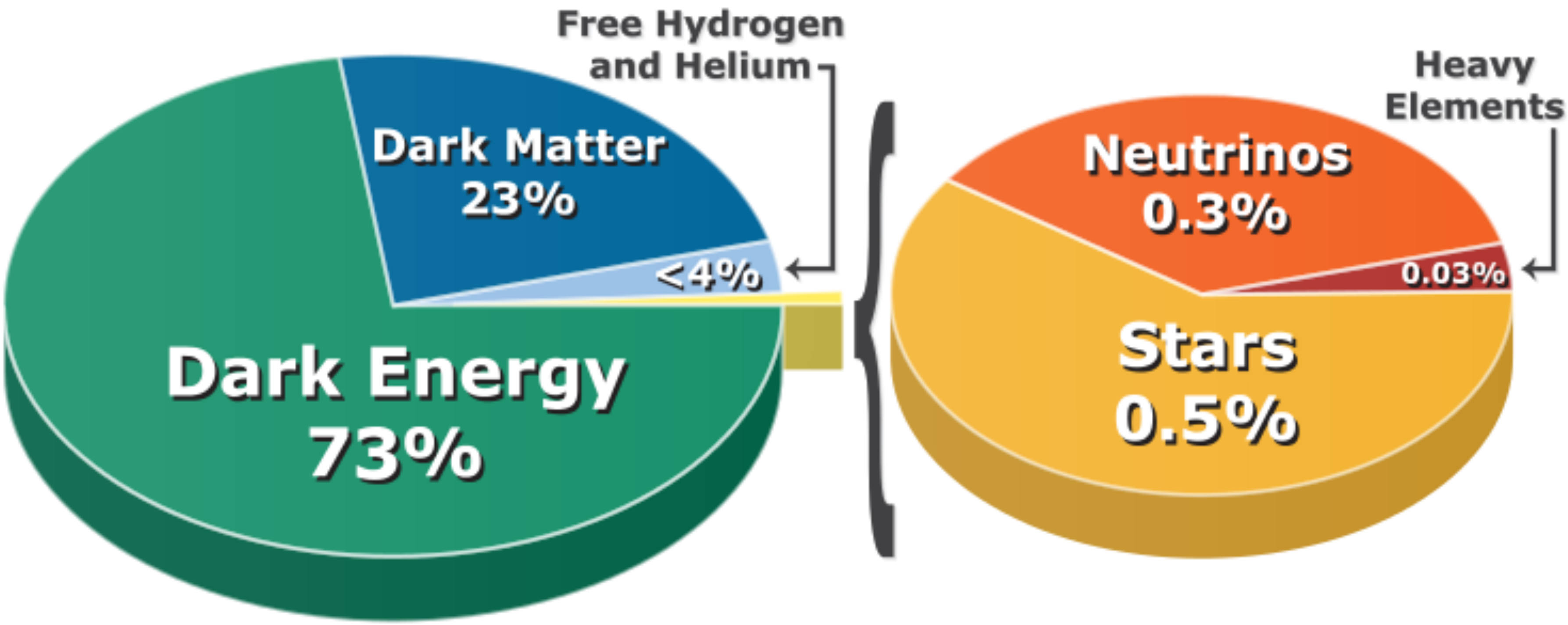
Based on the following works

- *Camalich, **Geng**, Vacas, PRD82(2010)074504*
- ***Geng**, Ren, Camalich, Weise, PRD84(2011)074024;*
- *Ren, **Geng**, Camalich, Meng, Toki, JHEP12(2012)073;*
- *Ren, **Geng**, Meng, Toki, PRD87(2013)074001*
- *Ren, **Geng**, Meng, EPJC74:2754,2014*
- *Ren, **Geng**, Meng, PRD91 (2015) 051502*
- *Ren, Alvarez-Ruso, **Geng**, Ledwig, Meng, Vicente Vacas, 1606.03820*

Motivation: why sigma terms

Energy-matter composition of the universe

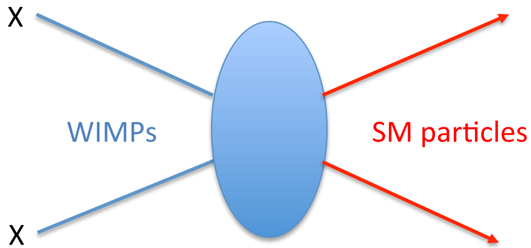
Plank: 1303.5062



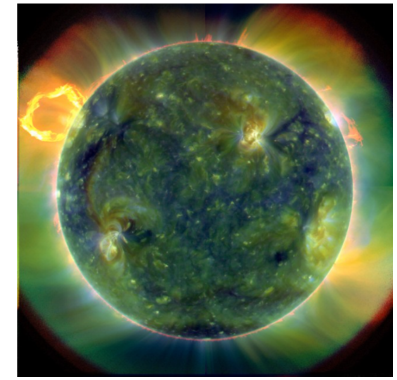
- **Weakly Interacting Massive Particles (WIMPS)**

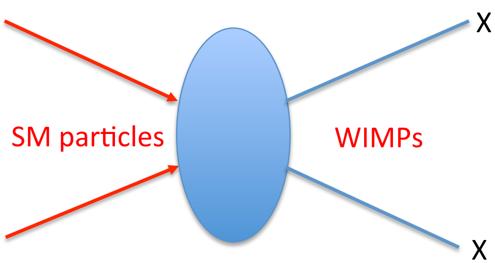
e.g., Neutrilino in MSSM.

Particle searches for WIMPs

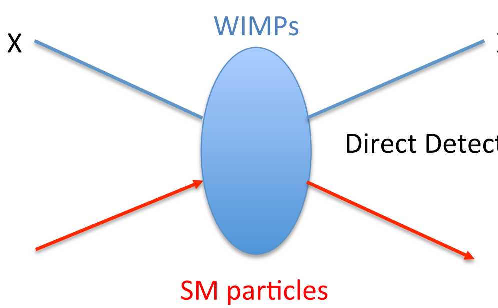


Indirect Detection



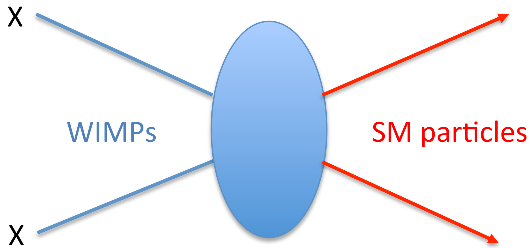


Collider Searches

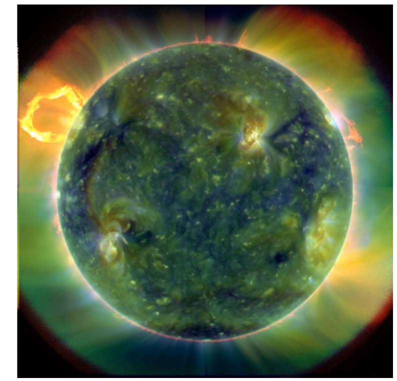


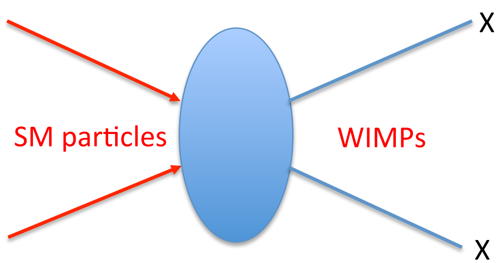
Direct Detection

Particle searches for WIMPs

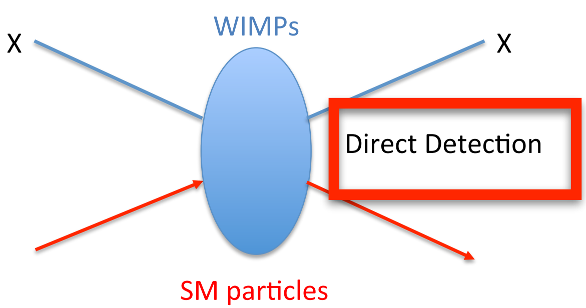


Indirect Detection



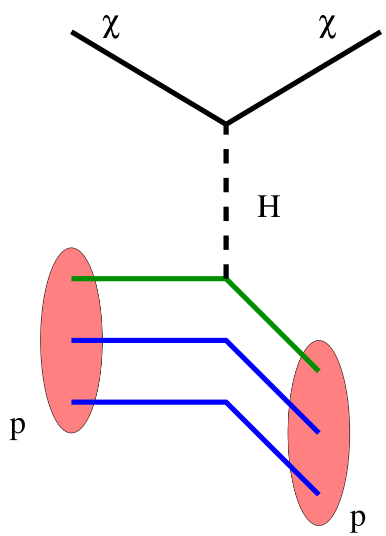


Collider Searches



MSSM

Spin-independent neutrilino-nucleon scattering



$$\mathcal{L}_{int} = \lambda_N \bar{n} n \bar{\chi}\chi \rightarrow \mathcal{L}_{int} = \lambda_q \bar{q} q \bar{\chi}\chi$$

$$\lambda_N \longrightarrow \sum_{q=1}^{6} f_q^N \lambda_q$$

Spin indep. WIMP-N X-section

$$\sigma_{SI} = \frac{4M^2}{\pi}\left[Z f_P + (A - Z) f_N\right]$$

with

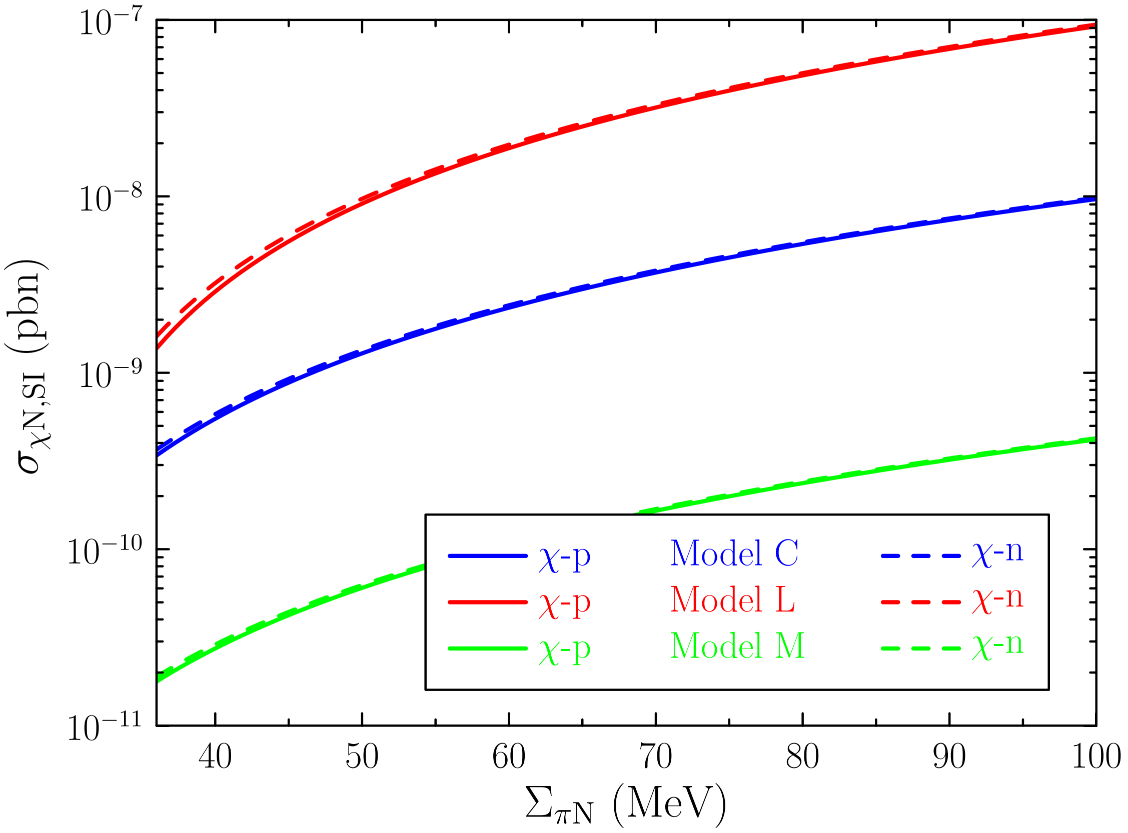
$$\frac{f_N}{M_N} = \sum_q f_q^N \frac{\lambda_q}{m_q}$$

pion- and strangeness sigma terms

$$\boxed{f_{ud}^N M_N = \sigma_{\pi N} = m_q \langle N | u\bar{u} + d\bar{d} | N \rangle}$$

$$\boxed{f_s^N M_N = \sigma_{sN}/2 = m_s \langle N | s\bar{u} | N \rangle}$$

Strong dependence on the strangeness sigma term

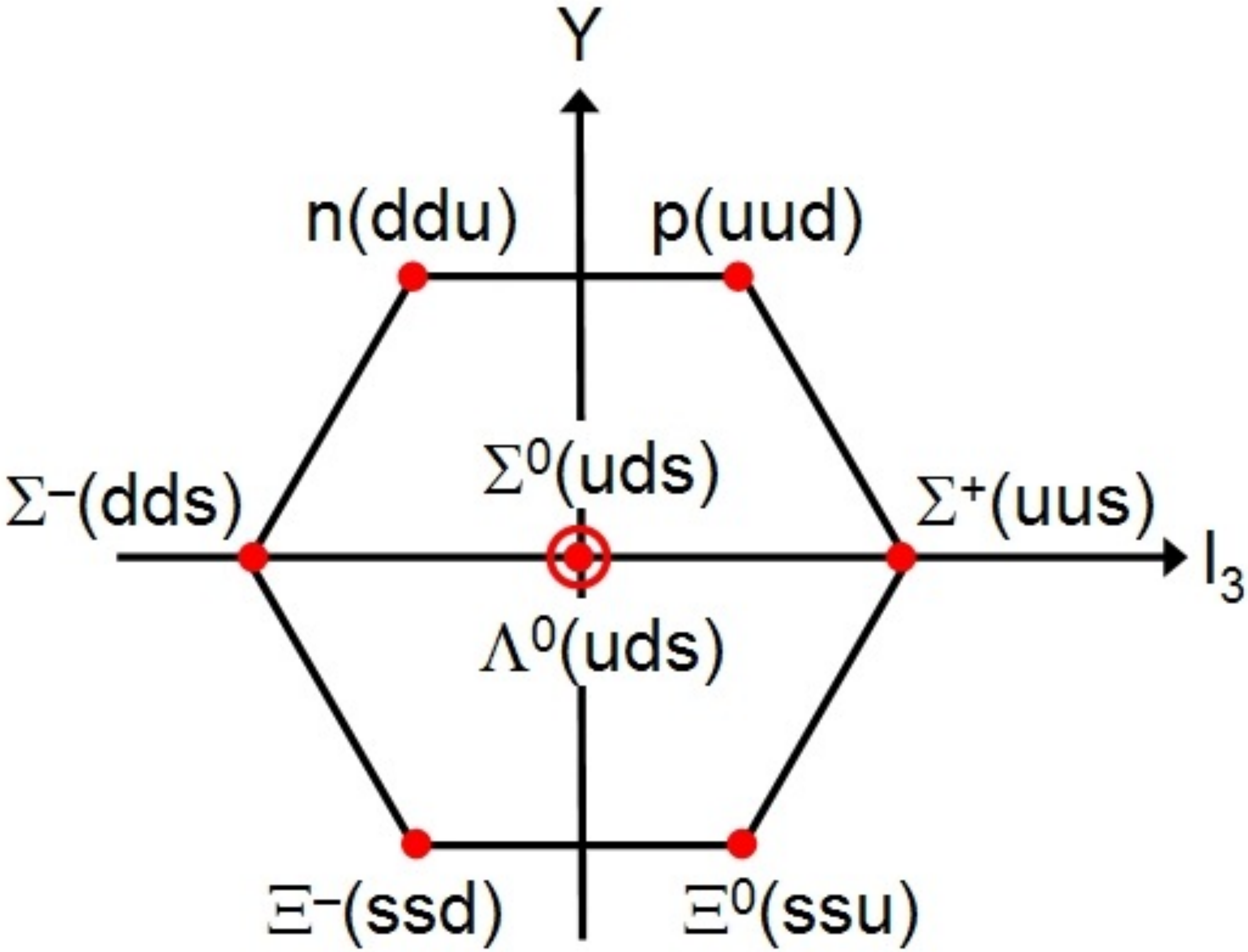


$$\sigma_{\chi p,SI} \sim (\Sigma_{\pi N} - \sigma_0)^2.$$

$$\sigma_{sN} \propto \Sigma_{\pi N} - 36$$

Ellis, Olive, Savage, PRD77(2008)065026

Quark-flavor structure of octet baryons



Naive quark model

Quark-flavor structure of the proton

- Naive quark model—minimal quark contents

$$|p\rangle = |uud\rangle$$

- **In reality,** $|p\rangle = |uud\rangle(1 + |u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)$
  - to the spin
    – deep-inelastic lepton scattering
  - to the electromagnetic formfactors
    - parity-violating electron-proton scattering
  - to the mass
    – scalar strangeness content, cannot be measured directly $\langle N|s\bar{s}|N\rangle$

Quark-flavor structure of the proton

- Naive quark model—minimal quark contents

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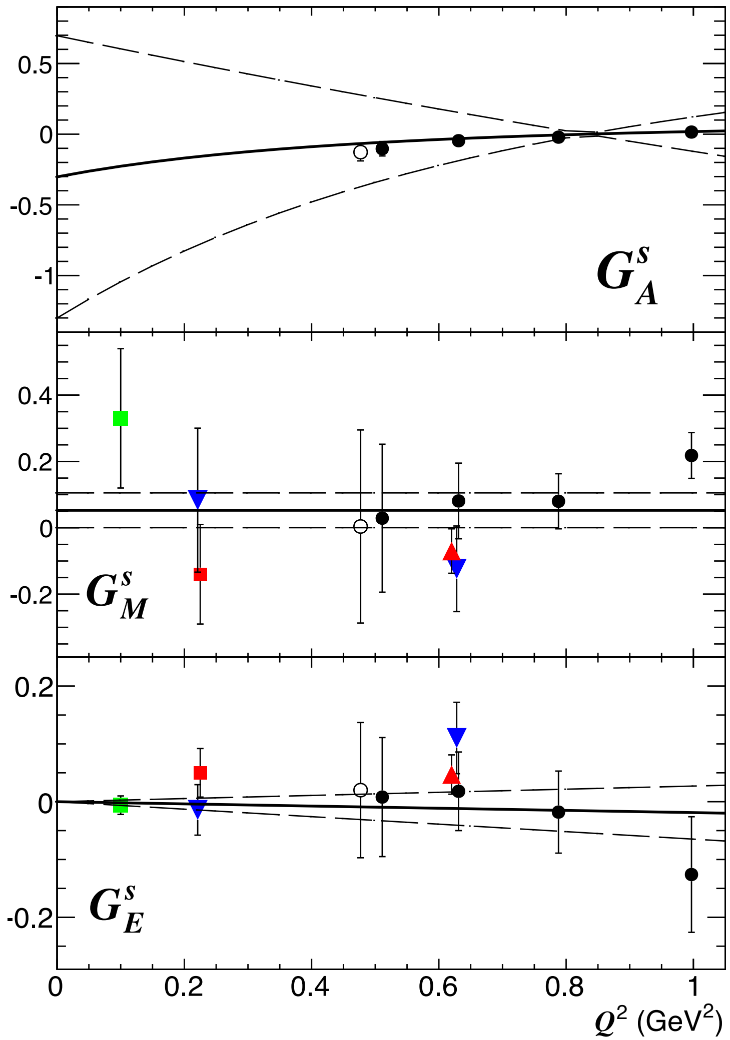
How to obtain the scalar strangeness content of the nucleon from the LQCD masses using Chiral Perturbation Theory

Global fit of the strangeness vector and axial vector form factors of the nucleon

arXiv:1308.5694



Parameter	Fit value
ρ_s	-0.071 ± 0.096
μ_s	0.053 ± 0.029
ΔS	-0.30 ± 0.42
Λ_A	1.1 ± 1.1
S_A	0.36 ± 0.50

- The electric and magnetic form factors are consistent with **zero, but not** the axial-vector form factor

How to determine sigma terms

Pion-nucleon sigma term

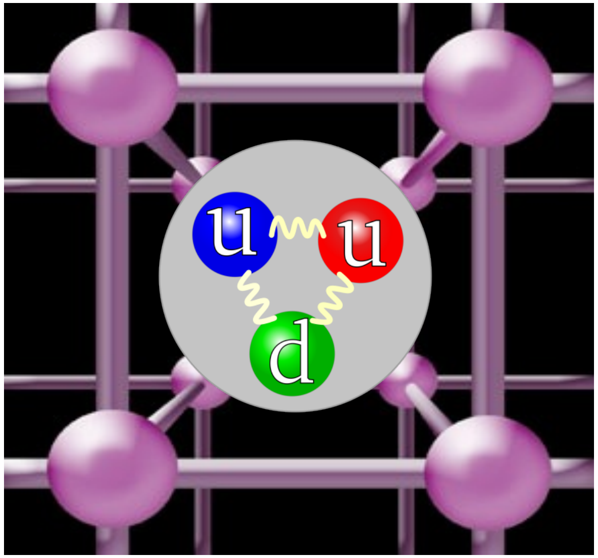
- Experimentally, **the pion-nucleon sigma term** can be inferred from pion-nucleon scattering data at Cheng-Dashen point $(s = u = m_N^2, t = 2M_\pi^2)$

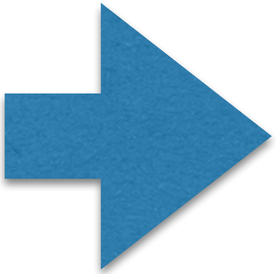
$\sigma_{\pi N} = 45 \pm 8 MeV$	**J. Gasser et al., PLB253,252**
$\sigma_{\pi N} = (59.1 \pm 1.9 \pm 3.0)$ MeV	**Hoferichter et al., PRL115, 092301**
$\sigma_{\pi N} = 59(7)$ MeV	**Alarcon et al., PRD 85, 051503(R)**
$\sigma_{\pi N} = 45 \pm 6$ MeV	**Chen et al., PRD 87, 054019**
	$\sigma_{\pi N} = 52 \pm 7$ MeV

See also Yao et al. [JHEP 1605 (2016) 038] and the talk by Dr. Siemens

strangeness sigma term

- Because of lack of kaon-nucleon scattering data, the strangeness-sigma term cannot be obtained this way
- **Lattice QCD** might be our hope to predict it from first principles





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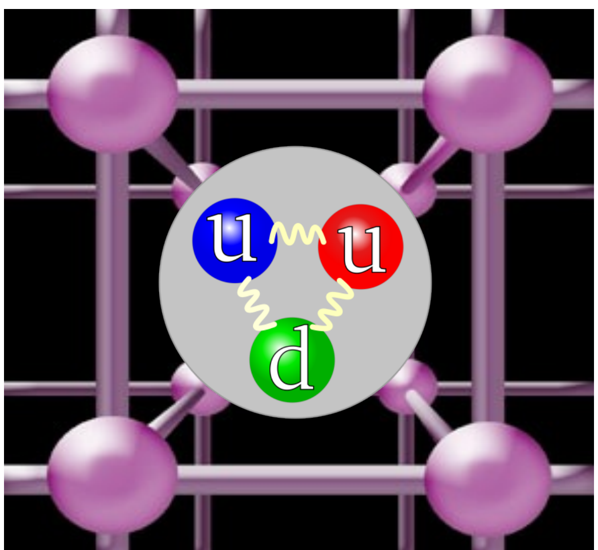
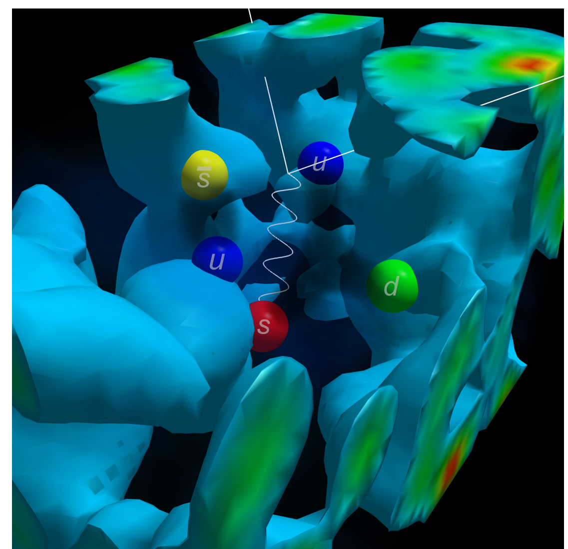
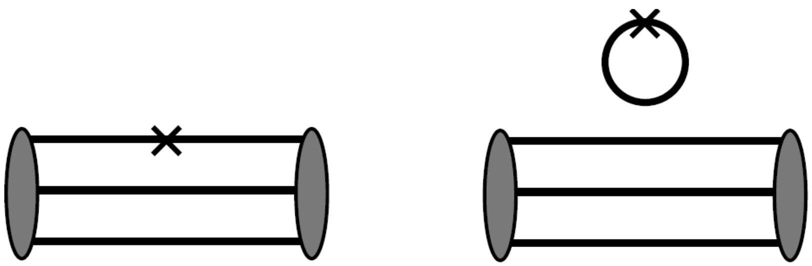




LQCD determination of sigma terms

- **Direct method**—calculates the 3-point connected and disconnect diagrams



  - JLQCD, PRD83,114506 (2011)
  - R. Babich *et al.*, PRD85,054510 (2012)
  - QCDSF, PRD85, 054502 (2012)
  - ETMC, JHEP 1208,037(2012)
  - M. Engelhardt *et al.*, PRD86, 114510 (2012)
  - JLQCD, PRD87, 034509 (2013)
  - ETMC, PRL 116, 252001 (2016)
  - RQCD, PRD 93, 094504 (2016)
  - χ QCD, PRD94, 054503 (2016)

- **Spectrum method**-calculates the baryon masses, and relates the sigma terms to their quark mass dependence via the Feynman Hellmann theorem

$$\sigma_{\pi B} = m_l \langle B(p)|\bar{u}u + \bar{d}d|B(p)\rangle = m_l \frac{\partial M_B}{\partial m_l}$$

$$\sigma_{sB} = m_s \langle B(p)|\bar{s}s|B(p)\rangle = m_s \frac{\partial M_B}{\partial m_s}.$$

  - JLQCD, PRD83,114506 (2011)
  - R. Babich *et al.*, PRD85,054510 (2012)
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  - M. Engelhardt *et al.*, PRD86, 114510 (2012)
  - JLQCD, PRD87, 034509 (2013)
  - BMW, PRL 116, 172001 (2016).

Our aim

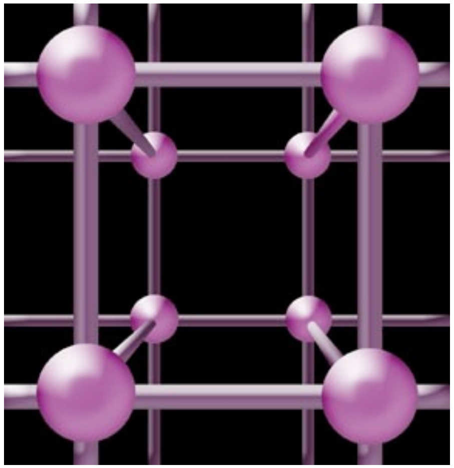
- To apply the Feynman-Hellmann theorem to predict the baryon sigma terms using the covariant (EOMS) baryon chiral perturbation theory

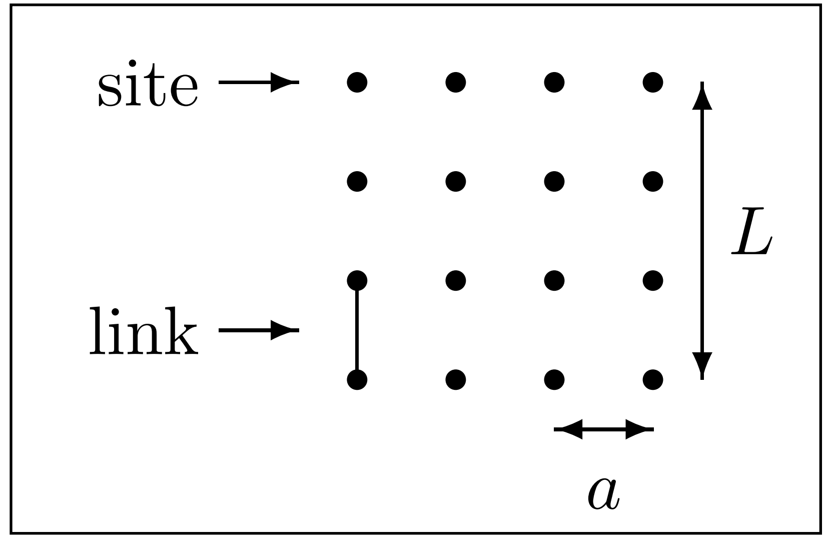
$$\sigma_{\pi B} = m_l \langle B(p)|\bar{u}u + \bar{d}d|B(p)\rangle = m_l \frac{\partial M_B}{\partial m_l}$$

$$\sigma_{sB} = m_s \langle B(p)|\bar{s}s|B(p)\rangle = m_s \frac{\partial M_B}{\partial m_s}.$$

- To fix the unknown low-energy constants of BChPT, we rely on the lQCD simulations of baryon masses

Lattice QCD



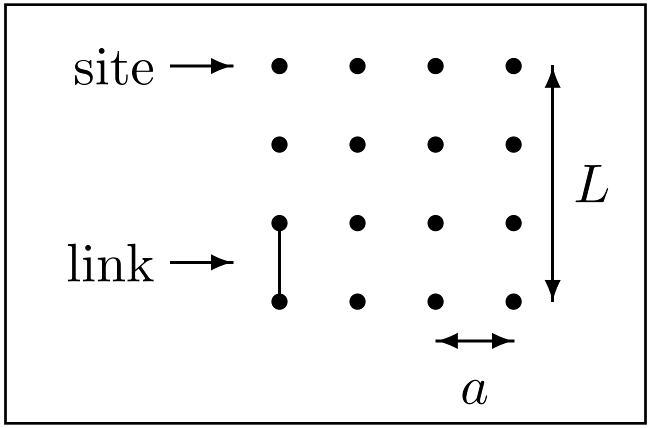


Basic idea： discretize space-time and solve non-perturbative strong interaction physics in a finite hypercube, utilizing monte carlo sampling techniques

Parameters and simulation costs

- light quark masses: m_u/m_d
- lattice spacing: a
- lattice volume： $V=L^4$

$$\text{cost} \propto \left(\frac{L}{a}\right)^4 \frac{1}{a} \frac{1}{m_\pi^2 a}$$



- To reduce cost: employ larger than physical light quark masses, finite lattice spacing and volume.
- To obtain physical quantities, multiple extrapolations are needed

Multiple extrapolations

- **Chiral extrapolations:** **light quark masses to their physical values**

$$m_q \to m_q(\mathrm{Phys.})$$

- **Finite volume corrections:** **infinite space-time**

$$L \to \infty$$

- **Continuum extrapolations:** **zero lattice spacing**

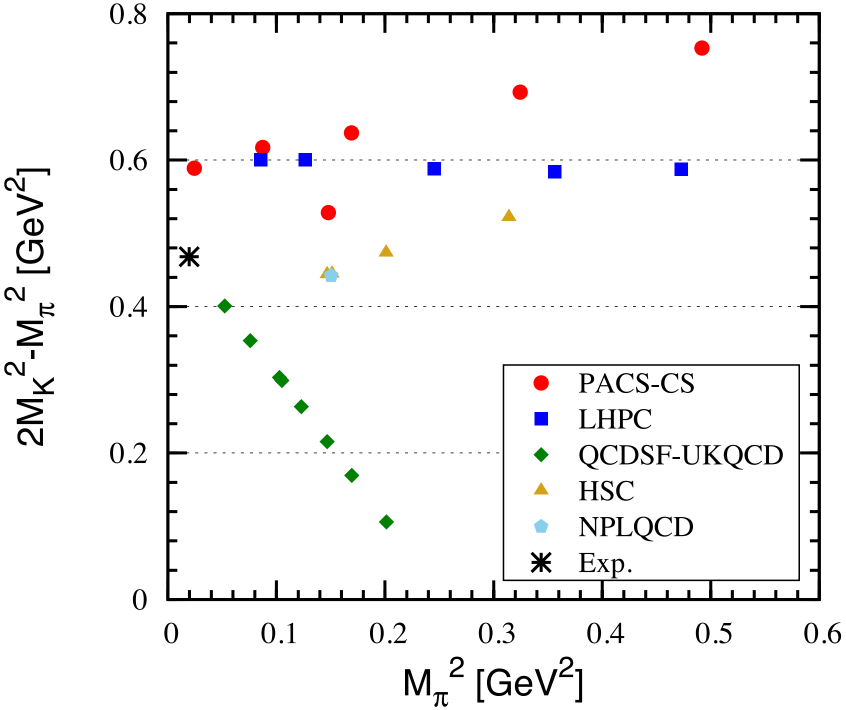
$$a \to 0$$

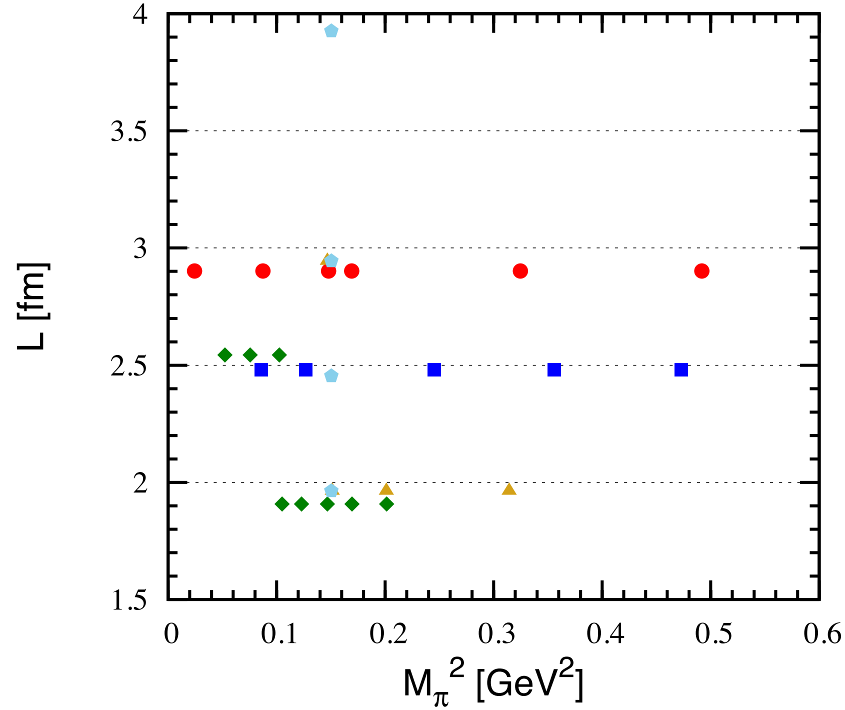
Two key factors for a reliable determination of the baryon sigma terms

- Lattice QCD simulations of baryon masses at various quark masses, volumes, and lattice spacings, and with various fermion/gauge actions

- A reliable formulation of ChPT, which not only can well describe the LQCD data, but also needs to satisfy all symmetry and analyticity constrains

landscape of latest 2+1 f LQCD simulations of g.s. octet baryon masses

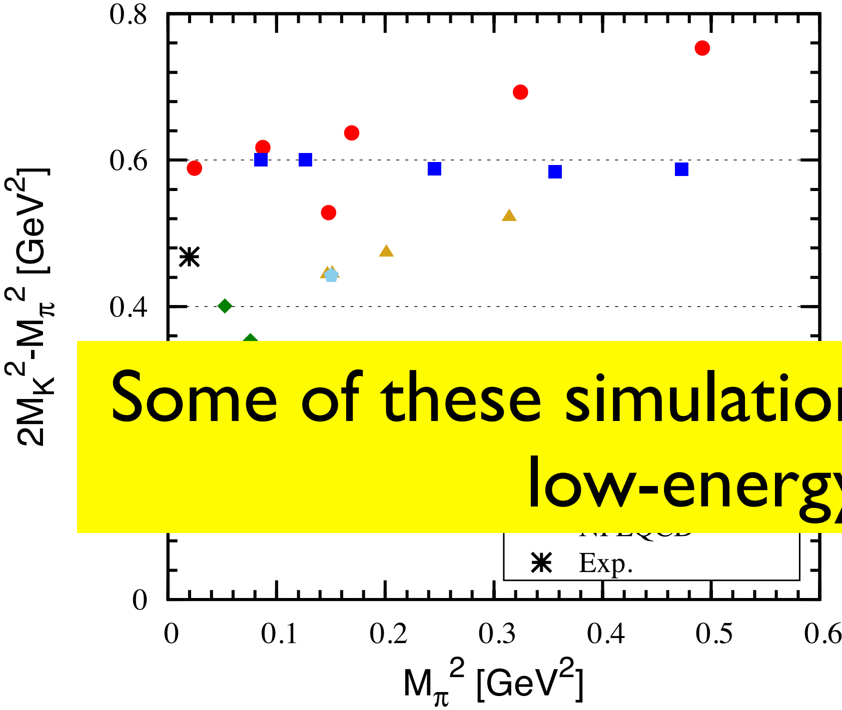


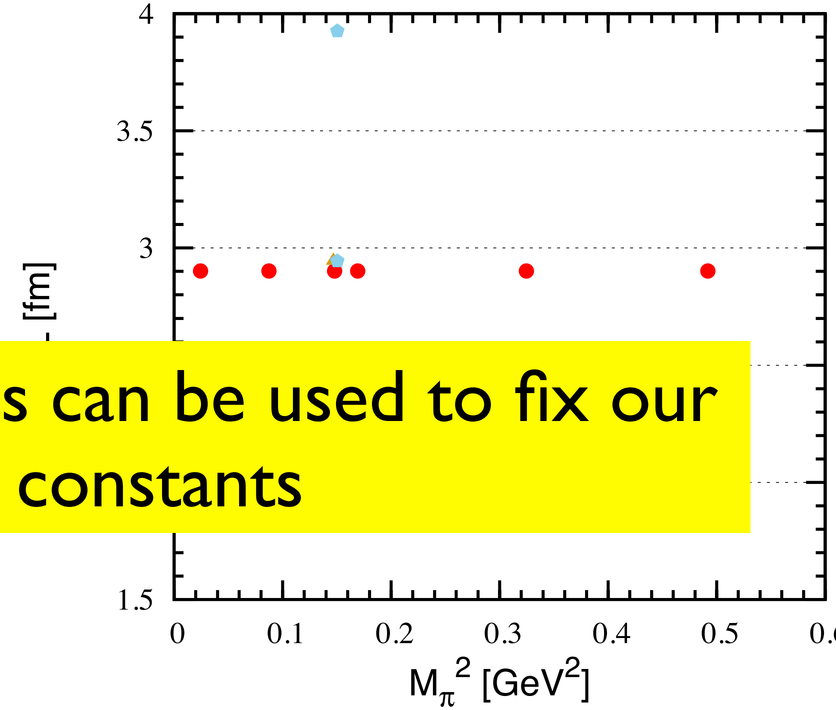


To obtain g.s. baryon masses in the physical world

- Extrapolate to the continuum: $a \to 0$
- Extrapolate to physical light quark masses: $m_q \to m_q(\mathrm{Phys.})$
- Extrapolate to infinite space-time: $L \to \infty$

landscape of latest 2+1 f LQCD simulations of g.s. octet baryon masses





Some of these simulations can be used to fix our low-energy constants

To obtain g.s. baryon masses in the physical world

- Extrapolate to the continuum: $a \to 0$
- Extrapolate to physical light quark masses: $m_q \to m_q(\mathrm{Phys.})$
- Extrapolate to infinite space-time: $L \to \infty$

HB vs. Infrared vs. EOMS

- Heavy baryon (HB) ChPT
  - non-relativistic
  - breaks analyticity of loop amplitudes
  - converges slowly (particularly in three-flavor sector)
  - strict PC and simple nonanalytical results
- Infrared BChPT
  - breaks analyticity of loop amplitudes
  - converges slowly (particularly in three-flavor sector)
  - analytical terms the same as HBChPT
- Extended-on-mass-shell (EOMS) BChPT
  - satisfies all symmetry and analyticity constraints
  - converges relatively faster--an appealing feature

Systematic study of the LQCD data with the EOMS BChPT

- *NNLO EOMS BChPT study of the PACS-CS and LHPC data: Camalich, Geng, Vacas, PRD82(2010)074504*
- *Finite volume corrections: Geng, Ren, Camalich, Weise, PRD84(2011)074024;*
- *First systematic study of all publically available LQCD data: Ren, Geng, Camalich, Meng, Toki, JHEP12(2012)073;*
- *Effects of virtual decuplet baryons: Ren, Geng, Meng, Toki, PRD87(2013)074001*
- *Continuum extrapolations: Ren, Geng, Meng, Eur.Phys.J. C74:2754,2014*

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The EOMS BChPT can be trusted to predict the baryon sigma terms

A careful selection of LQCD data

- All n_f=2+1 LQCD simulations
  - PACS-CS, LHPC, QCDSF-UKQCD, HSC, NPLQCD, BWM
  - BMW—not publicly available
  - HSC/NPLQCD—Low statistics/single combination of quark masses
- We take PACS-CS, LHPC, QCDSF-UKQCD

m_π<500 MeV
$m_\phi L$>4

An accurate determination of baryon sigma terms

- Scale setting: mass independent (given by the LQCD simulations or self-consistently determined) vs. mass dependent (r_0, r_1, X_π)
- Isospin breaking effects: better constrain the LQCD LECs—consistent with the latest BMW study [Science 347 (2015) 1452)]
- Theoretical uncertainties caused by truncating chiral expansions: NNLO vs. N3LO; EOMS vs. FRR

Scale-setting effects on the determination of baryon sigma terms

arXiv:1301.3231
P.E. Shanahan, A.W. Thomas and R.D. Young*

- **Lattice-scale setting**
  - **PACS-CS data with mass independent scale-setting:**

$$\sigma_{sN} = 59 \pm 7 \ (\mathrm{MeV})$$

  - **PACS data with mass dependent (r_0) scale-setting:**

$$\sigma_{sN} = 21 \pm 6 \ (\mathrm{MeV})$$

- **Whether other LQCD data will show the same trend?**

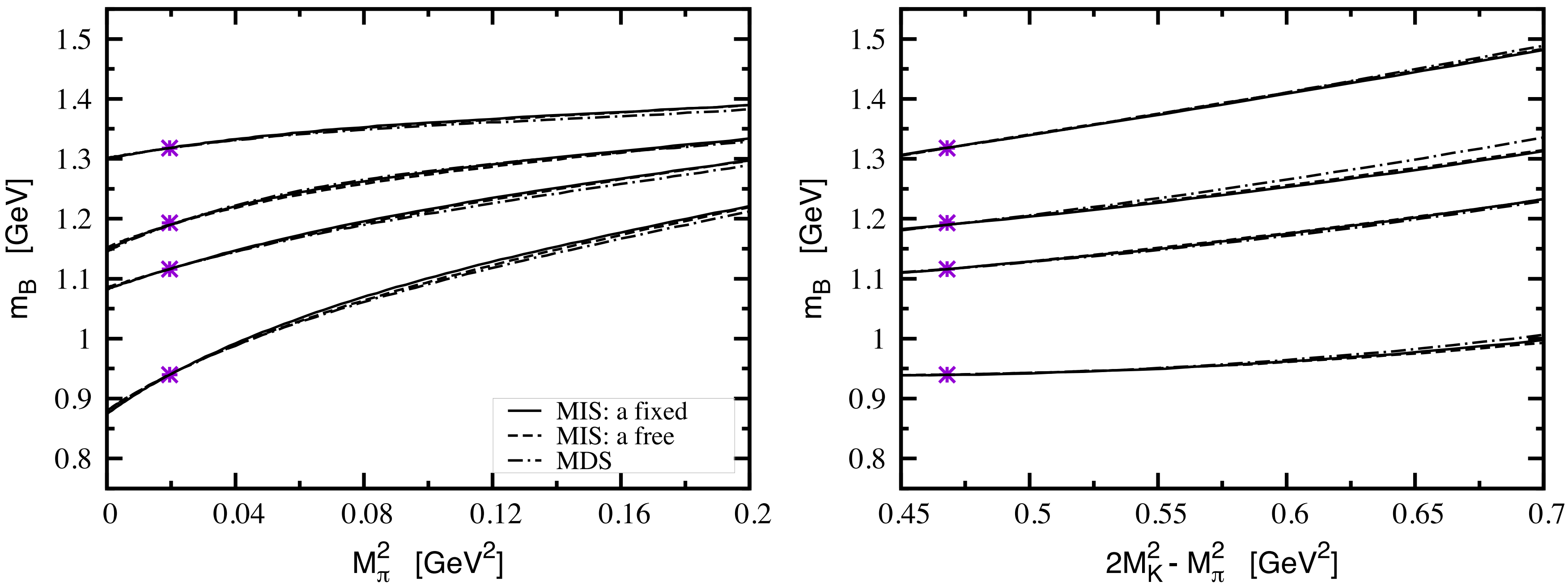
Three different fits at N^3LO

	MIS		MDS
	a fixed	a free	
m_0 [MeV]	884(11)	877(10)	887(10)
b_0 [GeV^{-1}]	$-0.998(2)$	$-0.967(6)$	$-0.911(10)$
b_D [GeV^{-1}]	0.179(5)	0.188(7)	0.039(15)
b_F [GeV^{-1}]	$-0.390(17)$	$-0.367(21)$	$-0.343(37)$
b_1 [GeV^{-1}]	0.351(9)	0.348(4)	$-0.070(23)$
b_2 [GeV^{-1}]	0.582(55)	0.486(11)	0.567(75)
b_3 [GeV^{-1}]	$-0.827(107)$	$-0.699(169)$	$-0.553(214)$
b_4 [GeV^{-1}]	$-0.732(27)$	$-0.966(8)$	$-1.30(4)$
b_5 [GeV^{-2}]	$-0.476(30)$	$-0.347(17)$	$-0.513(89)$
b_6 [GeV^{-2}]	0.165(158)	0.166(173)	$-0.0397(1574)$
b_7 [GeV^{-2}]	$-1.10(11)$	$-0.915(26)$	$-1.27(8)$
b_8 [GeV^{-2}]	$-1.84(4)$	$-1.13(7)$	0.192(30)
d_1 [GeV^{-3}]	0.0327(79)	0.0314(72)	0.0623(116)
d_2 [GeV^{-3}]	0.313(26)	0.269(42)	0.325(54)
d_3 [GeV^{-3}]	$-0.0346(87)$	$-0.0199(81)$	$-0.0879(136)$
d_4 [GeV^{-3}]	0.271(30)	0.230(24)	0.365(23)
d_5 [GeV^{-3}]	$-0.350(28)$	$-0.302(50)$	$-0.326(66)$
d_7 [GeV^{-3}]	$-0.435(10)$	$-0.352(8)$	$-0.322(7)$
d_8 [GeV^{-3}]	$-0.566(24)$	$-0.456(30)$	$-0.459(33)$
χ^2/d.o.f.	0.87	0.88	0.53

- **Mass independent**
  - Lattice spacing a fixed to the published value
  - Lattice spacing a determined self-consistently
- **Mass dependent**
  - r_0 for PACS-CS
  - r_1 for LHPC
  - X_π for QCDSF-UKQCD

Ren, Geng, Meng, PRD91 (2015) 051502

Evolution of baryon masses with u/d and s quark masses

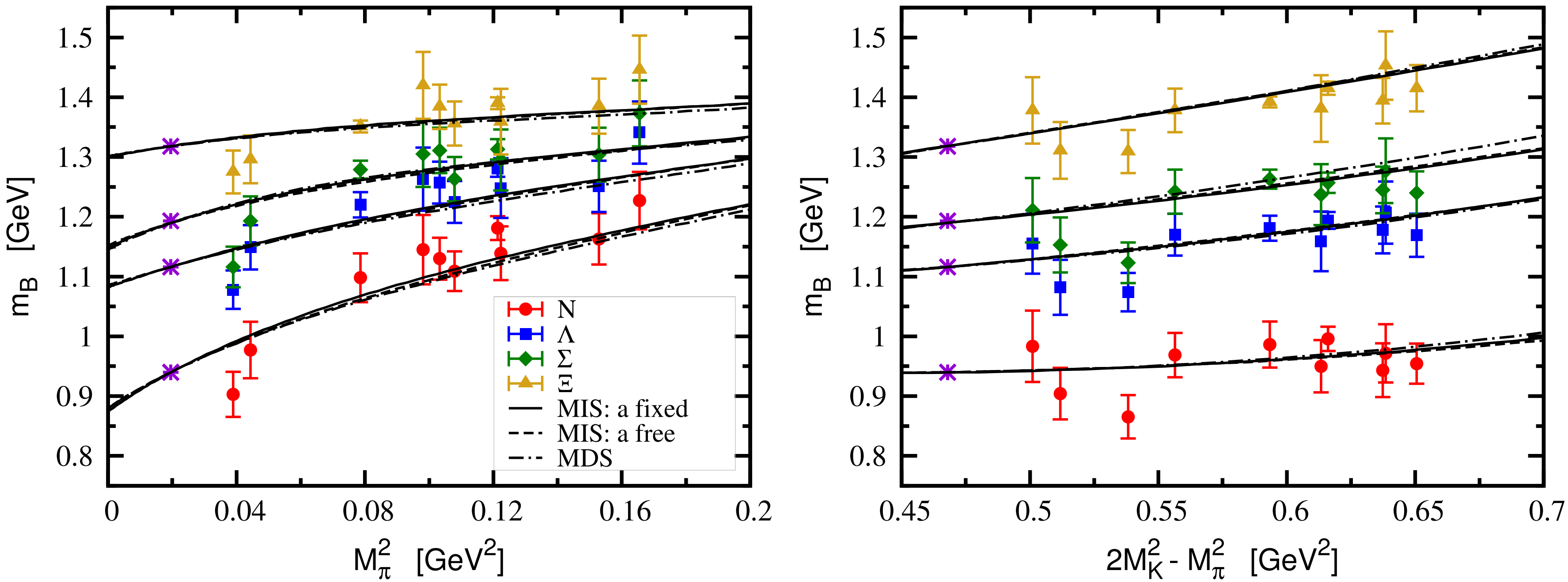


Only central values are shown!

Ren, Geng, Meng, PRD91 (2015) 051502

Ren, Geng, Meng, PRD91 (2015) 051502

In comparison with the BMW13 data



S. Durr et al., BMW collaboration, Phys.Rev. D85 (2012) 014509

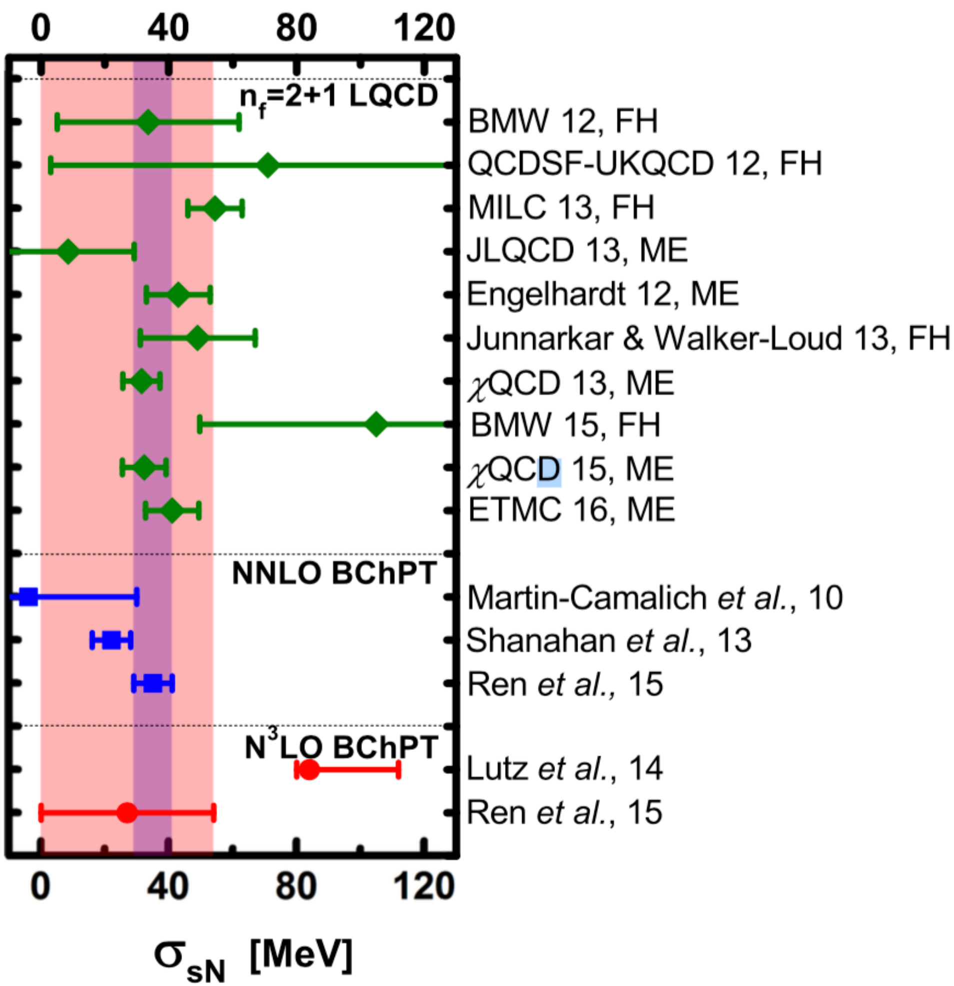
Baryon sigma terms from N^3LO BChPT

	MIS		MDS
	a fixed	a free	
$\sigma_{\pi N}$	55(1)(4)	54(1)	51(2)
$\sigma_{\pi\Lambda}$	32(1)(2)	32(1)	30(2)
$\sigma_{\pi\Sigma}$	34(1)(3)	33(1)	37(2)
$\sigma_{\pi\Xi}$	16(1)(2)	18(2)	15(3)
σ_{sN}	27(27)(4)	23(19)	26(21)
$\sigma_{s\Lambda}$	185(24)(17)	192(15)	168(14)
$\sigma_{s\Sigma}$	210(26)(42)	216(16)	252(15)
$\sigma_{s\Xi}$	333(25)(13)	346(15)	340(13)

- All three scale-setting methods yield similar baryon sigma terms

Ren, Geng, Meng, PRD91 (2015) 051502

Comparison with other studies

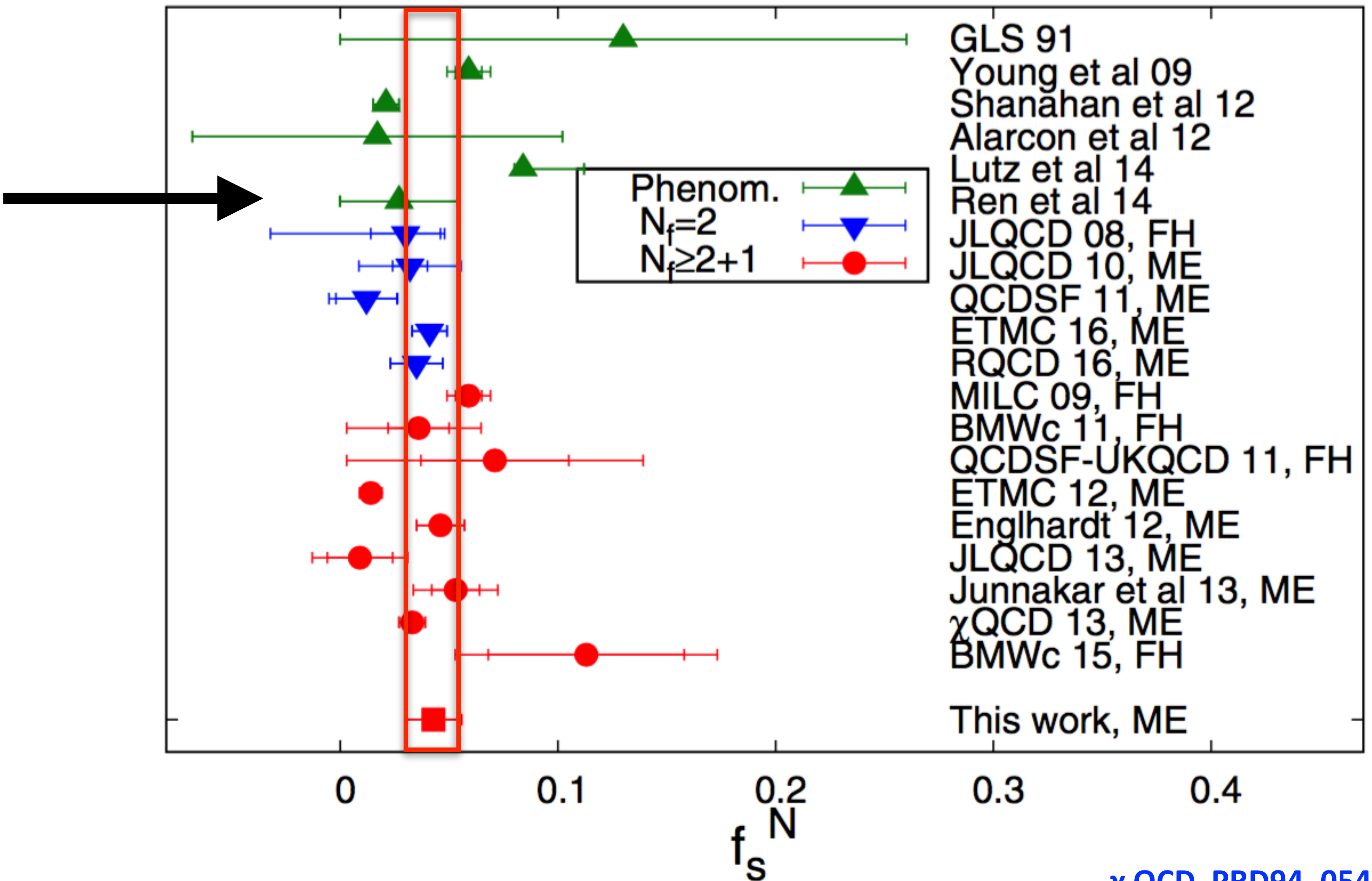


- **Consistent with most recent LQCD studies and those of NNLO ChPT, e.g., that of Young and Shanahan**
- **Uncertainties at N^3LO substantially larger, because of the extra LECs**

Nucleon Strangeness Sigma Term

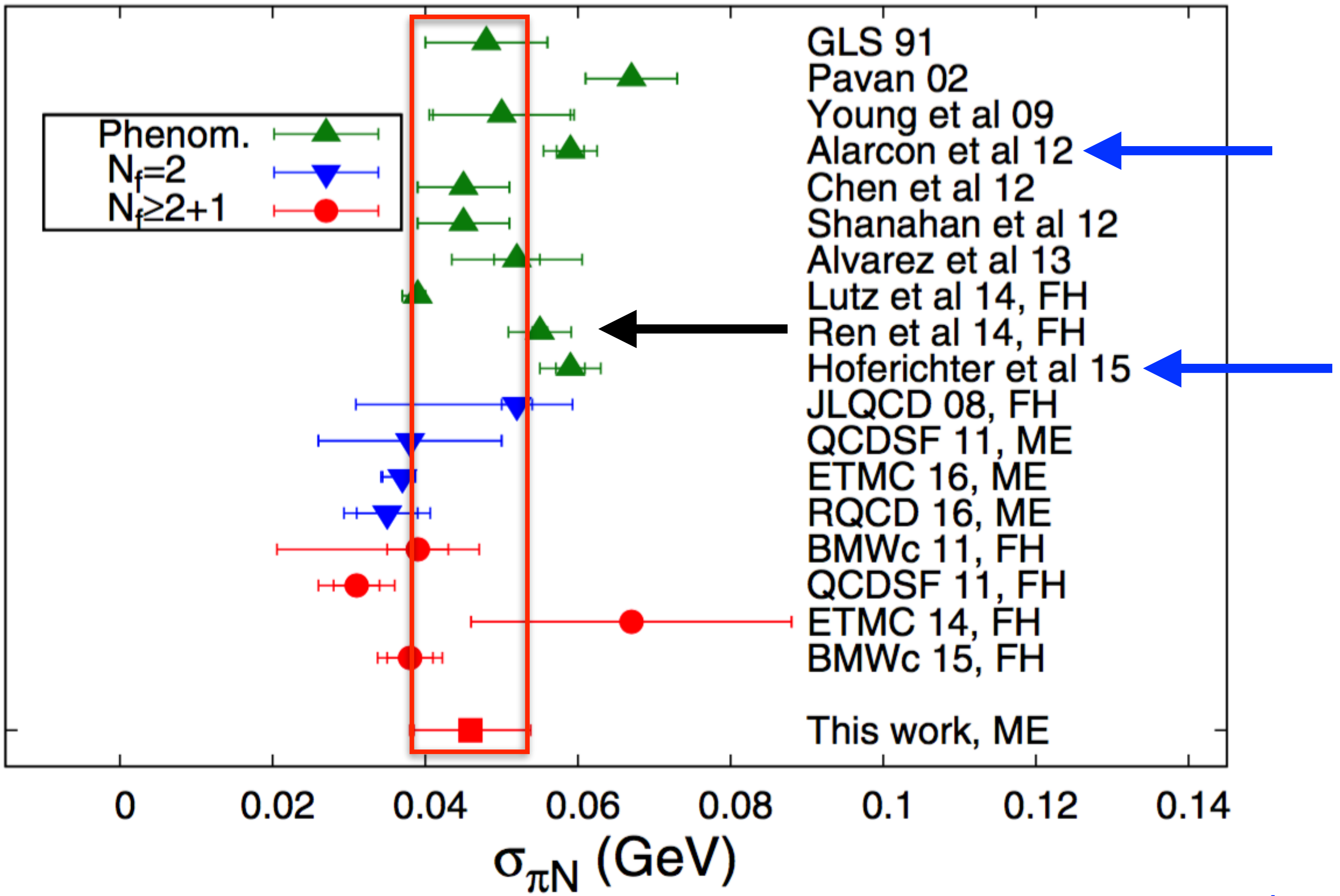
Ren, Geng, Meng, PRD91 (2015) 051502

strangeness-nucleon sigma term



χ QCD, PRD94, 054503 (2016)

pion-nucleon sigma term



χ QCD, PRD94, 054503 (2016)

Further checks

- We have studied most relevant lattice artifacts: chiral extrapolation, finite volume effects, finite lattice spacing effects , effects of heavier virtual states, and used all publicly available results

- What else is still missing?— an explicit check on the validity of SU(3) baryon chiral perturbation theory
  - large kaon mass leads to concerns about convergence of SU(3) BChPT, as seen in early failures of heavy-baryon BChPT and infrared BChPT

Prominent examples

- Octet baryon magnetic moments and chiral extrapolation of nucleon magnetic moments
  - V. Pascalutsa et al., Phys.Lett.B600:239-247,2004.
  - LSG, J. Martin Camalich , L. Alvarez-Ruso, M.J. Vicente Vacas, Phys.Rev.Lett.101:222002,2008

- lattice QCD baryon masses at leading one-loop order in HBChPT
  - LHPC (A. Walker-Loud et al.), Phys.Rev.D79:054502, 2009.
  - PACS-CS (K.-I. Ishikawa), Phys.Rev.D80:054502, 2009.
  - Camalich, Geng, Vacas, PRD82(2010)074504

The application of the EOMS formulation seems to remove or at least alleviate the problem

Match SU(3) to SU(2)

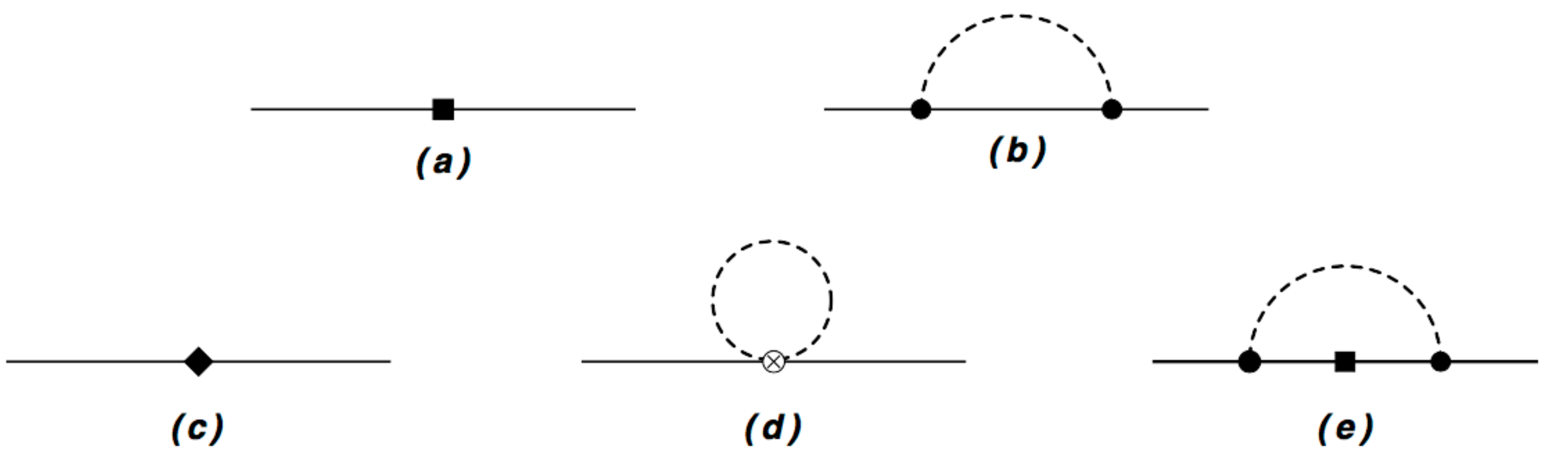
- **Take the strange quark mass as a heavy scale and perform an expansion in terms of $m_{u/d}/m_s$ of the SU(3) results and compare them with the results of the SU(2) study**
  - Alvarez-Ruso, Ledwig, Camalich, and Vicente-Vacas, PRD88, 054507 (2013)

- **An earlier study similar in spirit, but with no quantitative analysis, tried to constrain the SU(3) LECs with SU(2) inputs**
  - M. Frink and U.-G. Meissner, JHEP 0407, 028 (2004)

The procedure

- **In SU(3) up to O(p⁴)**

$$
\begin{aligned}
M_N^{\mathrm{SU}(3)} &= m_0 + m_N^{(2)} + m_N^{(3)} + m_N^{(4)} \\
&= m_0 + \xi_{N\pi}^{(a)} m_\pi^2 + \xi_{NK}^{(a)} m_K^2 + \xi_{N\pi}^{(c)} m_\pi^4 + \xi_{NK}^{(c)} m_K^4 + \xi_{N\pi K}^{(c)} m_\pi^2 m_K^2 \\
&\quad + \frac{1}{(4\pi F_\phi)^2} \sum_{\phi=\pi,\, K,\, \eta} \left[\xi_{N\phi}^{(b)} H_N^{(b)} + \xi_{N\phi}^{(d)} H_N^{(d)} + \sum_{B=N,\, \Lambda,\, \Sigma} \xi_{NB\phi}^{(e)} H_{NB}^{(e)} \right]
\end{aligned}
$$



The procedure

- **Isolate the strange quark contribution**

$$m_{s\bar{s}}^2 = 2B_0 m_s$$

- **Leading-order ChPT**

$$m_K^2 = \frac{1}{2}(m_\pi^2 + m_{s\bar{s}}^2), \quad m_\eta^2 = \frac{1}{3}(m_\pi^2 + 2m_{s\bar{s}}^2)$$

- **Expand the kaon and eta contributions in terms of** $m_\pi / m_{s\bar{s}}$

$$\Sigma_{K,\,\eta}^{(i)} = A_{K,\,\eta}^{(i)} + B_{K,\,\eta}^{(i)} m_\pi^2 + C_{K,\,\eta}^{(i)} m_\pi^4 + \mathcal{O}\left(\frac{m_\pi}{m_{s\bar{s}}}\right)^5$$

- **SU(2) equivalent nucleon mass**

$$M_N = m_0^{\rm eff} - 4c_1^{\rm eff} m_\pi^2 + \alpha^{\rm eff} m_\pi^4 + \beta^{\rm eff} m_\pi^4 \log\frac{\mu^2}{m_\pi^2} + \frac{1}{(4\pi F_\phi)^2}\frac{3}{2}(D+F)^2\left[H_N^{(b)}(m_0,m_\pi) + \frac{1}{2}H_N^{(e)}(m_0,m_\pi,\Delta m_N,\mu)\right]$$

- **To be compared with**

$$M_N^{\rm SU(2)} = M_0 - 4c_1 m_\pi^2 + \frac{1}{2}\alpha m_\pi^4 + \frac{1}{(4\pi f_\pi)^2}\frac{3}{8}\left[2(-8c_1+c_2+4c_3)+c_2\right]m_\pi^4 - \frac{1}{(4\pi f_\pi)^2}\frac{3}{4}(8c_1-c_2-4c_3)m_\pi^4\log\frac{\mu^2}{m_\pi^2} + \frac{1}{(4\pi f_\pi)^2}\frac{3}{2}g_A^2\left[H_N^{(b)}(M_0,m_\pi) + \frac{1}{2}H_N^{(e)}(M_0,m_\pi,(-4c_1m_\pi^2),\mu)\right],$$

Alvarez-Ruso, Ledwig, Camalich, and Vicente-Vacas, PRD88, 054507 (2013)

- **SU(2) equivalent nucleon mass**

$$M_N = m_0^{\rm eff} - 4c_1^{\rm eff} m_\pi^2 + \alpha^{\rm eff} m_\pi^4 + \beta^{\rm eff} m_\pi^4 \log\frac{\mu^2}{m_\pi^2} + \frac{1}{(4\pi F_\phi)^2}\frac{3}{2}(D+F)^2\left[H_N^{(b)}(m_0, m_\pi) + \frac{1}{2}H_N^{(e)}(m_0, m_\pi, \Delta m_N, \mu)\right]$$

- **To be compared with**

$$M_N^{\rm SU(2)} = M_0 - 4c_1 m_\pi^2 + \alpha^{\rm SU(2)} m_\pi^4 + \beta^{\rm SU(2)} m_\pi^4 \log\frac{\mu^2}{m_\pi^2} + \frac{1}{(4\pi f_\pi)^2}\frac{3}{2}g_A^2\left[H_N^{(b)}(M_0, m_\pi) + \frac{1}{2}H_N^{(e)}(M_0, m_\pi, (-4c_1 m_\pi^2), \mu)\right]$$

$$\alpha^{\rm SU(2)} = \frac{1}{2}\alpha - \frac{1}{(4\pi f_\pi)^2}\frac{3}{4}\left[(8c_1 - c_2 - 4c_3) - \frac{1}{2}c_2\right]$$

$$\beta^{\rm SU(2)} = -\frac{3}{4(4\pi f_\pi)^2}(8c_1 - c_2 - 4c_3).$$

Ren, Alvarez-Ruso, **Geng**, Ledwig, Meng, Vicente Vacas, 1606.03820

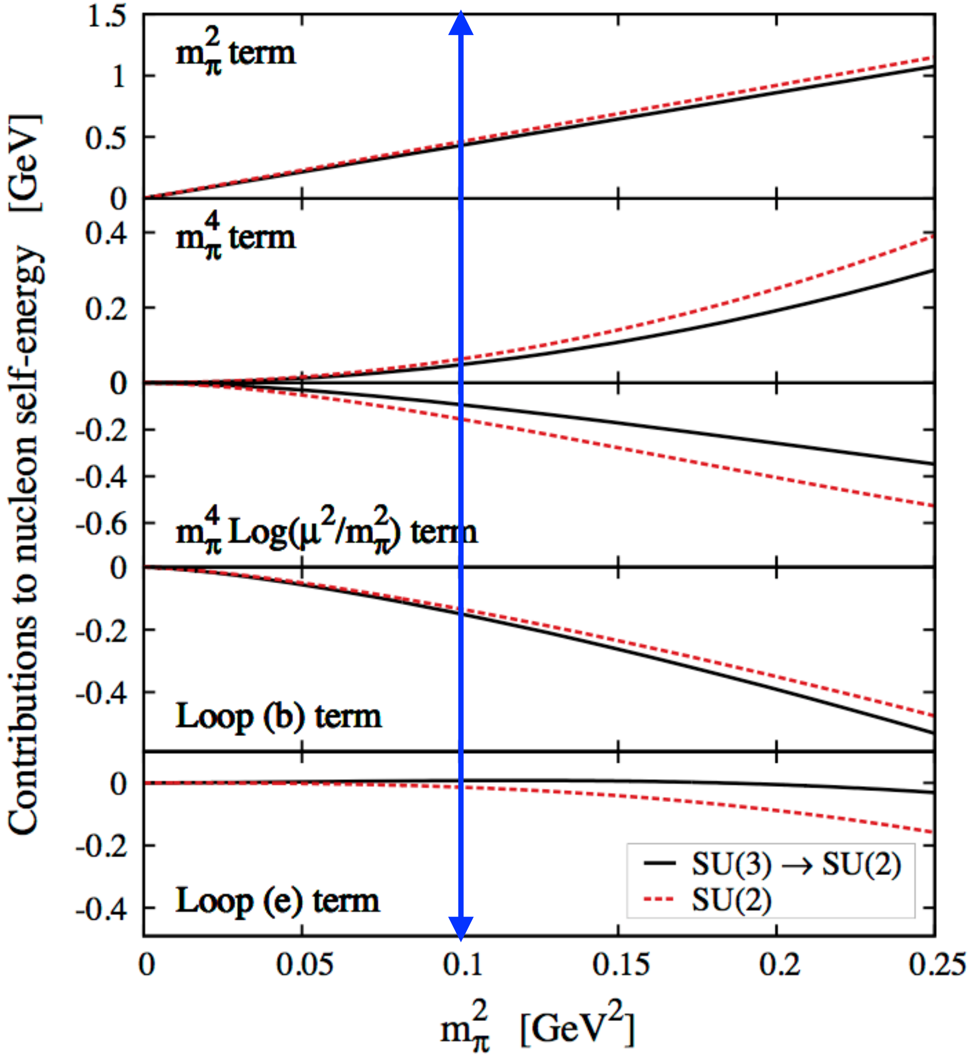
Comparison of effective parameters

SU(3)→SU(2)	SU(2)
$m_0^{\text{eff}} = 875(10)$ MeV	$M_0 = 870(3)$ MeV
$c_1^{\text{eff}} = -1.07(4)\ \text{GeV}^{-1}$	$c_1 = -1.15(3)\ \text{GeV}^{-1}$
$\alpha^{\text{eff}} = 4.81(9)\ \text{GeV}^{-3}$	$\alpha^{\text{SU(2)}} = 6.27(1.98)\ \text{GeV}^{-3}$
$\beta^{\text{eff}} = -4.02(20)\ \text{GeV}^{-3}$	$\beta^{\text{SU(2)}} = -7.62(93)\ \text{GeV}^{-3}$

- SU(3): Ren, Geng, Meng, PRD91 (2015) 051502
- SU(2): Alvarez-Ruso, Ledwig, Camalich, and Vicente-Vacas, PRD88, 054507 (2013)

Ren, Alvarez-Ruso, **Geng**, Ledwig, Meng, Vicente Vacas, 1606.03820

Decomposition of different contributions

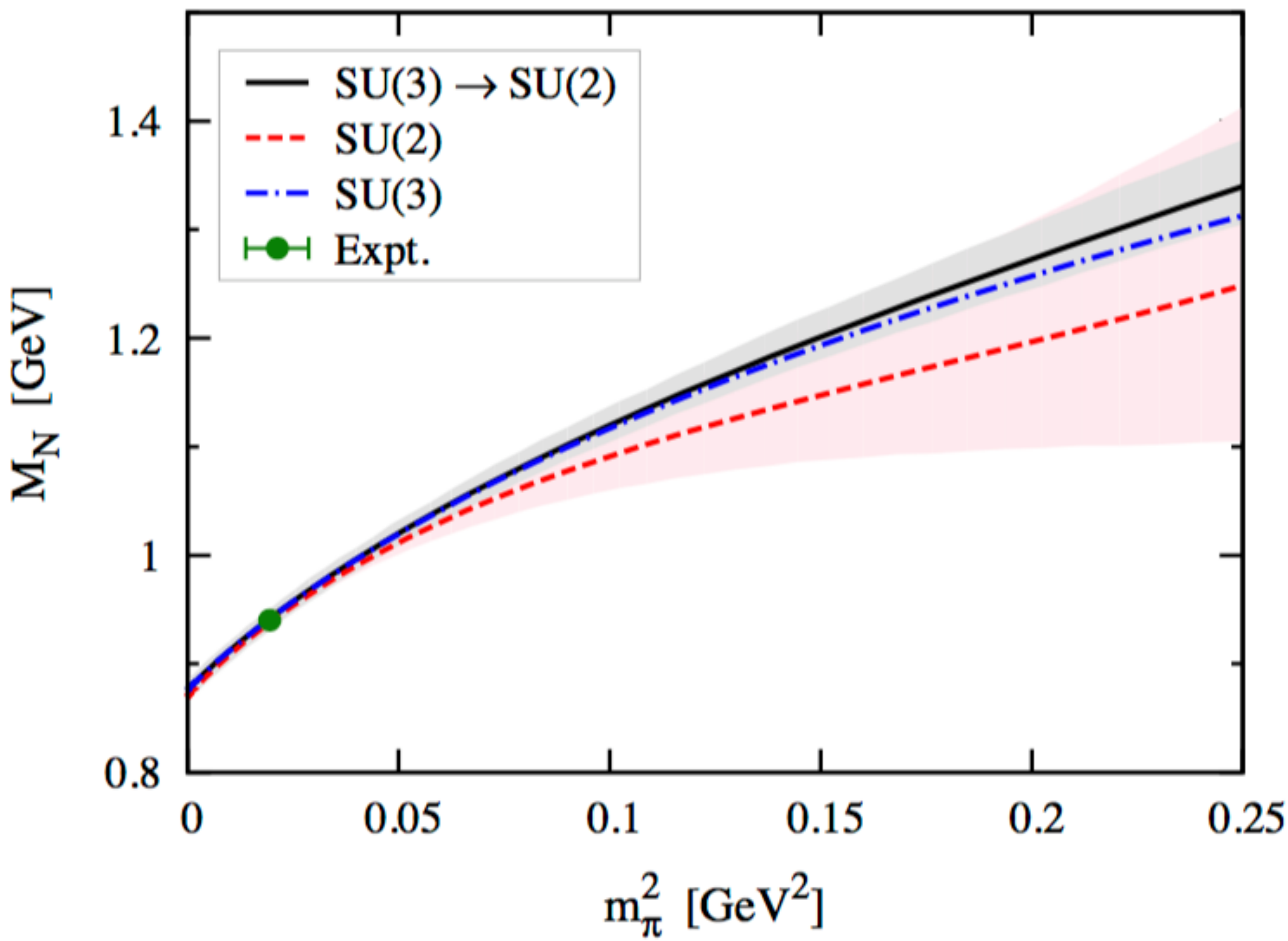


agree at small m_π<300 MeV

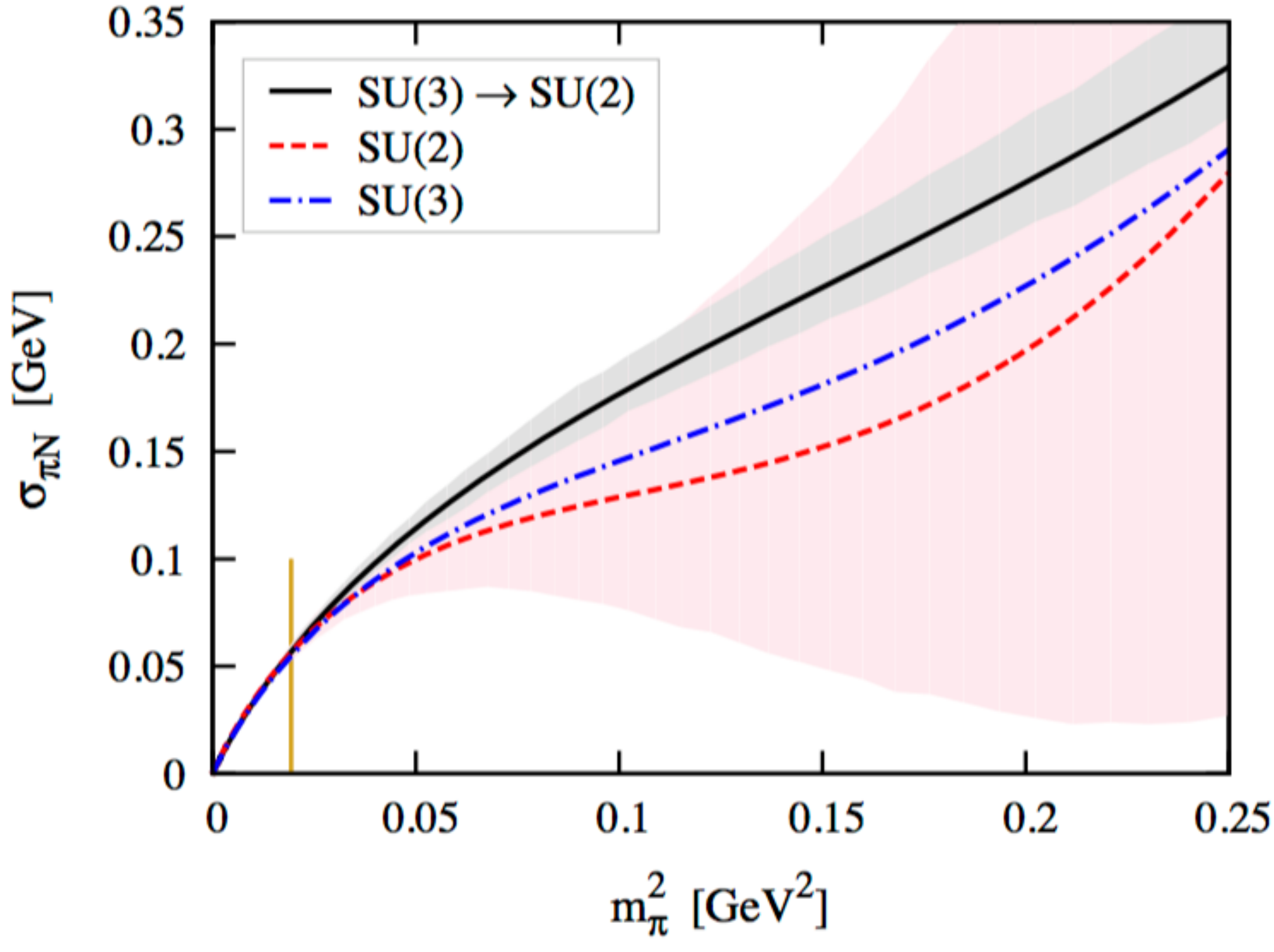
differ at larger m_π

*Ren, Alvarez-Ruso, **Geng**, Ledwig, Meng, Vicente Vacas, 1606.03820*

Chiral extrapolation of nucleon mass

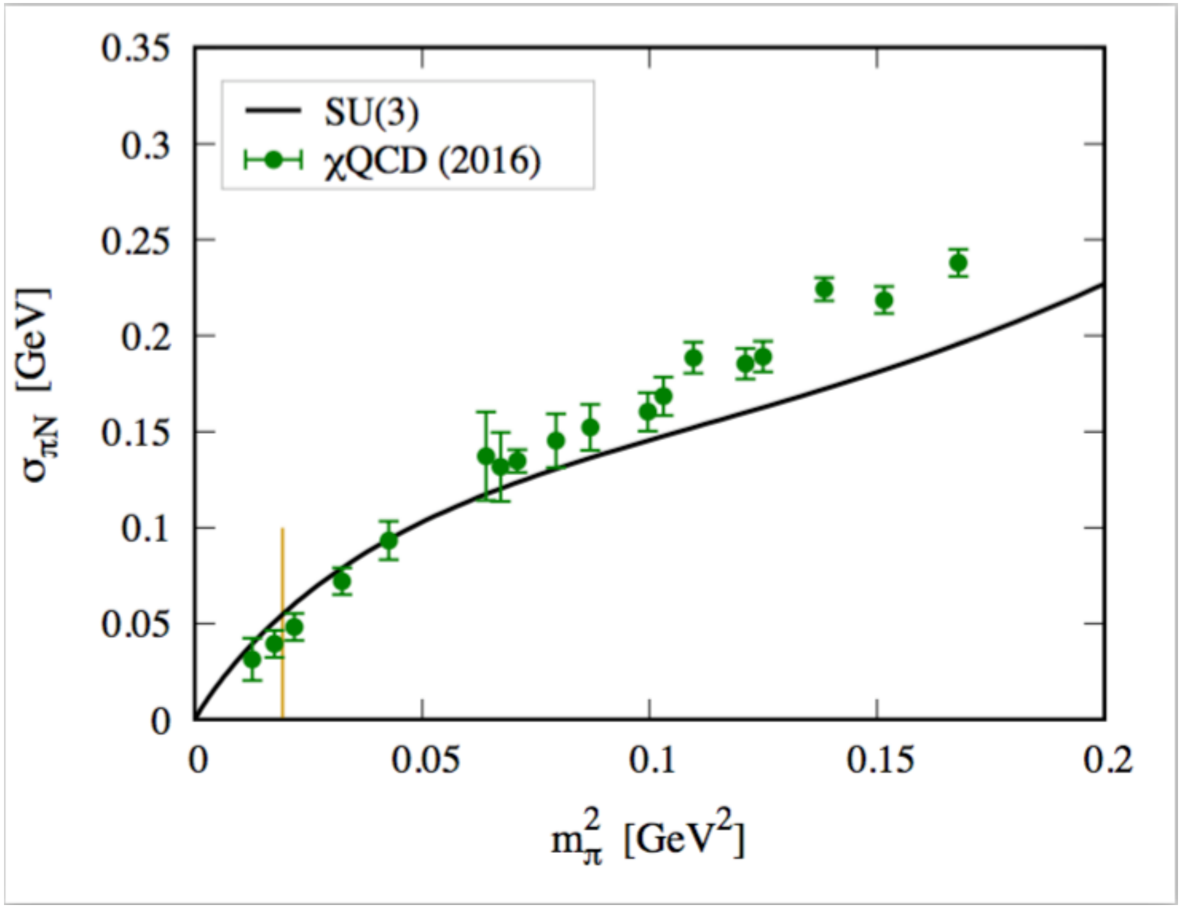


Light quark dependence of pion-nucleon sigma term



*Ren, Alvarez-Ruso, **Geng**, Ledwig, Meng, Vicente Vacas, 1606.03820*

In comparison with χQCD16



χ QCD, PRD94, 054503 (2016)

Summary

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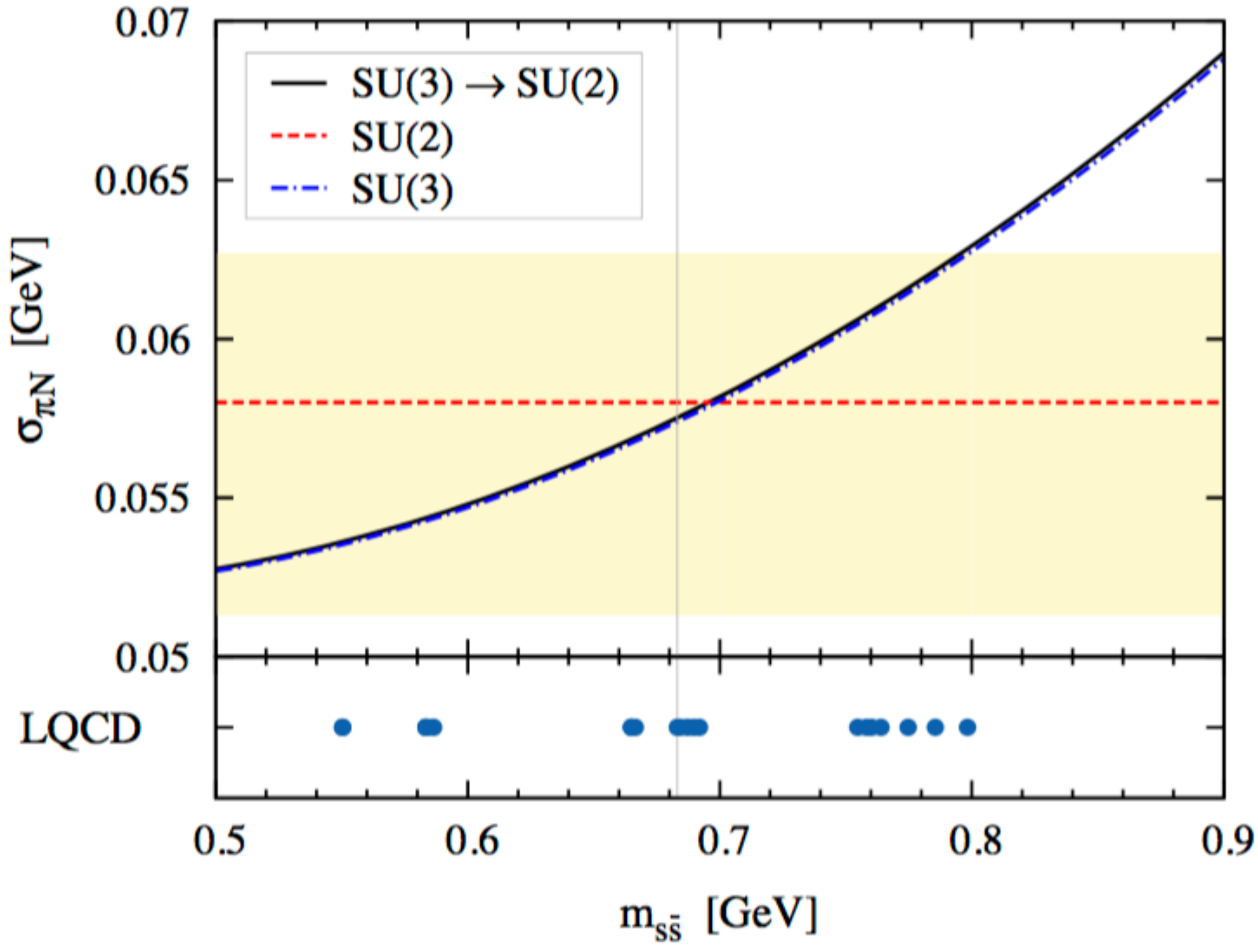
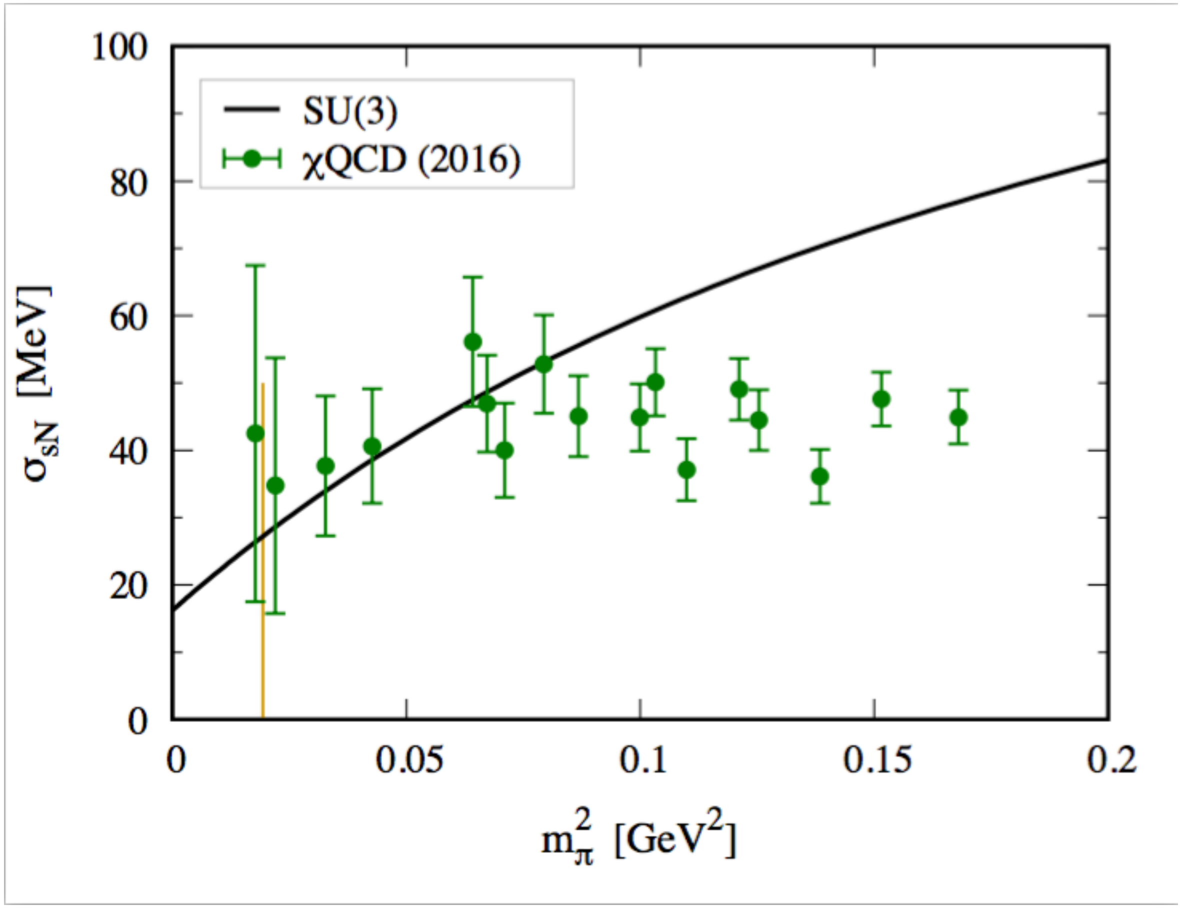
- ✤ It is to be stressed that we have taken into account as much as possible lattice artifacts and checked the validity of SU(3) BChP
- ✤ We should bear in mind, however, that an obvious caveat is that our results are tied to the reliability and accuracy of the available lQCD simulations

Thank you very much for your attention!

Strange quark mass dependence of pion-nucleon sigma term



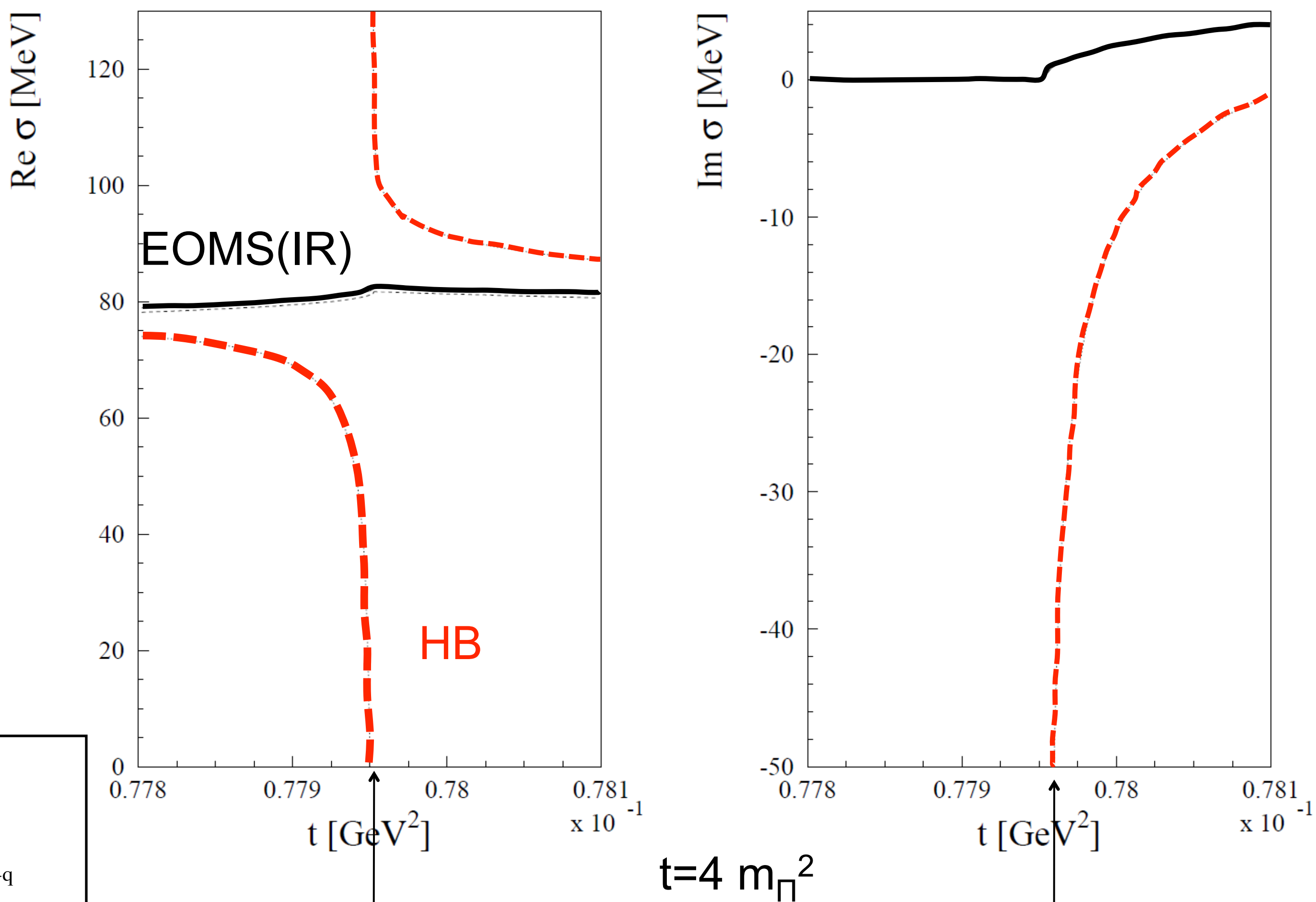
in comparison with χQCD16

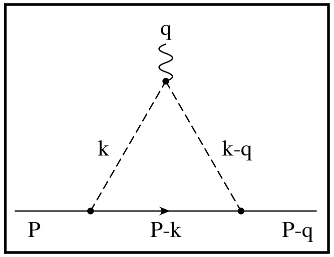


The nucleon scalar form factor at q^3

$$\langle p(p',s')|\mathcal{H}_{\rm sb}(0)|p(p,s)\rangle = \bar{u}(p',s')u(p,s)\sigma(t), \quad t=(p'-p)^2$$

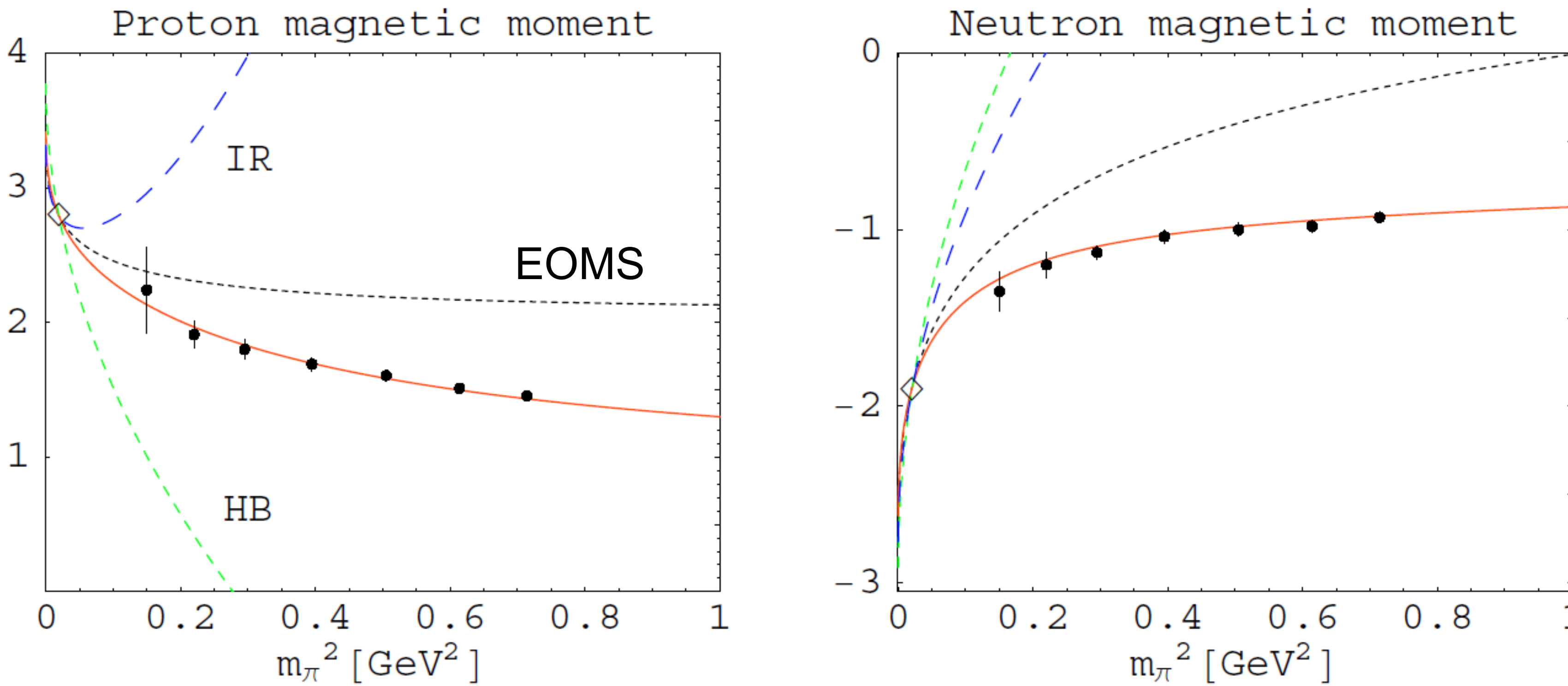
$$\mathcal{H}_{\rm sb} = \hat{m}(\bar{u}u+\bar{d}d)$$





S. Scherer, Prog.Part.Nucl.Phys.64:1-60,2010

Proton and neutron magnetic moments: chiral extrapolation



V. Pascalutsa et al., Phys.Lett.B600:239-247,2004.

Octet baryon magnetic moments at NLO BChPT

$$\chi^2 = \sum(\mu_{th} - \mu_{exp})^2$$

		p	n	Λ	Σ^-	Σ^0	Σ^+	Ξ^-	Ξ^0	$\Lambda\Sigma^0$	χ^2
LO	C-G	2.56	-1.60	-0.80	-0.97	0.80	2.56	-0.97	-1.60	1.38	**0.46**
NLO	HB	3.01	-2.62	-0.42	-1.35	0.42	2.18	-0.52	-0.70	1.68	**1.01**
	IR	2.08	-2.74	-0.64	-1.13	0.64	2.41	-1.17	-1.45	1.89	**1.83**
	EOMS	2.58	-2.10	-0.66	-1.10	0.66	2.43	-0.95	-1.27	1.58	**0.18**
	Exp.	2.79	-1.91	-0.61	-1.16	—	2.46	-0.65	-1.25	1.61	—

- Contribution of the chiral series [LO(1+NLO/LO)]:

$$\mu_p = 3.47(1-0.257), \quad \mu_n = -2.55(1-0.175), \quad \mu_\Lambda = -1.27(1-0.482),$$

$$\mu_{\Sigma^-} = -0.93(1+0.187), \quad \mu_{\Sigma^+} = 3.47(1-0.300), \quad \mu_{\Sigma^0} = 1.27(1-0.482),$$

$$\mu_{\Xi^-} = -0.93(1+0.025), \quad \mu_{\Xi^0} = -2.55(1-0.501), \quad \mu_{\Lambda\Sigma^0} = 2.21(1-0.284).$$

LSG, J. Martin Camalich , L. Alvarez-Ruso, M.J. Vicente Vacas, Phys.Rev.Lett. 101:222002,2008

Problems reported in SU(3) HBChPT (1)

LHPC (A. Walker-Loud et al.), Phys.Rev.D79:054502, 2009.

TABLE XVII. Results from NLO bootstrap χ extrapolations of the octet baryon masses, using mixed action (MA) and $SU(3)$ heavy baryon χPT.

C=1.2(2), D=0.715(50), F=0.453(50)

FIT: NLO	Range	M_0 (GeV)	σ_M (GeV^{-1})	α_M (GeV^{-1})	β_M (GeV^{-1})	C	D	F	χ^2	d.o.f.
M_N, M_Λ,	007–020: MA	1.087(51)	−0.03(5)	−0.72(8)	−0.62(4)	0.15(9)	0.33(4)	0.14(3)	6.0	5
M_Σ, M_Ξ	007–020: $SU(3)$	1.014(32)	−0.07(4)	−0.77(10)	−0.56(5)	0.18(9)	0.30(6)	0.19(4)	5.5	5
	007–030: MA	1.149(57)	0.01(4)	−0.79(11)	−0.67(7)	0.12(9)	0.38(6)	0.16(3)	14.4	9
	007–030: $SU(3)$	1.091(66)	−0.04(3)	−0.99(28)	−0.73(19)	0.1(1)	0.44(14)	0.24(7)	11.9	9
	007–040: MA	1.147(52)	0.01(3)	−0.78(10)	−0.68(6)	0.13(9)	0.39(6)	0.16(3)	14.9	13
	007–040: $SU(3)$	1.090(61)	−0.04(3)	−0.99(26)	−0.73(18)	0.1(1)	0.45(13)	0.25(6)	12.5	13

TABLE XX. Results from NLO bootstrap χ extrapolations of the decuplet masses, using mixed action (MA) and $SU(3)$ heavy baryon χPT.

FIT: NLO	Range	$M_{T,0}$ (GeV)	$\bar{\sigma}_M$ (GeV^{-1})	γ_M (GeV^{-1})	C	H	χ^2	d.o.f.
M_Δ, M_{Σ^*},	007–020: MA	1.68(10)	−0.04(3)	1.2(3)	0.00(07)	1.2(2)	18.9	7
M_{Ξ^*}, M_{Ω^-}	007–020: $SU(3)$	1.52(05)	−0.20(4)	1.3(3)	0.00(15)	1.4(3)	20.3	7
	007–030: MA	1.64(08)	−0.05(2)	1.1(2)	0.00(07)	1.1(2)	21.0	11
	007–030: $SU(3)$	1.52(04)	−0.19(4)	1.3(3)	0.00(15)	1.4(3)	21.1	11
	007–040: MA	1.73(08)	−0.01(1)	1.2(2)	0.00(06)	1.2(2)	32.8	15
	007–040: $SU(3)$	1.57(04)	−0.18(4)	1.4(3)	0.00(14)	1.6(2)	34.8	15

mixed action heavy baryon chiral perturbation theory. Both the three-flavor and two-flavor functional forms describe our lattice results, although the low-energy constants from the next-to-leading order $SU(3)$ fits are inconsistent with their phenomenological values. Next-to-next-to-leading order $SU(2)$ continuum

Problems reported in SU(3) HBChPT (II)

PACS-CS (K.-I. Ishikawa), Phys.Rev.D80:054502, 2009.

PACS-CS (S. Aoki et al.), Phys.Rev.D79:034503, 2009.

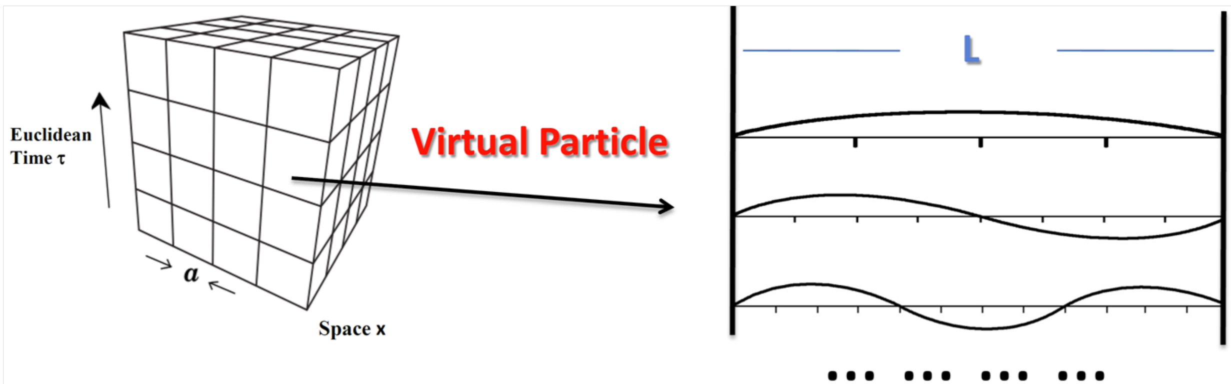
TABLE VI. Fit results with the SU(3) HBChPT for the octet baryon masses. NLO results are obtained with (case 2) and without (case 1) fixing D, F, and $\mathcal{C}$ at the phenomenological estimate.

		NLO		
	LO	Case 1	Case 2	Phenomenological
m_B	0.410(14)	0.391(39)	−0.15(9)	
α_M	−2.262(62)	−2.62(62)	−15.3(2.0)	
β_M	−1.740(58)	−2.6(1.5)	−21.3(3.0)	
σ_M	−0.53(12)	−0.71(34)	−9.6(1.4)	
D		$0.000(16) \times 10^{-8}$	0.80 fixed	0.80
F		$0.000(9) \times 10^{-8}$	0.47 fixed	0.47
$\mathcal{C}$		0.36(30)	1.5 fixed	1.5
χ^2/dof	1.10(63)	1.39(77)	153(82)	

We investigate the quark mass dependence of baryon masses in 2 + 1 flavor lattice QCD using SU(3) heavy baryon chiral perturbation theory up to one-loop order. The baryon mass data used for the analyses are obtained for the degenerate up-down quark mass of 3 to 24 MeV and two choices of the strange quark mass around the physical value. We find that the SU(3) chiral expansion fails to describe both the octet and the decuplet baryon data if phenomenological values are employed for the meson-baryon couplings. The SU(2) case is also examined for the nucleon. We observe that higher order terms are controlled only around the physical point. We also evaluate finite size effects using SU(3) heavy baryon chiral perturbation theory, finding small values of order 1% even at the physical point.

Finite volume corrections

- **Physical origin: existence of boundary conditions**



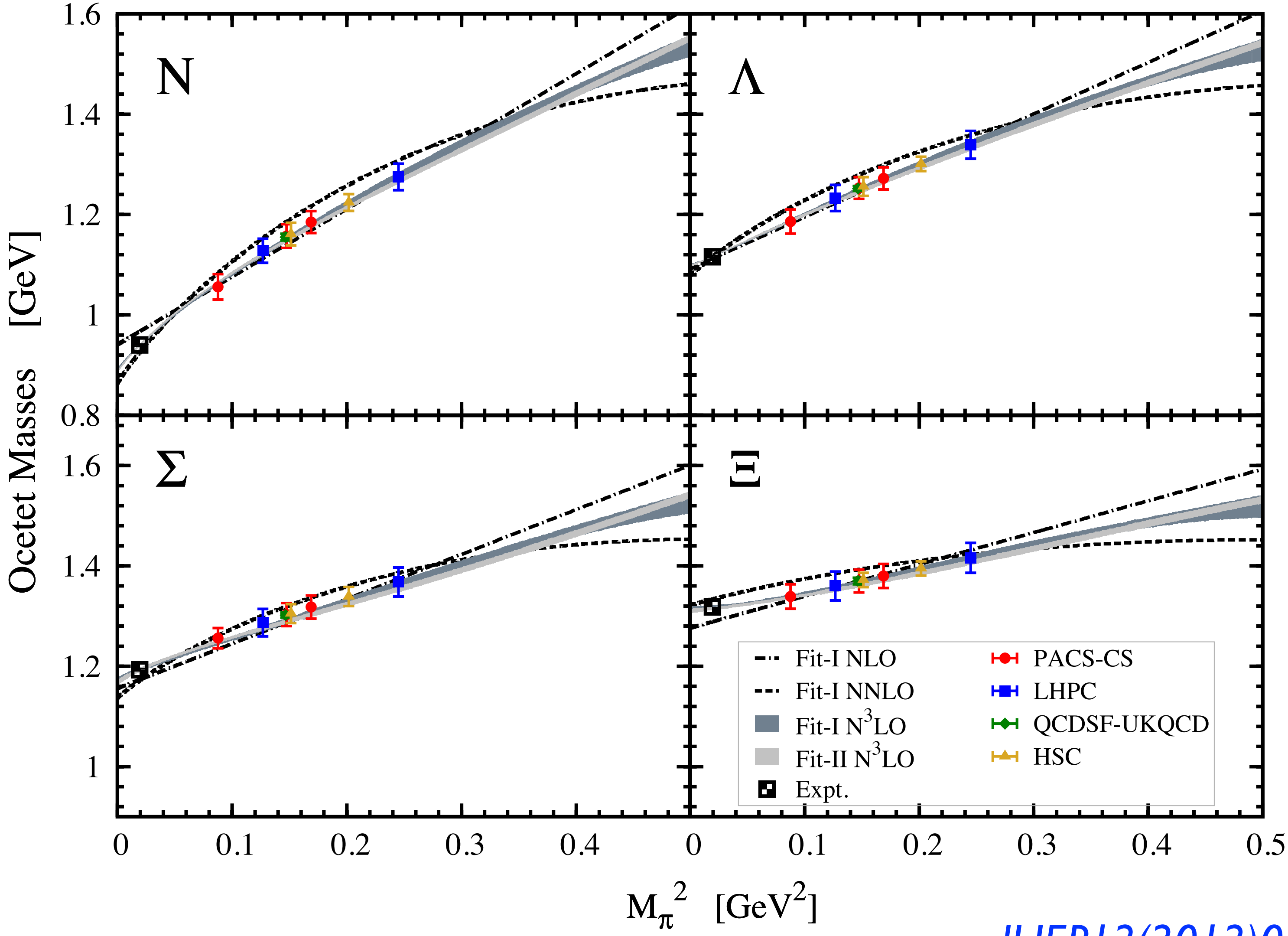
- **Momenta of virtual particles are discretized**

$$k_i = 2\pi \frac{n_i}{L},\ (i = 0, 1, 2, 3)$$

$$\int_{-\infty}^{\infty} dk \Rightarrow \sum_{n=-\infty}^{\infty} \left(\frac{2\pi}{L}\right) \cdot n.$$

Geng, Ren, Martin-Camalich, Weise, PRD84(2011)074024

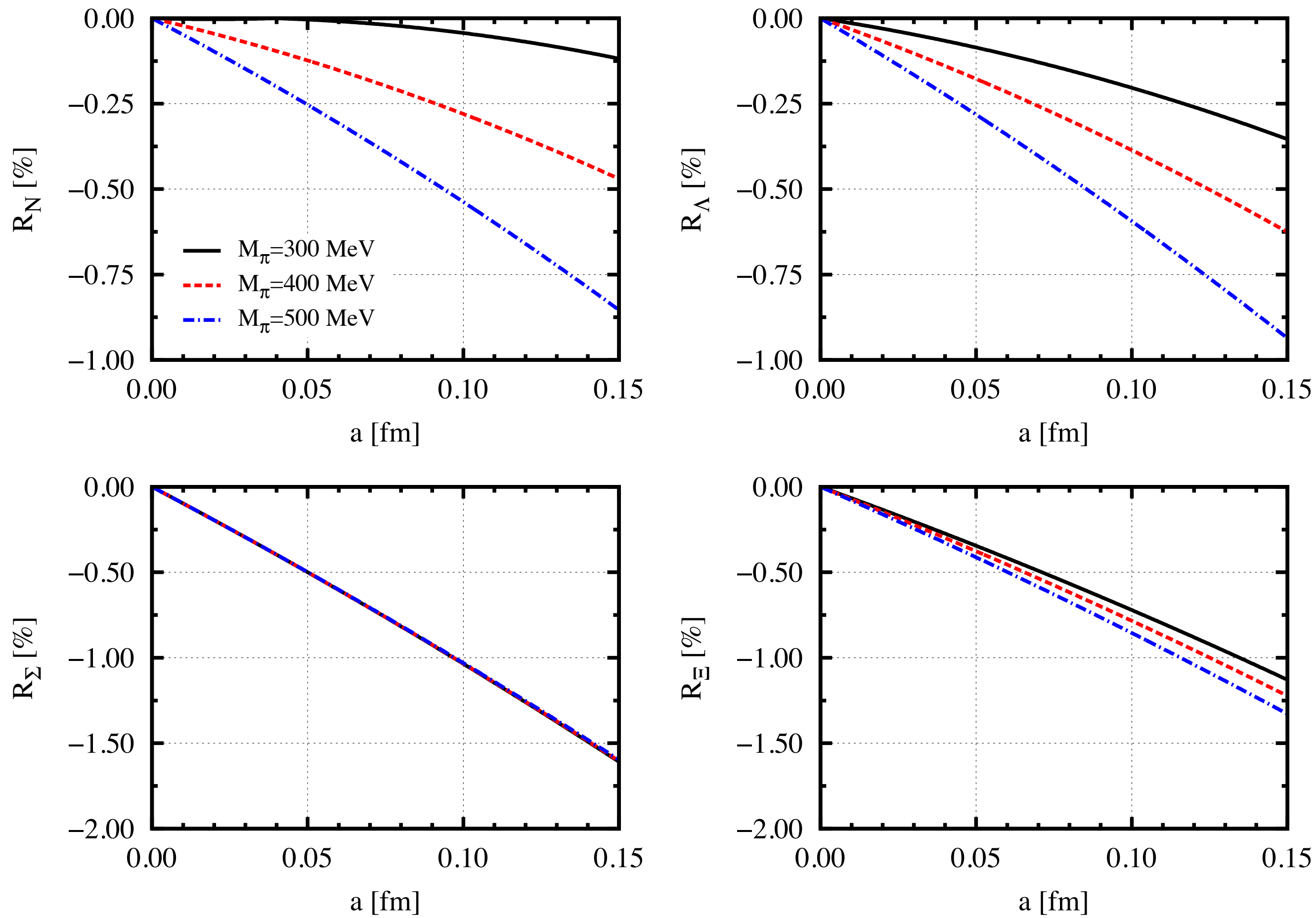
Chiral extrapolations upto N3LO in BChPT



JHEP12(2012)073;

Ren, Geng, Meng, EPJC74 (2014) 2754

Lattice spacing evolutions



$$R_B = m_B^{(a)}/m_B$$

- For LQCD simulations with m_π<500 MeV and a<0.15 fm, discretization effects are about 1 to 2 percent

NNLO fits

TABLE I. Values of the LECs obtained from the best fits to the LQCD simulations and the experimental octet baryon masses and the corresponding χ^2/d.o.f.. The underlined numbers denote the values at which they are fixed.

	EOMS		FRR	
	Fit-I	Fit-II	Fit-III	Fit-IV
m_0 [MeV]	757(7)	808(1)	829(7)	805(9)
b_0 [GeV^{-1}]	$-0.907(6)$	$-0.710(2)$	$-0.820(7)$	$-0.922(20)$
b_D [GeV^{-1}]	0.0582(22)	0.0570(22)	0.101(2)	0.116(3)
b_F [GeV^{-1}]	$-0.508(2)$	$-0.411(11)$	$-0.464(2)$	$-0.510(8)$
f_0 [GeV]	$\underline{0.0871}$	0.105(3)	$\underline{0.0871}$	$\underline{0.0871}$
Λ or μ [GeV]	$\underline{1.0}$	$\underline{1.0}$	$\underline{1.0}$	1.24(5)
χ^2/d.o.f.	3.0	1.6	2.4	1.8

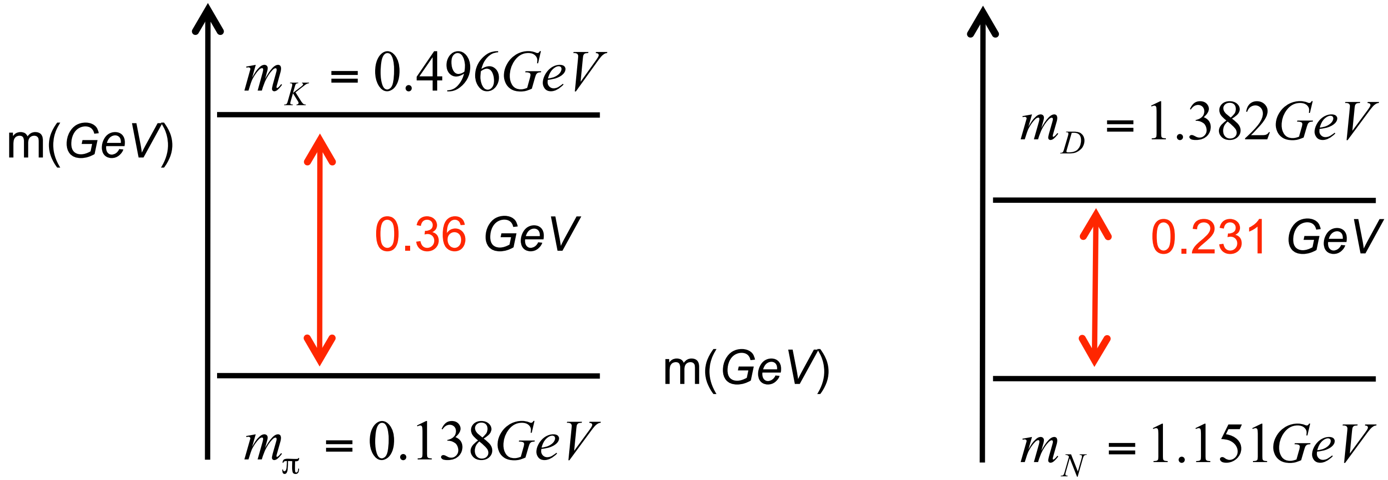
NNLO sigma terms

TABLE II. Sigma terms of the octet baryons at the physical point, predicted by the NNLO BChPT with the LECs of Table I.

	EOMS		FRR	
	Fit-I	Fit-II	Fit-III	Fit-IV
$\sigma_{\pi N}$ [MeV]	$56(0)$	$47(1)$	$47(0)$	$53(1)$
$\sigma_{\pi\Lambda}$ [MeV]	$35(1)$	$30(1)$	$31(1)$	$34(1)$
$\sigma_{\pi\Sigma}$ [MeV]	$32(0)$	$27(1)$	$25(0)$	$27(1)$
$\sigma_{\pi\Xi}$ [MeV]	$13(1)$	$12(1)$	$13(1)$	$13(1)$
σ_{sN} [MeV]	$35(6)$	$27(7)$	$21(6)$	$20(7)$
$\sigma_{s\Lambda}$ [MeV]	$147(7)$	$152(7)$	$162(7)$	$153(7)$
$\sigma_{s\Sigma}$ [MeV]	$218(7)$	$222(7)$	$226(7)$	$214(7)$
$\sigma_{s\Xi}$ [MeV]	$295(7)$	$313(8)$	$332(7)$	$312(8)$

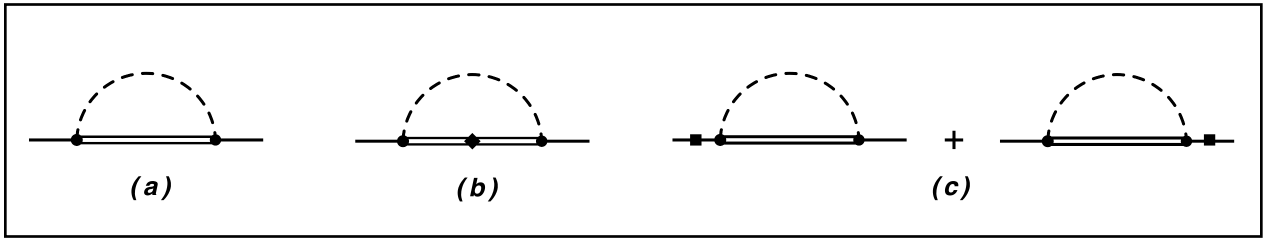
Effects of dynamical decuplet baryons

- ChPT relies on the assumption that all high-energy degrees of freedom can be integrated out--not necessarily true for SU(3) BChPT



Feynman diagrams/Lagrangians-no new unknown LECs

- Feynman diagrams



- Lagrangians
  - Octet-Decuplet-Pseudoscalr coupling

$$\mathcal{L}^{(1)}_{\phi BT} = \frac{i\mathcal{C}}{m_D F_\phi}\varepsilon^{abc}(\partial_\alpha \bar{T}^{ade}_\mu)\gamma^{\alpha\mu\nu}B^e_c\partial_\nu\phi^d_b + \text{H.c.},$$

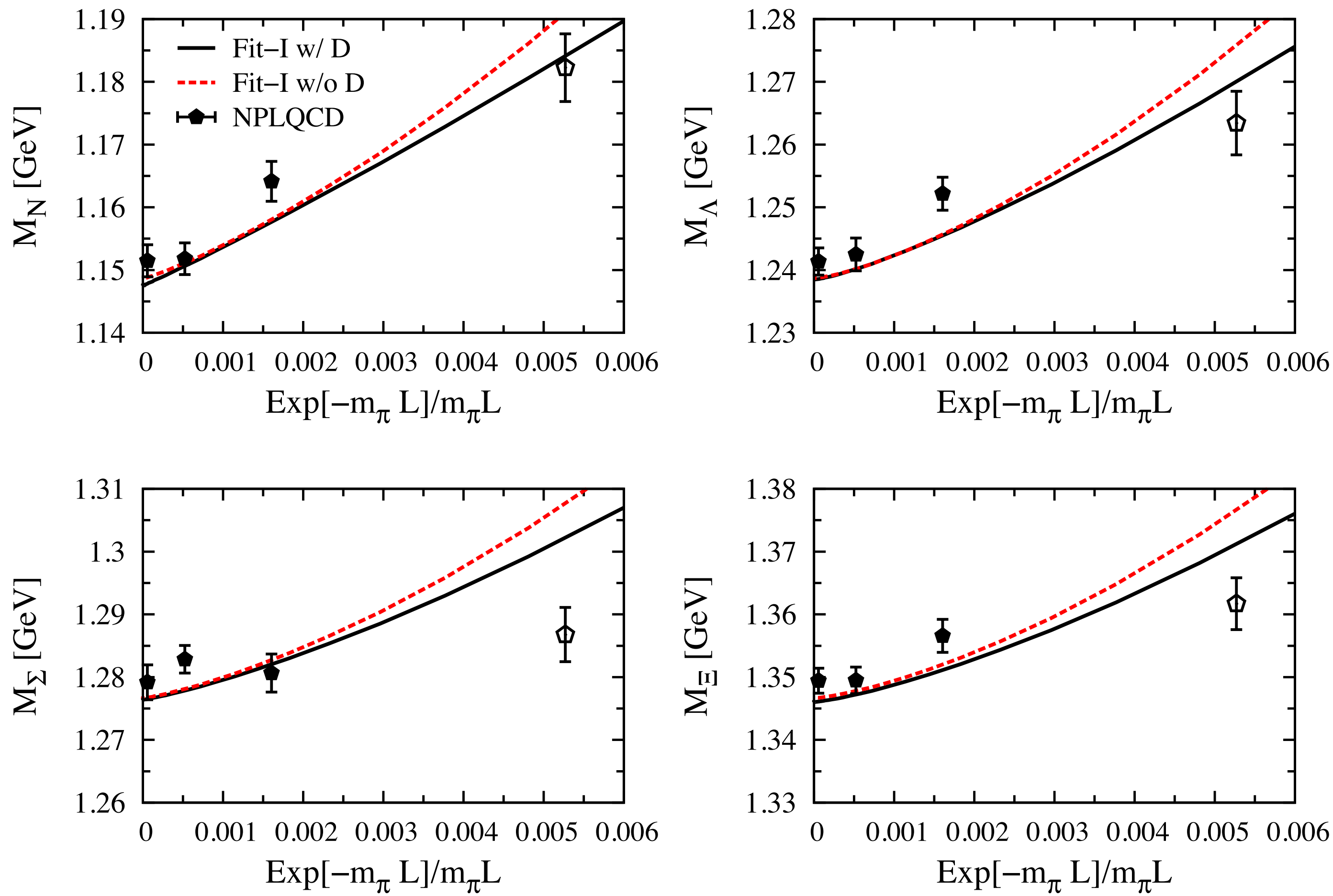
fixed from decay of a decuplet into an octet baryon and a pseudoscalar

  - mass corrections

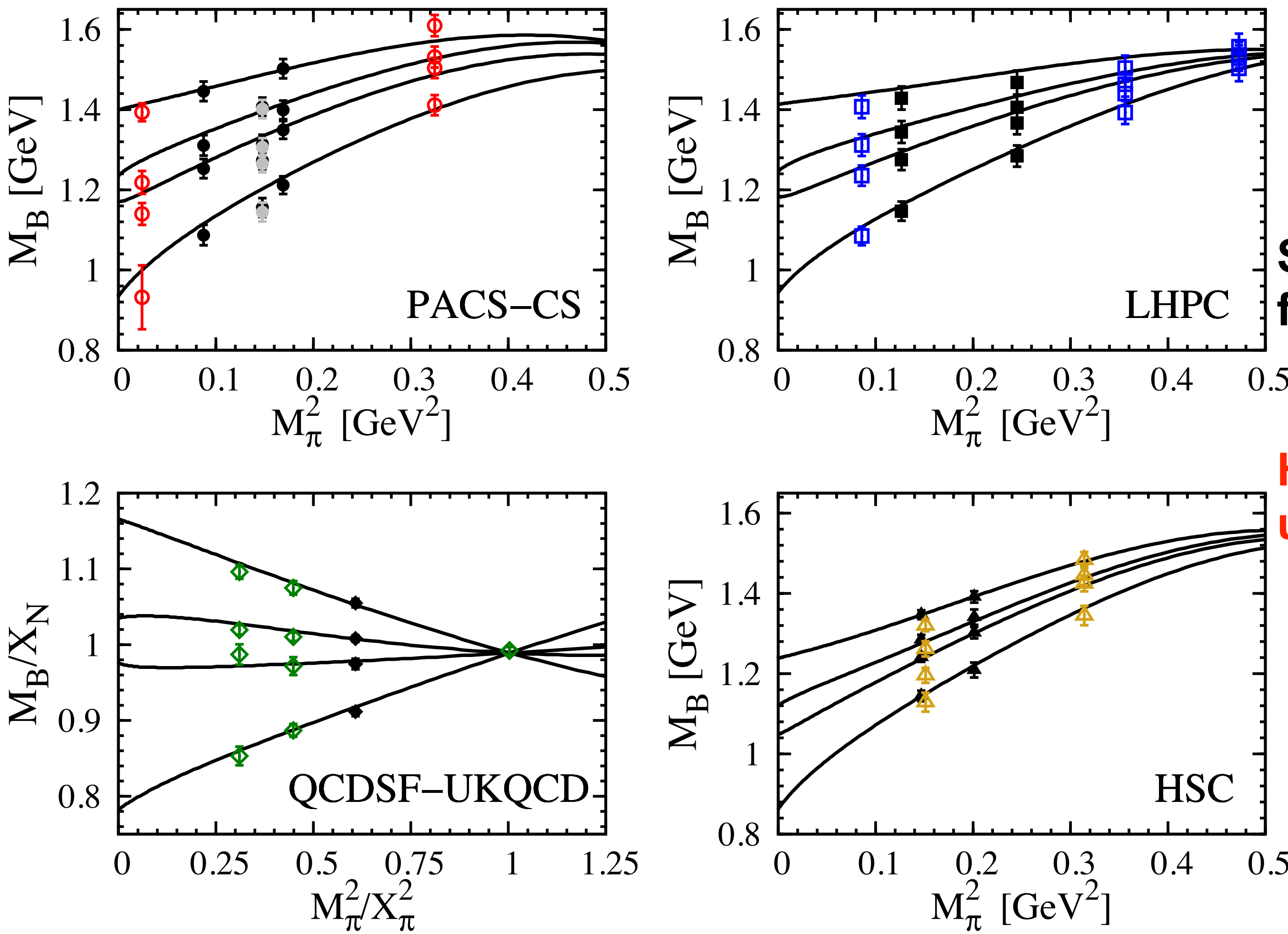
$$\mathcal{L}^{(2)}_T = \frac{t_0}{2}\bar{T}^{abc}_\mu g^{\mu\nu}T^{abc}_\nu\langle\chi_+\rangle + \frac{t_D}{2}\bar{T}^{abc}_\mu g^{\mu\nu}(\chi_+, T_\nu)^{abc},$$

fixed from the experimental decuplet masses

Slightly better description of the volume dependence of the NPLQCD data



Unfitted data can also reasonably well described



Solid black: fitted

Hollow colored: unfitted

Baryon Pion and Strangeness Sigma terms

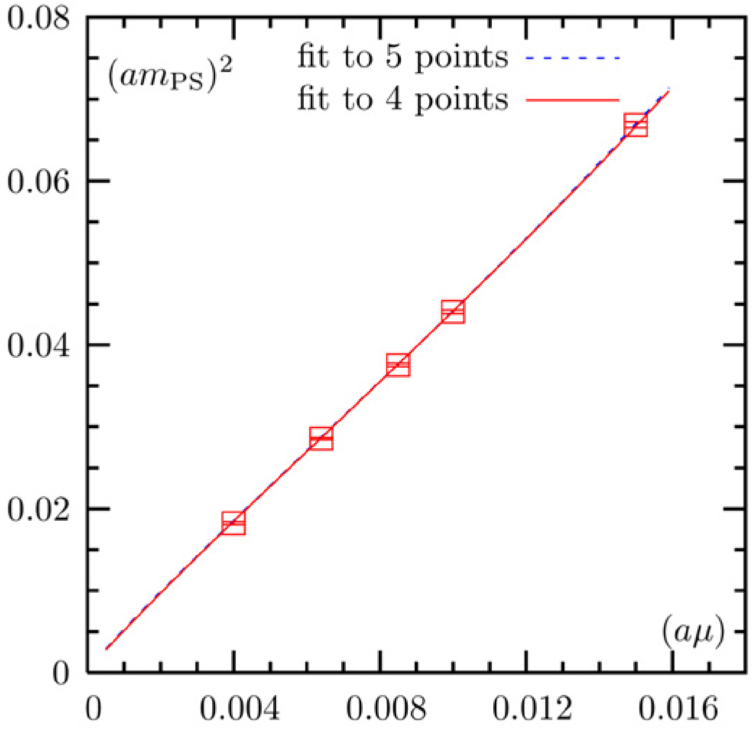
- **Feynman-Hellmann theorem states**

$$\sigma_{\pi B} = m_l \langle B(p)|\bar{u}u + \bar{d}d|B(p)\rangle = m_l \frac{\partial M_B}{\partial m_l}$$
$$\sigma_{sB} = m_s \langle B(p)|\bar{s}s|B(p)\rangle = m_s \frac{\partial M_B}{\partial m_s}.$$

- **Using leading-order ChPT meson masses**

$$\sigma \pi B = \frac{m_\pi^2}{2}\left(\frac{1}{m_\pi}\frac{\partial}{\partial m_\pi} + \frac{1}{2m_K}\frac{\partial}{\partial m_K} + \frac{1}{3m_\eta}\frac{\partial}{\partial m_\eta}\right) m_B$$
$$\sigma_s = \left(m_K^2 - \frac{m_\pi^2}{2}\right)\left(\frac{1}{2m_K}\frac{\partial}{\partial m_K} + \frac{2}{3m_\eta}\frac{\partial}{\partial m_\eta}\right) m_B$$

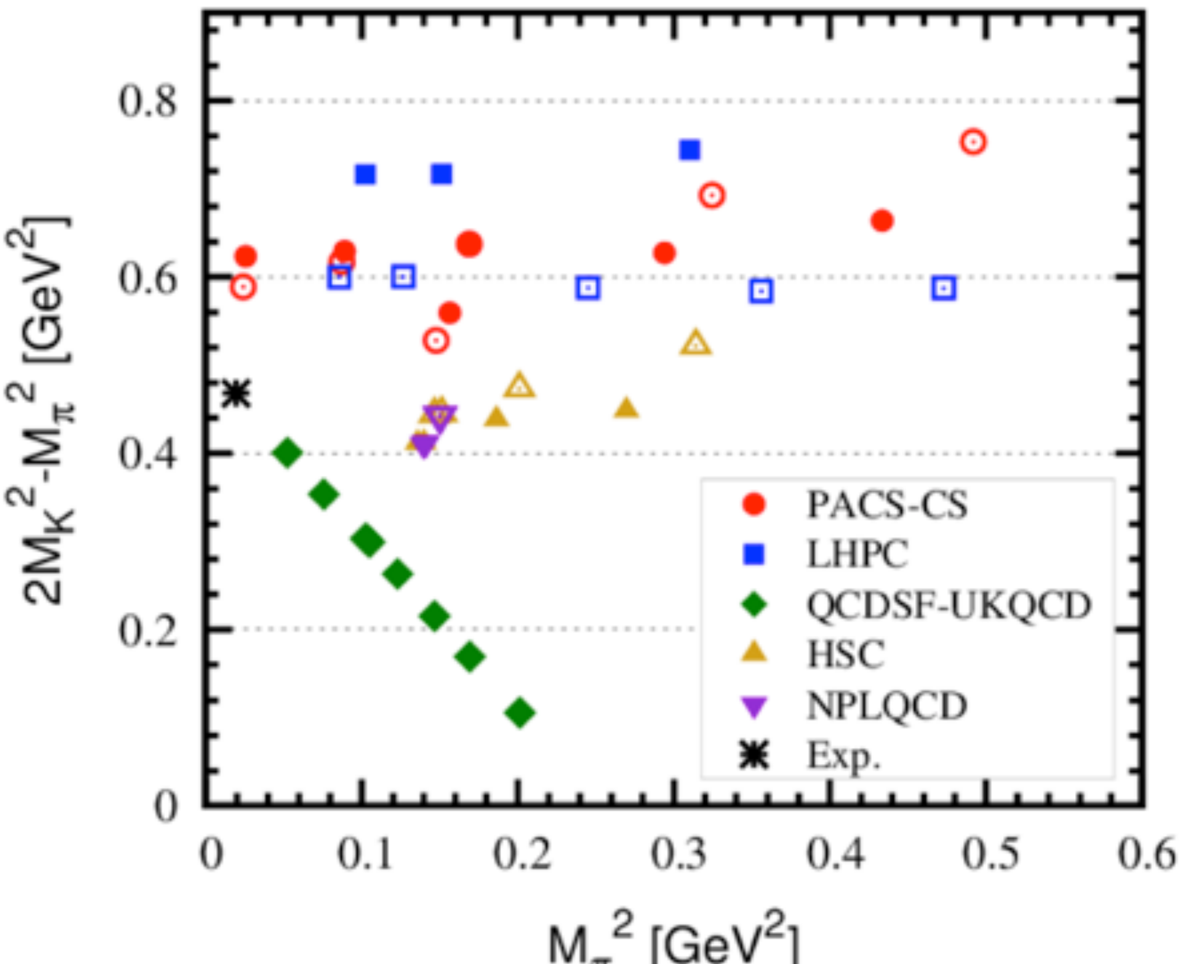
Pion mass vs. light quark mass



ETM collaboration, hep-lat/0701012

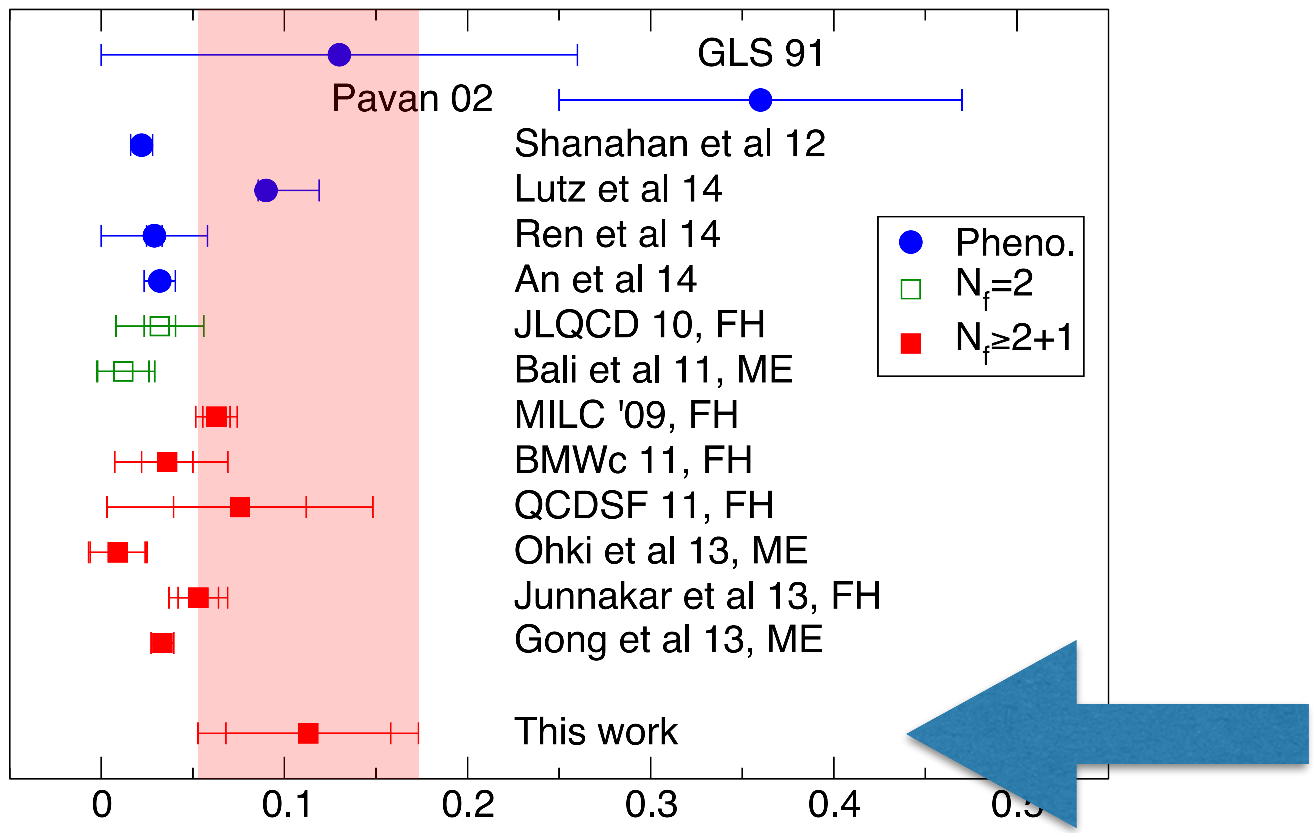
$$m_\pi^2 \propto m_q$$

Scale-setting effects on the octet baryon masses



- **Full symbols:** scale dependent
- **Hollow symbols:** scale independent

latest result from BWM collaboration



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