Puzzles in low-energy QCD

H. Leutwyler
University of Bern

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Pions

- Many of the quantities of interest at the precision frontier of particle physics require a good understanding of the strong interaction at low energies.
- In this context, the lightest hadrons are the most important

$$\pi^+ \quad \pi^0 \quad \pi^-$$

- It is essential that we know why the pions are so light. This understanding relies on symmetry.

Isospin symmetry

- Heisenberg 1932: strong interaction is invariant under isospin rotations – this is why $M_p \simeq M_n$.

$\Rightarrow$ Mass difference must be due to the e.m. interaction.

- Puzzle: e.m. field around the proton is stronger, makes the proton heavier than the neutron.
- Numerous unsuccessful attempts at solving this puzzle.
- If QCD describes the strong interaction correctly, then m_u must be very different from m_d. Gasser & L. 1975

  $m_u/m_d \simeq 0.67$, $m_s/m_d \simeq 22.5$ first crude estimate

Isospin symmetry

- $m_u/m_d \simeq 0.67$, $m_s/m_d \simeq 22.5$ first crude estimate 1975
- Masses of the pseudoscalar mesons confirm the picture:
  $M_{K^+} < M_{K^0}$ also requires a contribution due to $m_u < m_d$
  that is larger than the e.m. self-energy difference
  $m_u/m_d \simeq 0.56$, $m_s/m_d \simeq 20.1$ Weinberg 1977
- Current lattice estimates
  $m_u/m_d = 0.46 \pm 0.03$, $m_s/m_d = 20.0 \pm 0.4$
  FLAG, arXiv:1607.00299

Chiral symmetry

- Since m_u is very different from m_d: how come that isospin is a nearly perfect symmetry of the strong interaction ?
- QCD explains this very neatly: for yet unknown reasons, it so happens that m_u and m_d are very small.
- If m_u and m_d are set equal to zero $\Rightarrow$ QCD becomes invariant under independent flavour rotations of the right- and left-handed u, d-fields.
- Symmetry group: $\mathrm{SU(2)_R \times SU(2)_L}$
- This symmetry was discovered before QCD: Nambu 1960.
    - strong interaction has an approximate chiral symmetry
    - chiral symmetry is hidden, spontaneously broken
    - spontaneous symmetry breakdown generates massless bosons
    - the pions are the massless bosons of chiral symmetry
    - are not exactly massless, because the symmetry is not exact

Mass of the pion

- For $m_u = m_d = 0$ the pions are massless (Nambu-Goldstone bosons of an exact, spontaneously broken symmetry).
- For small values of m_u, m_d: M_π^2 is proportional to $m_u + m_d$:

$$M_\pi^2 = (m_u \underset{\substack{\Uparrow \\ \text{explicit}}}{+} m_d) \times |\langle 0|\, \underset{\substack{\Uparrow \\ \text{spontaneous}}}{\bar{u}u}\, |0\rangle| \times \frac{1}{F_\pi^2}$$

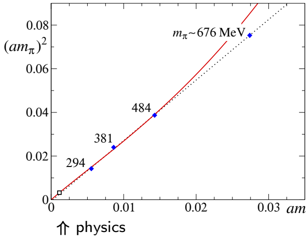
symmetry breaking

Gell-Mann, Oakes, Renner 1968

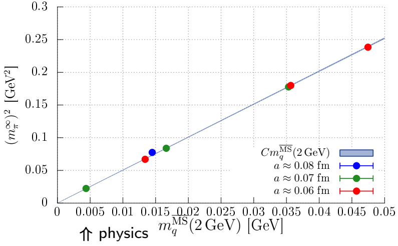
- Only $m_u + m_d$ counts.
- F_π is known from $\pi^+ \to \mu^+\nu$, but $|\langle 0|\, \bar{u}u\, |0\rangle| = ?$
  Non-perturbative method required to calculate $|\langle 0|\, \bar{u}u\, |0\rangle|$.

Lattice results for M_π

- GMOR formula is beautifully confirmed on the lattice: can determine M_π as a function of $m_u = m_d = m$.



Lüscher Lattice conference 2005



RQCD collaboration, arXiv:1603.00827

- Proportionality of M_π^2 to m_{ud} holds out to about $m_{ud} \simeq 10\times$ physical value of $\frac{1}{2}(m_u + m_d)$. Dürr, arXiv:1412.6434

Corrections to the GMOR relation

- Switch the electroweak interactions off, consider pure QCD.
  $M_\pi = M_\pi(\Lambda_{\text{QCD}}, m_u, m_d, m_s, m_c, m_b, m_t)$
- Chiral expansion, chiral perturbation theory (χPT):
  expand M_π in powers of m_u, m_d.
  The formula of GMOR gives the leading term:
  $$M^2 \equiv (m_u + m_d)B \qquad B = \lim_{m_u, m_d \to 0} \frac{|\langle 0|\, \bar{u}u\, |0\rangle|}{F_\pi^2}$$
  B is independent of m_u, m_d.
- χPT shows that the next term in the expansion is given by
  $$M_\pi^2 = M^2 \left\{ 1 - \frac{M^2}{2(4\pi F_\pi)^2} \bar{\ell}_3 + O(M^4) \right\}$$
  $$\bar{\ell}_3 = \ln \frac{\Lambda_3^2}{M^2} \quad \text{depends logarithmically on } M$$

Corrections to the GMOR relation

$$M_\pi^2 = M^2\left\{1 - \frac{M^2}{2(4\pi F_\pi)^2}\bar{\ell}_3 + O(M^4)\right\} \qquad \bar{\ell}_3 = \ln\frac{\Lambda_3^2}{M^2}$$

- Chiral symmetry does not determine the scale Λ_3.
  Lattice calculations reduced the uncertainty very significantly.
  Review of Bijnens and Ecker: arXiv:1405.6488
  $\bar{\ell}_3 = 3.0 \pm 0.8 \leftrightarrow \Lambda_3 \simeq 600$ MeV.

$\Rightarrow$ Correction in M_π is tiny: $\dfrac{M_\pi^2}{2(4\pi F_\pi)^2}\bar{\ell}_3 \simeq 0.024$

- Not a surprize: m_u, m_d are small, of the order of a few MeV.
  SU(2)×SU(2) should be a nearly perfect symmetry !

Why is the strong interaction nearly isospin invariant ?

- m_u, m_d small $\Rightarrow$ SU(2)×SU(2) a nearly perfect symmetry.
- Isospin is a subgroup of SU(2)×SU(2).

$\Rightarrow$ Isospin is a nearly perfect symmetry.

The strong interaction is nearly invariant under isospin rotations because m_u, m_d are small.

- But: the fact that SU(2)×SU(2) symmetry is broken is clearly seen: $M_\pi \neq 0$
  Why is the breaking of isospin symmetry so well hidden ?
  Why is M_{π^0} nearly equal to M_{π^+} ?
- The Nambu-Goldstone bosons are shielded from isospin breaking: leading term in $\mathcal{L}_{eff}$ only knows about $m_u + m_d$.

$\Rightarrow$ Expansion of $M^2_{\pi^+} - M^2_{\pi^0}$ in powers of m_u, m_d does not contain a term $\propto m_u - m_d$. Leading contribution is of order $(m_u - m_d)^2 \Rightarrow$ in QCD, $M_{\pi^+} - M_{\pi^0}$ is tiny.

Mass of the kaon

- Kaons are not protected from isospin breaking,
  are also NG bosons, become massless if m_s is sent to zero
- π^+: $u\bar{d}$ K^+: $u\bar{s}$ K^0: $d\bar{s}$
  Leading terms in the expansion in powers of m_u, m_d, m_s:
  $M^2_{\pi^+} = (m_u + m_d)B$
  $M^2_{K^+} = (m_u + m_s)B$
  $M^2_{K^0} = (m_d + m_s)B \quad \Rightarrow M^2_{K^+} - M^2_{K^0} = (m_u - m_d)B$
- B drops out in the ratios

$\Rightarrow \dfrac{M^2_{K^+}}{M^2_{\pi^+}} = \dfrac{m_u + m_s}{m_u + m_d}$ up to higher order contributions

- Masses of the NG bosons are very sensitive to m_u, m_d, m_s
- m_u, m_d, m_s break chiral symmetry

$\Rightarrow$ Explains why laws of nature contain approximate symmetries.

- M_π, M_K measure the strength of chiral symmetry breaking.

Convergence of the chiral perturbation series ?

- $M_K \gg M_\pi$

$\Rightarrow$ m_s is much larger than m_u, m_d.

$\Rightarrow$ SU(3)×SU(3) broken more strongly than SU(2)×SU(2).

$\Rightarrow$ Expansion in m_s converges more slowly.

- Chiral expansion is not an ordinary Taylor series:
  Leading order: Nambu-Goldstone bosons are massless.

$\Rightarrow$ Infrared singularities at next-to-leading order and beyond.
Strength of singularities determined by leading terms in $\mathcal{L}_{eff}$.

- Typical size of the corrections from higher orders:
  unless the expansion contains strong infrared singularities
  - SU(2)×SU(2), isospin: a few percent
  - SU(3)×SU(3), eightfold way: 20 percent

- ∃ quantities where the higher order contributions exceed the typical size, but in all cases I know, the reason is well-understood: infrared singularities with large coefficients.

Isospin limit

- If u and d are given the same mass m_{ud} and $e = 0$, there are three degenerate isospin multiplets: M_π, M_K, M_η
- At leading order of the chiral expansion, the masses obey the Gell-Mann-Okubo formula $\Rightarrow$ M_η determined by M_K, M_π
  $M_\eta^2 = \frac{1}{3}(4M_K^2 - M_\pi^2)$
  predicted: $M_\eta = 566$ MeV, observed $M_\eta = 548$ MeV

$\Rightarrow$ Correction amounts to 3%, surprisingly small.

- The relative size of M_K and M_π is determined by the relative size of m_{ud} and m_s

$$\frac{M_K^2}{M_\pi^2} = \frac{m_{ud} + m_s}{2m_{ud}}$$

Isospin limit

$$\frac{M_K^2}{M_\pi^2} = \frac{m_{ud} + m_s}{2m_{ud}}$$ valid at LO

- How large are the contributions from the higher orders ?
  Denote these by Δ_M:
  $$\frac{M_K^2}{M_\pi^2} = \frac{m_{ud} + m_s}{2m_{ud}}(1 + \Delta_M)$$
- Lattice result for quark masses: $m_s/m_{ud} = 27.3(3)$ FLAG
  Δ_M = -0.05(1)

$\Rightarrow$ Corrections are remarkably small also here.

- More typical case: $\frac{F_K}{F_\pi} = 1 + \Delta_F$ $\quad \Delta_F = 0.193(3)$ FLAG

Effects from $m_u \neq m_d$

Two sources of isospin breaking: $m_u \neq m_d$ and e^2.
First discuss the symmetry breaking due to $m_u \neq m_d$

- As mentioned already, the vacuum shields the pions from isospin breaking within QCD.
- For the kaons, there is a low-energy theorem Gasser & L. 1985

$$\left.\frac{M_{K^0}^2 - M_{K^+}^2}{M_K^2 - M_\pi^2} \cdot \frac{M_\pi^2}{M_K^2}\right|_{\text{QCD}} = \frac{m_d^2 - m_u^2}{m_s^2 - m_{ud}^2}(1 + \delta_M)$$

  Similar to the one for M_K^2/M_π^2, but there is a difference: δ_M is of NNLO, hence expected to be very small.
- For small quantities like δ_M, details matter.
  Identify M_π^2, M_K^2 with the mean squared masses of the two multiplets and evaluate the e.m. self-energies of the neutral particles with the numbers quoted by FLAG.

Low-energy theorem for $M^2_{K^0} - M^2_{K^+}$

- Recent lattice results:

	BMW	MILC
m_u/m_d	0.485(20)	0.455(13)
m_s/m_{ud}	27.53(22)	27.36(10)
δ_M	0.08(7)	-0.01(5)

Results agree within about 1 σ, are consistent with $\delta_M = 0$.

$\Rightarrow$ Low energy theorem is confirmed.

e.m. self-energies

- The e.m. self-energy of the pion obeys a low-energy theorem which neatly explains the magnitude of $M_{\pi^+} - M_{\pi^0}$.

  Das, Guralnik, Low, Mathur & Young 1967

- This theorem does not rely on the expansion in powers of m_s

$\Rightarrow$ Holds up to corrections of order $e^2 M_\pi^2$.

- Dashen theorem: at LO of the expansion in m_u, m_d, m_s: $M^2_{K^+}$ gets the same contribution from the e.m. interaction as $M^2_{\pi^+}$, while $M^2_{\pi^0}, M^2_{K^0}, M^2_{\bar{K}^0}, M^2_\eta$, do not get anything at all.

$\Rightarrow$ $(M^2_{K^+} - M^2_{K^0})_{\text{QED}} = (M^2_{\pi^+} - M^2_{\pi^0})_{\text{QED}}$

- Dashen theorem only holds at leading order of χPT, denote the corrections of $O(m_u, m_d, m_s)$ by ϵ.

  $(M^2_{K^+} - M^2_{K^0})_{\text{QED}} = (M^2_{\pi^+} - M^2_{\pi^0})_{\text{QED}} \times (1 + \epsilon)$

  How large are the higher order contributions in this case ?

Dashen theorem

$$(M_{K^+}^2 - M_{K^0}^2)_{\text{QED}} = (M_{\pi^+}^2 - M_{\pi^0}^2)_{\text{QED}} \times (1+\epsilon)$$

- Oven fresh lattice determinations:
  $\epsilon = 0.73(18)$ BMW arXiv:1604.07112
  $\epsilon = 0.73(14)$ MILC arXiv:1606.01228

$\Rightarrow$ In the self-energies, the higher order effects are very large.

- Why is that ? Does the semi-quantitative rule fail here ?
  Explanation was given long ago: Langacker and Pagels 1973

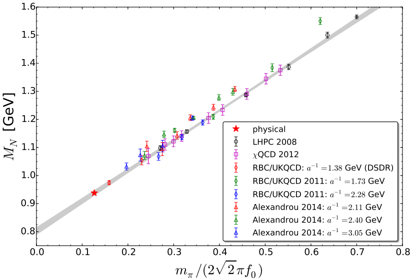
The self-energies contain very strong IR singularities at NLO. Contributions from these are as large as the LO term.

Mass of the nucleon

- M_N, M_π are determined by $\Lambda_{\text{QCD}}, m_u, m_d, \ldots, m_t$.
- Set m_u and m_d equal, common mass m_{ud}.
  Vary m_{ud}, keeping all other parameters fixed.

$\Rightarrow$ Values of M_N, M_π only depend on m_{ud}.
Conversely, m_{ud} is determined by M_π.

$\Rightarrow$ Value of M_N determined by value of M_π.



'Ruler plot' of
André Walker-Loud
I thank Claude Bernard
for providing this update
(see PoS(CD15)004)

- Lattice results shown are roughly on a straight line:

$$M_N = M_0 + c\,M_\pi$$

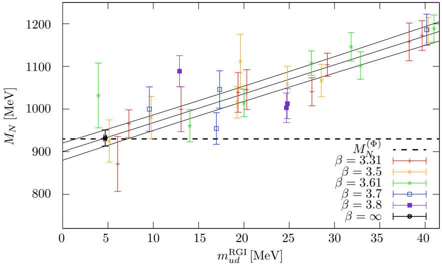
Mass of the nucleon

- Lattice results shown are roughly on a straight line:
  $M_N = M_0 + c\,M_\pi$
- In QCD, the Taylor series starts with
  $M_N = M_0 + c_1 M_\pi^2 + c_2 M_\pi^3 + c_3 M_\pi^4 \ell n(c_4 M_\pi) + O(M_\pi^5)$
  A term proportional to M_π does not occur.
  $M_\pi^2 \propto m_{ud} \Rightarrow M_\pi \propto \sqrt{m_{ud}}$

$\Rightarrow$ ruler fit is puzzling.

Mass of the nucleon

- New data from BMW



I thank Stephan Dürr for this plot (see arXiv:1510.08013)

- In the range shown, the data are consistent with

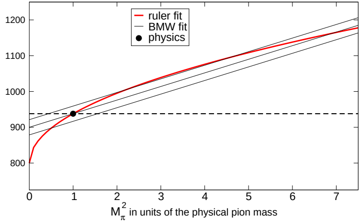
$$M_N = M_0 + k_1\, m_{ud}$$
$$M_\pi^2 = k_2\, m_{ud}$$

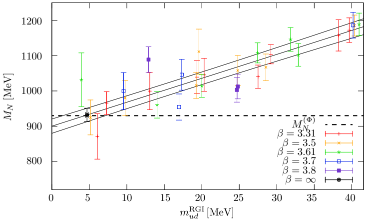
$\Rightarrow$ BMW data are well described by

$$M_N = M_0 + c\, M_\pi^2$$

Mass of the nucleon

- Comparison of ruler fit with BMW fit





$\Rightarrow$ No evidence for a term linear in M_π.

- Recent lattice results allow a determination of the σ-term matrix elements (focus on σ-term in isospin limit):

$$\sigma_{\pi N} = \frac{m_{ud}}{2M_N}\langle p|\bar{u}u + \bar{d}d|p\rangle$$

$$y = \frac{2\langle p|\bar{s}s|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle}$$

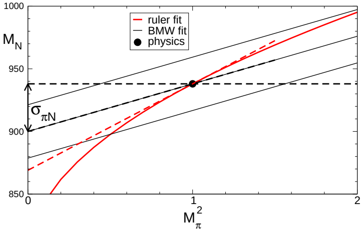
Significance of $\sigma_{\pi N}$

- Feynman-Hellman theorem: $\sigma_{\pi N} = m_{ud} \left.\frac{\partial M_N}{\partial m_{ud}}\right|_{\text{at physical } m_{ud}}$
- Since the physical value of m_{ud} is small, it is in the region where $M_\pi^2 = k_2\, m_{ud}$ holds to high accuracy.

$\Rightarrow$ $\sigma_{\pi N} = M_\pi^2 \left.\frac{\partial M_N}{\partial M_\pi^2}\right|_{\text{at physical } M_\pi}$

$\Rightarrow$ In the plot of M_N versus M_π^2, the σ-term measures the slope at the physical point.

$\Rightarrow$ $\sigma_{\pi N} \simeq M_N - M_0$



BMW: $\sigma_{\pi N} \simeq 38$ MeV

ruler fit: $\sigma_{\pi N} \simeq 70$ MeV

Significance of y

- y measures the size of $\langle p|\bar{s}s|p\rangle$.
  Violates the Okubo-Zweig-Iizuka-rule, vanishes for $N_c \to \infty$.
- y is relevant for matrix element of the octet operator:
  $$\sigma_0 = \frac{m_{ud}}{2M_N}\langle p|\bar{u}u + \bar{d}d - 2\,\bar{s}s|p\rangle = \sigma_{\pi N}\,(1-y)$$

Recent lattice results for $\sigma_{\pi N}$ and y

	$\sigma_{\pi N}$ (MeV)	y	archiv
BMW	38(3)(3)	0.20(8)(8)	1510.08013
χQCD	44.4(3.2)(5.5)	0.058(6)(8)	1511.09089
ETM	$37.22(2.57)(^{+0.99}_{-0.63})$	0.075(16)	1601.01624
RQCD	35(6)	0.104(51)	1603.00827
blind average	38.2(2.0)	0.064(8)	

- Two independent methods are used:
  - Feynman-Hellman-theorem.
  - Direct determination of the σ-term matrix elements.
- The results are consistent with one another.

$\Rightarrow$ Data indicate a σ-term around 38 MeV and a small value of y.

- Blind average over the four lattice results yields $\sigma_0 = 35.7(1.9)$ MeV

$\Rightarrow$ σ_0 smaller than $\sigma_{\pi N} = 38.2(2.0)$, but only slightly.

Low-energy theorem for σ_0

- For $m_u = m_d = m_s$, SU(3) is an exact symmetry of QCD.

$\Rightarrow$ N, Σ, Λ, Ξ have the same mass.

- $m_s - m_{ud}$ removes the degeneracy, breaks SU(3) (disregard from isospin breaking, take $m_u = m_d$). Expand in powers of $m_s - m_{ud}$.
- $2M_N + 2M_\Xi = 3M_\Lambda + M_\Sigma$ Gell-Mann-Okubo-formula valid to $O(m_s - m_{ud})$. Works very well, also for the baryons.
- Mass splitting is determined by the matrix element

$$M_\Sigma + M_\Xi - 2M_N = \frac{m_s - m_{ud}}{2M_N} \langle p|\bar{u}u + \bar{d}d - 2\bar{s}s|p\rangle$$

$\Rightarrow$ This leads to a low-energy theorem for σ_0:

$$\sigma_0 = \frac{m_{ud}}{m_s - m_{ud}}(M_\Sigma + M_\Xi - 2M_N)\left\{1 + O(m_s - m_{ud})\right\}$$

Low-energy theorem for σ_0

- $$\sigma_0 = \frac{m_{ud}}{m_s - m_{ud}} (M_\Sigma + M_\Xi - 2M_N) \left\{1 + O(m_s - m_{ud})\right\}$$
  Numerically, the leading term amounts to $\sigma_0 \simeq 25$ MeV.
- The NLO corrections were analyzed long ago.
  They do contain juicy infrared singularities. These amplify the corrections, increasing the value of σ_0 by about 10 MeV:

  $\sigma_0 = 35 \pm 5$ MeV   Gasser 1981

$\Rightarrow$ The quoted lattice results beautifully confirm this prediction.

Low-energy theorems for πN scattering

- At low energies, the most important contribution to the πN scattering amplitude is the Born term, which is proportional to the square of the coupling constant $g_{\pi N}$.
- Goldberger-Treiman relation predicts the value of $g_{\pi N}$:

$$g_{\pi N}^{GT} = \frac{M_N}{F_\pi} g_A$$

Goldberger & Treiman 1957

Low-energy theorem:

$$g_{\pi N} = g_{\pi N}^{GT}\{1 + O(m_{ud})\}$$

Experiment: $g_A = 1.2723(23)$, $F_\pi = 92.28(9)$ MeV

$\Rightarrow$ Prediction: $g_{\pi N}^{GT} = 12.95(3)$

Experiment: $g_{\pi N}^{exp} = 13.12(9)$ Hoferichter et al., arXiv:1510.06039

$\Rightarrow$ $g_{\pi N}^{exp}/g_{\pi N}^{GT} = 1.013(8)$ GT relation is obeyed very accurately.

Low-energy theorem for D^+

- The theorem concerns the isospin limit (QCD, $m_u = m_d$) and states that the leading term in the expansion of the isospin even πN scattering amplitude

  $\Sigma = F_\pi^2 \bar{D}^+\big|_{s=u,\, t=2M_\pi^2}$ ←'Cheng-Dashen point'

  in powers of m_{ud} is given by $\sigma_{\pi N}$.

⇒ If the common mass of the two lightest quarks is turned off, both Σ and $\sigma_{\pi N}$ tend to 0 and the ratio $\Sigma / \sigma_{\pi N}$ tends to 1.

- Relying on the dispersive analysis of Höhler et al. (Karlsruhe-Helsinki collaboration), we obtained $\sigma_{\pi N} = 45$ MeV — Gasser, L. & Sainio 1991
- This was compatible with $\sigma_0 = \sigma_{\pi N}(1 - y) = 35(5)$ MeV, provided a modest violation of the OZI-rule was allowed for: $y = 0.2$ — Gasser, L. & Sainio 1991
- The picture thus looked coherent, but the πN data showed serious inconsistencies. For this reason we were not able to attach meaningful uncertainties to the above estimates.

Roy-Steiner equations

- In the meantime, the lattice results have confirmed the value of σ_0, but indicate that the violation of the OZI-rule is smaller $\Rightarrow$ lattice values for $\sigma_{\pi N}$ cluster below 45 MeV.
- There is very significant progress in the dispersive analysis.
  Hoferichter, de Elvira, Kubis & Meissner, 2015, 2016
- Solutions of the Roy-Steiner equations for the πN scattering amplitude are now available. The extension from $\pi\pi$ to πN is a highly nontrivial achievement, because not all three channels involve the same physics: while the s- and u-channels carry the quantum numbers of πN, the t-channel concerns the transition $\pi\pi \leftrightarrow N\bar{N}$.
- Spin is a nontrivial complication: 4 amplitudes are needed. For $\pi\pi$ scattering a single amplitude suffices.

σ-term puzzle

- Outcome of the Roy-Steiner analysis:
  $\sigma_{\pi N} = 59.1(3.5)$ MeV Hoferichter et al., arXiv:1506.04142
- I find this result very puzzling because of two prejudices:
  1. SU(3) is a decent approximate symmetry, also for the matrix elements of the operator $\bar{q}\lambda^a q$ in the baryon octet.
  2. The rule of Okubo, Zweig and Iizuka is approximately valid.

  If $\sigma_{\pi N}$ is above 50 MeV $\Rightarrow$ at least one of these is wrong.
  The lattice results are consistent with both of them.
- Clash between two independent determinations of $\sigma_{\pi N}$:

Matrix elements of $\bar{q}q$	πN scattering
Lattice	Roy-Steiner
38 MeV	59 MeV

σ-term puzzle

- The clash is not new – many references deal with the subject.
  see for instance Pavan et al. 2002, Stahov et al. 2013, Matsinos & Rasche 2015
- New results accentuate the problem:
  Model dependence of the analysis is reduced.
  Uncertainty estimates have become small.
- Can the discrepancy be resolved with χPT ?

  Alarcon, Alvarez-Ruso, V. Bernard, de Elvira, Epelbaum, Gasparyan, Gegelia
  Geng, Hoferichter, Krebs, Kubis, Ledwig, Leinweber, Martin Camalich, Meißner,
  Meng, Oller, Ren, Shanahan, Siemens, Thomas, Vicente Vacas, Yao, Young
- A reliable lattice determination of the LECs relevant for the masses of the meson and baryon octets would be most welcome, but is not easy to achieve.
- The lattice results depend on extra parameters related to the regularization used. This may be the reason why the values of $\sigma_{\pi N}$ obtained by analyzing lattice data with χPT differ from those found by the collaborations responsible for the data.

  (recall comparison of ruler fit with BMW fit.)

σ-term puzzle

- πN analysis relies on data taken in the world as it is.
- Lattice calculations can be done in a much simpler framework: QCD with $m_u = m_d$.
  - For $\sigma_{\pi N}$, 4 flavours should yield a very accurate result.

  $\Rightarrow$ Theory can be specified in terms of M_N, M_π, M_K, M_D.
  - Isospin limit of M_π is a matter of convention (fixes m_{ud}).
  - In view of $\sigma_{\pi N} \propto M_\pi^2$, the value $\sigma_{\pi N} \simeq 38$ MeV for $M_\pi = M_{\pi^0}$ increases by about 2.6 MeV if M_π is identified with M_{π^+} (convention usually adopted in πN scattering)

small drop on a hot stone ...

Potential sources of error

1. πN data need to be corrected for isospin breaking effects.
2. CD point $\notin$ physical region, extrapolation needed.

1. Isospin breaking

- The σ-term is small, hides behind Born term and $\Delta(1232)$.
- Beautiful experiments on level-shift and line-width of πH and πD provide an excellent handle on the S-wave scattering lengths.
- Caveat: the numerical values of the scattering lengths $a^{\frac{1}{2}}, a^{\frac{3}{2}}$ quoted in some of the recent literature do not concern QCD with $m_u = m_d$, but merely represent auxiliary quantities: $a^{\frac{1}{2}}_{\not\pi}, a^{\frac{3}{2}}_{\not\pi}$.

$\Rightarrow$ The values obtained for $I = \frac{1}{2}, \frac{3}{2}$ would be of considerable interest !

- Experience from $\pi\pi$ scattering: if isospin breaking is neglected, some of the low-energy theorems for quantities that break chiral symmetry are in flat disagreement with experiment.

for a thorough discussion see Gasser, PoS EFT **09** (2009) 029

- More work needed to clarify the role of isospin breaking in determinations of $\sigma_{\pi N}$ from πN scattering

but I doubt that this can yield more than another small drop on the hot stone

Potential sources of error

2. Extrapolation

- Dispersion relations for $\bar{D}^+$ involve two subtraction constants. These can be identified with a_{0+}^+ (S-wave) and a_{1+}^+ (P-wave).

⇒ Low-energy theorem takes the form:

$$\sigma_{\pi N} = c_1\, a_{0+}^+ + c_2\, a_{1+}^+ + c_3 + O(m_{ud}^2)$$

Gasser, L., Locher, Sainio 1988

- c_1, c_2, c_3 can accurately be pinned down with dispersion relations for the S- and P-waves, using partial wave representations exclusively in the experimentally well explored region ($q > M_\pi$).

⇒ $\sigma_{\pi N}$ can be expressed in terms of measurable quantities.

- If isospin breaking is understood, a_{0+}^+ can accurately be calculated from level-shift and line-width of pionic atoms.
- Crucial remaining parameter for $\sigma_{\pi N}$: a_{1+}^+.
  Does represent an observable, but very high precision is required: 1% error in a_{1+}^+ affects the result for $\sigma_{\pi N}$ by more than 3 MeV.

  Threshold parameter of P-wave ⇒ not accessible in pionic atoms.

Potential sources of error

- In all recent determinations of the σ-term from data on πN scattering (including the Roy-Steiner analysis), the subtraction constant a^+_{1+} merely represents one of the many variables needed to parameterize the amplitude – these are simultaneously determined by minimizing discrepancies.
- New element in the RS-calculation: the t-channel singularities are analyzed by solving a Muskhelishvili-Omnès problem.
- RS-equations require fewer subtractions $\Rightarrow$ can calculate a^+_{1+}. But: with fewer subtractions, the high energy behaviour becomes more important. Can it be shown that the MO solution describes the physics at the necessary accuracy ?
- In my opinion, this represents the weakest part of the currently best determination of $\sigma_{\pi N}$ from πN scattering.

Conclusions

Mesons

- The quark masses are mysterious, but the mass spectrum of the lightest hadrons is well-understood in terms of these.
- Key point: m_u, m_d, m_s are small, can expand and retain only the first few terms, i.e. use χPT.
- χPT predictions for the dependence of M_π on m_{ud} confirmed.
- χPT predictions for the ratios $M_\pi : M_K : M_\eta$ confirmed.
- If isospin breaking is disregarded, the mass pattern of the lightest mesons is controlled by the quark mass ratio m_s/m_{ud}, which happens to be large.
- The mass difference between π^+ and π^0 is due almost exclusively to the e.m. interaction and is understood on the basis of a low-energy theorem that does not require an expansion in m_s.

Conclusions

- The mass difference between K^+ and K^0 is dominated by the contribution proportional to $m_u - m_d$.
- There is a low-energy theorem for this contribution, valid to NNLO of χPT. The lattice results confirm this prediction.
- The e.m. self-energy of the K^+ is small and strongly modified by non-leading orders of the expansion in powers of m_s. Their size is determined quite well on the lattice, but more work is needed to comprehend the numerical results.

Conclusions

Baryons

- Significant progress on the lattice.
  - The results are consistent with the chiral expansion.
  - In particular, the values obtained for σ_0 confirm the old estimate obtained from the expansion of the baryon masses.
  - Violations of the OZI-rule are found to be small.
- Significant progress in dispersive analysis of πN scattering.
  - New analysis of t-channel dispersion relations.
  - Outcome for $\sigma_{\pi N}$ is puzzling.
  - Disagrees with the lattice results and calls for exorbitant violations of SU(3)-symmetry in the matrix elements of $\bar{q}\lambda^a q$.

Open ends: πN

- There is a wealth of data on πN scattering.
- Comparison with Roy-Steiner analysis will be most interesting.

  Can the experimental inconsistencies be resolved ?

  In particular: $\pi^- p \to \pi^0 n$, $\pi^0 p \to \pi^+ n$?
- Are the basic theoretical constraints obeyed ?
  Goldberger-Treiman relation (ties $g_{\pi N}$ to g_A) $\surd$
  Adler-Weisberger sum rule (ties g_A to the total cross sections) ?
- Are the predictions for the contributions from the t-channel singularities consistent with experiment ?

Open ends: σ-term matrix elements

- Determine the matrix elements of $\bar{u}u$, $\bar{d}d$, $\bar{s}s$ for other members of the meson and baryon octets.
- 'Scalar charge' $g_S = \frac{1}{2m}\langle p|\bar{u}d|n\rangle \overset{\text{isospin}}{\Downarrow}{=} \frac{1}{2m}\langle p|\bar{u}u - \bar{d}d|p\rangle$

  Relevant for the mass difference between p and n in QCD.

  González-Alonso & Martin Camalich, arXiv:1309.4434

  Bhattacharya, Cirigliano et al., arXiv:1606.07049
- Any evidence for strong violations of SU(3) in scalar matrix elements ?

$\sigma_{\pi N}$ not the only puzzle worth thinking about ...

- Proton charge radius
- Standard Model prediction for magnetic moment of the muon

$$\vdots$$