

Chiral effective field theory for dark matter direct detection

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HC2NP

Hadronic Contributions to New Physics Searches

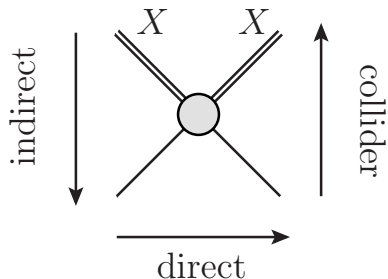
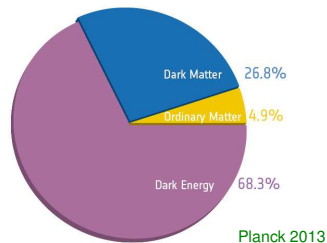
Puerto de la Cruz, September 25, 2016

PLB 746 (2015) 410, PRD 94 (2016) 063505 with P. Klos, J. Menéndez, A. Schwenk

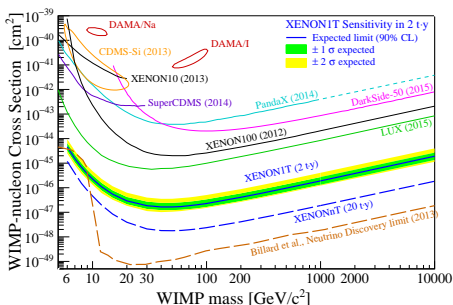
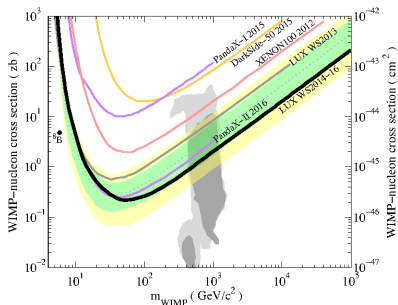
- 1 Direct detection of dark matter
- 2 Chiral effective field theory
- 3 Corrections beyond standard nuclear response
 - Subleading one-body responses
 - Two-body currents
 - Radius corrections
- 4 Conclusions

How to search for dark matter?

- Search strategies: direct, indirect, collider
- Assume DM particle is WIMP
- **Direct detection**: search for **WIMPs** scattering off nuclei in the large-scale detectors
- Ingredients for interpretation:
 - **DM halo**: velocity distribution
 - **Nucleon matrix elements**: WIMP–nucleon couplings
 - **Nuclear structure factors**: embedding into target nucleus



Direct detection of dark matter: schematics



- **Nuclear recoil** in WIMP–nucleus scattering

- **Flux factor** Φ : DM halo and velocity distribution
- **WIMP–nucleus cross section**

- **Spin-independent**: coherent $\propto A^2$

- **Spin-dependent**: $\propto \langle \mathbf{S}_p \rangle$ or $\langle \mathbf{S}_n \rangle$

- Information on BSM physics encoded in normalization at $q = 0$

↔ for SI case: $\sigma_{\chi N}^{\text{SI}}$

- **Rate**

$$\frac{dR}{dq^2} = \frac{\rho M}{m_A m_\chi} \int_{v_{\min}}^{v_{\text{esc}}} d^3v |\mathbf{v}| f(|\mathbf{v}|) \frac{d\sigma_{\chi\mathcal{N}}}{dq^2}$$

- **Halo-independent methods** Drees, Shan 2008, Fox, Liu, Weiner 2010, ...

- **Nuclear structure factors** Engel, Pittel, Vogel 1992

$$\frac{d\sigma_{\chi\mathcal{N}}}{dq^2} = \frac{8G_F^2}{(2J+1)v^2} [S_A(q) + S_S(q)]$$

- Normalization at $|\mathbf{q}| = 0$:

$$S_S(0) = \frac{2J+1}{4\pi} |c_0 A + c_1(Z-N)|^2$$

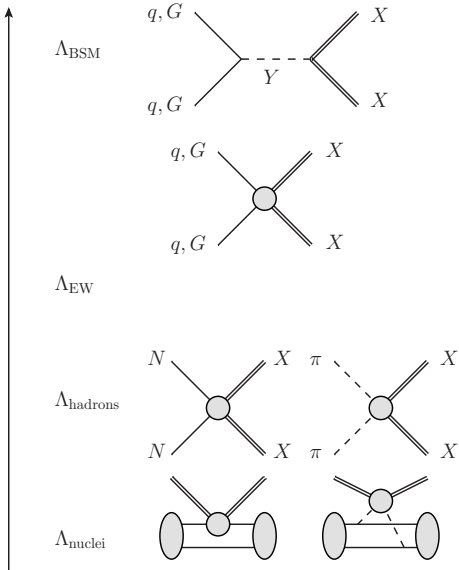
$$S_A(0) = \frac{(2J+1)(J+1)}{4\pi J} |(a_0 + a_1)\langle \mathbf{S}_p \rangle + (a_0 - a_1)\langle \mathbf{S}_n \rangle|^2$$

- Assume $c_1 = 0$ and SI scattering

$$\frac{d\sigma_{\chi\mathcal{N}}^{\text{SI}}}{dq^2} = \frac{\sigma_{\chi\mathcal{N}}^{\text{SI}}}{4v^2 \mu_N^2} \mathcal{F}_{\text{SI}}^2(\mathbf{q}^2)$$

↪ phenomenological **Helm form factor** $\mathcal{F}_{\text{SI}}^2(\mathbf{q}^2)$

Effective field theories for the direct detection of dark matter



1 **BSM scale** $\Lambda_{\text{BSM}}: \mathcal{L}_{\text{BSM}}$

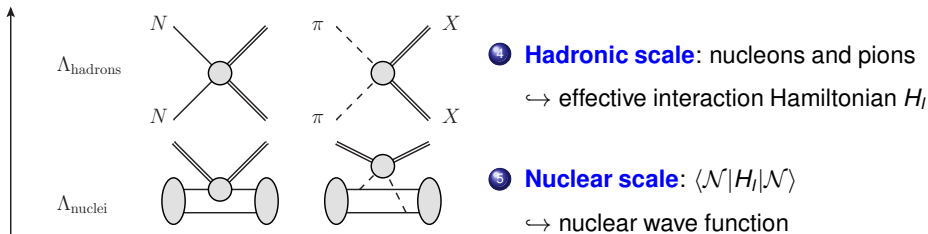
2 **Effective Operators:** $\mathcal{L}_{\text{SM}} + \sum_{i,k} \frac{1}{\Lambda_{\text{BSM}}^i} \mathcal{O}_{i,k}$

3 Integrate out **EW physics**

4 **Hadronic scale:** nucleons and pions
 \hookrightarrow effective interaction Hamiltonian H_I

5 **Nuclear scale:** $\langle \mathcal{N} | H_I | \mathcal{N} \rangle$
 \hookrightarrow nuclear wave function

Direct detection of dark matter: scales



- Typical WIMP–nucleon **momentum transfer**

$$|\mathbf{q}_{\text{max}}| = 2\mu_{\mathcal{N}\chi} |\mathbf{v}_{\text{rel}}| \sim 200 \text{ MeV} \quad |\mathbf{v}_{\text{rel}}| \sim 10^{-3} \quad \mu_{\mathcal{N}\chi} \sim 100 \text{ GeV}$$

- QCD constraints: spontaneous breaking of chiral symmetry

\Rightarrow **Chiral effective field theory for WIMP–nucleon scattering**

Prézeau et al. 2003, Cirigliano et al. 2012, 2013, Menéndez et al. 2012, Klos et al. 2013, MH et al. 2015

Chiral EFT: a modern approach to nuclear forces

- Traditionally: meson-exchange potentials
- Chiral effective field theory
 - Based on **chiral symmetry** of QCD
 - **Power counting**
 - **Low-energy constants**
 - Hierarchy of multi-nucleon forces
 - Consistency of NN and $3N$

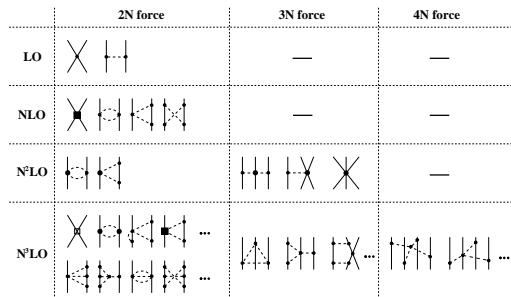
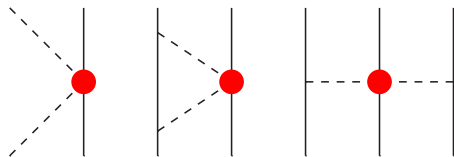


Figure taken from 1011.1343

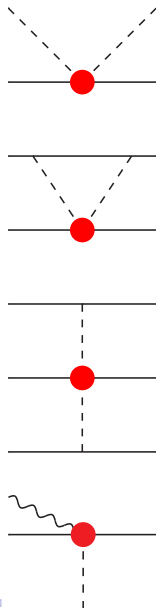
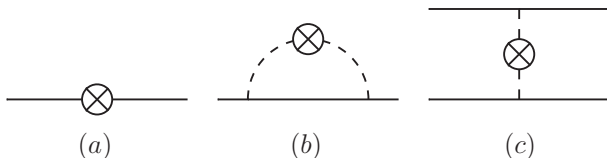
↔ modern theory of nuclear forces

- Long-range part related to **pion–nucleon scattering**

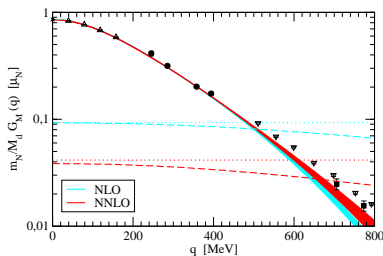
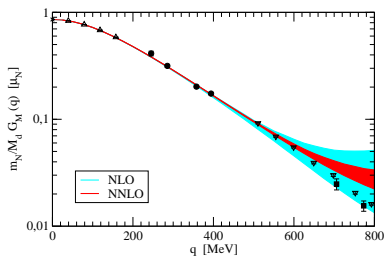


Chiral EFT: currents

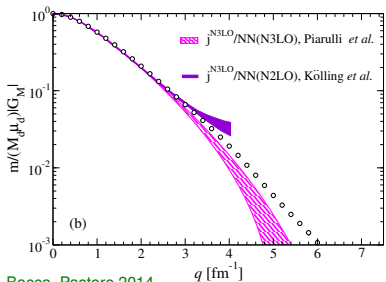
- Coupling to **external sources** $\mathcal{L}(v_\mu, a_\mu, s, p)$
- Same LECs appear in **axial current**
 - $\hookrightarrow \beta$ decay, neutrino interactions, dark matter
- Vast literature for v_μ and a_μ , up to one-loop level
 - With unitary transformations: Kölling et al. 2009, 2011, Krebs et al. to appear
 - Without unitary transformations: Pastore et al. 2008, Park et al. 2003, Baroni et al. 2015
- For **dark matter** further currents: s, p , tensor, spin-2, θ_μ^μ



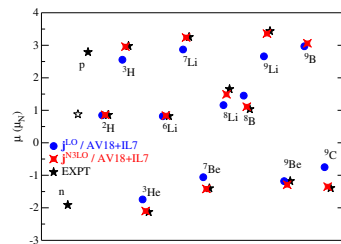
Vector current in chiral EFT: deuteron form factors, magnetic moments



Kölling, Epelbaum, Phillips 2012

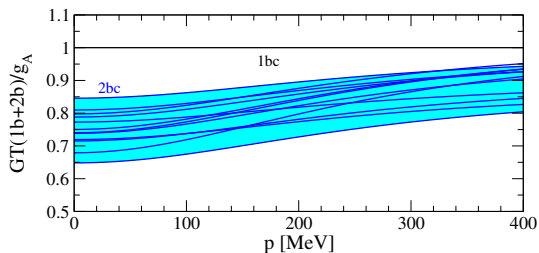


Bacca, Pastore 2014



Pastore *et al.* 2013

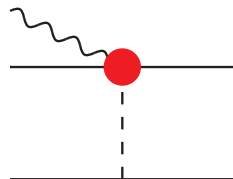
Axial-vector current in chiral EFT: ν -less double β decay

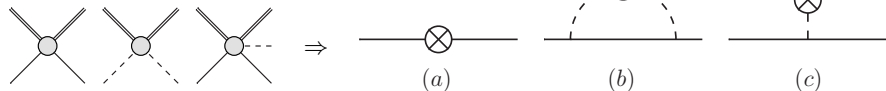


Menéndez, Gazit, Schwenk 2011

- Normal ordering over Fermi sea \Rightarrow effective one-body currents
- **Two-body currents** contribute to **quenching of g_A** in Gamov–Teller operator

$$g_A \sigma \tau^-$$





- Expansion around **chiral limit** of QCD
 - ↔ simultaneous expansion in momenta and quark masses
- Three classes of corrections:
 - **Subleading one-body responses** (a) [Fan et al. 2010](#), [Fitzpatrick et al. 2012](#), [Anand et al. 2013](#)
 - **Radius corrections** (b)
 - **Two-body currents** (c), (d)

- Starting point: **effective WIMP Lagrangian** Goodman et al. 2010

$$\begin{aligned} \mathcal{L}_\chi &= \frac{1}{\Lambda^3} \sum_q \left[C_q^{SS} \bar{\chi} \chi m_q \bar{q} q + C_q^{PS} \bar{\chi} i \gamma_5 \chi m_q \bar{q} q + C_q^{SP} \bar{\chi} \chi m_q \bar{q} i \gamma_5 q + C_q^{PP} \bar{\chi} i \gamma_5 \chi m_q \bar{q} i \gamma_5 q \right] \\ &+ \frac{1}{\Lambda^2} \sum_q \left[C_q^{VV} \bar{\chi} \gamma^\mu \chi \bar{q} \gamma_\mu q + C_q^{AV} \bar{\chi} \gamma^\mu \gamma_5 \chi \bar{q} \gamma_\mu q + C_q^{VA} \bar{\chi} \gamma^\mu \chi \bar{q} \gamma_\mu \gamma_5 q + C_q^{AA} \bar{\chi} \gamma^\mu \gamma_5 \chi \bar{q} \gamma_\mu \gamma_5 q \right] \\ &+ \frac{1}{\Lambda^3} \left[C_g^S \bar{\chi} \chi \alpha_s G_{\mu\nu}^a G_a^{\mu\nu} \right] \end{aligned}$$

- Chiral power counting**

$$\partial = \mathcal{O}(p), \quad m_q = \mathcal{O}(p^2) = \mathcal{O}(M_\pi^2), \quad a_\mu, v_\mu = \mathcal{O}(p), \quad \frac{\partial}{m_N} = \mathcal{O}(p^2)$$

↪ construction of effective Lagrangian for nucleon and pion fields

↪ organize in terms of **chiral order** ν , $\mathcal{M} = \mathcal{O}(p^\nu)$

Chiral counting: summary

	Nucleon	V		A	
WIMP		t	\mathbf{x}	t	\mathbf{x}
	1b	0	1 + 2	2	0 + 2
V	2b	4	2 + 2	2	4 + 2
	2b NLO	—	—	5	3 + 2
	1b	0 + 2	1	2 + 2	0
A	2b	4 + 2	2	2 + 2	4
	2b NLO	—	—	5 + 2	3

	Nucleon	S	P
WIMP			
	1b	2	1
S	2b	3	5
	2b NLO	—	4
	1b	2 + 2	1 + 2
P	2b	3 + 2	5 + 2
	2b NLO	—	4 + 2

- +2 from NR expansion of WIMP spinors, terms can be dropped if $m_\chi \gg m_N$
- **Red**: all terms up to $\nu = 3$
- Two-body currents: AA [Menéndez et al. 2012](#), [Klos et al. 2013](#), SS [Prézeau et al. 2003](#), [Cirigliano et al. 2012](#), but **new currents in AV and VA channel** [1503.04811](#)

Example: chiral counting in scalar channel

- Leading pion–nucleon Lagrangian

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi} \left[i\gamma_\mu (\partial^\mu - i\nu^\mu) - m_N + \frac{g_A}{2} \gamma_\mu \gamma_5 \left(2a^\mu - \frac{\partial^\mu \pi}{F_\pi} \right) + \dots \right] \Psi$$

↪ **no scalar source!**

<hr/>		
	Nucleon	S
WIMP		
	1b	2
S	2b	3

Example: chiral counting in scalar channel

- Leading pion–nucleon Lagrangian

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi} \left[i\gamma_\mu (\partial^\mu - i\nu^\mu) - m_N + \frac{g_A}{2} \gamma_\mu \gamma_5 \left(2a^\mu - \frac{\partial^\mu \pi}{F_\pi} \right) + \dots \right] \Psi$$

↪ **no scalar source!**

- Scalar coupling

$$f_N = \frac{m_N}{\Lambda^3} \sum_{q=u,d,s} C_q^{SS} f_q^N + \dots \quad \langle N | m_q \bar{q}q | N \rangle = f_q^N m_N$$

↪ for $q = u, d$ related to **pion–nucleon σ -term** $\sigma_{\pi N}$

- Chiral expansion

$$\sigma_{\pi N} = -4c_1 M_\pi^2 - \frac{9g_A^2 M_\pi^3}{64\pi F_\pi^2} + \mathcal{O}(M_\pi^4) \quad \dot{\sigma} = \frac{5g_A^2 M_\pi}{256\pi F_\pi^2} + \mathcal{O}(M_\pi^2)$$

↪ slow convergence

	Nucleon	S
WIMP		
	1b	2
S	2b	3

Matching to nonrelativistic EFT

- Operator basis in NREFT Fan et al. 2010, Fitzpatrick et al. 2012, Anand et al. 2013

$$\begin{aligned}\mathcal{O}_1 &= \mathbb{1} & \mathcal{O}_2 &= (\mathbf{v}^\perp)^2 & \mathcal{O}_3 &= i\mathbf{S}_N \cdot (\mathbf{q} \times \mathbf{v}^\perp) & \mathcal{O}_4 &= \mathbf{S}_X \cdot \mathbf{S}_N \\ \mathcal{O}_5 &= i\mathbf{S}_X \cdot (\mathbf{q} \times \mathbf{v}^\perp) & \mathcal{O}_6 &= \mathbf{S}_X \cdot \mathbf{q} \mathbf{S}_N \cdot \mathbf{q} & \mathcal{O}_7 &= \mathbf{S}_N \cdot \mathbf{v}^\perp & \mathcal{O}_8 &= \mathbf{S}_X \cdot \mathbf{v}^\perp \\ \mathcal{O}_9 &= i\mathbf{S}_X \cdot (\mathbf{S}_N \times \mathbf{q}) & \mathcal{O}_{10} &= i\mathbf{S}_N \cdot \mathbf{q} & \mathcal{O}_{11} &= i\mathbf{S}_X \cdot \mathbf{q}\end{aligned}$$

- Matching to chiral EFT (f_N, \dots : Wilson coefficients + nucleon form factors)

$$\begin{aligned}\mathcal{M}_{1,\text{NR}}^{SS} &= \mathcal{O}_1 f_N(t) & \mathcal{M}_{1,\text{NR}}^{SP} &= \mathcal{O}_{10} g_5^N(t) & \mathcal{M}_{1,\text{NR}}^{PP} &= \frac{1}{m_X} \mathcal{O}_6 h_5^N(t) \\ \mathcal{M}_{1,\text{NR}}^{VV} &= \mathcal{O}_1 \left(f_1^{V,N}(t) + \frac{t}{4m_N^2} f_2^{V,N}(t) \right) + \frac{1}{m_N} \mathcal{O}_3 f_2^{V,N}(t) + \frac{1}{m_N m_X} (t\mathcal{O}_4 + \mathcal{O}_6) f_2^{V,N}(t) \\ \mathcal{M}_{1,\text{NR}}^{AV} &= 2\mathcal{O}_8 f_1^{V,N}(t) + \frac{2}{m_N} \mathcal{O}_9 \left(f_1^{V,N}(t) + f_2^{V,N}(t) \right) \\ \mathcal{M}_{1,\text{NR}}^{AA} &= -4\mathcal{O}_4 g_A^N(t) + \frac{1}{m_N^2} \mathcal{O}_6 g_P^N(t) & \mathcal{M}_{1,\text{NR}}^{VA} &= \left\{ -2\mathcal{O}_7 + \frac{2}{m_X} \mathcal{O}_9 \right\} h_A^N(t)\end{aligned}$$

- Conclusions

- \mathcal{O}_2 , \mathcal{O}_5 , and \mathcal{O}_{11} do not appear at $\nu = 3$, not all \mathcal{O}_i independent
- 2b operators of similar or even greater importance than some of the 1b operators
- Next: **phenomenological implications**

- Six distinct nuclear responses

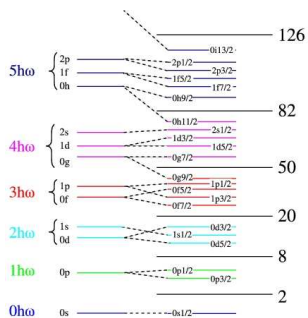
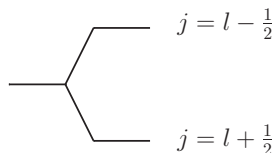
Fitzpatrick et al. 2012, Anand et al. 2013

- $M \leftrightarrow \mathcal{O}_1 \leftrightarrow SI$
- $\Sigma', \Sigma'' \leftrightarrow \mathcal{O}_4, \mathcal{O}_6 \leftrightarrow SD$
- $\Phi'' \leftrightarrow \mathcal{O}_3 \leftrightarrow$ quasi-coherent, spin-orbit operator
- $\Delta, \tilde{\Phi}'$: not coherent

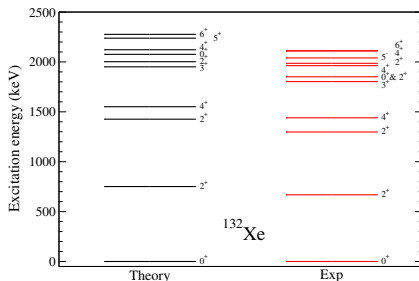
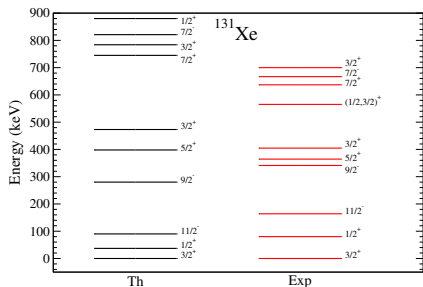
- **Quasi-coherence** of Φ''

- Spin-orbit splitting
- Coherence until mid-shell
- About 20 coherent nucleons in Xe
- Interference $M-\Phi'' \leftrightarrow \mathcal{O}_1-\mathcal{O}_3$

- Further coherent M -responses from $\mathcal{O}_5, \mathcal{O}_8, \mathcal{O}_{11}$, but no interference with \mathcal{O}_1 due to sum over \mathbf{S}_X

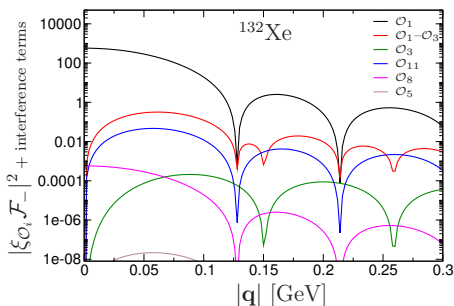
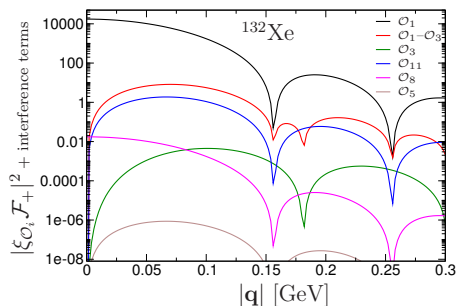


Spectra and shell-model calculation



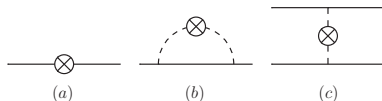
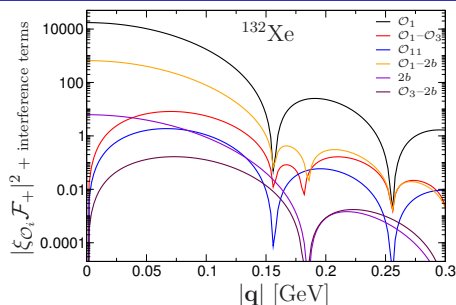
- **Shell-model diagonalization** for Xe isotopes with ^{100}Sn core
- **Uncertainty estimates**: currently phenomenological shell-model interaction
 ↪ chiral-EFT-based interactions in the future

Consequences for the structure factors



- $\xi_{\mathcal{O}_i}$ kinematic factors for \mathcal{O}_i , e.g. $\xi_{\mathcal{O}_1} = 1$, $\xi_{\mathcal{O}_3} = \frac{\mathbf{q}^2}{2m_N^2}$
- \mathcal{O}_{11} assumes $m_\chi = 2 \text{ GeV}$
 \hookrightarrow much stronger suppressed for heavy WIMPs
- Structure factors imply **hierarchy** as long as coefficients do not differ strongly

Two-body currents: SI case

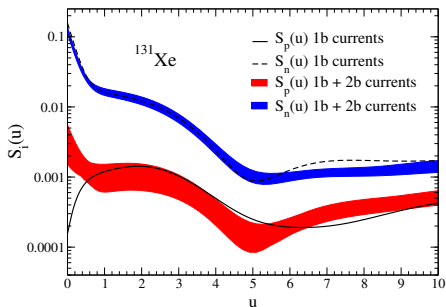
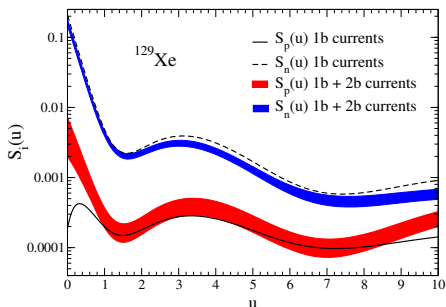


- Finite at $|\mathbf{q}| = 0$
- Most important next to IS and IV O_1
- Sensitive to **new combination of Wilson coefficients**, e.g. for scalar channel

$$f_N = \frac{m_N}{\Lambda^3} \left(\sum_{q=u,d,s} C_q^{SS} f_q^N - 12\pi f_Q^N C_9^{S'} \right) \quad f_\pi = \frac{M_\pi}{\Lambda^3} \sum_{q=u,d} \left(C_q^{SS} + \frac{8\pi}{9} C_9^{S'} \right) f_q^\pi \quad \dots$$

- Typically (5–10)% effect, enhanced whenever cancellations occur: **blind spots**, **heavy WIMP limit**

Two-body currents: SD case

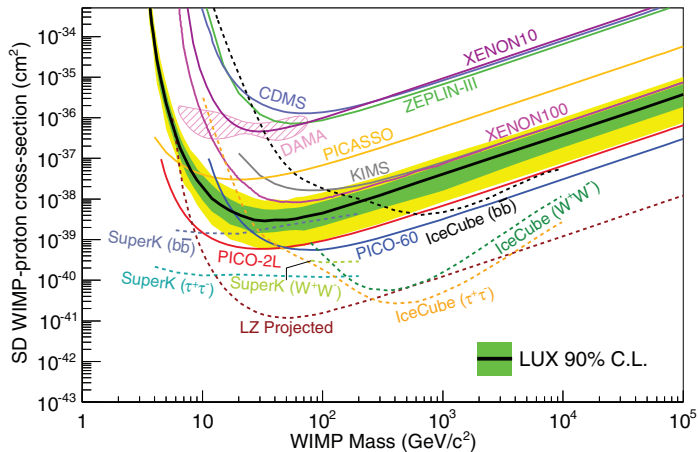


- Nuclear structure factors for **spin-dependent interactions**

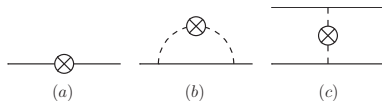
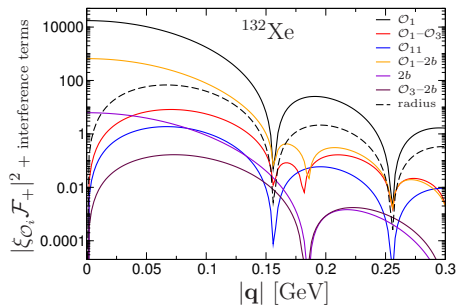
Klos et al. 2013

- Based on chiral EFT currents (1b+2b)
- Shell model
- $u = q^2 b^2 / 2$ related to momentum transfer
- 2b currents absorbed into redefinition of 1b current

Two-body currents: SD case



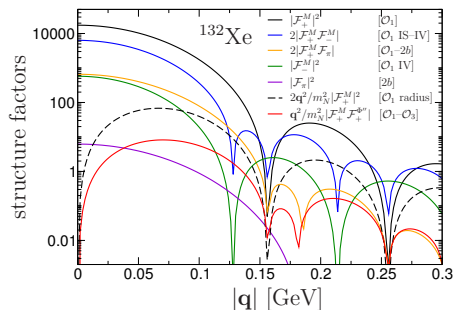
Radius corrections



- Set scale as \mathbf{q}^2/m_N^2
- Strong suppression at small $|\mathbf{q}|$, but potentially relevant later
- Yet another new combination

$$i_N = \frac{m_N}{\Lambda^3} \left(\sum_{q=u,d,s} C_q^{SS} i_q^N - 12\pi i_Q^N C_g^{S'} \right)$$

Full set of coherent contributions



- Parameterize cross section as

$$\frac{d\sigma_{\chi N}^{\text{SI}}}{dq^2} = \frac{1}{4\pi\mathbf{v}^2} \left| \left(c_+^M - \frac{q^2}{m_N^2} \dot{c}_+^M \right) \mathcal{F}_+^M(q^2) + \left(c_-^M - \frac{q^2}{m_N^2} \dot{c}_-^M \right) \mathcal{F}_-^M(q^2) + c_\pi \mathcal{F}_\pi(q^2) \right. \\ \left. + \frac{q^2}{2m_N^2} \left[c_+^{\Phi''} \mathcal{F}_+^{\Phi''}(q^2) + c_-^{\Phi''} \mathcal{F}_-^{\Phi''}(q^2) \right] \right|^2$$

- Single-nucleon cross section: $\sigma_{\chi N}^{\text{SI}} = \mu_N^2 |c_+^M|^2 / \pi$
- c related to Wilson coefficients and nucleon form factors

- Parameters ($\zeta = 1(2)$ for Dirac (Majorana)):

$$c_{\pm}^M = \frac{\zeta}{2} [f_p \pm f_n + f_1^{V,p} \pm f_1^{V,n}] \quad c_{\pi} = \zeta f_{\pi} \quad c_{\pm}^{\Phi''} = \frac{\zeta}{2} (f_2^{V,p} \pm f_2^{V,n})$$

$$\dot{c}_{\pm}^M = \frac{\zeta m_N^2}{2} \left[\dot{f}_p \pm \dot{f}_n + \dot{f}_1^{V,p} \pm \dot{f}_1^{V,n} + \frac{1}{4m_N^2} (f_2^{V,p} \pm f_2^{V,n}) \right]$$

- Couplings

$$f_N = \frac{m_N}{\Lambda^3} \left(\sum_{q=u,d,s} c_q^{SS} f_q^N - 12\pi f_Q^N c_g^{S} \right) \quad f_{\pi} = \frac{M_{\pi}}{\Lambda^3} \sum_{q=u,d} \left(c_q^{SS} + \frac{8\pi}{9} c_g^{S} \right) f_q^{\pi} \quad \dots$$

- Conclusions

- Different c probe **different linear combinations** of Wilson coefficients
- Ideally: global analysis of different experiments
- One-operator-at-a-time strategy**: producing limits e.g. on c_{-}^M and c_{π} in addition to c_{+}^M would provide additional information on BSM parameter space

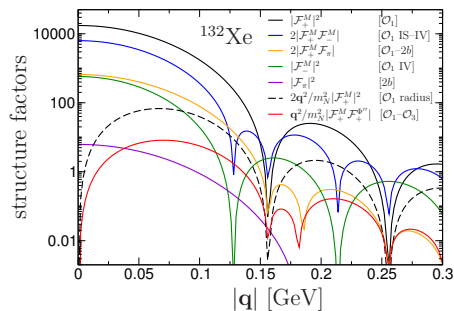
• Analysis of direct detection searches

including

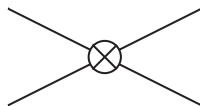
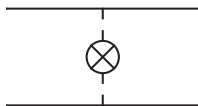
- 1 standard SI isoscalar WIMP–nucleon interaction
- 2 its isovector counterpart
- 3 two-body currents
- 4 radius corrections
- 5 quasi-coherent response associated with the Φ'' operator

↔ canonical generalization of SI searches

↔ captures all coherent contributions up to third chiral order



- Scalar source also suppressed for $(N^\dagger N)^2$
 - ↔ **long-range contribution dominant** (in Weinberg counting)
- Typical size **(5–10)%**
 - ↔ reflected by results for structure factors
 - ↔ more important in case of cancellations
- Contact terms ↔ nuclear σ -terms [Beane et al. 2014](#)



Spin-2 and coupling to the energy-momentum tensor

- Effective Lagrangian truncated at dim-7, but if WIMP heavy $m_\chi/\Lambda = \mathcal{O}(1)$

↪ heavy-WIMP EFT [Hill, Solon 2012, 2014](#)

$$\mathcal{L} = \frac{1}{\Lambda^4} \left\{ \sum_q C_q^{(2)} \bar{\chi} \gamma_\mu i \partial_\nu \chi \frac{1}{2} \bar{q} \left(\gamma^{\{\mu} i D_-^{\nu\}} - \frac{m_q}{2} g^{\mu\nu} \right) q + C_g^{(2)} \bar{\chi} \gamma_\mu i \partial_\nu \chi \left(\frac{g_{\mu\nu}}{4} G_{\lambda\sigma}^a G_a^{\lambda\sigma} - G_a^{\mu\lambda} G_{a\lambda}^\nu \right) \right\}$$

↪ leading order: **nucleon pdfs**

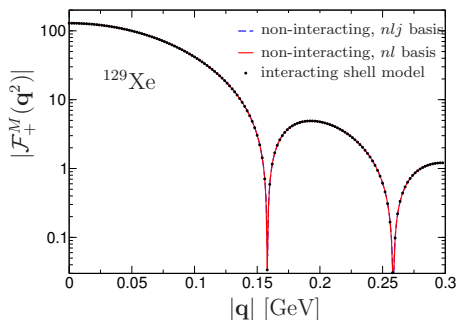
↪ similar two-body current as in scalar case, pion pdfs, EMC effect

- Coupling of trace anomaly θ_μ^μ to $\pi\pi$

$$\theta_\mu^\mu = \sum_q m_q \bar{q} q + \frac{\beta_{\text{QCD}}}{2g_s} G_{\mu\nu}^a G_a^{\mu\nu} \quad \Leftrightarrow \quad \langle \pi(p') | \theta_{\mu\nu} | \pi(p) \rangle = p_\mu p'_\nu + p'_\mu p_\nu + g_{\mu\nu} (M_\pi^2 - p \cdot p')$$

↪ probes gluon Wilson coefficient C_g^S

Some details on the implementation



- **Shell-model diagonalization** for Xe isotopes with ^{100}Sn core
- Correlations among valence nucleons, j -coupling small
 - ↪ treat two-body currents in the same way
- **Uncertainty estimates**: currently phenomenological shell-model interaction
 - ↪ ChEFT based interactions in the future