Damping of Pseudo-Goldstone Fields from Schwinger-Keldysh Effective Field Theory

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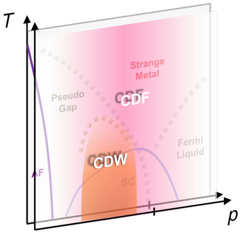
Motivation & preview

- Spontaneous symmetry breaking leads to massless Goldstone fields, included in low-energy effective field theories
- Typically symmetries are **approximate**, and also involve some small explicit breaking $\Rightarrow$ **pinned** Goldstone fields with **small mass**
- Zero temperature EFTs well studied [Weinberg], but crucial to include finite temperature dissipative effects

Motivation & preview

- Understand the structure of **hydrodynamic EFTs with pseudo-spontaneous symmetry breaking**, particulary systems with broken **translations**



Temperature vs doping [Arpaia,Ghiringhelli]

- Dynamical charge fluctuations with translational order in phase diagram of cuprate High-T_c Superconductors [Seibold et al.]

Motivation & preview

- Pinning leads to **damping** of Goldstones, i.e. Josephson relation takes the form

$$\boxed{\dot{\phi} = -\mu - \Omega\phi + \cdots}$$

- Typically due to mobile topological defects, such as superfluid vortices [Anderson; Delacrétaz,Goutéraux,Hartnoll,Karlsson]
  - Here focus only on phase relaxation originating from **pinning**

Main result

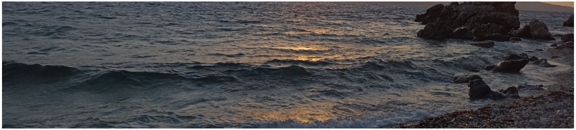
Damping rate $\sim$ (Pinning mass)$^2 \times$ Diffusivity

Motivation & preview

- Initially observed in:
  - Various holographic models of pseudo-spontaneous breaking of translations [Amoretti,Areán,Goutéraux,Musso; Ammon,Baggioli,Jiménez-Alba; Donos,Martin,Pantelidou,VZ]
  - Holographic superfluids [Donos,Kailidis,Pantelidou; Ammon,Aréan,Baggioli,Gray,Grieninger]
  - QCD with approximate chiral symmetry [Grossi,Soloviev,Teaney,Yan]
- We now understand it from various points of view
  - Locality of hydrodynamics [Delacrétaz,Goutéraux,VZ]
  - **Schwinger-Keldysh EFTs for finite temperature hydrodynamics**
  - Second law of thermodynamics [Armas,Jain,Lier]
- Not a coincidence or artifact: **consistency of effective field theory**
- Practical application of holography!

Overview

Hydrodynamics



- Hydrodynamics describes **late-time, long wavelength** behavior of thermalizing systems compared to local equilibration scale $\ell_{th} \sim T^{-1}$ [Landau,Lifshitz; Kovtun]

Universality:

- QGP, Heavy-Ion collisions, neutron star mergers, early universe, high-temperature superconductors, strange metals, graphene, charge density waves, Wigner crystals,...

Hydrodynamics

- (i) **Identify slow modes** (conserved charges, Goldstone modes, order parameters) and corresponding **conservation laws**

$$\dot{n}_a + \nabla j_a = 0$$

- (ii) **Constitutive relations** for currents in **derivative expansion**

$$j_a = \alpha_{ab} n_b + \sigma_{ab} \nabla n_b + \lambda_{ab} \nabla^2 n_b + \cdots$$

  with transport coefficients determined by UV theory

- (iii) Equations of motion

$$\dot{n}_a(q,t) + M_{ab}(q) n_b(q,t) = 0$$

- (iv) (Phenomenological) **restrictions** on M:
  - Isotropy, Galilean/Lorentzian boosts,...
  - Time-reversal $\Rightarrow$ Onsager relations $G_{ab}(\omega, q; B) = \eta_a \eta_b G_{ba}(\omega, -q; -B)$
  - Positivity of entropy production $\nabla_\mu J^\mu_S \geq 0$
  - Existence of equilibrium on arbitrary backgrounds
  - ...

  $\Longrightarrow$ **(in)equality conditions** between transport coefficients

- (v) Retarded Green's functions [Kadanoff,Martin]

$$G^R_{ab} = M_{ac}(-i\omega + M)^{-1}_{cd}\chi_{db}\,, \qquad \chi_{ab} = -\frac{\delta^2 f}{\delta\mu_a\delta\mu_b}$$

- Physical modes correspond to poles of Green's functions

$$\det(-i\omega + M) = 0$$

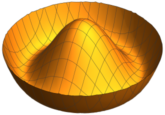
Limitations:

- Classical formulation at the level of **equations of motion**: how to write **actions for dissipative systems**?
- Beyond linear response: how to systematically include **interactions** between hydro modes, and with thermal bath? $\Longrightarrow$ long-time tails, stochastic hydrodynamics, ...
- Direct access to G^A, G^S **and higher point TO correlators**? Fluctuation-dissipation theorem not enough
- Fundamental **derivation** of phenomenological restrictions?

Overcome by **Schwinger-Keldysh** construction!

Superfluid hydrodynamics

- **Isolate condensate** $\Rightarrow$ Hydrodynamic dofs:
  - $U(1)$ charge density n
  - Conjugate phase (Goldstone) ϕ
- ϕ shifts under the symmetry $\Rightarrow$ only **gradients** appear in f



$$f = \frac{f_s}{2}(\nabla\phi)^2 - \frac{\chi_{nn}}{2}\delta\mu^2 + \cdots$$

  - Constitutive relation

$$j = f_s \nabla\phi - D_n \nabla n$$

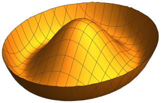
  - Current conservation & Josephson relation

$$\dot{n} + \nabla \cdot j = 0\,, \qquad \dot{\phi} = -\frac{1}{\chi_{nn}} n + D_\phi \nabla^2 \phi$$

  - Second sound mode

$$\omega = \pm c_s q - \frac{i}{2}(D_n + D_\phi) q^2\,, \qquad c_s^2 = \frac{f_s}{\chi_{nn}}$$

Pinning the Goldstone field



- **Break the symmetry weakly** $\Longrightarrow$ (lower-gradient **mass**) term breaking shift symmetry and introducing new length scale $1/q_o$

$$f = \frac{f_s}{2}[(\nabla\phi)^2 + q_o^2\phi^2] - \frac{\chi_{nn}}{2}\delta\mu^2 + \cdots$$

  - Charge conservation is also **weakly broken**

$$\dot{n} + \nabla \cdot j = -\Gamma n + f_s q_o^2 \phi + \cdots$$

  - Josephson relation gets **phase relaxation** term

$$\dot{\phi} \simeq -\Omega\phi - \frac{1}{\chi_{nn}} n + D_\phi \nabla^2 \phi + \cdots$$

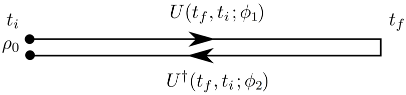
  - Sound mode acquires gap and resonance

$$\omega = \pm c_s q_o - \frac{i}{2}(\Gamma + \Omega) + \mathcal{O}(q)$$

  - Imposing **locality** leads to $\boxed{\Omega \simeq q_o^2 D_\phi}$ [Delacrétaz,Goutéraux,VZ]

Schwinger-Keldysh EFT

- **Action principle** for hydrodynamics on **SK closed time path contour** [Crossley,Glorioso,Liu; Haehl,Loganayagam,Rangamani; Liu,Glorioso]



- Compute *n*-point TO correlators from generating functional

$$e^{W[s_1,s_2]} = \int D\psi_1 D\psi_2 e^{iS[\psi_1,s_1]-iS[\psi_2,s_2]} \simeq \int D\phi_1 D\phi_2 e^{iS_{EFT}[\phi_1,s_1,\phi_2,s_2]}$$

- **Doubling** of the fields $\Rightarrow$ ϕ_r: **physical**, ϕ_a: **stochastic**

$$\phi_r = \frac{1}{2}(\phi_1 + \phi_2) \qquad \phi_a = \phi_1 - \phi_2$$

Schwinger-Keldysh EFT

Systematic procedure for constructing S_{EFT}: [Liu,Glorioso]

- (i) **Identify** the low-energy dynamical **dofs** and write down the effective action in **derivative expansion**
- (ii) Impose **unitarity constraints**
  - $S^*_{EFT}[\phi_r, \phi_a] = -S_{EFT}[\phi_r, -\phi_a]$
  - $\mathrm{Im} S_{EFT} \geq 0$
  - $S_{EFT}[\phi_r, \phi_a = 0] = 0$
- (iii) Impose **dynamical KMS condition**
  - $S_{EFT}[\phi_r, \phi_a] = S_{EFT}[\Theta\phi_r, \Theta\phi_a + iT^{-1}\Theta\dot{\phi}_r]$

Fundamental symmetry principles $\Longrightarrow$ Systematic inclusion of interactions and fluctuations, leads to Onsager relations and 2nd law, manifestly local and consistent,...

- Beyond linear response, $\exists$ stochastic coefficients appearing in higher-point correlators and loop corrections invisible to classical hydrodynamics [Jain,Kovtun]

Write down **superfluid SK effective action**:

- Shift symmetry $\Rightarrow \phi_{a,r}$ enter with derivatives

$$S_{eff} = \chi_{nn} \int \left(\dot{\phi}_a \dot{\phi}_r - c_s^2 \nabla \phi_a \nabla \phi_r \right) + \left(D_n \nabla^2 \phi_a \dot{\phi}_r + \frac{D_\phi}{c_s^2} \ddot{\phi}_a \dot{\phi}_r \right)$$
$$+ iT \left[D_n (\nabla \phi_a)^2 + \frac{D_\phi}{c_s^2} \dot{\phi}_a^2 \right] + \mathcal{O}\left(\partial^4, \phi_a^3, \phi_a^2 \phi_r, \dots \right)$$

- Current conservation derived as **eom** for ϕ_a

$$J^\mu \equiv \frac{\delta S_{eff}}{\delta(\partial_\mu \phi_a)} \qquad \partial_\mu J^\mu = 0$$

- Explicitly

$$J^0 = -\chi_{nn} \left(\dot{\phi}_r - \frac{D_\phi}{c_s^2} \ddot{\phi}_r \right)$$
$$J^i = \chi_{nn} \left(c_s^2 \partial^i \phi_r + D_n \partial^i \dot{\phi}_r \right)$$

Schwinger-Keldysh EFT for superfluids

- Josephson relation appears as the natural choice of **hydrodynamic frame**:

$$\mu = -\dot{\phi}_r + \frac{D_\phi}{c_s^2}\ddot{\phi}_r$$

leading to the conventional form of constitutive relations

$$J^0 = \chi_{nn}\mu$$
$$J^i = \chi_{nn}\left(c_s^2\partial^i\phi_r - D_n\partial^i\mu\right)$$

- Retarded Green's function

$$G^R_{\phi\phi} = \langle\phi_r\phi_a\rangle \sim \left(c_s^2 q^2 - \omega^2 - iD_n\omega q^2 - i\frac{D_\phi}{c_s^2}\omega^3\right)^{-1}$$

gives second sound pole

Schwinger-Keldysh EFT for superfluids

- Now **explicitly break** the symmetry $\Longrightarrow$ We can write **only two** new terms

$$\delta S_{eff} = -\chi_{nn} \int q_o^2 c_s^2 \phi_a \phi_r + \Gamma \phi_a \dot{\phi}_r - iT\Gamma \phi_a^2 + \cdots$$

- From $S_{eff} + \delta S_{eff}$ we find

$$G^R_{\phi\phi}(\omega, q = 0) \sim \left(q_o^2 c_s^2 - \omega^2 - i\omega\Gamma - i\frac{D_\phi}{c_s^2}\omega^3 \right)^{-1}$$

leading directly to the pinned sound mode

$$\omega = \pm c_s q_o - \frac{i}{2}\left(\Gamma + q_o^2 D_\phi\right) + \mathcal{O}(q)$$

Schwinger-Keldysh EFT for superfluids

Recover conventional hydrodynamics:

- Modified conservation equation

$$\partial_0 J^0 + \partial_i J^i = \chi_{nn} q_o^2 c_s^2 \phi_r + \Gamma \dot{\phi}_r$$

$\Longrightarrow$ conventional frame choice gives the **Josephson relation**

$$\mu = -\dot{\phi}_r + \frac{D_\phi}{c_s^2}\ddot{\phi}_r \simeq -\dot{\phi}_r - D_\phi q_o^2 \phi_r + \cdots$$

from where we read off

$$\boxed{\Omega \simeq q_o^2 D_\phi}$$

- Note that SK effective action **manifestly local** up to hydrodynamic UV cutoff!

    - Coupling to external gauge fields is ambiguous when the symmetry is not exact $\Rightarrow$ further coefficients which do not affect modes, but may affect Green's functions [Armas,Jain,Lier; Delacrétaz,Goutéraux,VZ]

Holography for broken $U(1)$ symmetry

Simpler case of $U(1)$ charge relaxation: how to obtain Γ in holography?

- Bulk Maxwell field $\Leftrightarrow$ dual QFT with conserved $U(1)$ current

$$\partial_\mu j^\mu = 0$$

- Softly **break bulk gauge symmetry** $\Leftrightarrow$ QFT with charge relaxation

$$\partial_\mu j^\mu \simeq -\Gamma n$$

$\Rightarrow$ We consider bulk **Proca theory** [ongoing work w/ Baggioli,Bu]

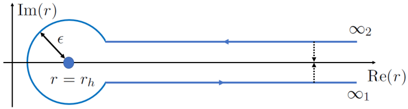
$$S_{\text{bulk}} = -\int d^5x\sqrt{-g}\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A_\mu A^\mu\right]$$

Holography for broken $U(1)$ symmetry

- Schwarzschild-AdS bulk geometry $\Leftrightarrow$ neutral, thermal state in QFT

$$ds^2 = 2drdv - r^2\left(1 - r_h^4/r^4\right)dv^2 + r^2\delta_{ij}dx^i dx^j$$

- Holographic prescription for SK contour [Skenderis,van Rees; de Boer,Heller,Pinzani-Fokeeva]
  - **Complexify radial coordinate** and **analytically continue** around the horizon [Glorioso,Crossley,Liu]



- We then (partially) solve the bulk eoms, and derive the **finite temperature SK action** for the hydrodynamics of broken $U(1)$ symmetry

- We find $\boxed{\Gamma \sim m^2 r_h}$
  - Transport coefficients given by **horizon quantities** $\Rightarrow$ horizon encodes dissipation [Kovtun,Son,Starinets; Iqbal,Liu; Donos,Gauntlett]

Outlook

- **Applications**: QCD, nematic/hexatic liquid crystals, (anti-)ferromagnets, Wigner crystal/Charge density waves, Strange metallic transport [Delacrétaz,Goutéraux,VZ]

Future directions:

- Further applications to:
  - **order parameter** fluctuations near phase transitions [Kapustin,Mrini]
  - **higher-form symmetries** [Glorioso,Son]
  - Systems with (pseudo-)spontaneous breaking of **translations**
- SK action for **pinned holographic superfluids**
- Effects of pinning **beyond linear response** [Chen-Lin,Delacrétaz,Hartnoll]
- Include **dynamical topological defects** [Delacrétaz,Goutéraux,Hartnoll,Karlsson]
- Implications for **strange metal phenomenology**

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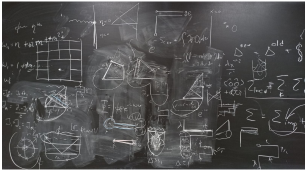
Thank You!



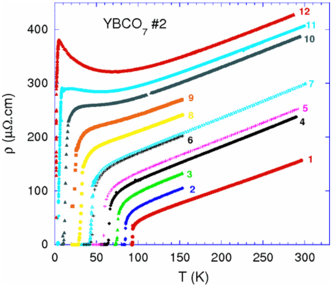
Extra slides: Strange metallic transport

- Expect diffusivities in strongly-correlated materials to saturate **Planckian bound** [Hartnoll]

$$D \simeq \frac{\hbar}{k_B T} c_s^2$$

- Resistivity for CDWs

$$\rho_{\rm dc} = \frac{m^\star}{ne^2}\left(\Gamma_\pi + \frac{q_o^2 c_s^2}{\Omega}\right) \simeq \frac{m^\star}{ne^2}\left(\Gamma_\pi + \frac{k_B T}{\hbar}\right)$$



Resistivity vs temperature for irradiated single crystal $YBCO_7$ [Rullier-Albenque,Alloul,Tourbot]

- Γ_π from conventional scattering (Umklapp, disorder, el-ph interactions)
- Slope of the linear term **independent** from disorder
- Natural mechanism for T-**linear** resistivity