α′ Corrections to Stringy BHs

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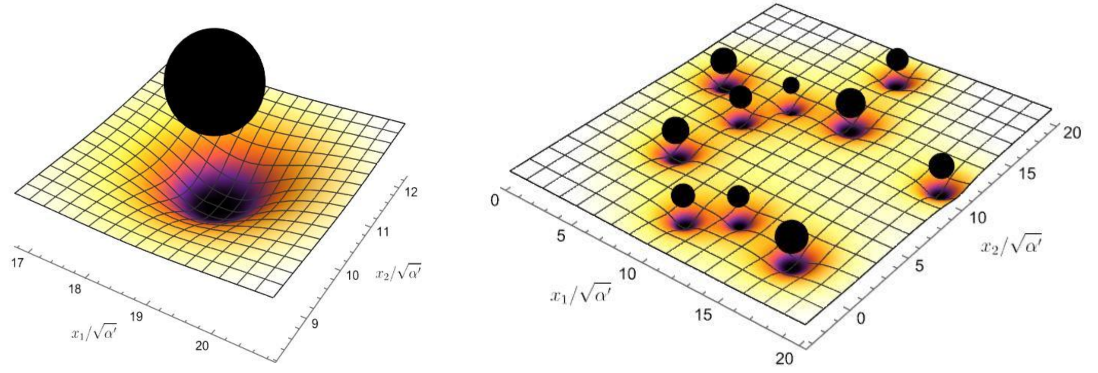
In a Nutshell

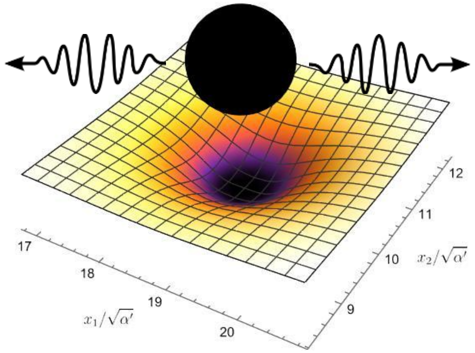
Compute unknown α’ corrections to known BH solutions of heterotic string theory (HST)

[P. Cano, T. Ortín, A. Ruipérez, M.Z., 21; 22]

Extremal 5d/4d, SUSY/~~SUSY~~



Non Extremal 5d



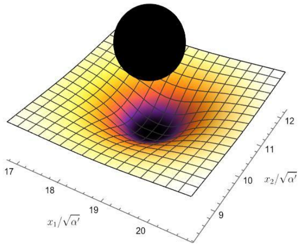
Framework: HST

$$S = \frac{g_s^2}{16\pi G_N^{(10)}} \int d^{10}x \sqrt{|g|}\, e^{-2\phi} \left\{ R - 4(\partial\phi)^2 + \frac{1}{12} H^{(1)\,2} - \frac{\alpha'}{8} R_{(-)\mu\nu}{}^{a}{}_{b} R_{(-)}{}^{\mu\nu b}{}_{a} \right\}$$

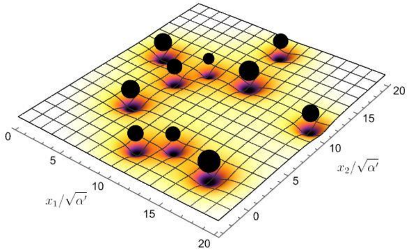
Higher Order Derivatives terms at $O(\alpha')$ are proportional to the Zeroth Order EOMs

[E. Bergshoeff, M. de Roo, 89]

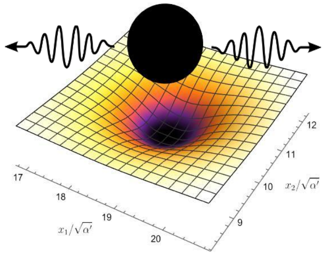
Motivations



- Test SMARR Formula
- Test Swampland Conjectures



- Cancellation of Forces beyond probe approximation



- Test First Law of Thermodynamics

Extremal BHs 5d: Ansatz

Fields Ansatz for $\mathrm{T}^5 = \mathrm{S}^1 \times \mathrm{T}^4$ compactification

$$
\begin{aligned}
d\hat{s}^2 = & \frac{1}{\mathcal{Z}_+\mathcal{Z}_-}dt^2 - \mathcal{Z}_0(d\rho^2 + \rho^2 d\Omega^2_{(3)}) \\
& - \frac{k_\infty^2 \mathcal{Z}_+}{\mathcal{Z}_-}\left[dz + \beta_+ k_\infty^{-1}\left(\mathcal{Z}_+^{-1} - 1\right)dt\right]^2 - dy^m dy^m \,,
\end{aligned}
$$

$$
\hat{H} = d\left[\beta_- k_\infty \left(\mathcal{Z}_-^{-1} - 1\right) dt \wedge dz\right] + \beta_0 \rho^3 \mathcal{Z}_0' \omega_{(3)} \,,
$$

$$
e^{-2\hat{\phi}} = e^{-2\hat{\phi}_\infty} \mathcal{Z}_- / \mathcal{Z}_0 \,.
$$

Unknown functions Ansatz

$$
\mathcal{Z}_{\pm,0} = 1 + \frac{q_{\pm,0}}{\rho^2} + \alpha' \delta \mathcal{Z}_{\pm,0}(\rho)
$$

Extremal BHs 5d: Solution

$$\mathcal{Z}_0 = 1 + \frac{q_0}{\rho^2} + \alpha' \frac{q_0^2}{\rho^2(\rho^2 + q_0)^2} + \mathcal{O}(\alpha'^{\,2})$$

$$\mathcal{Z}_+ = 1 + \frac{q_+}{\rho^2} - \alpha'(1 + \beta_+\beta_-)\frac{q_+ q_-}{\rho^2(\rho^2 + q_-)(\rho^2 + q_0)} + \mathcal{O}(\alpha'^{\,2})\,,$$

$$\mathcal{Z}_- = 1 + \frac{q_-}{\rho^2} + \mathcal{O}(\alpha'^{\,2})\,,$$

Extremal BHs 5d: Microstates & Thermodynamics

	t	z	y^1	y^2	y^3	y^4	x^1	x^2	x^3	x^4	#	sign
F1	$\times$	$\times$	$\sim$	$\sim$	$\sim$	$\sim$	$-$	$-$	$-$	$-$	q_-	β_-
W	$\times$	$\times$	$\sim$	$\sim$	$\sim$	$\sim$	$-$	$-$	$-$	$-$	q_+	β_+
S5	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$-$	$-$	$-$	$-$	q_0	β_0

$$M = \frac{\pi}{4G_N^{(5)}}\left(q_+ + q_- + q_0\right)$$

$$S = \frac{\pi^2}{2G_N^{(5)}}\sqrt{q_+q_-\left[q_0 + \alpha'(2+\beta_+\beta_-)\right]}$$

Extremal Multi BH: Solutions

$$\mathcal{Z}_+ = \mathcal{Z}_+^{(0)} - \alpha' \left\{ \frac{(1+\beta_+\beta_-)}{4} \frac{\partial_m \mathcal{Z}_+^{(0)} \partial_m \mathcal{Z}_-^{(0)}}{\mathcal{Z}_0^{(0)} \mathcal{Z}_-^{(0)}} + \mathcal{H}_+ \right\} + \mathcal{O}(\alpha'^2)$$

$$\mathcal{Z}_0 = \mathcal{Z}_0^{(0)} + \alpha' \left\{ \frac{1}{4} \frac{\partial_m \mathcal{Z}_0^{(0)} \partial_m \mathcal{Z}_0^{(0)}}{\left(\mathcal{Z}_0^{(0)}\right)^2} + \mathcal{H}_0 \right\} + \mathcal{O}(\alpha'^2)$$

$$\mathcal{Z}_- = \mathcal{Z}_-^{(0)} + \mathcal{O}(\alpha'^2)$$

Non Extremal BHs 5d: Ansatz

Fields Ansatz for $\mathrm{T}^5 = \mathrm{S}^1 \times \mathrm{T}^4$ compactification

$$d\hat{s}^2 = \frac{1}{\mathcal{Z}_+ \mathcal{Z}_-} W_{tt} dt^2 - \mathcal{Z}_0 (W_{\rho\rho}^{-1} d\rho^2 + \rho^2 d\Omega_{(3)}^2)$$

$$- \frac{k_\infty^2 \mathcal{Z}_+}{\mathcal{Z}_-} \left[dz + \beta_+ k_\infty^{-1} \left(\mathcal{Z}_+^{-1} - 1\right) dt\right]^2 - dy^m dy^m \,, \qquad m = 1, \ldots, 4 \,,$$

$$\hat{H}^{(1)} = \beta_- d \left[k_\infty \left(\mathcal{Z}_{h-}^{-1} - 1\right) dt \wedge dz\right] + \beta_0 \rho^3 \mathcal{Z}'_{h0} \omega_{(3)} \,,$$

$$e^{-2\hat{\phi}} = -\frac{2c_{\hat{\phi}}}{\rho^3} \frac{\mathcal{Z}_{h-}^2}{\mathcal{Z}_0 \mathcal{Z}_- \mathcal{Z}'_{h-}} \sqrt{W_{tt}/W_{\rho\rho}} \,.$$

Blackening factor Ansatz

$$W_{tt} = 1 + \frac{\omega}{\rho^2} + \alpha' \delta W_{tt} \qquad W_{\rho\rho} = 1 + \frac{\omega}{\rho^2} + \alpha' \delta W_{\rho\rho}$$

Finding Non Extremal BH Solutions

- 7 unknown functions → EOMs cannot be solved directly
- NEW approach

  - Expand in $1/\rho$ the unknown functions $$\delta\mathcal{Z}_i = \sum_{n=1} \frac{d_i^{(2n)}}{\rho^{2n}}$$
  - Replace in the EOMs → obtain algebraic constraints for the $d_i^{(2n)}$
  - Find the generating functions of the series (using Mathematica)

$$\left\{0, 0, 0, q_0^2 - q_0 w + \frac{5w^2}{16}, -2q_0^3 + 2q_0^2 w - \frac{7q_0 w^2}{40} - \frac{9w^3}{40}, \ldots\right\}$$

Non Extremal BHs 5d: Solution

Let us now comment on how to solve the equations. In the case of $\tilde{z}_0$, the Bianchi identity provides us with a decoupled equation for this variable that can be integrated immediately, and it yields

$$\tilde{z}_0 = -\frac{6m\rho^4 + 9m\rho^2 Q_0 + mQ_0^2 - 2Q_0^3}{2\rho^2 Q_0\left(\rho^2+Q_0\right)^2} + \frac{3m}{Q_0^2}\log\left(1+\frac{Q_0}{\rho^2}\right), \tag{2.16}$$

where we have already imposed the boundary condition $\tilde{z}_0 = \mathcal{O}(\rho^{-4})$ when $\rho \to \infty$.

The resolution for the rest of the functions is much more involved, since they satisfy coupled equations that can be taken to be the components tt, tz, zz and $\rho\rho$ of Einstein's equations together with the dilaton's equation. In order to find the solution, we perform a $1/\rho$ expansion of these functions, using the boundary conditions we have already discussed. Analyzing the coefficients of these expansions, it is possible to find a general pattern and to sum the whole series. One can check a posteriori that the result is in fact an exact solution. With the analytic solution at hands, we observe that imposing regularity the the horizon $\rho = \sqrt{m}$ leads to the following values of c_0 and c_-:

$$c_0 = -\frac{3m^2\left(3m+4Q_-+2Q_+\right)\left(3m+2Q_0\right)}{\left(m+Q_0\right)^2\left(3m^2+4m\left(Q_-+Q_++Q_0\right)+4Q_-Q_++4\left(Q_-+Q_+\right)Q_0\right)}, \tag{2.17}$$

$$c_- = -\frac{6m^2\left(Q_--Q_+\right)\left(3m+2Q_0\right)}{\left(m+Q_0\right)^2\left(3m^2+4m\left(Q_-+Q_++Q_0\right)+4Q_-Q_++4\left(Q_-+Q_+\right)Q_0\right)}. \tag{2.18}$$

In that case, the solution reads explicitly as follows:

$$\begin{aligned}
z_0 =& \frac{m\log\left(1+\frac{Q_-}{\rho^2}\right)\left(\epsilon_-\sqrt{\frac{Q_-(m+Q_-)Q_+}{m+Q_+}}+Q_-\epsilon_+\right)}{2Q_-^2\epsilon_+\left(Q_--Q_0\right)} - \frac{m^3\mathrm{Li}_2\left(-\frac{Q_0}{\rho^2}\right)}{8Q_0^2\left(m+Q_0\right)^2}\\
&+\frac{1}{8\rho^2\left(m+Q_0\right)^2}\Bigg[\frac{4m\epsilon_-\sqrt{\frac{Q_-(m+Q_-)Q_+}{m+Q_+}}\left(m+Q_0\right)^2}{Q_-\epsilon_+Q_0}+\frac{8Q_0\left(m+Q_0\right)^3}{\left(\rho^2+Q_0\right)^2}+m\left(3m+4Q_0\right)\\
&-\frac{3m^2\left(3m+4Q_-+2Q_+\right)\left(3m+2Q_0\right)}{3m^2+4Q_-Q_++4\left(Q_-+Q_+\right)Q_0+4m\left(Q_-+Q_++Q_0\right)}\Bigg]\\
&+\frac{m\log\left(1+\frac{Q_0}{\rho^2}\right)}{8\rho^2\epsilon_+Q_0^2\left(m+Q_0\right)^2\left(-Q_-+Q_0\right)}\Bigg(2m^2\epsilon_+\left(Q_--Q_0\right)Q_0+\\
&4\rho^2\epsilon_-\sqrt{\frac{Q_-\left(m+Q_-\right)Q_+}{m+Q_+}}\left(m+Q_0\right)^2+\rho^2\epsilon_+\left(4Q_-Q_0^2+m^2\left(Q_-+3Q_0\right)+mQ_0\left(3Q_-+5Q_0\right)\right)\Bigg)
\end{aligned} \tag{2.19}$$

$$\begin{aligned}
w =& \frac{m^2\log\left(1+\frac{Q_-}{\rho^2}\right)\left(\epsilon_-\sqrt{\frac{Q_-(m+Q_-)Q_+}{m+Q_+}}+Q_-\epsilon_+\right)}{2\rho^2Q_-^2\epsilon_+\left(Q_--Q_0\right)} - \frac{m^4\mathrm{Li}_2\left(-\frac{Q_0}{\rho^2}\right)}{8\rho^2Q_0^2\left(m+Q_0\right)^2}\\
&+\frac{m^2\log\left(1+\frac{Q_0}{\rho^2}\right)}{8\rho^2\epsilon_+Q_0^2\left(m+Q_0\right)^2\left(-Q_-+Q_0\right)}\Bigg(4\epsilon_-\sqrt{\frac{Q_-\left(m+Q_-\right)Q_+}{m+Q_+}}\left(m+Q_0\right)^2\\
&+\epsilon_+\left(4Q_-Q_0^2-m^2\left(-5Q_-+Q_0\right)+2m\rho^2\left(-Q_-+Q_0\right)+mQ_0\left(7Q_-+Q_0\right)\right)\Bigg)\\
&+\frac{m}{4\rho^4\left(\rho^2+Q_-\right)Q_0\left(\rho^2+Q_0\right)}\Bigg[\frac{2\epsilon_-}{Q_-\epsilon_+}\sqrt{\frac{Q_-\left(m+Q_-\right)Q_+}{m+Q_+}}\left(2\rho^2Q_-Q_0+m\left(\rho^4-Q_-Q_0+\rho^2\left(Q_-+Q_0\right)\right)\right)\\
&+\frac{1}{\left(m+Q_0\right)^2\left(3m^2+4Q_-Q_++4\left(Q_-+Q_+\right)Q_0+4m\left(Q_-+Q_++Q_0\right)\right)}\Bigg(\\
&-3mQ_-\left(m+2Q_-\right)Q_+Q_0^2\left(3m+2Q_0\right)-m\rho^6\left(3m^3+12Q_-Q_0^2+4mQ_+\left(Q_-+Q_0\right)\right.\\
&+2mQ_0\left(11Q_-+3Q_0\right)+m^2\left(4\left(Q_-+Q_+\right)+13Q_0\right)\big)+m\rho^4\left(6m^4+m^3\left(5Q_-+8Q_++17Q_0\right)\right.\\
&-4Q_0^2\left(Q_-\left(3Q_-+Q_+\right)+\left(Q_--2Q_+\right)Q_0\right)+m^2\left(4Q_-\left(-Q_-+Q_+\right)+\left(7Q_-+11Q_+\right)Q_0+9Q_0^2\right)\\
&+2m\left(-2Q_-^2Q_+-Q_-\left(11Q_-+5Q_+\right)Q_0+\left(-2Q_-+7Q_+\right)Q_0^2+Q_0^3\right)\big)\\
&+\rho^2\left(6m^5\left(Q_-+2Q_0\right)+16Q_-Q_0^3\left(Q_-Q_++\left(Q_-+Q_+\right)Q_0\right)+m^4\left(8Q_-\left(Q_-+Q_+\right)\right.\right.\\
&+\left(45Q_-+16Q_+\right)Q_0+40Q_0^2\big)+4mQ_0^2\left(7Q_-^2Q_++Q_-\left(11Q_-+15Q_+\right)Q_0+4\left(2Q_-+Q_+\right)Q_0^2\right)\\
&+2m^2Q_0\left(5Q_-^2Q_++Q_-\left(25Q_-+38Q_+\right)Q_0+\left(47Q_-+21Q_+\right)Q_0^2+8Q_0^3\right)\\
&+m^3\left(8Q_-^2Q_++Q_-\left(36Q_-+43Q_+\right)Q_0+\left(97Q_-+39Q_+\right)Q_0^2+44Q_0^3\right)\big)\Bigg)\Bigg]
\end{aligned} \tag{2.20}$$

$$\begin{aligned}
\tilde{z}_- =& \frac{m^3\mathrm{Li}_2\left(-\frac{Q_0}{\rho^2}\right)Q_-}{8\rho^2Q_0^2\left(m+Q_0\right)^2}+\frac{\log\left(1+\frac{Q_-}{\rho^2}\right)\left(m\epsilon_-\sqrt{\frac{Q_-(m+Q_-)Q_+}{m+Q_+}}+mQ_-\epsilon_+\right)}{-2\rho^2Q_-^2\epsilon_++2\rho^2Q_-\epsilon_+Q_0}\\
&-\frac{m\log\left(1+\frac{Q_0}{\rho^2}\right)Q_-}{8\rho^2\epsilon_+Q_0^2\left(m+Q_0\right)^2\left(-Q_-+Q_0\right)}\Bigg[4\epsilon_-\sqrt{\frac{Q_-\left(m+Q_-\right)Q_+}{m+Q_+}}\left(m+Q_0\right)^2\\
&+\epsilon_+\left(4Q_-Q_0^2-m^2\left(-5Q_-+Q_0\right)+2m\rho^2\left(-Q_-+Q_0\right)+mQ_0\left(7Q_-+Q_0\right)\right)\Bigg]\\
&-\frac{1}{4\rho^4\epsilon_+Q_0\left(m+Q_0\right)^2\left(3m^2+4Q_-Q_++4\left(Q_-+Q_+\right)Q_0+4m\left(Q_-+Q_++Q_0\right)\right)}\times\\
&\times\Bigg[2m\epsilon_-\left(m+Q_0\right)^2\sqrt{\frac{m+Q_-}{m+Q_+}}\Bigg(3m^2\sqrt{Q_-Q_+}+4m\sqrt{Q_-Q_+}Q_0+4m\sqrt{Q_-^3Q_+}\\
&+4m\sqrt{Q_-Q_+^3}+4\sqrt{Q_-^3Q_+}Q_0+4\sqrt{Q_-Q_+^3}Q_0+4Q_-Q_+\sqrt{Q_-Q_+}\Bigg)\\
&+mQ_-\epsilon_+\left(6m^4+m^3\left(-3\rho^2+8\left(Q_-+Q_+\right)+20Q_0\right)-4m\rho^2\left(Q_-Q_++\left(Q_-+Q_+\right)Q_0\right)\right.\\
&+8Q_0^2\left(Q_-Q_++\left(Q_-+Q_+\right)Q_0\right)+2mQ_0\left(8Q_-Q_++9Q_-Q_0+15Q_+Q_0+4Q_0^2\right)\\
&+m^2\left(8Q_-Q_++15Q_-Q_0+33Q_+Q_0+22Q_0^2-4\rho^2\left(Q_-+Q_++Q_0\right)\right)\big)\Bigg]
\end{aligned} \tag{2.21}$$

$$\begin{aligned}
z_- =& \frac{m^3\mathrm{Li}_2\left(-\frac{Q_0}{\rho^2}\right)Q_-}{8\rho^2Q_0^2\left(m+Q_0\right)^2}-\frac{m\sqrt{Q_-}\sqrt{m+Q_-}\epsilon_-\sqrt{Q_+}}{\sqrt{m+Q_+}2\rho^2\epsilon_+Q_0\left(\rho^2+Q_0\right)}+\frac{m}{2\rho^2\left(Q_0-Q_-\right)}\log\left(1+\frac{Q_-}{\rho^2}\right)\Bigg(1\\
&+\frac{\sqrt{m+Q_-}\epsilon_-\sqrt{Q_+}}{\sqrt{Q_-}\sqrt{m+Q_+}\epsilon_+}\Bigg)+\frac{m\log\left(1+\frac{Q_0}{\rho^2}\right)Q_-}{8\rho^2Q_0^2\left(-Q_-+Q_0\right)}\Bigg(-\frac{4\sqrt{Q_-}\sqrt{m+Q_-}\epsilon_-\sqrt{Q_+}}{\sqrt{m+Q_+}\epsilon_+}\\
&+\frac{2m\rho^2\left(Q_--Q_0\right)-4Q_-Q_0^2+m^2\left(-5Q_-+Q_0\right)-mQ_0\left(7Q_-+Q_0\right)}{\left(m+Q_0\right)^2}\Bigg)\\
&+\frac{m}{4\rho^2Q_0\left(m+Q_0\right)^2\left(\rho^2+Q_0\right)\left(3m^2+4Q_-Q_++4\left(Q_-+Q_+\right)Q_0+4m\left(Q_-+Q_++Q_0\right)\right)}\times\\
&\Big[-6m^4Q_--m^3Q_-\left(8\left(Q_-+Q_+\right)+17Q_0\right)-8Q_-Q_0^2\left(Q_-Q_++\left(Q_-+Q_+\right)Q_0\right)\\
&-2mQ_0\left(6Q_-^2Q_++10Q_-\left(Q_-+Q_+\right)Q_0+\left(7Q_--3Q_+\right)Q_0^2\right)-m^2\left(8Q_-^2Q_+\right.\\
&+20Q_-\left(Q_-+Q_+\right)Q_0+9\left(3Q_--Q_+\right)Q_0^2\big)+m\rho^2\left(3m^2Q_-+4Q_-^2Q_+\right.\\
&+4mQ_-\left(Q_-+Q_+\right)+4Q_-\left(Q_-+Q_+\right)Q_0+m\left(-5Q_-+9Q_+\right)Q_0+6\left(-Q_-+Q_+\right)Q_0^2\big)\Big]
\end{aligned} \tag{2.22}$$

$$\begin{aligned}
z_+ =& \frac{m^3\mathrm{Li}_2\left(-\frac{Q_0}{\rho^2}\right)Q_+}{8\rho^2Q_0^2\left(m+Q_0\right)^2}+\frac{m\log\left(1+\frac{Q_-}{\rho^2}\right)\left(\epsilon_-Q_+^{3/2}\sqrt{\frac{Q_-(m+Q_-)}{m+Q_+}}+Q_-Q_+\epsilon_+\right)}{2\rho^2Q_-^2\epsilon_+\left(-Q_-+Q_0\right)}\\
&+\frac{m\log\left(1+\frac{Q_0}{\rho^2}\right)Q_+}{8\rho^2\epsilon_+Q_0^2\left(m+Q_0\right)^2\left(-Q_-+Q_0\right)}\Bigg[-\frac{4\epsilon_-\sqrt{Q_-\left(m+Q_-\right)Q_+\left(m+Q_+\right)}\left(m+Q_0\right)^2}{m+Q_+}\\
&+\epsilon_+\left(2m\rho^2\left(Q_--Q_0\right)-4Q_-Q_0^2+m^2\left(-5Q_-+Q_0\right)-mQ_0\left(7Q_-+Q_0\right)\right)\Bigg]\\
&-\frac{1}{4m\rho^2Q_-\left(\rho^2+Q_-\right)\epsilon_+Q_0\left(m+Q_0\right)^2\left(\rho^2+Q_0\right)}\Bigg[2m\epsilon_-\sqrt{\frac{Q_-\left(m+Q_-\right)Q_+}{m+Q_+}}\left(m+Q_0\right)^2\times\\
&\times\left(m\left(\rho^2+Q_-\right)Q_++2Q_-Q_+Q_0+m\left(Q_-+Q_+\right)Q_0\right)+mQ_-Q_+\epsilon_+\left(4Q_-Q_0^3\right.\\
&+2m^3\left(\rho^2+Q_-+2Q_0\right)+2mQ_0^2\left(\rho^2+5Q_-+2Q_0\right)+m^2\left(-\rho^2\left(\rho^2+Q_-\right)\right.\\
&+\left(3\rho^2+7Q_-\right)Q_0+8Q_0^2\big)\big)\Bigg]
\end{aligned} \tag{2.23}$$

Non Extremal BHs 5d: Solution (with T-duality)

$$\delta Z_{h0} = \frac{2q_0^3 + \omega\left(q_0^2 + 9q_0\rho^2 + 6\rho^4\right)}{2q_0\rho^2(q_0+\rho^2)^2} - \frac{3\omega}{q_0^2}\log Z_0\,, \tag{3.29a}$$

$$\delta Z_0 = \frac{8q_0^6 - 24q_0^5\omega + 2q_0^4\omega\left(7\omega - 4\rho^2\right) + q_0^3\omega\left(-4\rho^4 - 2\rho^2\omega + \omega^2\right)}{4q_0^2\rho^2\left(q_0+\rho^2\right)^2\left(2q_0^2 - 3q_0\omega + \omega^2\right)}$$
$$+ \frac{q_0^2\rho^2\omega^2\left(2\rho^2 + 9\omega\right) + q_0\rho^4\omega^2\left(2\rho^2 + 3\omega\right) - \rho^6\omega^3}{4q_0^2\rho^2\left(q_0+\rho^2\right)^2\left(2q_0^2 - 3q_0\omega + \omega^2\right)}$$
$$- \frac{q_0(2q_- - \omega)(2q_+ - \omega)}{\omega(q_- - q_+)(2q_0 - \omega)\rho^2}d_-^{(2)} + \frac{\omega^2 q_0\left(2\rho^2 + \omega\right) - \rho^2\omega^2\left(\rho^2 + 2\omega\right)}{4q_0^2\rho^2(q_0-\omega)^2}\log\left(Z_0/W\right), \tag{3.29b}$$

$$\delta Z_{h-} = \frac{\omega q_-}{2q_0\rho^4} + \frac{q_-^2(2q_+ - \omega)}{\omega(q_- - q_+)\rho^4}d_-^{(2)}\,, \tag{3.29c}$$

$$\delta Z_- = \delta Z_{h-} + Z_-\left[\Delta_k - \frac{\Delta_C}{\beta_+} + \frac{\rho^3}{4}\left(\frac{\delta Z'_{h-}}{q_-} - \frac{\mathrm{T}\left[\delta Z'_{h-}\right]}{q_+}\right)\right], \tag{3.29d}$$

$$\delta Z_+ = -Z_+\frac{\Delta_C}{\beta_+} + \mathrm{T}\left[\delta Z_{h-}\right],$$

$$\delta W_{tt} = \frac{\omega^2}{2q_0\rho^4} - \frac{\beta_-(W + \beta_+\beta_-) + W(\beta_+ + \beta_-)}{8\beta_+ Z_0 Z_-}Z'_- W'$$
$$+ \frac{(2q_+ + \rho^2)(2q_- - \omega)}{(q_- - q_+)\rho^4}d_-^{(2)},$$

$$\delta W_{\rho\rho} = \frac{\omega\left[-4q_0^3 + q_0^2\left(2\omega - 4\rho^2\right) + q_0\omega\left(3\rho^2 + \omega\right) - 2\rho^2\omega\left(\rho^2 + \omega\right)\right]}{4q_0^2\rho^2\left(q_0 + \rho^2\right)\left(q_0 - \omega\right)}$$
$$- \frac{(2q_- - \omega)(2q_+ - \omega)}{\omega(q_- - q_+)\rho^2}d_-^{(2)} + \frac{\omega^2\left(2\rho^4 + 3\rho^2\omega + \omega^2\right)}{4q_0^2\rho^2(q_0-\omega)^2}\log\left(Z_0/W\right),$$

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Non Extremal BHs 5d: Thermodynamics

- Charge, Mass NO α’ Corrections
- Entropy, Temperature YES
- HST entropy formula proposed by [Z. Elgood, T. Ortín, D. Pereñíguez, 20] satisfies the First Law and Smarr Formula

$$M = \frac{3\pi}{8G_N^{(5)}}\left\{-\omega + \tfrac{2}{3}\left(q_+ + q_- + q_0\right)\right\}$$

$$S_{BH}^{(1)} = \frac{\pi^2}{2G_N^{(5)}}\sqrt{q_+ q_- q_0}\left[1 + \alpha' \frac{8q_0 + \omega + 4q_0\beta_+\beta_-}{8q_0^2}\right]$$

Future Directions

- Multiple 4d black hole with corrected mass
- Non Extremal 4d black hole
- Microstates interpretation in the non extremal case

Thank you for the attention!

Framework: HST

$$S = \frac{g_s^2}{16\pi G_N^{(10)}} \int d^{10}x \sqrt{|g|}\, e^{-2\phi} \left\{ R - 4(\partial\phi)^2 + \tfrac{1}{12} H^{(1)\,2} - \frac{\alpha'}{8} R_{(-)\,\mu\nu}{}^{a}{}_{b} R_{(-)}{}^{\mu\nu\,b}{}_{a} \right\}$$

(1) $$\Omega^{(0)\;a}_{(\pm)\;\;b} = \omega^{a}{}_{b} \pm \tfrac{1}{2} H^{(0)a}_{\mu}{}_{b} dx^{\mu}$$

(2) $$\omega^{\mathrm{L}\,(0)}_{(\pm)} = d\Omega^{(0)\;a}_{(\pm)\;\;b} \wedge \Omega^{(0)\;b}_{(\pm)\;\;a} - \tfrac{2}{3} \Omega^{(0)\;a}_{(\pm)\;\;b} \wedge \Omega^{(0)\;b}_{(\pm)\;\;c} \wedge \Omega^{(0)\;c}_{(\pm)\;\;a}$$

(3) $$R^{(0)\;a}_{(\pm)\;\;b} = d\Omega^{(0)\;a}_{(\pm)\;\;b} - \Omega^{(0)\;a}_{(\pm)\;\;c} \wedge \Omega^{(0)\;c}_{(\pm)\;\;b}$$

(4) $$H^{(1)} = dB + \frac{\alpha'}{4} \omega^{\mathrm{L}\,(0)}_{(-)}$$

Extremal BHs 4d: Ansatz

Fields Ansatz for $\mathrm{T}^6 = \mathrm{S}^1 \times \mathrm{S}^1 \times \mathrm{T}^4$ compactification

$$\begin{aligned} d\hat{s}^2 =& \frac{1}{\mathcal{Z}_+\mathcal{Z}_-}dt^2 - \mathcal{Z}_0\mathcal{H}\left(dr^2 + r^2 d\Omega^2_{(2)}\right) - \ell^2_\infty \frac{\mathcal{Z}_0}{\mathcal{H}}\left(dw + \beta\ell^{-1}_\infty q\cos\theta d\varphi\right)^2 \\ & - k^2_\infty \frac{\mathcal{Z}_+}{\mathcal{Z}_-}\left[dz + \beta_+ k^{-1}_\infty\left(\mathcal{Z}^{-1}_+ - 1\right)dt\right]^2 - dy^i dy^i \,, \\ \hat{H} =& d\left[\beta_- k_\infty\left(\mathcal{Z}^{-1}_- - 1\right)dt\wedge dz\right] + \beta_0 \ell_\infty r^2 \mathcal{Z}'_{0h}\,\omega_{(2)}\wedge dw\,, \\ e^{2\hat{\phi}} =& -\frac{C_\phi \mathcal{Z}_0\mathcal{Z}'_- r^2}{\mathcal{Z}_- q_-}\,. \end{aligned}$$

Unknown functions Ansatz

$$\mathcal{Z}_{\pm,0} = 1 + \frac{q_{\pm,0}}{r} + \alpha'\delta\mathcal{Z}_{\pm,0} \qquad \mathcal{Z}_{0h} = 1 + \frac{q_0}{r} + \alpha'\delta\mathcal{Z}_{0h} \qquad \mathcal{H} = 1 + \frac{q}{r} + \alpha'\delta\mathcal{H}$$

Extremal BHs 4d: Microstates & Thermodynamics

	t	z	y^1	y^2	y^3	y^4	w	x^1	x^2	x^3	#
F1	$\times$	$\times$	$\sim$	$\sim$	$\sim$	$\sim$	$\sim$	$-$	$-$	$-$	q_-
W	$\times$	$\times$	$\sim$	$\sim$	$\sim$	$\sim$	$\sim$	$-$	$-$	$-$	q_+
S5	$\times$	$\sim$	$\times$	$\times$	$\times$	$\times$	$\times$	$-$	$-$	$-$	q_0
KK6	$\times$	$\sim$	$\times$	$\times$	$\times$	$\times$	$\times$	$-$	$-$	$-$	q

$$M = \frac{1}{4G_N^{(4)}} \left\{ q_+ + q_- + q_0 + q - \alpha' \frac{(1-\beta_0\beta)}{4(q_0-q)^5} \left[(q_0^2 - q^2)(q_0^2 + q^2 - 8q_0 q) + 12 q_0^2 q^2 \log\left(q_0/q\right) \right] \right\}$$

$$S = \frac{\pi}{G_N^{(4)}} \sqrt{q_+ q_- (q_0 q + 2 + \beta_+\beta_- + \beta_0\beta)}$$

Extremal Multi BH: Thermodynamics

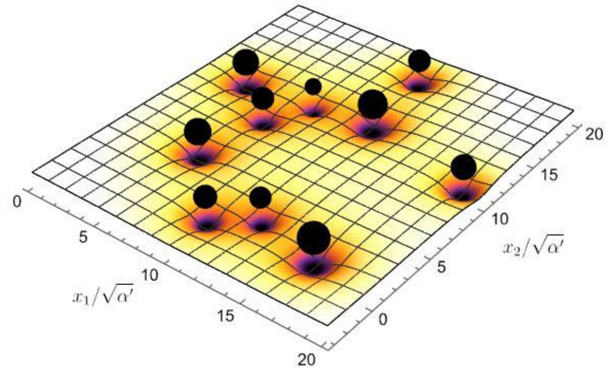
$$M = \sum M_k^{\text{ext}}$$

$$S = \sum S_k^{\text{ext}}$$

$$q_i = \sum q_{i,k}^{\text{ext}}$$

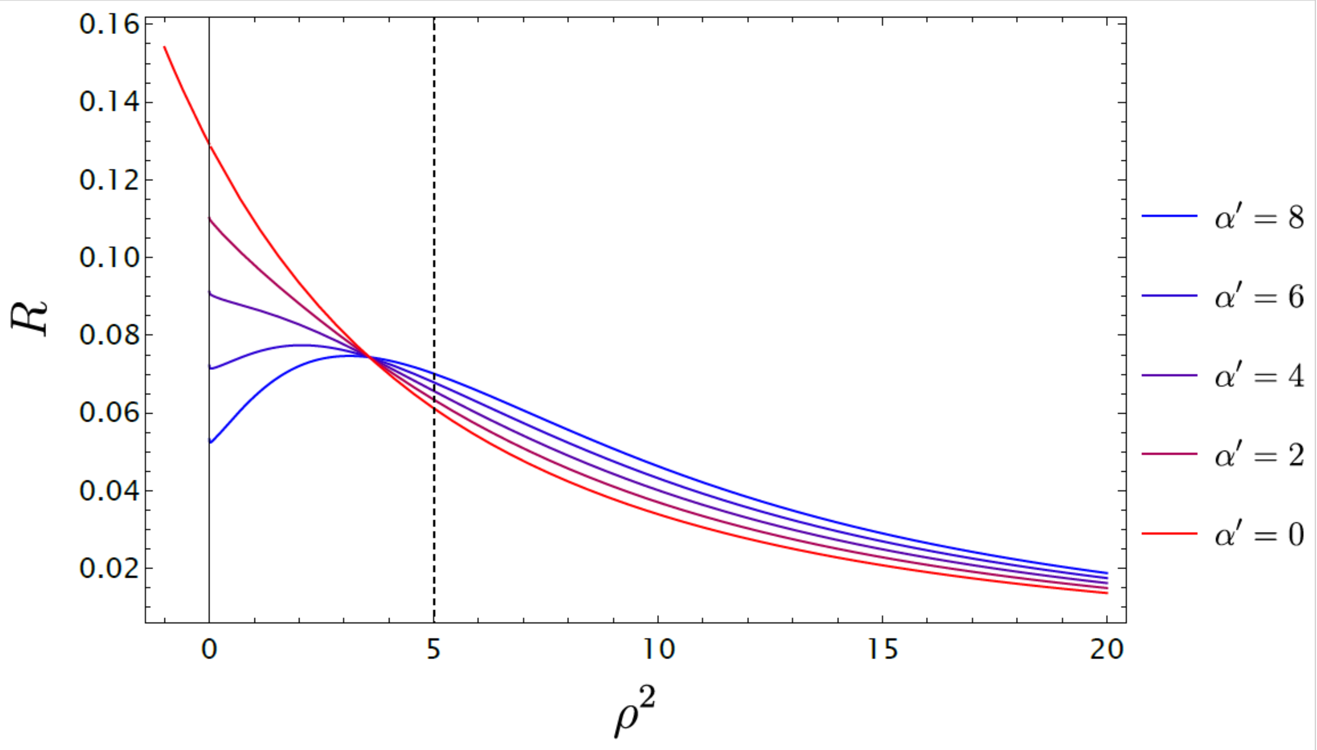
$$T_k = 0$$

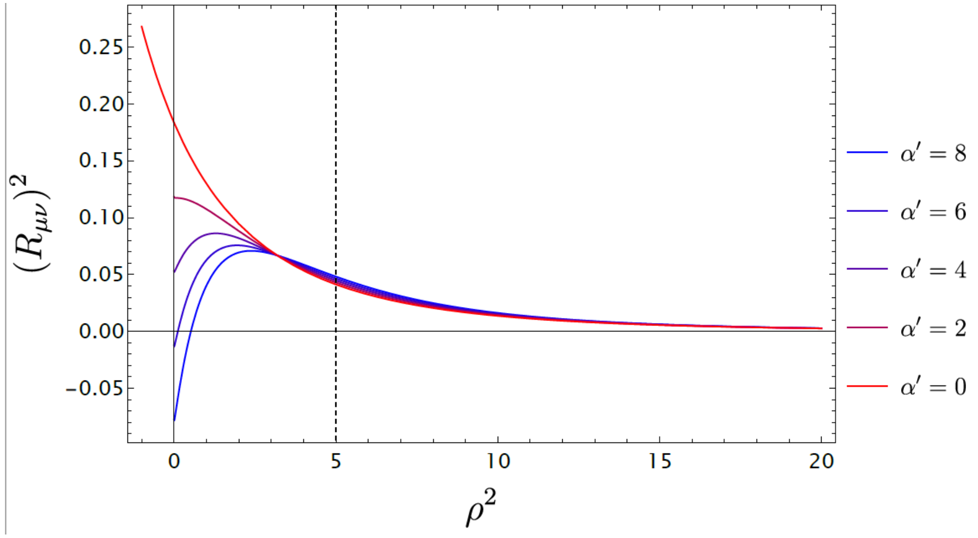
$\Longrightarrow$

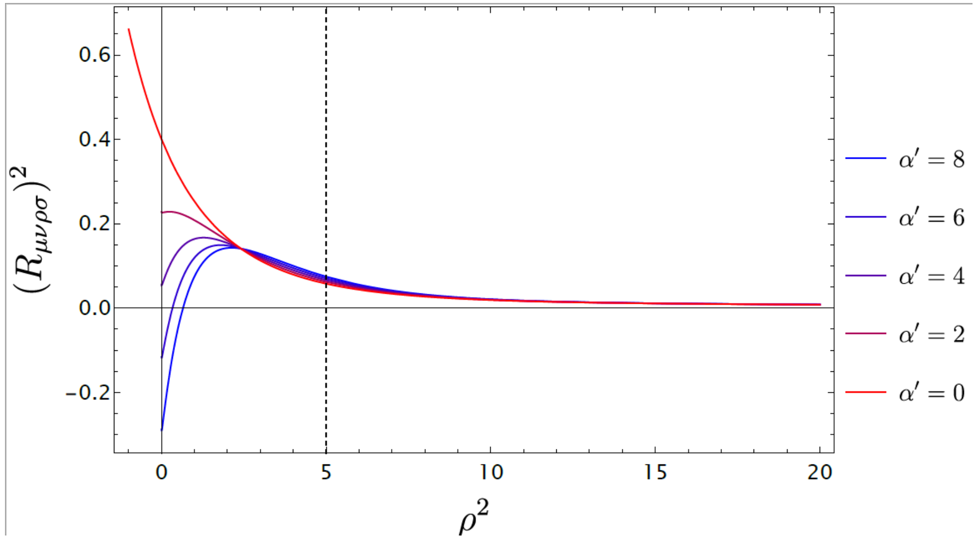


Cancellation of Forces between ~~SUSY~~ Extremal BHs with α’ Corrections

Non Extremal BHs 5d: Solution







Non Extremal BHs 5d: Thermodynamics (Part 2)