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Multipartite information in conformal field theories

Alejandro Vilar López

Université Libre de Bruxelles (ULB)

[2209.14311 – César A. Agón, P. Bueno, Óscar Lasso, AVL]

Iberian Strings 2023 - Murcia

Introduction

QFT from **vacuum correlation functions**

$$\langle 0|\phi_i(x)|0\rangle$$

$$\langle 0|\phi_i(x)\phi_j(y)|0\rangle$$

$$\dots$$

[Wightman program of axiomatic QFT]

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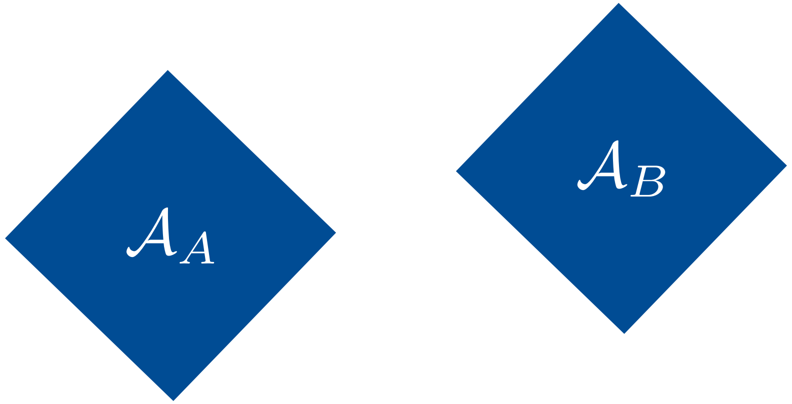
QFT from **information theoretic measures?**



[Algebraic QFT language]

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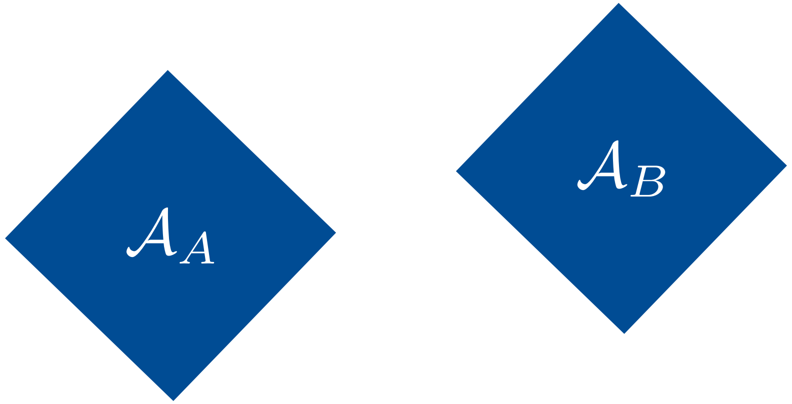

[Algebraic QFT language]

We will **focus on CFTs** $\longrightarrow$ (Δ_i, s_i, C_{ijk})

Introduction

Basic quantity associated to a region / subalgebra: **entanglement entropy (EE)**

$$S(A) \equiv S(\rho_{\mathcal{A}}) = c_0 \left(\frac{L}{\epsilon_{UV}} \right)^{d-2} + \ldots$$

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$$I_2(A,B) = S(A) + S(B) - S(A \cup B) \geq 0$$

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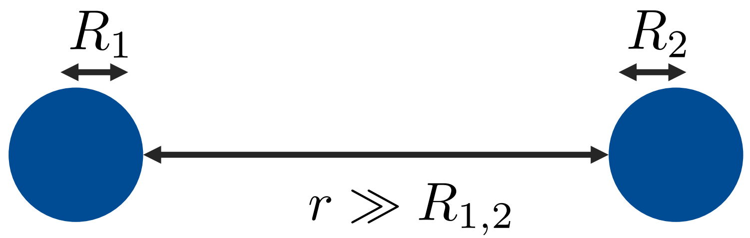
➡ Large separation expansion between two spheres in any CFT

[1511.07462 – Agón, Faulkner]
[1304.7985 – Cardy]

$$I \sim \frac{\sqrt{\pi}}{4} \frac{\Gamma(2\Delta+1)}{\Gamma(2\Delta+3/2)} \frac{(R_1 R_2)^{2\Delta}}{r^{4\Delta}} + \ldots$$

Lowest conformal dimension (scalar)



Introduction

Generalize by including more subregions (**N-partite information**)

$$I_N(A_1,\dots,A_N) \equiv -\sum_{\sigma}(-1)^{|\sigma|}S(\sigma)\ , \qquad \sigma\subset\{A_1,\dots,A_N\}$$

- $I_3(A_1,A_2,A_3) = S(A_1)+S(A_2)+S(A_3)-S(A_1A_2)-S(A_1A_3)-S(A_2A_3)+S(A_1A_2A_3)$
- $I_N(\cdot,A_{N-1},A_N) = I_{N-1}(\cdot,A_{N-1}) + I_{N-1}(\cdot,A_N) - I_{N-1}(\cdot,A_{N-1}A_N)$
- $I_N(A_1,\dots,A_N) \lessgtr 0$

These quantities have not been studied much (only in holographic theories a bit)

Outline

1. N-partite information as a correlator of twist fields & long distance expansion
2. Spherical regions: exact results up to $N = 4$
3. Free scalar in $d = 3$: checks against lattice computations
4. Comments on connections to holographic results

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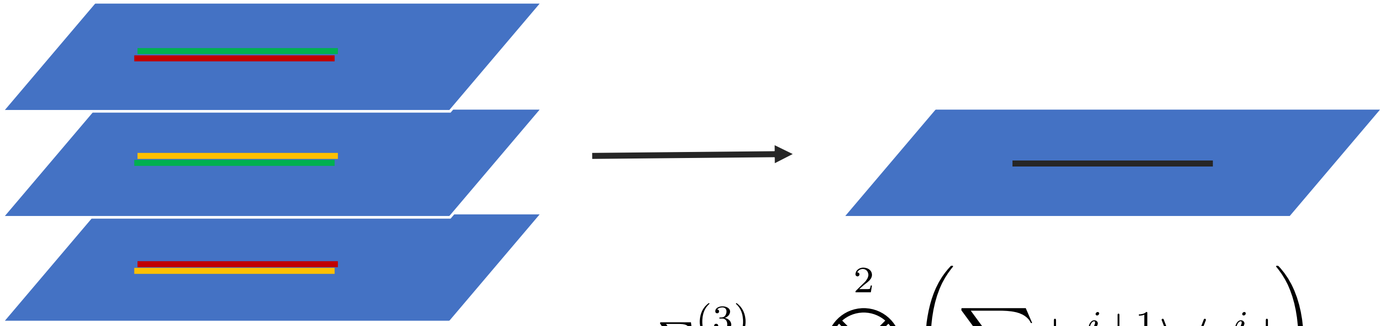
The replica trick and twist operators

$$S(A) = \lim_{n\to 1} \frac{1}{1-n} \log\left[\frac{Z(\mathcal{C}_A^{(n)})}{Z^n}\right] = \lim_{n\to 1} \frac{1}{1-n} \log\left[\langle \Sigma_A^{(n)} \rangle_{\mathrm{CFT}^{\otimes n}}\right]$$

[0405152 – Calabrese, Cardy]

[1011.5482 – Calabrese, Cardy, Tonni]

$$\mathcal{C}_A^{(3)} =$$



$$\Sigma_A^{(3)} = \bigotimes_{i=0}^{2} \left(\sum_{a\in A} |e_a^{i+1}\rangle \langle e_a^i| \right) \otimes \mathbf{1}_{\bar{A}}$$

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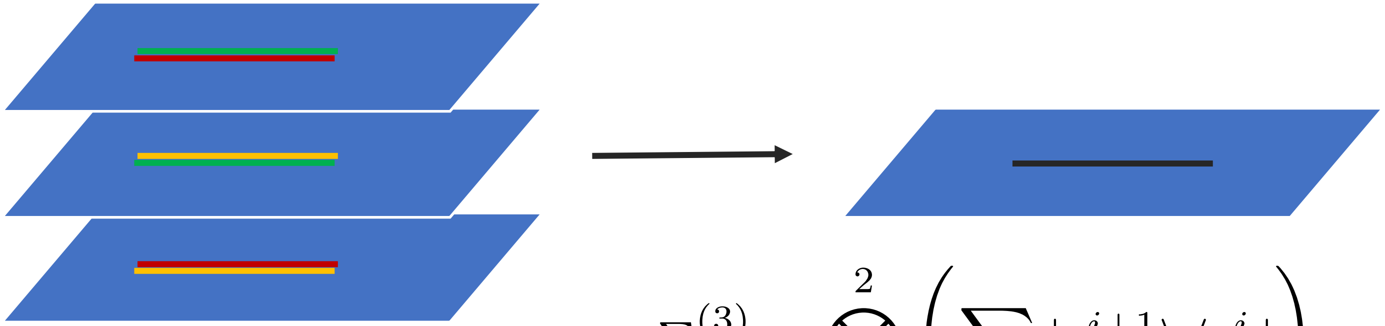
$\mathcal{C}_A^{(3)} =$ 

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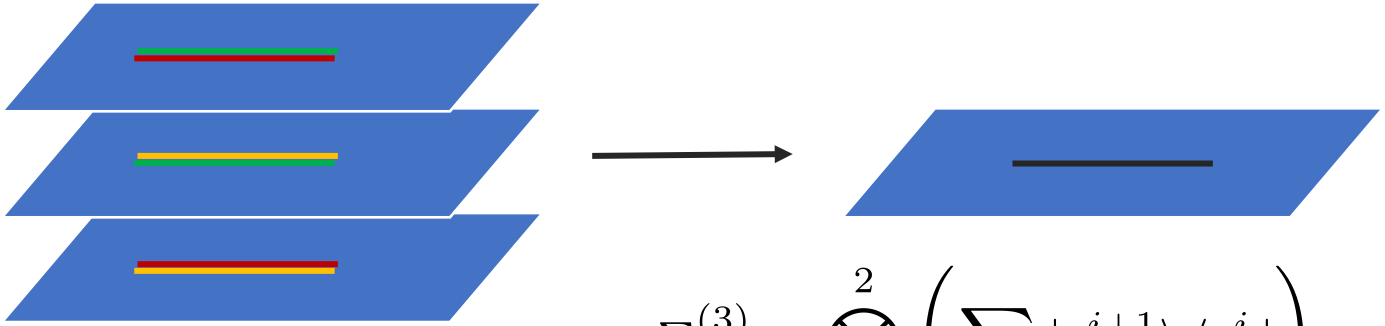
$\mathcal{C}_A^{(3)} =$ 

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The long distance expansion

OPE-like expansion of the twist operators for each region

[1006.0047 – Headrick]
[1011.5482 – Calabrese, Cardy, Tonni]
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$\mathrm{CFT}^{\otimes n}$

$$\langle \Sigma_A^{(n)} \rangle \left(1 + \tilde{\Sigma}_A^{(n)}\right) = \bigotimes_{i=0}^{n-1} \left(\sum_{a \in A} |e_a^{i+1}\rangle \langle e_a^i| \right) \otimes \mathbf{1}_{\bar{A}} \longrightarrow \tilde{\Sigma}_A^{(n)} = \sum_{\{k_j \neq \mathbf{1}\}} C^A_{\{k_j\}} \prod_{j=0}^{n-1} \Phi_{k_j}^{(j)}(x_A)$$

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At large separation between regions, keep only the lowest scaling dimension operator

$$\tilde{\Sigma}_A^{(n)} = \sum_i C_i^A \mathcal{O}^i(x_A) + \dots + \sum_{i<j} C_{ij}^A \mathcal{O}^i(x_A)\mathcal{O}^j(x_A) + \dots$$

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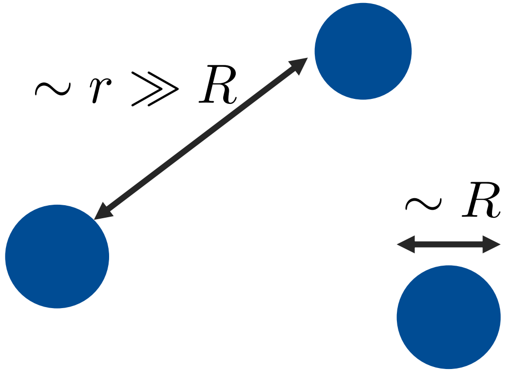
$\sim (n-1)$

We will assume a scalar

The long distance expansion

This already gives the **long distance behaviour** of the N-partite information

$$I_N(\{A_i\}) \sim \sum C^{A_1}_{i_1 j_1} \dots C^{A_N}_{i_N j_N} \langle \mathcal{O}^{i_1}_{A_1} \mathcal{O}^{j_1}_{A_1} \dots \mathcal{O}^{i_N}_{A_N} \mathcal{O}^{j_N}_{A_N} \rangle \sim \left(\frac{R}{r}\right)^{2N\Delta}$$



$$\left[\langle \mathcal{O}^i(x) \mathcal{O}^j(y) \rangle = \frac{\delta^{ij}}{|x-y|^{2\Delta}} \right]$$

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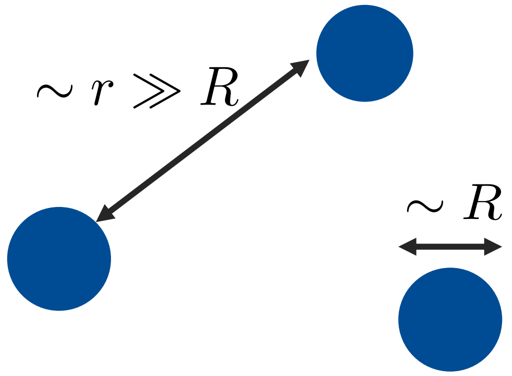

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Coefficients can be computed from correlators:

$$\left[\langle \mathcal{O}^i(x) \mathcal{O}^j(y) \rangle = \frac{\delta^{ij}}{|x-y|^{2\Delta}} \right]$$

- $$C^A_{ij} = \lim_{x\to\infty} |x - x_A|^{4\Delta} \langle \tilde{\Sigma}^{(n)}_A \mathcal{O}^i(x) \mathcal{O}^j(x) \rangle \equiv R^{2\Delta}_A C_{ij}$$
- For **spheres**, there is a trick relating the correlator to that of two modular-evolved operators

[2103.15857 – Casini, Testé, Torroba]

$$C_{ij} \underset{n\to 1}{\sim} \frac{1}{\sin^{2\Delta}\left(\frac{\pi(i-j)}{n}\right)}$$

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Spheres at long distances

Organize the correlator in terms of the **number of sheets with non-trivial insertions**:

$$I_N(\{A_i\}) \sim R^{2N\Delta} \sum C_{i_1 j_1} \ldots C_{i_N j_N} \langle \mathcal{O}^{i_1}_{A_1} \mathcal{O}^{j_1}_{A_1} \ldots \mathcal{O}^{i_N}_{A_N} \mathcal{O}^{j_N}_{A_N} \rangle$$

➜ I_2

$$I_2 = c^{(2)}_{2:2} \frac{R_1^{2\Delta} R_2^{2\Delta}}{r^{4\Delta}} \qquad c^{(2)}_{2:2} = \lim_{n\to 1} \frac{1}{n-1} \sum_{i<j} C_{ij}^2 = \frac{\sqrt{\pi}}{4} \frac{\Gamma(2\Delta+1)}{\Gamma(2\Delta+3/2)}$$

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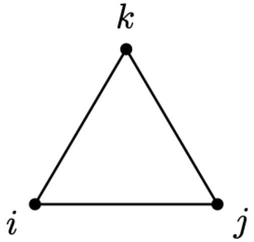
- I_3

$$c^{(3)}_{3:2} \sim \sum_{i<j} C_{ij}^3$$



$$c^{(1,1,1)}_{3:3} \sim \sum_{i<j<k} C_{ij} C_{jk} C_{ki}$$

$$I_3 = \left[c^{(1,1,1)}_{3:3} - c^{(3)}_{3:2} \left(C_{\mathcal{O}\mathcal{O}\mathcal{O}} \right)^2 \right] \left(\frac{R}{r} \right)^{6\Delta}$$

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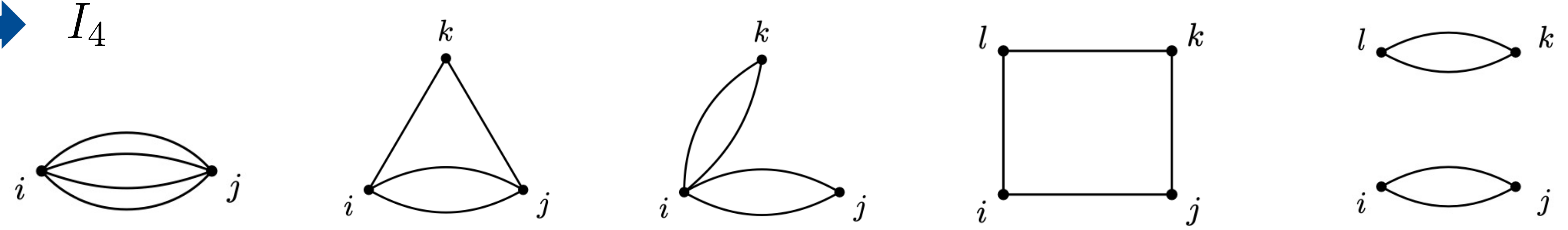
Spheres at long distances

I_4



$$\frac{I_4}{R^{8\Delta}} = \left[\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4\rangle - \frac{3}{r^{4\Delta}}\right]^2 \frac{2^{8\Delta}\Gamma(4\Delta+1)^2}{2\Gamma(8\Delta+2)} + \cdots + [\ldots]\, c_{4:4}^{(1,1,1,1)}$$

Spheres at long distances

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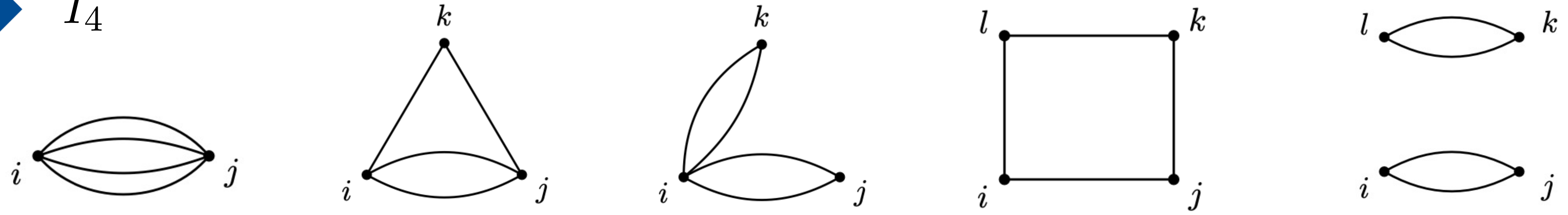

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Computed analytically for conformal dimensions ½ and 1, otherwise numerically

$$c_{4:4}^{(1,1,1,1)} \sim \int_{-\infty}^{\infty} \mathrm{d}p\, \mathrm{d}q\, \mathrm{d}r\, B_p(\Delta) B_q(\Delta) B_r(\Delta) \frac{B_{p+q+r}(\Delta)}{(e^{p+r}-1)(e^{p+q}-1)}$$

Recap

- By analyzing a long distance expansion of twist operators, we showed that for **any CFT** and in **any dimension** the long distance behaviour of the N-partite information is

$$I_N(\{A_i\}) \underset{r\gg R}{\sim} \left(\frac{R}{r}\right)^{2N\Delta}$$

! Lowest dimensional operator assumed to be a scalar

! Regions do not have to be spherical

! There might be cancellations in the prefactor

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! There might be cancellations in the prefactor

- If the regions considered are spheres, we can be more explicit about the prefactor. The leading piece of the **N-partite information** encodes the **N-point function** of the operator.

$$I_2 \longrightarrow \Delta \qquad I_3 \longrightarrow C_{\mathcal{O}\mathcal{O}\mathcal{O}} \qquad I_4 \longrightarrow \langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle$$

Going to higher N is technically involved, but there is a systematic procedure.

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The free scalar CFT

When considering a free scalar theory:

- Results should be generically applicable if a CFT has a free scalar sector which provides the lowest-dimensional operator
- Correlators factorize, and are non-zero only if they involve an even number of fields
- $$\Delta_{\text{free scalar}} = \frac{d-2}{2}$$

We will compare our results with lattice computations in d = 3

The free scalar CFT

For a free scalar theory in the lattice, EE can be obtained from two-point correlators

$$S(A) = \mathrm{Tr}\left[\left(C_A + \frac{1}{2}\right)\log\left(C_A + \frac{1}{2}\right) - \left(C_A - \frac{1}{2}\right)\log\left(C_A - \frac{1}{2}\right)\right]$$

[0212631 – Peschel]
[0905.2562 – Casini, Huerta]

$$C_A = \sqrt{X_A P_A}\,, \quad X_{ij} = \mathrm{Tr}(\rho\phi_i\phi_j)\,, \quad P_{ij} = \mathrm{Tr}(\rho\pi_i\pi_j)$$

$$X_{(0,0),(i,j)} = \frac{1}{8\pi^2}\int_{-\pi}^{\pi}\mathrm{d}x\int_{-\pi}^{\pi}\mathrm{d}y\,\frac{\cos(jy)\cos(ix)}{\sqrt{2(1-\cos(x))+2(1-\cos(y))}}$$

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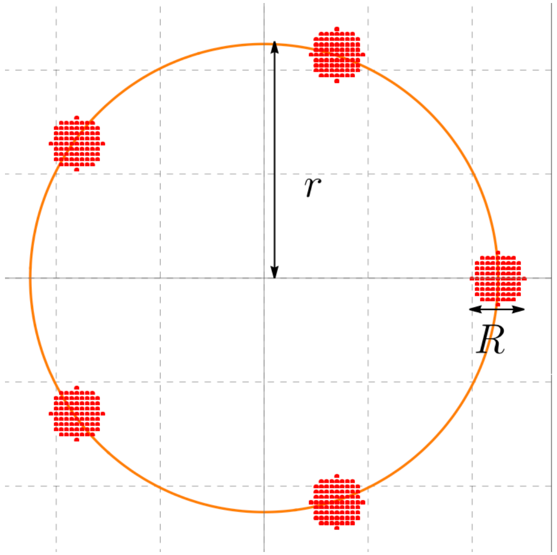

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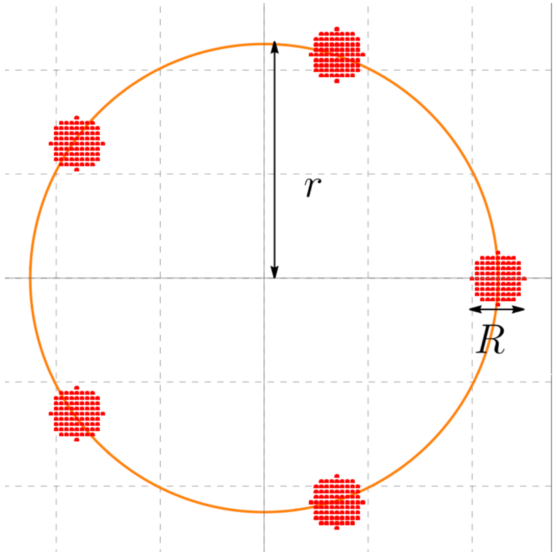

By analizing more general subregions we **verified the long distance expansion** up to N = 6

The free scalar CFT

There is very good matching between our analytical and numerical results

- $$I_2|_{d=3} = \frac{1}{48}\left(\frac{R}{r}\right)^2 \approx 0.08333\left(\frac{R}{r}\right)^2 \qquad I_2|_{d=3}^{\text{latt}} = 0.0832\left(\frac{R}{r}\right)^2$$
- $$I_3|_{d=3} = \frac{1}{12\sqrt{3}\,\pi}\left(\frac{R}{r}\right)^3 \approx 0.01531\left(\frac{R}{r}\right)^3 \qquad I_3|_{d=3}^{\text{latt}} = 0.0155\left(\frac{R}{r}\right)^3$$
- $$I_4|_{d=3} = \left(\frac{1}{180}+\frac{1}{6\pi^2}\right)\left(\frac{R}{r}\right)^4 \approx 0.0224\left(\frac{R}{r}\right)^4 \qquad I_4|_{d=3}^{\text{latt}} = 0.0207\left(\frac{R}{r}\right)^4$$

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$$I_4|_{d=3} = \left(\frac{1}{180}+\frac{1}{6\pi^2}\right)\left(\frac{R}{r}\right)^4 \approx 0.0224\left(\frac{R}{r}\right)^4 \qquad I_4|_{d=3}^{\text{latt}} = 0.0207\left(\frac{R}{r}\right)^4$$

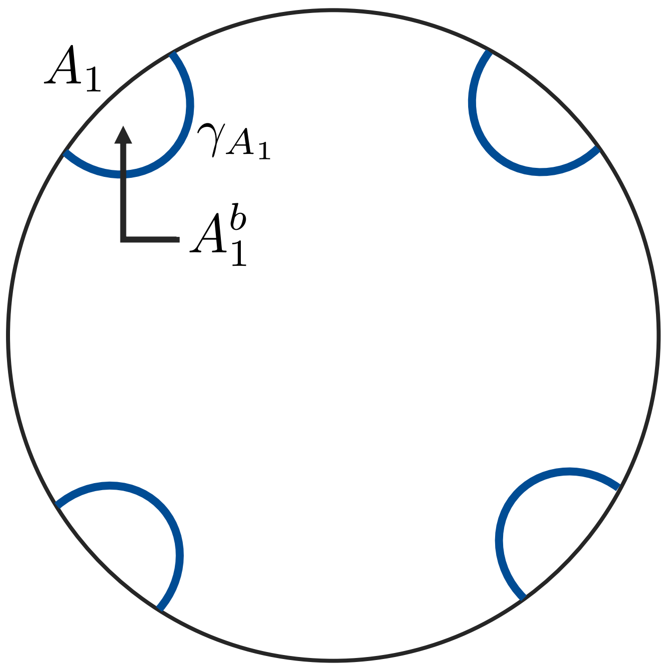
Analytical results for other d up to N = 4 and numerical ones for higher N < 7 suggest that

$$I_{N,d}|^{\text{free scalar}} > 0$$

Comments for holographic theories

In the large separation regime, the multipartite information vanishes at large N due to

$$S(A_1,\ldots,A_\alpha) = \min_{\gamma} \frac{\text{Area}(\gamma_{A_1,\ldots,A_\alpha})}{4G_N} = S(A_1)+\cdots+S(A_\alpha)$$



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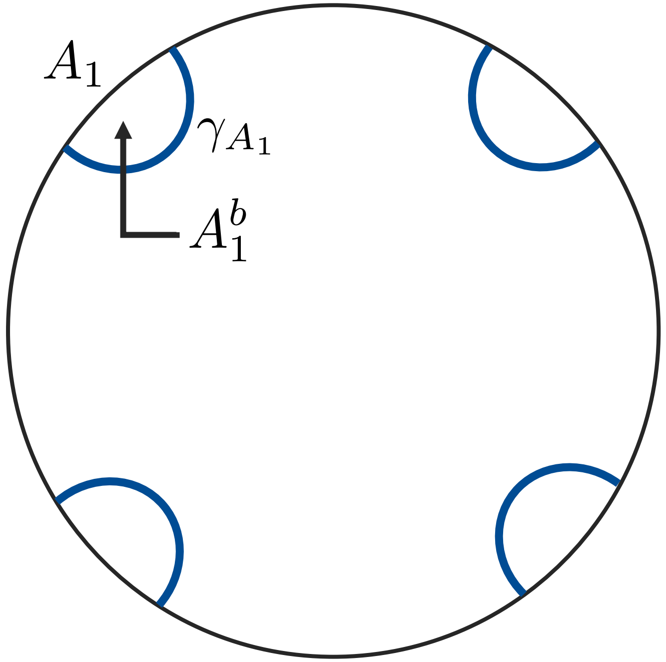
We must include the first correction in 1/N

[1307.2892 – FLM]

$$S(A)=\min_\gamma \frac{\text{Area}(\gamma_A)}{4G_N}+S_b(A^b)$$

$$I_N(A_1,\ldots,A_N)=I_N^b(A_1^b,\ldots,A_N^b)$$



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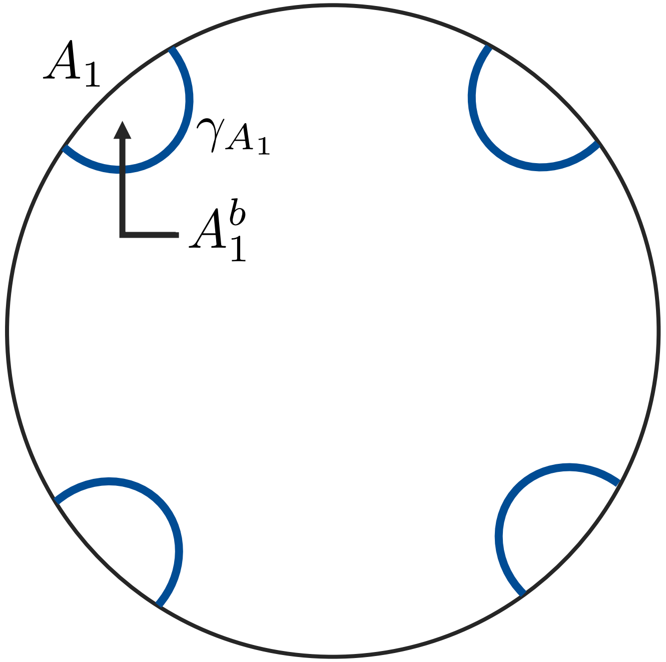

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We can prove this for **well separated spherical regions** in the boundary in an independent way, by using the bulk modular flow

Comments for holographic theories

- Extrapolate dictionary at large separation:

$$\langle \phi(x_1, z_1), \dots \phi(x_n, z_n) \rangle \underset{|x_i - x_j| \gg |z_k|}{=} \alpha_\Delta^N z_1^\Delta \dots z_N^\Delta \langle \mathcal{O}(x_1) \dots \mathcal{O}(x_N) \rangle$$

- There is an expansion of twist operators in the bulk analogous to the boundary one:

[1511.07462 – Agón, Faulkner]

$$\tilde{\Sigma}^{(n)}_{A^b} = \sum_{i<j} C^{A^b}_{ij} \phi^i(X_{A^b}) \phi^j(X_{A^b}) + \dots$$

$$C^{A^b}_{ij} = \lim_{|x - x_A| \to \infty} G(X, X_{A^b})^{-2} \langle \tilde{\Sigma}^{(n)}_{A^b} \phi^i(x) \phi^j(x) \rangle \ , \quad G(X, X_{A^b}) \underset{\text{extr.}}{\sim} \alpha_\Delta^2 \frac{z^\Delta z^\Delta_{A^b}}{|x - x_a|^{2\Delta}}$$

- The coefficients are given by a two-point function of modular evolved fields (under control for bulk hemispheres)

$$C^{A^b}_{ij} = \frac{C_{ij}}{\alpha_\Delta^2 z^2_{A^b}} \Rightarrow \tilde{\Sigma}^{(n)}_{A^b} \underset{\text{extr.}}{\sim} \tilde{\Sigma}^{(n)}_{A} \Rightarrow I^b_N(A^b_1, \dots, A^b_N) = I_N(A_1, \dots, A_N)$$

Conclusions and open questions

- We provided a proof of the long distance behaviour of the N-partite information in a CFT for general regions and in any dimension.
- For spherical regions, we showed how to systematically obtain the coefficient of the leading order term at long distances. It characterizes the lowest dimensional operator of the theory.
- We checked our results for a $d = 3$ free scalar on the lattice and for holographic theories.

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- We provided a proof of the long distance behaviour of the N-partite information in a CFT for general regions and in any dimension.
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There are several questions still to be understood:

- How to reconstruct the CFT data? Presumably, we will need to include subleading terms.
- Non-scalar lowest dimensional operators? Exact results beyond spheres?
- Is there a meaning of I_N as a bound on correlators?

$$I_2 \geq \frac{\left(\langle \mathcal{O}_1 \mathcal{O}_2 \rangle - \langle \mathcal{O}_1 \rangle \langle \mathcal{O}_2 \rangle\right)^2}{2 \|\mathcal{O}_1\|^2 \|\mathcal{O}_2\|^2}$$

¡Muchas gracias!