Conformal dualities from unoriented gauge theories

Fabio Riccioni

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INFN
Istituto Nazionale di Fisica Nucleare
Sezione di Roma

SAPIENZA
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Based on:
A. Antinucci, S. Mancani, FR, PLB 811 (2020)
A. Antinucci, M. Bianchi, S. Mancani, FR, NPB 976 (2022)
A. Amariti, M. Bianchi, M. Fazzi, S. Mancani, FR, S. Rota, JHEP 09 (2022)
A. Amariti, M. Bianchi, M. Fazzi, S. Mancani, FR, S. Rota, hep-th/2212.03913

AdS/CFT correspondence

IIB string theory on $\mathrm{AdS}_5 \times S^5$ background dual to $\mathcal{N}=4$ SYM with gauge group $SU(N)$, describing a stack of N D3-branes in flat space

[Maldacena, 1997]

More generally, for regular D3-branes probing the tip of a CY cone, the world-volume CFT is dual to IIB on $\mathrm{AdS}_5 \times \mathrm{H}_5$ background, where H_5 is Sasaki-Einstein and is the base of the cone

[Morrison, Plesser, 1998]

Gauge group: $\mathrm{G} = \prod SU(N)$, with either $\mathcal{N}=1$ or $\mathcal{N}=2$ SUSY

We will consider mainly $\mathcal{N}=1$ theories

AdS/CFT correspondence

For chiral operators, the superconformal symmetry fixes the dimension to be

$$\Delta = \frac{3}{2} R$$

There is a unique superconformal R-symmetry given by the local maximum of

$$a = \frac{3}{32} (\mathrm{Tr} R^3 - \mathrm{Tr} R)$$

[Intriligator, Wecht, 2003]

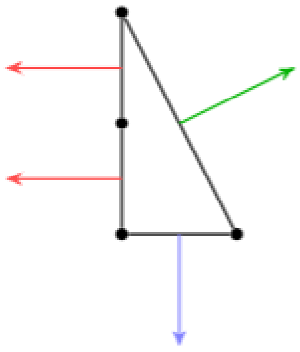
This charge is related to the inverse of the volume of the SE manifold

[Gubser, 1998]

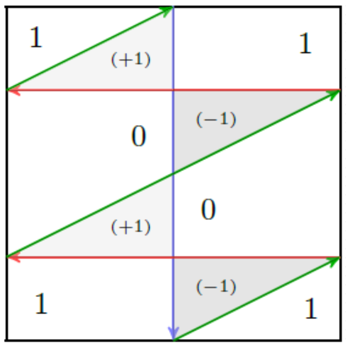
Toric geometry & dimers

We consider toric CY cones. Geometric information is encoded in the toric diagram.

Example: $\mathbb{C}^3/\mathbb{Z}_2$ orbifold

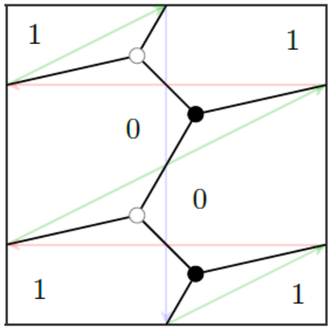


The isometry is rank 3 (at least $U(1)^3$). Performing 2 T-dualities, the configuration is mapped to D5s on a 2-torus with NS5 on top.



Brane tiling or dimer

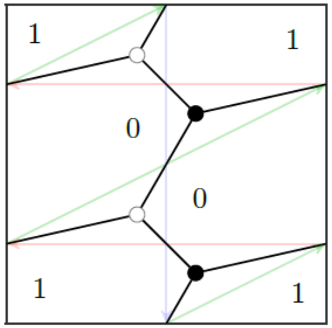
[Franco,Hanany,Kennaway,Vegh,Wecht,2005]



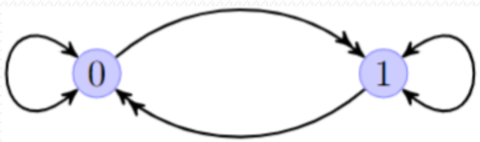
Dimers

Dictionary:

Face $\longrightarrow$ Gauge group
Edge $\longrightarrow$ Matter field
Node $\longrightarrow$ Interaction term



Quiver:



Gauge group: $SU(N)\times SU(N)$

Superpotential:

$$W_{\mathbb{C}^3/\mathbb{Z}_2} = \phi_0(X^1_{01}X^2_{10} - X^2_{01}X^1_{10}) + \phi_1(X^1_{10}X^2_{01} - X^2_{10}X^1_{01})$$

Orientifolds

Orientifolds reverse the orientation of strings. The resulting theory is unoriented.

Why orientifolds?

- They allow for SO, Sp gauge groups and tensor matter fields [Bianchi,Pradisi,Sagnotti,1990s]

- Present in all attempts to reproduce the MSSM [Wijnholt,2007]

- Change the qualitative feature of RG flow and IR dynamics [Argurio,Bertolini,2017]

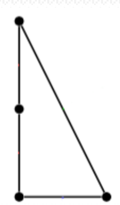
Example:
$\mathrm{O3}^{\pm}$plane, near-horizon space $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_2$. Gauge theory $\mathcal{N}=4$ with gauge group $\mathrm{U}Sp(N)$, $SO(N)$ [Witten,1998]

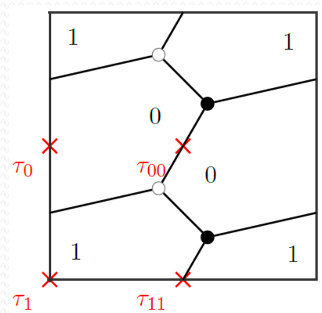
Dimers & orientifolds

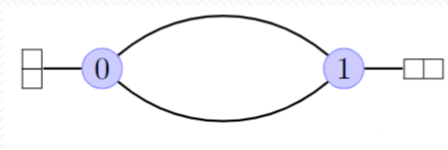
In the gauge theory, the orientifold acts as a $\mathbb{Z}_2$ involution that can be realized as fixed loci on the dimer

[Franco,Hanany,Krefl,Park,Uranga,Vegh,2007]

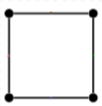
Fixed points:
$\mathbb{C}^3/\mathbb{Z}_2$ example.
Gauge group $G=SO(N_0)\times USp(N_1)$

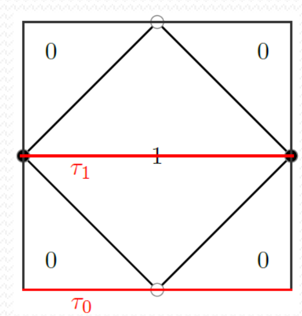


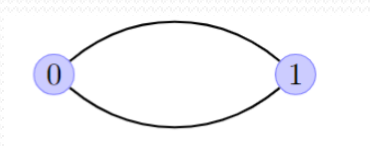




Fixes lines:
Conifold example.
Gauge group $G=SO(N_0)\times USp(N_1)$







Orientifold CFTs?

What is the fate of the conformal invariance after the orientifold involution?

-Usually broken

-'Restored' by the presence of flavour branes
[Bianchi,Inverso,Morales,Pacifici,2014]

-Same R-charges and same of the parent (at large N). In this case the central charge a is half its value in the parent (at large N).

There is actually a third possibility…

PdP_{3b} & PdP_{3c}

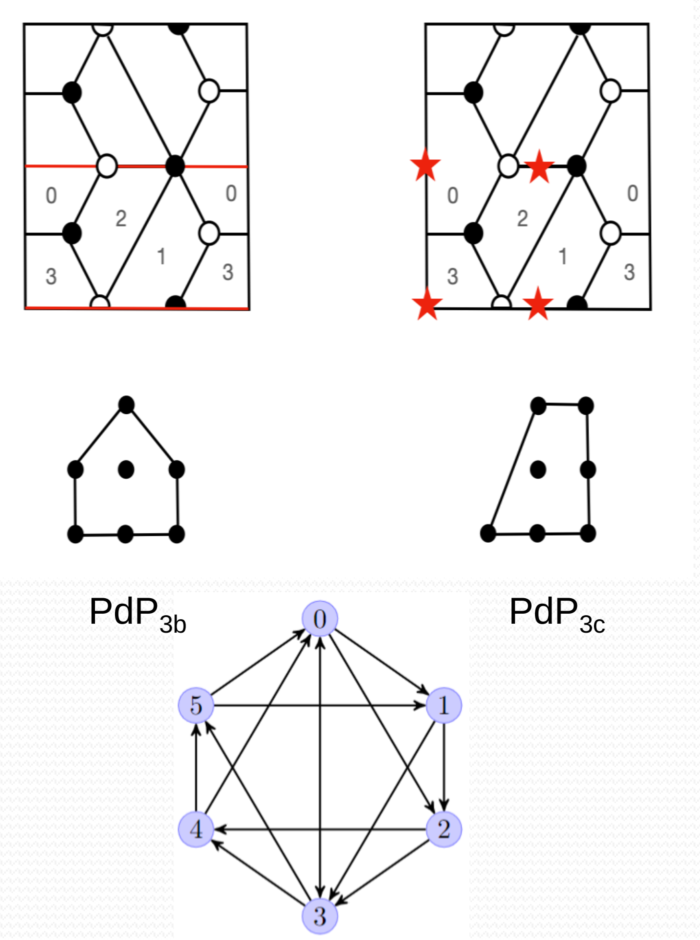
Consider a fixed-line orientifold for PdP_{3b} and a fixed-point orientifold for PdP_{3c}

In the latter case one has two possible choices for the signs denoted Ω_A and Ω_B

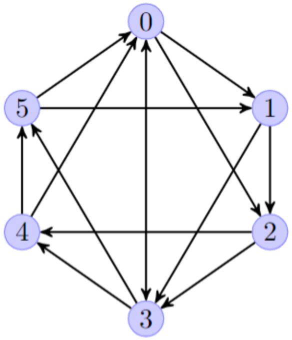
For Ω_A you recover the same R-charges of the parent theory at large N.

For Ω_B you flow to a different fixed point! The R-charges are identical to those of the PdP_{3b} orientifold Ω (even at finite N).

[Antinucci,Mancani,FR,2020]



PdP$_{3b}$ & PdP$_{3c}$



$$\Omega_A = (+,-,-,+)$$
$$SO(N)\times SU(N)\times SU(N+2)\times USp(N+2)$$
$$R_{03} = 2 - 2\frac{\sqrt{3}}{3}$$
$$R_{12} = R_{22} = R_{11} = 1 - \frac{\sqrt{3}}{3}$$
$$R_{01} = R_{02} = R_{13} = R_{23} = \frac{\sqrt{3}}{3}$$
$$a_{\Omega_A} = \frac{3\sqrt{3}}{8}N^2$$

$$\Omega_B = (-,+,-,+)$$
$$USp(N-2)\times SU(N)\times SU(N)\times SO(N+2)$$
$$R_{12} = 7 - 3\sqrt{5}$$
$$R_{02} = R_{03} = R_{13} = 3 - \sqrt{5}$$
$$R_{01} = R_{23} = R_{22} = R_{11} = 2\sqrt{5} - 4$$
$$a_{\Omega_B} = a_{\Omega} = \frac{27}{8}(5\sqrt{5} - 11)N^2$$

In Ω_B one flavour $U(1)$ becomes anomalous.

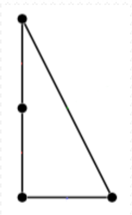
Ω_B and Ω only differ because of the superpotential.

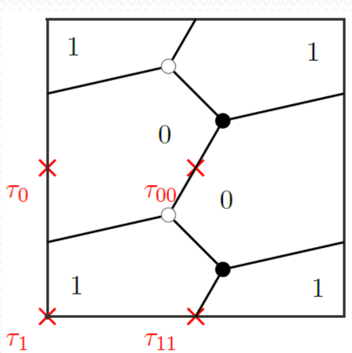
→ Conformal duality

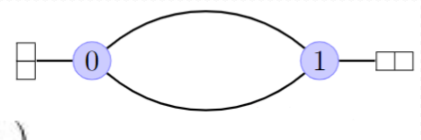
Non-chiral models

Can we find other examples? YES!

Parent: $\mathbb{C}^3/\mathbb{Z}_2$

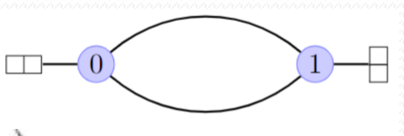






$$G = SO(N) \times USp(N-2) \qquad \Omega_A = (+,-,-,+)$$

$$W^A_{\mathbb{C}^3/\mathbb{Z}_2} = A_0 X^1_{01} X^2_{10} - S_1 X^2_{10} X^1_{01} \qquad \mathcal{N}=2 \quad a_{\Omega_A} = \frac{1}{4} N^2$$



$$G = SO(N) \times USp(N-2) \qquad \Omega_B = (+,+,-,-)$$

$$W^B_{\mathbb{C}^3/\mathbb{Z}_2} = S_0 X^1_{01} X^2_{10} - A_1 X^2_{10} X^1_{01} \qquad \mathcal{N}=1 \quad a_{\Omega_B} = \frac{27}{128} N^2$$

Non-chiral models

Compare the Ω_B orientifold with the orientifold of the conifold

$$a_{\Omega_B} = \frac{27}{128} N^2$$
$$R_S = R_A = 1$$
$$R_{01} = R_{10} = \frac{1}{2}$$
$$G = SO(N) \times USp(N-2)$$
$$W^B_{\mathbb{C}^3/\mathbb{Z}_2} = S_0 X^1_{01} X^2_{10} - A_1 X^2_{10} X^1_{01}$$

$$a_{\mathcal{C}_\Omega} = \frac{27}{128} N^2$$
$$R_{01} = R_{10} = \frac{1}{2}$$
$$G = SO(N) \times USp(N-2)$$
$$W^\Omega_{\mathcal{C}} = X^1_{01} (X^2_{01})^T X^1_{01} (X^2_{01})^T - X^1_{01} (X^1_{01})^T X^2_{01} (X^2_{01})^T$$

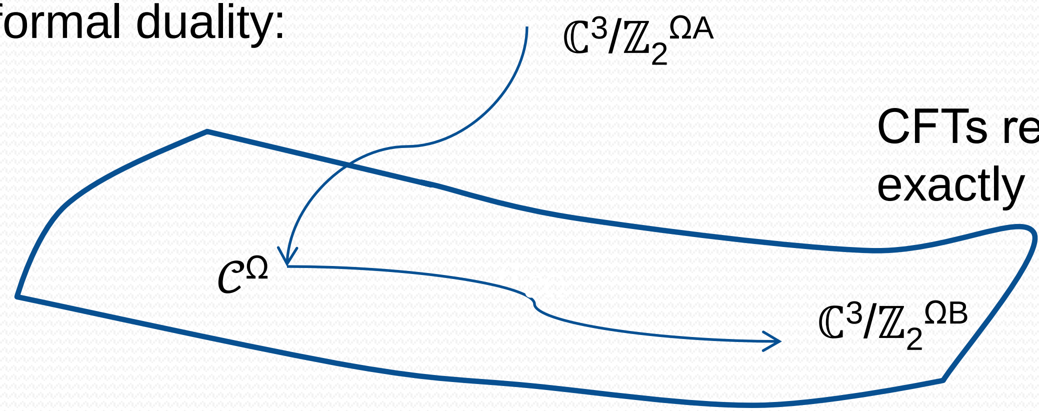
Matter content differs by tensor fields with R-charge 1. The two models have same a charge, `t Hooft anomalies and superconformal index.

The conifold arises as a mass deformation of the $\mathbb{C}^3/\mathbb{Z}_2$ orbifold [Klebanov,Witten,1998]

Same is true for Ω_A orientifold and the orientifold of the conifold. Ratio of a charges is 27/32 [Tachikawa,Wecht,2009]

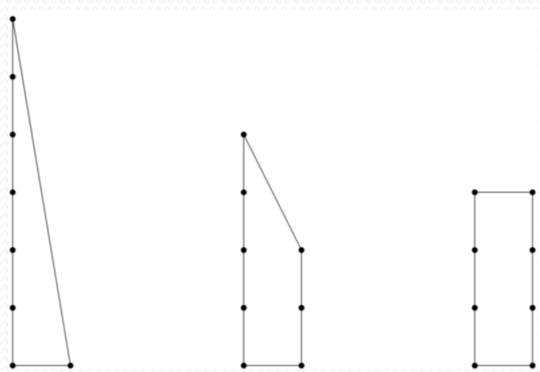
Non-chiral models

Conformal duality:

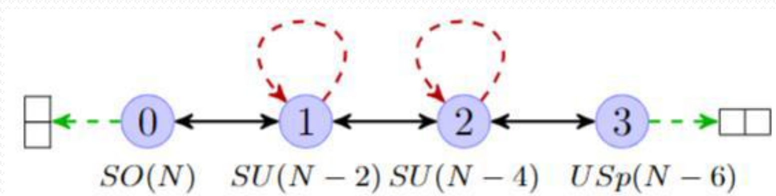


CFTs related by a `conformal mass' exactly marginal deformation

Can be generalized to an infinite class on models (elliptic models or $L^{a,b,a}$)



$L^{0,6,0}$ $L^{2,4,2}$ $L^{3,3,3}$



[Antinucci,Bianchi,Mancani,FR,2021]
[Amariti,Fazzi,Rota,Segati,2021]

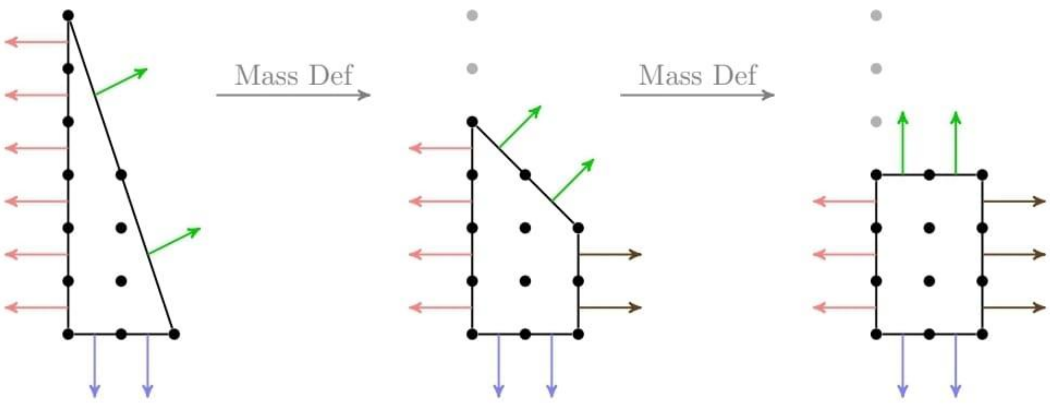
Chiral models

The results can be further extended to orientifolds of chiral $\mathbb{Z}_2$ orbifolds or the $L^{a,b,a}$ models.

The parent theories are related by chains of mass deformations

We consider $a+b=2k$, with k fixed along the chain

We consider two different types of orientifolds. In one case (A) all the groups are unitary and we only project fields.



$L^{0,6,0}/\mathbb{Z}_2$ $L^{2,4,2}/\mathbb{Z}_2$ $L^{3,3,3}/\mathbb{Z}_2$

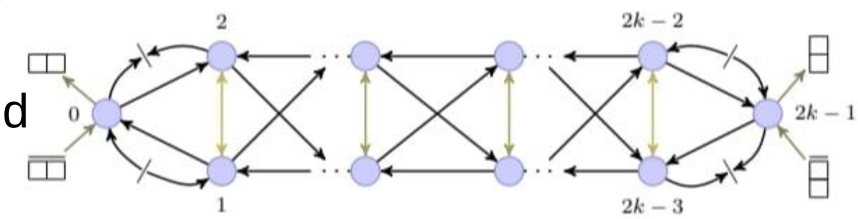
In the second case (B) four gauge groups are projected.

[Amariti,Bianchi,Fazzi,Mancani,FR,Rota,2022]

Chiral models

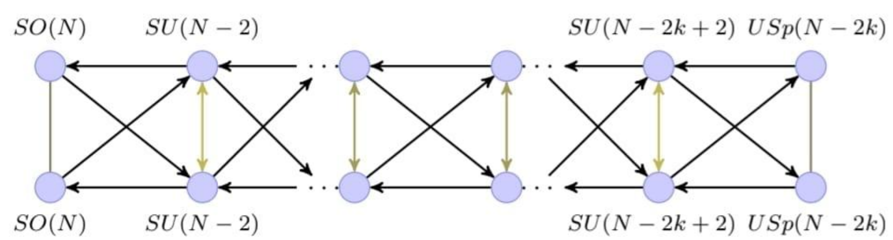
In both cases the light-coloured chiral fields have R-charge 1.

Case A: the groups all have the same rank. The last model, namely the orientifold of $L^{k,k,k}/\mathbb{Z}_2$, is realized as a glide orientifold projection



[García-Valdecasas,Meynet,Pasternak,Tatitscheff,2021]

Case B: the last model is realized as a fixed-line orientifold

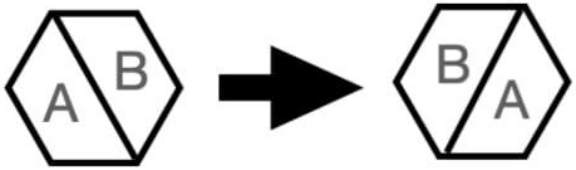


Back to where we started

Apart from the first PdP example, all the models we have discussed have fields with R-charge 1.

Recall the basic feature of the PdP example: two toric models give rise to the same quiver.

This occurs because of a flip of the diagonal of an exagon.



We want to generalize this: we want to find more general `multi-planarizable' quivers, that is quivers that arise from different dimers.

Remarkably, this generalization will allow us to better understand why the models with the same central charge that we construct are actually conformally dual.

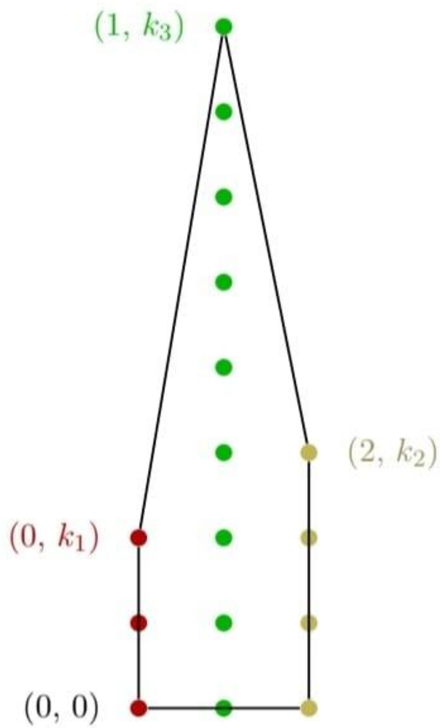
[Amariti,Bianchi,Fazzi,Mancani,FR,Rota,hep-th/2212.03913]

Back to where we started

Consider the toric diagram characterized by the integers (k_1,k_2,k_3). The PdP case corresponds to (1,1,2) and (0,2,2).

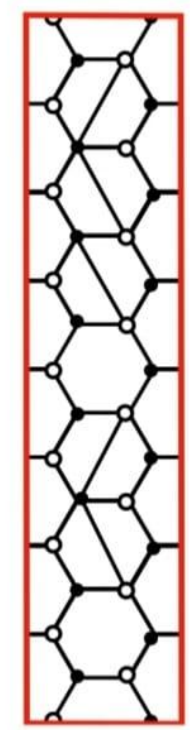
There are k_1 hexagons cut in the NW-SE direction, and k_2 in the NE-SW direction.

We require $k_3 \geq k_1 + k_2$, which is saturated when there are no hexagons in the central column.



Flipping the diagonal of each hexagon leaves the quiver invariant. It clearly modifies the superpotential.

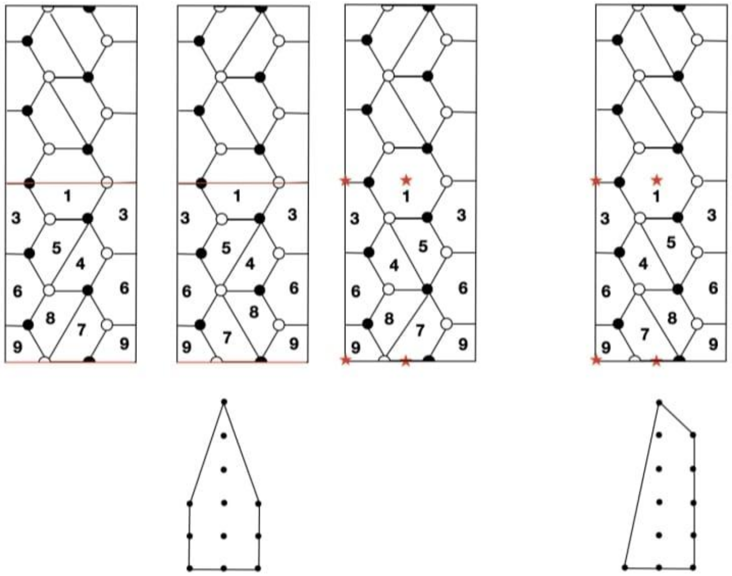
We find that in general after performing Seiberg dualities the superpotentials only differ by some signs.



(2,2,5)-(0,4,5) example

In this example there are different toric phases of the (2,2,5)-theory that are no longer dual after the orientifold.

We show that all these models are conformally dual by showing that after suitable Seiberg dualities the superpotentials only differ by some signs. No field redefinition can reabsorb the relative sign.



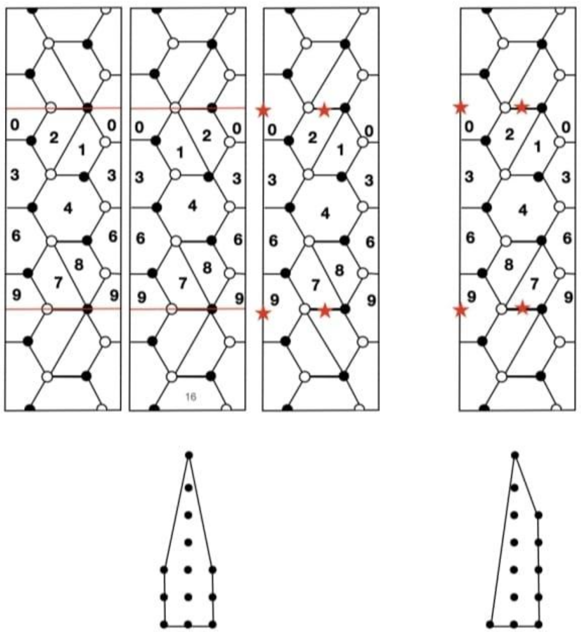
E.g. for the first two models, after dualizing w.r.t. group 5 one gets

$$W^{\Omega_1 \text{f.l.}}_{2,2,5} = (\ldots) + M_{46}X_{68}X_{84} - X_{46}X_{67}X_{78}X_{84}$$

$$W^{\Omega_2 \text{f.l.}}_{2,2,5} = (\ldots) + M_{46}X_{67}X_{78}X_{84} - X_{46}X_{68}X_{84}$$

which differ by a sign after making the field redefinition $N^{\pm}_{46} = X_{46} \pm M_{46}$

(2,2,6)-(0,4,6) example



This is another example in which different dual toric phases lead to conformally dual models after the orientifold.

The `ranks' of the gauge groups are

$$N_0 = N \qquad N_1 = N_2 = N - 2 \qquad N_3 = N - 4$$
$$N_4 = N - 6 \qquad N_6 = N - 8 \qquad N_7 = N_8 = N - 10$$
$$N_9 = N - 12$$

The projected groups are $SO(N_0)$ and $USp(N_9)$

Summary and conclusions

-We find infinite families of sets of different conformally dual unoriented theories.

- The models are related by exactly marginal deformations which are either \`conformal mass' deformations for fields of R-charge 1 or generalizations of beta deformations (sign flips in some terms of the superpotential)

- The PdP case is the most subtle because it requires the use of deconfinement tricks

- In one family of conformally dual theories, a crucial role is played by a new type of orientifold without fixed points (glide orientifold)

- We would like to have a less case-by-case understanding on the gauge-theory side

- More importantly, we would like to have some geometrical understanding of this mechanism on the gravity side