Flavored anisotropic black holes

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(Based on 2208.04958, in collaboration with A. Garbayo, C. Hoyos, N. Jokela and A. V. Ramallo)

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1st version of the gauge/gravity correspondence (Maldacena '97):

$$\mathcal{N} = 4 \text{ SYM}, SU(N) \Leftrightarrow \text{type IIB strings on } AdS_5 \times S^5$$

• Only adjoint matter

Fundamentals? $\Rightarrow$ Add N_f flavor D-branes (Karch '01)

- N_f small $\rightarrow$ Flavors as probes $\rightarrow$ Non-dynamical, infinitely massive quarks
- N_f large $\rightarrow$ Backreaction of flavor branes! $\rightarrow S_{SUGRA} + S_{branes}$

• flavor branes = sources to sugra eoms $\rightarrow$ violation of Bianchi id. for fluxes $dF \neq 0$

• If the sources are localized $\rightarrow dF \sim \delta(x)$ Challenging equations!

$\Rightarrow$ Use smeared sources: a continuous distribution of branes $\rightarrow$ avoids $\delta(x)$

D3-D5 setup

	0	1	2	3	4	5	6	7	8	9
D3	x	x	x	x	-	-	-	-	-	-
D5	x	x	x	-	x	x	x	-	-	-

- Defect in (x^0, x^1, x^2) where fundamentals live
- (2+1)-d fundamentals coupled to gauge theory in (3+1)-d
- Veneziano limit: $N_c \to \infty, N_f \to \infty, \frac{N_f}{N_c} \sim \text{finite}$
- D3 branes on the tip of a cone over a Sasaki-Einstein space
- Flavor backreaction $\to \mathcal{M}_5$ deformation $ds^2_{SE} = ds^2_{KE} + [...](d\tau + A)^2$

Previous works

• 1607.04998 (Conde, Lin, J. M. P., Ramallo, Zoakos): massless anisotropic bckg. Scaling solution:

$$ds^2 = \frac{r^2}{R^2}\left[dx^2_{1,2} + \left(\frac{4Q_f}{3}\right)^{\frac{4}{3}} \frac{(dx_3)^2}{r^{\frac{4}{3}}}\right] + R^2\frac{dr^2}{r^2} + \bar{R}^2\left[ds^2_{KE} + \frac{9}{8}(d\tau + A)^2\right] \qquad (1)$$

• 1710.00548 (J. M. P., Ramallo, Zoakos): massless black hole:

$$ds^2 = \frac{r^2}{R^2}\left[-\left(1-\frac{r_h^{\frac{10}{3}}}{r^{\frac{10}{3}}}\right)(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + \left(\frac{4Q_f}{3}\right)^{\frac{4}{3}}\frac{(dx_3)^2}{r^{\frac{4}{3}}}\right]$$
$$+R^2\left(1-\frac{r_h^{\frac{10}{3}}}{r^{\frac{10}{3}}}\right)^{-1}\frac{dr^2}{r^2} + \bar{R}^2\left[ds^2_{KE} + \frac{9}{8}(d\tau + A)^2\right] \quad (2)$$

• 1901.02020 (Jokela, J. M. P., Ramallo, Zoakos): massive bckg. $Q_f \to Q_f p(r)$. $p(r < r_q) = 0$. r_q 'cavity' $\sim$ quark mass

• 2001.08218 (Hoyos, Jokela, J. M. P., Ramallo): bckg with non-monotonic 'flavor' profiles

In this work we address

• Backreaction of a chemical potential

• Analytic solution for $N_f \to 0$

- Turn on worldvolume gauge field on D5s

$$\mathcal{F} = A_t'(\rho) d\rho \wedge dt \tag{3}$$

($\rightarrow$ dual to a baryon density)
$S = S_{IIB} + S_{branes}$

$$S_{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \Big(R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{e^{-\phi}}{2\cdot 3!} H_3^2 - \tfrac{e^{2\phi}}{2} F_1^2 - \tfrac{e^{2\phi}}{2} F_1^2$$
$$- \tfrac{e^{\phi}}{2\cdot 3!} F_3^2 - \tfrac{1}{4\cdot 5!} F_5^2 \Big) - \frac{1}{2\kappa_{10}^2} \int \tfrac{1}{2} C_4 \wedge H_3 \wedge F_3$$
$$S_{branes} = -T_5 \sum_{N_f} \int_{\mathcal{M}_6} d^6\xi\, e^{\frac{\phi}{2}} \sqrt{-det(\hat{g} - e^{-\frac{\phi}{2}} \mathcal{F})} + S_{WZ}$$
$$S_{WZ} = T_5 \int \Xi \wedge \left(C_6 - C_4 \wedge \mathcal{F} + \tfrac{1}{2!} C_2 \wedge \mathcal{F} \wedge \mathcal{F} - \tfrac{1}{3!} C_0 \mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F} \right) \tag{4}$$

Ξ **smearing form**, encondes distribution of the D5 charge

$$ds_{SE}^2 = ds_{KE}^2 + (d\tau + \mathcal{A})^2 \tag{5}$$

Canonical basis e^i for the KE. Kähler form:

$$J_{KE} = e^1 \wedge e^2 + e^3 \wedge e^4, \; J_{KE} = \frac{d\mathcal{A}}{2}, \;\; \hat{\Omega}_2 = e^{3i\tau}(e^1 + ie^2) \wedge (e^3 + ie^4) \tag{6}$$

Ansatz

$$ds_{10}^2 = h^{-\frac{1}{2}}[-bdt^2 + (dx^1)^2 + (dx^2)^2 + \alpha^2(dx^3)^2] + h^{\frac{1}{2}}[\frac{F^2S^8\alpha^2}{b\rho^{10}}d\rho^2 + S^2 ds_{KE}^2 + F^2(d\tau + \mathcal{A})^2] \quad (7)$$

$$F_1 = H_3 = 0$$

$$F_5 = F_5^{(0)} + F_5^{cp} + \star F_5^{cp} \quad (8)$$

$$F_5^{(0)} = K(\rho)(1+\star)d^4x \wedge d\rho \quad (9)$$

$F_5^{(0)}$ closed fixes $K(\rho)$

$$F_5^{cp} = dC_4^{cp}, \quad C_4^{cp} = J(\rho)dx^1 \wedge dx^2 \wedge \mathrm{Re}(\hat{\Omega}_2) \quad (10)$$

D5s $\rightarrow F_3$

$$F_3 = Q_f dx^3 \wedge \mathrm{Im}(\hat{\Omega}_2) + F_{123}dx^1 \wedge dx^2 \wedge dx^3 \quad (11)$$

We obtain the smearing form:

$$dF_3 = 2\kappa_{10}^2 T_5 \Xi \quad (12)$$

$$2\kappa_{10}^2 T_5 \Xi = -3Q_f dx^3 \wedge \mathrm{Re}(\hat{\Omega}_2) \wedge (d\tau + \mathcal{A}) \quad (13)$$

- Zero chemical potential case: $J = A_t = F_{123} = 0$

• Strategy to solve the system of equations for the geometry:
$(\{g_{\mu\nu}, \mathcal{F}_2, F_1, H_3, F_5, \phi\} \rightarrow \{h, b, \phi, F, S, \alpha, J, A_t\})$?

$\Rightarrow$ Define perturbative parameters $\epsilon = \frac{Q_f}{5\rho_h}$ and δ such that

$$F_{123} = \epsilon\delta, \qquad J(\rho) = \epsilon\delta j(\rho) \tag{14}$$

• At $\epsilon = \delta = 0$, we impose $AdS_5 \times \mathcal{M}_5$

$\Rightarrow$ Two kinds of **analytic** solutions:

1. Black hole with $J = A_t = F_{123} = 0$
2. Black hole with non zero J, A_t, F_{123}

- **Case $J = A_t = F_{123} = 0$**: define $\tilde{\rho} = \frac{\rho}{\rho_h}$ and $\Omega(\tilde{\rho})$:

$$\Omega(\tilde{\rho}) \equiv \frac{5}{4}\left[2\arctan\tilde{\rho} + \log\left(\frac{\tilde{\rho}^4}{(\tilde{\rho}+1)^2(\tilde{\rho}^2+1)}\right) - \pi\right] . \qquad (15)$$

To first order in ϵ:

$$b = 1 - \frac{1}{\tilde{\rho}^4}\,, \quad \phi = \epsilon\,\Omega\,, \quad \alpha = 1 - \epsilon\,\Omega\,, \quad h = \frac{Q_c}{4\rho_h^4\tilde{\rho}^4}\left(1 - \epsilon\,\Omega\right)\,,$$

$$G \equiv b g_{\rho\rho} h^{-\frac{1}{2}} = 1 + \epsilon\left(\frac{1}{2}\,\tilde{\rho}^4\Omega + \frac{5}{8}\left(4\tilde{\rho}^3 - 1\right)\right)\,,$$

$$F = \rho_h\,\tilde{\rho}\left(1 + \epsilon\,F_1\right)\,, \qquad S = \rho_h\,\tilde{\rho}\left(1 + \epsilon\,S_1\right)\,, \qquad (16)$$

$$F_1 = \frac{3\sqrt{2}\,\pi}{8}\left(P(\tilde{\rho})\,\mathcal{I}_Q^{(1)}(\tilde{\rho}) + Q(\tilde{\rho})\,\mathcal{I}_P^{(1)}(\tilde{\rho})\right) + \frac{1}{2}\,\tilde{\rho}^3 - \frac{1}{8} + \frac{1}{10}\left(\tilde{\rho}^4 + 2\right)\Omega\,,$$

$$S_1 = -\frac{3\sqrt{2}\,\pi}{32}\left(P(\tilde{\rho})\,\mathcal{I}_Q^{(1)}(\tilde{\rho}) + Q(\tilde{\rho})\,\mathcal{I}_P^{(1)}(\tilde{\rho})\right) + \frac{1}{2}\,\tilde{\rho}^3 - \frac{1}{8} + \frac{1}{10}\left(\tilde{\rho}^4 + 2\right)\Omega\,.$$

$$\mathcal{I}_P^{(1)}(x) = \int_1^x z^2 P(z)\,dz\,, \qquad \mathcal{I}_Q^{(1)}(x) = \int_x^\infty z^2 Q(z)\,dz\,.$$

With: $P(\tilde{\rho}) = F[-\frac{1}{2}, \frac{3}{2}; 1, 1 - \tilde{\rho}^4]$, $\quad Q(\tilde{\rho}) = (2\tilde{\rho}^4 - 1)F[\frac{5}{4}, \frac{3}{4}; 2; (2\tilde{\rho}^4 - 1)^{-2}]$

- **Case $J = A_t = F_{123} \neq 0$**: $F_{123} = \epsilon\delta$, $J(\tilde\rho) = \epsilon\delta j(\tilde\rho)$. Define $\tilde\delta = \frac{Q_c}{10\sqrt{6}}\frac{\delta}{\rho_h^3}$

$$b = \left(1 - \tfrac{1}{\tilde\rho^4}\right)\left[1 - 4\,\epsilon\,\tilde\delta^2\left(\Omega + \tfrac{5}{\tilde\rho}\right)\right], \quad \phi = \epsilon\,\Omega - \epsilon\,\tilde\delta^2\left(\Omega + \tfrac{5}{\tilde\rho}\right),$$

$$\alpha = 1 - \epsilon\,\Omega - \epsilon\,\tilde\delta^2\left(\Omega + \tfrac{5}{\tilde\rho}\right), \quad h = \tfrac{Q_c}{4\rho_h^4\tilde\rho^4}\left[1 - \epsilon\,\Omega - 3\,\epsilon\,\tilde\delta^2\left(\Omega + \tfrac{5}{\tilde\rho}\right)\right],$$

$$G = 1 - \tfrac{1}{2}\epsilon\left(\tfrac{5}{4}\left(1 - 4\tilde\rho^3\right) - \tilde\rho^4\,\Omega\right) - \epsilon\,\tilde\delta^2\left(\tfrac{5}{8}\left(20\tilde\rho^3 - 5 + \tfrac{4}{\tilde\rho}\right) + \tfrac{5}{2}\,\tilde\rho^4\,\Omega\right),$$

$$F = \rho_h\,\tilde\rho\left(1 + \epsilon\,F_1 + \epsilon\,\tilde\delta^2\,F_2\right), \quad S = \rho_h\,\tilde\rho\left(1 + \epsilon\,S_1, + \epsilon\,\tilde\delta^2\,S_2\right), \qquad (17)$$

$$F_2 = -\tfrac{\pi}{4\sqrt{2}}\left(P(\tilde\rho)\,\mathcal{I}_Q^{(2)}(\tilde\rho) + Q(\tilde\rho)\,\mathcal{I}_P^{(2)}(\tilde\rho)\right) - \tfrac{5}{2}\,\tilde\rho^3 + \tfrac{5}{8} + \tfrac{9}{2\,\tilde\rho} + \tfrac{1}{2}(2 - \tilde\rho^4)\,\Omega,$$

$$S_2 = \tfrac{\pi}{16\sqrt{2}}\left(P(\tilde\rho)\,\mathcal{I}_Q^{(2)}(\tilde\rho) + Q(\tilde\rho)\,\mathcal{I}_P^{(2)}(\tilde\rho)\right) - \tfrac{5}{2}\,\tilde\rho^3 + \tfrac{5}{8} + \tfrac{9}{2\,\tilde\rho} + \tfrac{1}{2}(2 - \tilde\rho^4)\,\Omega.$$

Where: $\mathcal{I}_P^{(2)}(x) = \int_1^x \frac{P(z)}{z^2}\,dz$, $\mathcal{I}_Q^{(2)}(x) = \int_x^\infty \frac{Q(z)}{z^2}\,dz$

And for J:

$$j(\tilde\rho) = -\frac{Q_c\left[\Gamma\left(\frac{3}{4}\right)\right]^2}{8\sqrt{\pi}\,\rho_h}\left(J_1(\tilde\rho)\,\mathcal{I}_2(\tilde\rho) + J_2(\tilde\rho)\,\mathcal{I}_1(\tilde\rho)\right) . \tag{18}$$

where:

$$J_1(\tilde\rho) = F\left(-\tfrac{3}{4}, \tfrac{3}{4}; 1; 1-\tilde\rho^4\right) \;, \quad J_2(\tilde\rho) = \left(\tilde\rho^4 - 1\right)^{-3/4} F\left(\tfrac{3}{4}, \tfrac{3}{4}; \tfrac{5}{2}; \tfrac{1}{1-\tilde\rho^4}\right) .$$

$$\mathcal{I}_1(\tilde\rho) = \int_1^{\tilde\rho} \frac{J_1(z)}{z^2}dz \;, \quad \mathcal{I}_2(\tilde\rho) = \int_{\tilde\rho}^{\infty} \frac{J_2(z)}{z^2}dz .$$

For the gauge field A_t

$$A_t'(\tilde\rho) = 2\pi\alpha'\tilde\delta\,\left(a_0(\tilde\rho) + \epsilon\, a_1(\tilde\rho)\right) \tag{19}$$

$$a_0 = -\frac{\rho_h}{\sqrt{6}\pi\alpha'}\frac{1}{\tilde\rho^2} \;,$$

$$a_1 = -\frac{\sqrt{3}\,\rho_h}{16\,\alpha'\tilde\rho^2}\left(P(\tilde\rho)\,I_Q(\tilde\rho) + Q(\tilde\rho)\,I_P(\tilde\rho) - \frac{\sqrt{2}\left(3\left(1-4\tilde\rho^3\right)-\left(3\tilde\rho^4+1\right)\frac{4}{5}\Omega\right)}{3\pi} + \frac{80\sqrt{2}\,\rho_h}{\pi\,Q_c}j(\tilde\rho)\right) .$$

- What is the meaning of this expansion? $\rightarrow$ Let us define the:
- **quark density**: $n_q = -\frac{N_c}{4\pi^2 g_s \alpha'} F_{123}$
- **baryon density** $n_b = \frac{n_q}{N_c}$ (and defect density $n_f = \frac{N_f}{L_3}$)

$$\Rightarrow \quad \tilde{\delta} = -\frac{4\sqrt{2}\,\alpha' g_s}{\sqrt{3}} \frac{n_b}{n_f} \frac{1}{T^2}\left(1 + \mathcal{O}\left(\frac{n_f}{T}\right)\right), \quad \epsilon\tilde{\delta}^2 = \gamma \frac{n_q^2}{Q_f \rho_h^5} \tag{20}$$

Where T is the temperature (linearized gravity):

$$T = \frac{1}{2\pi}\frac{1}{\sqrt{g_{\rho\rho}}}\partial_\rho \sqrt{-g_{tt}}|_{\rho\to\rho_h} = \frac{2\rho_h}{\pi Q_c^{\frac{1}{2}}}\left(1 - \frac{15}{8}\epsilon - \frac{5}{8}\epsilon\hat{\delta}^2\right) \tag{21}$$

$\Rightarrow$ We can derive the **thermodynamics**:
- Entropy density s from Bekenstein-Hawking:

$$s = \frac{\mathrm{Vol}(\mathcal{M}_5)}{(2\pi)^6 \alpha'^4 g_s^2} Q_c^{\frac{1}{2}} \rho_h^3 \left(1 + \frac{15}{8}\epsilon + \frac{5}{8}\epsilon\hat{\delta}^2\right) \tag{22}$$

- Internal energy $\mathcal{E} = E_{ADM} V_3^{-1}$
- Free energy $f = \mathcal{E} - Ts$

We can rewrite the expansion parameters as:

$$\epsilon = \frac{\lambda^{\frac{1}{2}}}{20 v_T N_c \bar{a}^{\frac{1}{2}}} \frac{n_f}{T} + \ldots, \quad \epsilon \hat{\delta}^2 = \frac{4 v_T}{5\pi^4} \frac{\lambda^{\frac{1}{2}}}{N_c \bar{a}^{\frac{1}{2}}} \frac{n_b^2}{n_f T^5} + \ldots \tag{23}$$

with $\bar{a} = \frac{\pi^3}{\mathrm{Vol}(\mathcal{M}_5)}$, $\lambda = 4\pi g_s N_c$, $v_T = \frac{\mathrm{Vol}(\mathcal{M}_2)}{\mathrm{Vol}(\mathcal{M}_5)}$

• What if # D5 branes change? → allow n_f to vary. 1st law of thermo (Mateos, Trancanelli):

$$d\mathcal{E} = TdS + \Phi dn_f + \mu dn_q \tag{24}$$

μ: **baryon chemical potential**
Φ: **'brane' potential** measuring cost of adding flavor branes

$$df = -sdT + \Phi dn_f + \mu dn_q \quad \Rightarrow s = -(\partial_T f)_{n_f, n_q} \ \text{(consistency check)} \tag{25}$$

Similar computations for Φ, μ.
Pressures from Gibbs energy $g = f - \Phi n_f - \mu n_q$:

$$p_{xy} = -g - \Phi n_f, \quad p_z = -g \tag{26}$$

$$p_{xy} = \left(\tfrac{1}{3} + \tfrac{10}{9}(1+\hat{\delta}^2)\epsilon\right)\mathcal{E} = \frac{\pi^2 N_c^2}{8}\bar{a}T^4\left[1 + \frac{\lambda^{\frac{1}{2}}}{N_c \bar{a}^{\frac{1}{2}}}\left(\frac{n_f}{3v_T T} + \frac{8v_T n_b^2}{\pi^4 n_f T^5}\right)\right]$$

$$p_z = \left(\tfrac{1}{3} - \tfrac{20}{9}(1+\hat{\delta}^2)\epsilon\right)\mathcal{E} = \frac{\pi^2 N_c^2}{8}\bar{a}T^4 \tag{27}$$

Speeds of sound:

$$v_{xy,z}^2 = \partial_{\mathcal{E}}\, p_{xy,z} \tag{28}$$

Also: $\partial_\mu p_{xy} = n_q,\ \partial_\mu p_z = 0$

Cross-checks of thermodynamics:

- $TS^{on-shell} = \Omega \equiv f - \mu n_q$
- Energy density and pressures from Brown-York tensor
- Chemical potential μ agrees with UV value of A_t to leading order in $\hat{\delta}$

Hydrodynamics at zero μ

Let's use the 4-d reduction (1710.00548), perturb the metric and get shear mode. Impose a relation:

$$\omega = -iD_\eta q^2(1 + \tau_s D_\eta q^2) + \ldots \tag{29}$$

$$\frac{\eta}{s} = TD_\eta = \frac{1}{4\pi}, \quad \frac{\tau_s}{2\pi T} = 1 - \log 2 + \epsilon\frac{5}{2}(\pi - 3) \tag{30}$$

• η: shear viscosity
• Increase in τ relative to AdS_5 value ⇒ Intersection between D3-D5 → (2+1)-d CFT → AdS_4, ($\tau_s^{CFT_{2+1}}$ is larger than $\tau_s^{CFT_{3+1}}$)

Quark-antiquark potentials
Take fundamental string with endpoints lying at the UV bdy:

$$S_{NG} = \frac{1}{2\pi\alpha'} = \int_\Sigma d\tau d\sigma e^{\frac{\phi}{2}}\sqrt{-\det(g_2)} \tag{31}$$

Two configurations:

1. $\tau = x^0, \ \sigma = x^1, \ \rho = \rho(x^1)$, no dependence on x^2, x^3
2. $\tau = x^0, \ \sigma = x^3, \ \rho = \rho(x^3)$, no dependence on x^1, x^2

Integrals expanded as: $d_\| = d_\|^{(0)} + \epsilon d_\|^{(\epsilon)} + \epsilon\tilde{\delta}^2 d_\|^{(\epsilon\tilde{\delta}^2)}$, $V_{q\bar{q}}^\| = V_\|^{(0)} + \epsilon V_\|^{(\epsilon)} + \epsilon\tilde{\delta}^2 V_\|^{(\epsilon\tilde{\delta}^2)}$
• Regularized with 2 straight strings from ρ_h to UV ($\rho \to \infty$)

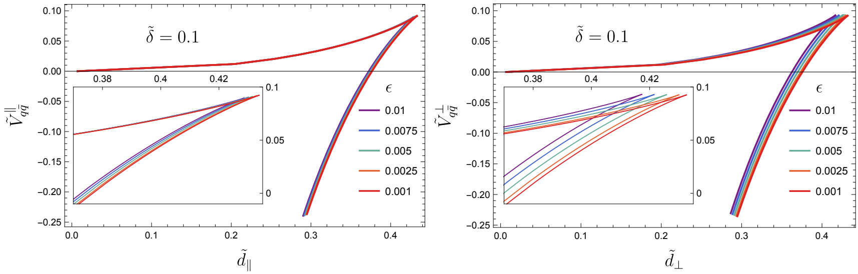

Figure: Quark-antiquark potential $\tilde{V}^a_{q\bar{q}} = \frac{\mathrm{Vol}(\mathcal{M}_5)^{\frac{1}{2}}}{\pi^{\frac{3}{2}}\,\lambda^{\frac{1}{2}}\,T}\,V^a_{q\bar{q}}$ vs $\tilde{d}_a = \pi\,T\,d_a$ $(a = \parallel, \perp)$, for fixed $\delta \sim n_q$, varying $\epsilon \sim n_f$

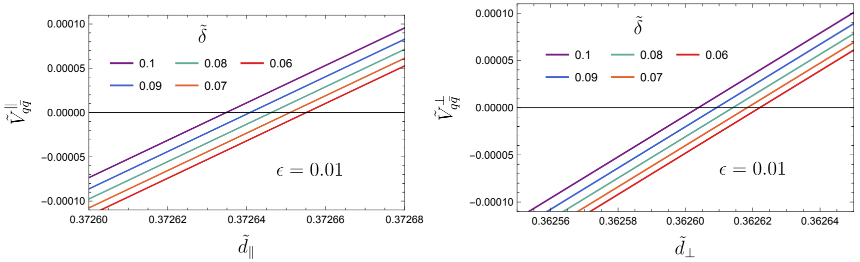

Figure: Quark-antiquark potential $\tilde{V}^a_{q\bar{q}} = \frac{\mathrm{Vol}(\mathcal{M}_5)^{\frac{1}{2}}}{\pi^{\frac{3}{2}}\,\lambda^{\frac{1}{2}}\,T}\,V^a_{q\bar{q}}$ vs $\tilde{d}_a = \pi\,T\,d_a$ ($a = \parallel, \perp$), for fixed $\epsilon \sim n_f$, varying $\delta \sim n_q$

Physical interpretation

- At d small, V follows the coloured curves until it reaches horizontal line $\Rightarrow$ disconnected configuration dominant and V is flat
- $n_f(\sim \epsilon)$ increases $\Rightarrow$ separation where charges are screened is smaller (both $\{\parallel, \perp\}$)
- Enhanced screening in both $\{\parallel, \perp\}$ due to more color non-singlet d.o.f. from fields at defects
- Effect more pronounced in transverse $\perp$ direction (string endpoints at D5 transverse)
- Increasing $n_b(\sim \tilde{\delta})$ increases screening, since also more d.o.f. contribute to screening
- Effect of (n_b) enhanced in $\parallel$ directions, along which charges localize

Entanglement entropy for slabs (EE)

Ryu-Takayanagi: 'holographic EE between spatial region A in the gauge theory and its compliment
$\Rightarrow$ find surface Σ with bdy coinciding with bdy of A minimizing S_A':

$$S_A = \frac{1}{4G_{10}} \int_\Sigma d^8\xi \sqrt{g_8} \tag{32}$$

Two configurations:

1. Parallel slab $A = \{-\frac{l_\parallel}{2} < x^1 < \frac{l_\parallel}{2}, -\infty < x^2, x^3 < \infty\}$,
   - Divergent quantity, needs regularization:

$$S_\parallel^{\rm div} = \frac{N_c^2}{2\pi} \bar{a} \frac{L_2 L_3}{\varepsilon_{UV}^2} + N_f\, N_c\, \bar{a}^{\frac{1}{2}} \frac{2}{15} \frac{\lambda^{\frac{1}{2}}}{v_\perp} \frac{L_2}{\varepsilon_{UV}} \tag{33}$$

   Flavor part diverges with area law for (2+1)-d theory from fields at the defect

2. Transverse slab $A = \left\{-\frac{l_\perp}{2} < x^3 < \frac{l_\perp}{2}, -\frac{L_1}{2} \leq x^1 < \frac{L_1}{2}, -\frac{L_2}{2} \leq x^2 < \frac{L_2}{2}\right\}$,

$$S_\perp^{\rm div} = \frac{N_c^2}{2\pi} \bar{a} \frac{L_1 L_2}{\varepsilon_{UV}^2} - N_c \bar{a}^{\frac{1}{2}} \frac{1}{30} \frac{\lambda^{\frac{1}{2}}}{v_\perp} \frac{n_f L_1 L_2}{\varepsilon_{UV}} \tag{34}$$

   Similar area law (2+1)-d behaviour, but negative contribution

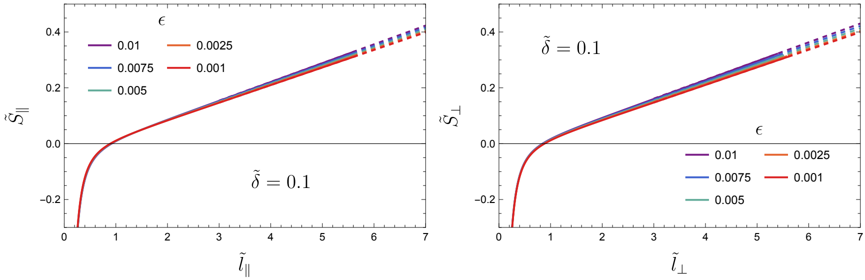

Figure: Rescaled $\parallel$ (left) and $\perp$ (right) EE: $\tilde{S} = \frac{\mathrm{Vol}(\mathcal{M}_5)}{8\,\pi^4\, L_2\, L_3\, Nc^2\, T^2} S^{\mathrm{reg}}$ vs rescaled width.

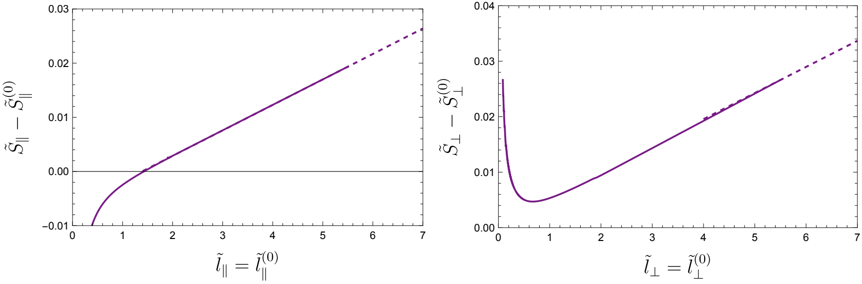

Figure: Differences in the finite contributions to the entanglement entropy between the flavored and unflavored theories. We have set $\epsilon = 0.01$ and $\tilde{\delta} = 0.1$.

⇒ 'Measure of correlation': Mutual information for regions A and B

$$I(A, B) = S(A) + S(B) - S(A \cup B) \tag{35}$$

⇒ Suggests that flavor d.o.f. are correlated in the transverse direction for small distances

Summary

- Backreacted D3-D5 intersection with massless quarks at finite T and chemical potential μ with smeared D5s
- Construction of analytic, peturbative solution in 2 parameters. Regime of validity

$$\frac{n_f}{T} \gg \frac{n_b}{T^3}, \quad \frac{N_c}{\lambda^{\frac{1}{2}}} \gg \frac{n_f}{T} \tag{36}$$

- In this regime we obtain consistent anisotropic thermodynamics and we computed quark-antiquark potentials, EE
- Results for $\mathcal{E}$, s and pressures show additional d.o.f. along the (2+1)-d intersection
- These d.o.f. also increase EE
- These d.o.f. also contribute to the screening of color charges in $q\bar{q}$ potentials
- Hydrodynamics show increase in transport coefficient. $\Rightarrow$ Tending to (2+1)-d dynamics
- EE show correlations between flavors in $\perp$ directions for small distances

What to do with this? Extensions

- Construct a numerical solution valid for any election of parameters
- Construct solutions interpolating between the black hole geometry in 1710.00548, non analytic in N_f and with Lifshitz-like scaling symmetry
- Classify all possible solutions studying different boundary conditions
- The regime of large μ and small T can be used to extract an equation of state to model neutron stars with anisotropic pressures (leading to more compact neutron stars. Connections with black hole no-hair relations)
- Add D7-brane probes to study anisotropic physics of holographic multilayer theories as in 1909.01864 (Gran, Jokela, Musso, Ramallo, Tornsö). Modelling 'graphene'
- Extend to other dimensions of defect and ambient theory: D2-D6 geometry of 1505.00210 (Faedo, Mateos, Tarrio) or D3-D3' geometry 2112.13677 (Jokela, J. M. P., Rigatos)

Thanks for your attention!