de Sitter space and braneworld holography

Juan F. Pedraza

IFT Madrid

Based on work with:

Emparan, Svesko, Tomašević & Visser [arXiv:2207.03302]
Aguilar-Gutierrez, Patra & Sasieta [W.I.P.]

Outline

- Motivation: backreaction in semi-classical gravity
- Braneworld holography I: AdS branes
- Braneworld holography II: dS branes
- Quantum black holes in dS_3
  - Thermodynamics & BH nucleation
- Doubly holographic interpretation
- ‘Wedge holography’ for dS and information transfer
- Outlook

Semi-classical gravity & backreaction

- Consider a theory of gravity coupled to matter, e.g.,

$$S = \int d^{d+1}x\sqrt{-g}\left(\frac{(R-2\Lambda)}{16\pi G_N} + \mathcal{L}_{\text{matter}}\right)$$

- We do not know how to quantize it, in general
- Often, we are interested in studying leading quantum effects
- Focus on semi-classical regime where

$$\ell_P \ll \ell \ll \ell_{\text{macro}}, \qquad \ell_P \sim (\hbar G_N)^{1/(d-1)}$$

- Treat geometry classically; quantize matter fields
- At zeroth order QFT in a fixed background. More generally,

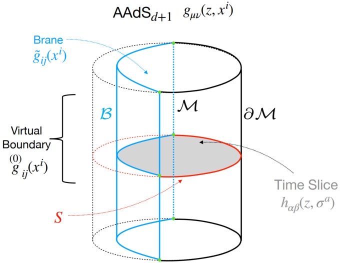
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N \langle T_{\mu\nu}\rangle$$

Technical problem: iterative calculation with increasing complexity

Braneworld holography I: AdS branes

- Start with pure AdS/CFT. Bulk AdS is dual to a CFT living on the boundary
- Introduce a pure tensional brane (analogous to [Karch-Randall]):

$$S = S_{\text{Bulk}}[\mathcal{M}] + S_{\text{GHY}}[\partial\mathcal{M}] + S_{\text{Brane}}[\mathcal{B}]\,, \qquad S_{\text{Brane}} = -\tau \int_{\mathcal{B}} d^d x \sqrt{-h}$$



- 'Integrate out' the UV [de Haro, Skenderis, Solodukhin]
- This pulls the CFT to the brane, coupled to dynamical gravity

Braneworld holography I: AdS branes

- This process yields:

$$\tilde{S}_{\text{Brane}} = S_{\text{Bgrav}}[\mathcal{B}] + S_{\text{CFT}}[\mathcal{B}]\,,$$

$$S_{\text{Bgrav}} = \frac{1}{16\pi G_d}\int_{\mathcal{B}} d^d x\sqrt{-h}\left[R - 2\Lambda_d + \ell^2(R^2\text{-terms}) + \cdots\right],$$

- $\ell \propto 1/\tau$ is a scale generated by integration
- S_{Bgrav} arises from integrating non-normalizable modes
- S_{CFT} arises from integrating normalizable modes
  $\rightarrow$ Depends on bulk state, thus left unspecified

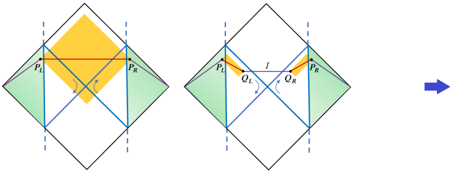
Important: Classical bulk solutions induce semi-classical solutions on the brane

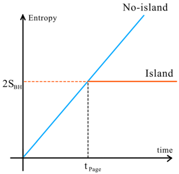
$$G_{\mu\nu} + \Lambda g_{\mu\nu} + \cdots = 8\pi G_N \langle T_{\mu\nu}\rangle$$

exactly to all orders in backreaction!

Braneworld holography I: AdS branes

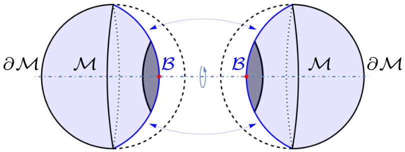
- Useful not only to derive consistent semi-classical solutions (e.g. [Emparan, Frassino, Way]), but also understand their semi-classical properties!
- Semi-classical corrections to complexity [Hernandez, Myers, Ruan; Emparan, Frassino, Sasieta, Tomašević]
- Semi-classical extended thermodynamics [Frassino, Pedraza, Svesko, Visser]
- BH evaporation [Emparan, Luna, Suzuki, Tomašević, Way]
- Entanglement islands in higher dimensions [Chen, Myers, Neuenfeld, Reyes, Sandor]. In 2D:

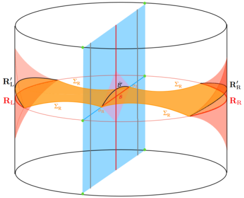




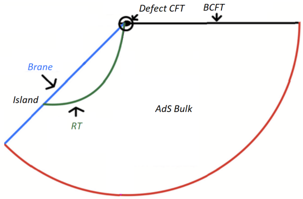
Braneworld holography I: AdS branes

- We can arrange for a similar setting by constructing a $\mathbb{Z}_2$-symmetric version, and put a BH on the brane. Islands emerge in higher dimensions:





- Via AdS/CFT, we have a 'double holographic' interpretation, with three equivalent descriptions of the same system Bulk/Brane/CFT:



Braneworld holography II: dS branes

- So far I have discussed the standard braneworld setting with AdS branes. Can we relax some of the conditions?
- Position of the brane determined by Israel junction conditions
  $\rightarrow \quad \Delta h_{ab} = 0, \quad \Delta K_{ab} - h_{ab}\Delta K = -8\pi G_N T_{ab}$
- Several ways to realize it. The picture I showed assumes a foliation of AdS_{d+1} with AdS_d slices, but we can likewise choose dS_d slices
- Start from Rindler AdS (consider a 4D bulk):

$$ds^2 = -\left(\frac{\rho^2}{\ell_4^2} - 1\right) dt_R^2 + \frac{d\rho^2}{\frac{\rho^2}{\ell_4^2} - 1} + \rho^2(d\vartheta^2 + \sinh^2\vartheta\, d\phi^2)$$

- This covers a Rindler patch with associated $T = \frac{1}{2\pi\ell_4}$.
- Next, implement the following bulk diffeo (and redefine $t = R_3 t_R/\ell_4$):

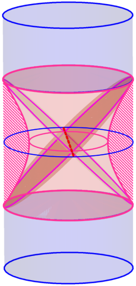
$$\frac{\hat{r}^2}{R_3^2} = \frac{\rho^2 \sinh^2\vartheta}{\rho^2\cosh^2\vartheta - \ell_4^2}\,, \quad \cosh\sigma = \frac{\rho}{\ell_4}\cosh\vartheta$$

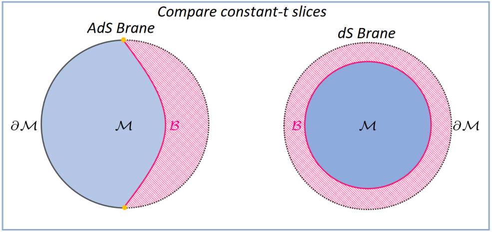
Braneworld holography II: dS branes

- One obtains AdS foliated with dS slices:

$$ds^2 = \ell_4^2 d\sigma^2 + \frac{\ell_4^2}{R_3^2}\sinh^2\sigma \left[-\left(1-\frac{\hat{r}^2}{R_3^2}\right)dt^2 + \left(1-\frac{\hat{r}^2}{R_3^2}\right)^{-1} d\hat{r}^2 + \hat{r}^2 d\phi^2 \right]$$

- The brane in this case is embedded as follows:





- In this case a positive Λ is induced on the brane theory

Adding structure: (Quantum) black holes in dS?

- Recall the Schwarzschild-dS solution:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-1}^2, \qquad f(r) = 1 - \frac{m}{r^{d-2}} - \frac{r^2}{R^2}$$

$$m = \frac{16\pi G_N M}{d-1} \text{Vol}(S^{d-1})$$

- $f(r)$ hast two real roots, corresponding to BH and cosmological horizons. Mass has a maximum value (Nariai limit).

- In 3D (d=2) the mass term is a trivial shift $m = 8G_N M$
  $\rightarrow$ No classical black holes in dS_3!

- Instead, a conical singularity with angle deficit $\delta = 2\pi\left(1 - \sqrt{1 - 8G_3 M}\right)$ (also with an upper bound on the mass)

- To see that, define $\gamma \equiv \sqrt{1 - 8G_3 M}$ and let $\{\tilde{t} = \gamma t\ ,\ \tilde{r} = \gamma^{-1} r\ ,\ \tilde{\phi} = \gamma\phi\}$:

$$ds^2 = -\left(1 - \frac{\tilde{r}^2}{R_3^2}\right) d\tilde{t}^2 + \left(1 - \frac{\tilde{r}^2}{R_3^2}\right)^{-1} d\tilde{r}^2 + \tilde{r}^2 d\tilde{\phi}^2$$

$\rightarrow$ dS metric but with $\tilde{\phi} \sim \tilde{\phi} + 2\pi\gamma$

Adding structure: (Quantum) black holes in dS?

- In 3D $G_N M$ is dimensionless. Extra scale required for BH horizon
- Claim: Quantum effects provide the extra scale, ℓ_P, and lead to black holes!
- Case study: conformal scalar in conical dS space

$$S = \frac{1}{16\pi G_3}\int d^3x\sqrt{-g}[R-2\Lambda] - \frac{1}{2}\int d^3x\sqrt{-g}\left[(\nabla\Phi)^2 + \frac{1}{8}R\Phi^2\right]$$

$$T_{\mu\nu} = \frac{3}{4}\nabla_\mu\Phi\nabla_\nu\Phi - \frac{1}{4}g_{\mu\nu}(\nabla\Phi)^2 - \frac{1}{4}\Phi\nabla_\mu\nabla_\nu\Phi + \frac{1}{4}g_{\mu\nu}\Phi\Box\Phi + \frac{1}{8}G_{\mu\nu}\Phi^2$$

- Compute $\langle T_{\mu\nu}\rangle$ by point splitting

$$\langle T_{\mu\nu}(x)\rangle = \lim_{x'\to x}\left(\frac{3}{4}\nabla^x_\mu\nabla^{x'}_\nu G - \frac{1}{4}g_{\mu\nu}g^{\alpha\beta}\nabla^x_\alpha\nabla^{x'}_\beta G - \frac{1}{4}\nabla^x_\mu\nabla^x_\nu G + \frac{1}{16R_3^2}g_{\mu\nu}G\right)$$

- Here $G(x,x')$ is the 2-pt function in conical dS

Adding structure: (Quantum) black holes in dS?

- In pure dS:

$$G(x,x') = \frac{1}{4\pi}\frac{1}{|x-x'|} + \frac{\lambda}{4\pi}\frac{1}{|x+x'|}$$

- Here $|x-x'|$ is the geodesic distance in $\mathbb{R}^{2,2}$ and λ picks particular BCs ($\lambda=0$ for transparent, $\lambda=1$ for Neumann and $\lambda=-1$ for Dirichlet)
- We pick $\lambda=0$ which corresponds to the Euclidean vacuum
- For conical dS, we use the method of images. We let $\gamma = 1/N$ for $N \in \mathbb{N}$ and at the end analytically continue to $N \in \mathbb{R}$

$$G_{\mathrm{CdS}_3}(x,x') = \sum_{n=-\infty}^{\infty} G_{\mathrm{dS}_3}(x, H^n x') = \frac{1}{4\pi}\sum_{n\in\mathbb{Z}} \frac{1}{|x - H^n x'|}$$

$$H \equiv \begin{pmatrix} \cos(2\pi\gamma) & \sin(2\pi\gamma) & 0 & 0 \\ -\sin(2\pi\gamma) & \cos(2\pi\gamma) & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Adding structure: (Quantum) black holes in dS?

- The calculation spits (recall $\gamma = \sqrt{1 - 8G_3 M}$):

$$\langle T^{\mu}_{\ \nu} \rangle = \frac{F(M)}{8\pi r^3} \mathrm{diag}(1, 1, -2)$$

$$F(M) = \hbar \frac{\gamma^3}{4\sqrt{2}} \sum_{n=1}^{N-1} \frac{3 + \cos(2\pi n\gamma)}{[1 - \cos(2\pi n\gamma)]^{3/2}}$$

- Finally, plugging this into the (linearized) Einstein's equations yields:

$$\delta g_{tt} = \delta g_{rr} = \frac{2\ell_P F(M)}{r}$$

- This generates an attractive potential, which may come from a modification of the blackening factor of the form:

$$f(r) = 1 - 8G_3 M - \frac{r^2}{R_3^2} - \frac{2\ell_P F(M)}{r}$$

- However, perturbative calculation cannot be trusted beyond leading order

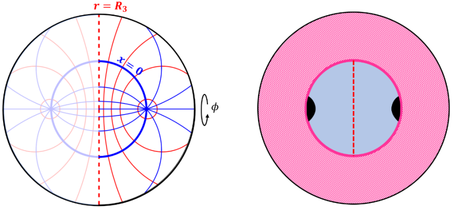
Quantum black holes in dS from braneworlds

- We can tackle this problem via braneworld holography!
- Start with an accelerated BH in AdS_4, which we take as the AdS_4 C-metric (a particular case of the Plebanski-Demianski type-D solutions):

$$ds^2 = \frac{\ell^2}{(\ell + xr)^2}\left[-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2\left(\frac{dx^2}{G(x)} + G(x)d\phi^2\right)\right]$$

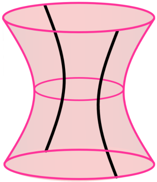
$$H(r) = 1 - \frac{r^2}{R_3^2} - \frac{\mu\ell}{r}\,, \qquad G(x) = 1 - x^2 - \mu x^3$$

- Describes two BHs accelerating back to back, but we excise one of them
- $x = 0$ satisfies 'umbilic condition' ($K_{ab} \propto h_{ab}$), trivializes junction conditions



Quantum black holes in dS from braneworlds

- The BHs localize on the brane, the spacetime picture is:



- Leads to a truly quantum BH on dS_3!

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\phi^2, \qquad f(r) = 1 - 8G_N M - r^2 H^2 - \frac{2c\ell_P F(M)}{r}$$

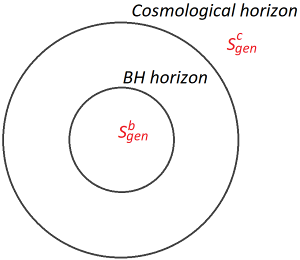
$$8G_N M = 1 - \frac{4x_1^2}{(3-x_1^2)^2}, \qquad F(M) = \frac{8(1-x_1^2)}{(3-x_1^2)^3}$$

- Same structure from perturbative analysis, but with a few key differences

Thermodynamics of quantum BHs in dS_3

- 4D horizon areas map to generalized entropies in higher order 3D gravities:

$$S_{\text{gen}}^{(b,c)} \equiv S_{\text{Wald}}^{(b,c)} + S_{\text{vN}}^{(b,c)}$$



- The horizons satisfy: $dM = T^b dS_{\text{gen}}^b$, $dM = -T^c dS_{\text{gen}}^c$, with

$$T^{(b,c)} = \kappa^{(b,c)}/2\pi$$

- Altogether:

$$T^b dS_{\text{gen}}^b + T^c dS_{\text{gen}}^c = 0$$

Entropy deficits and BH nucleation

- The probability of nucleating a small BH in dS is

$$\mathcal{P} \sim e^{-\Delta S}$$

- A brief calculation shows that for classical 4D dS BHs,

$$\Delta S = 2\pi R_4 M = \sqrt{S_0 s}$$

- [Susskind] recently argued that this behavior follows from Matrix theory, giving support to a conjecture for a dual of de Sitter
- More generally, for D-dimensional dS one finds:

$$\Delta S^{(D)} = \left(\frac{D-2}{2}\right) S_0^{\frac{1}{D-2}} s^{\frac{D-3}{D-2}}$$

- For our quantum dS BH solution in 3D we find:

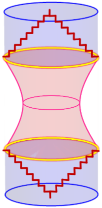
$$\Delta S^{(3)}_{\text{gen}} \sim \sqrt{S_{\text{gen},0}\, s_{\text{gen}}}$$

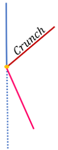
- For general quantum dS black holes in any dimension, we conjecture

$$\Delta S^{(d)}_{\text{gen}} \sim S_0^{\frac{1}{D-1}} s^{\frac{D-2}{D-1}}$$

Double holographic interpretation?

- Since our setup contains AdS/CFT we can try to see if we have a double holographic interpretation, as we do for AdS branes
- Our branes only reach the boundary at $\mathcal{I}^+$ and $\mathcal{I}^-$ (dS time). The defect is hence, a Euclidean CFT, in the same spirit as dS/CFT [Strominger,...]
- Probing this from the higher dimensional bulk seems hard. Contrary to the AdS case, our branes are accelerating, and hence emitting radiation at the quantum level. At $\mathcal{I}^+$ infinite amount of radiation is collected, which induces a crunch singularity (similarly for $\mathcal{I}^-$).

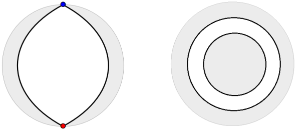




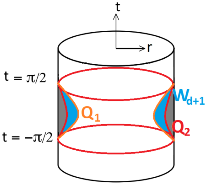
- One my try to circumvent this issue by preparing the in and out states (at $\mathcal{I}^+$ and $\mathcal{I}^-$) using a Euclidean path integral. Needs further study!

‘Wedge holography’ for dS and information transfer

- We want to study the information problem in dS [Sybesma; Aalsma, Sybesma; Hartman, Jiang, Shaghoulian; Shaghoulian; Balasubramanian, Kar, Ugajin...] in higher d. However, in our setup we have integrated out the UV and the radiation cannot be collected
- We introduce a second brane, that can act as a bath, in the spirit of ‘wedge holography’ or ‘codimension-two holography’ [Akal, Kusuki, Takayanagi, Wei; Geng, Karch, Perez-Pardavila, Raju, Randall, Riojas, Shashi]



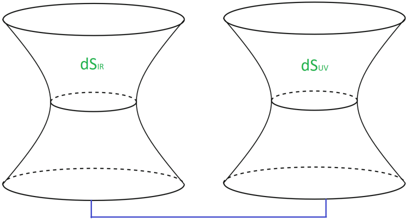
- In the full spacetime:



‘Wedge holography’ for dS and information transfer

Comments:

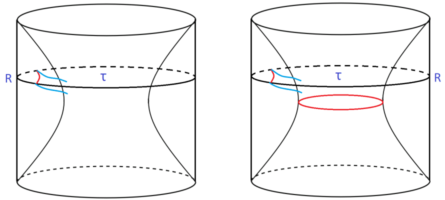
- Construction possible, provided $\tau_{IR} < 0$
- Setup describes two disconnected, but entangled universes, in the spirit of [Balasubramanian, Kar, Ugajin], with a twist: universes are explicitly coupled



- The coupling implies it is possible to send signals from one brane to another, as evident from the bulk perspective
- To study information transfer, we start by considering a physical brane connected to a non-gravitating bath. This is possible if one sends the UV brane to the boundary

'Wedge holography' for dS and information transfer

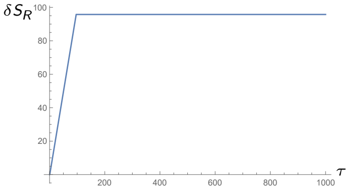
- We define a region R on the UV brane, where we are allowed to collect radiation, and find the HRT surfaces:



- There is a transition from a simple (disconnected) surface, to an 'island' surface connecting UV and IR branes.
- For large enough R the disconnected surface includes a portion that wraps the IR brane. This is analogous to global AdS black holes. This portion is needed to satisfy the homology constraint and is required to satisfy the Araki-Lieb inequality [Hubeny, Maxfield, Rangamani, Tonni]

'Wedge holography' for dS and information transfer

- Start with the original theory and partition IR and UV degrees of freedom $\mathcal{H} = \mathcal{H}_{IR} \otimes \mathcal{H}_{UV}$ (e.g. in momentum space [Balasubramanian, McDermott, Raamsdonk]). Via braneworld holography, the IR sector is geometrized and replaced by a gravitational theory in dS, while the UV sector describes a dS QFT with a gap
- Further, pick a spatial subregion in the UV sector such that $\mathcal{H}_{UV} = \mathcal{H}_R \otimes \mathcal{H}_{\bar{R}}$, and compute $S_R = -\text{Tr}[\rho_R \log \rho_R]$
- If R is the full space, then $S_R = S_{UV} = S_{IR} = S_{dS}$. This explains the disconnected term for large R
- For other R, we find a Page curve:



'Wedge holography' for dS and information transfer

Comments:

- In our setup, both τ_{Page} and the final entropy scale with the UV cutoff. This may sound strange, however S_R includes correlations between R and $\bar{R}$ and the Hilbert space of the UV dS QFT is infinite dimensional
- Further, in our setting, islands arise due to momentum space entanglement
- From the dS QFT perspective, the island phase arise due to the gap. In confining models of 'holographic QCD' a similar transition was observed since the early days of holography (e.g. [Klebanov, Kutasov, Murugan]), way before 'entanglement islands'
- Qualitatively the same in any dimension
- The case with dynamical gravity on both branes doesn't lead to interesting evolution. In this case HRT surfaces become trivial as one imposes Neumann BC on both branes [Geng, Karch, Perez-Pardavila, Raju, Randall]
- Currently investigating the case where the IR brane has a dS BH...

Outlook

We studied various aspects of dS braneworlds but there are still lots to explore!

- Other observables: correlation functions, complexity, etc
- Tests of dS/CFT?
- Adding structure: BH with charge and rotation...
- More general bulk foliations (e.g. FRW slices). Applications in cosmology? (see: [Ross; Raamsdonk, Swingle,...])

Outlook

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Thanks!