Loop corrections to soft graviton theorems

2205.11477 with Laura Donnay (SISSA) and Romain Ruzziconi (TU Vienna)

Kevin Nguyen

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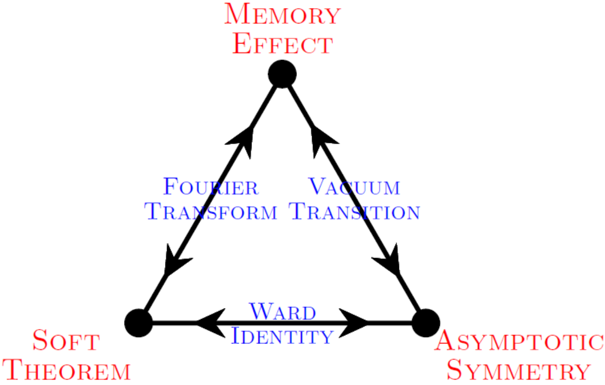
Iberian Strings '23
Murcia

The Infrared Triangle



- symmetry interpretation of soft theorems
- opportunity to get better control over gravitational phase-space

Previous work

- asymptotic symmetries include supertranslations and superrotations [Bondi-van der Burg-Metzner-Sachs '62, Barnich-Troessaert '10]

$$SL(2,\mathbb{C}) \ltimes T^4 \quad \longrightarrow \quad BMS = (Vir \times \overline{Vir}) \ltimes T^\infty$$

- leading soft graviton theorem [Weinberg '65]
- equivalence to Ward identity of supertranslations [Strominger '13, He-Lysov-Mitra-Strominger '14]
- subleading soft graviton theorem [Cachazo-Strominger '14] + loop correction [Bern-Davies-Nohle '14]
- equivalence to Ward identity of superrotations [Kapec-Lysov-Pasterski-Strominger '14]
- first proposal for one-loop corrections [He-Kapec-Raclariu-Strominger '17]

Roadmap

Main result

The equivalence

subleading soft graviton theorem $\leftrightarrow$ *superrotation Ward id.*

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Strategy:

- organize the asymptotic phase-space w.r.t. superrotations
- construct canonical generators of BMS symmetries $\rightarrow$ BMS fluxes
- express Ward identity in terms of fluxes
- extra input: supertranslation Goldstone controls IR divergences

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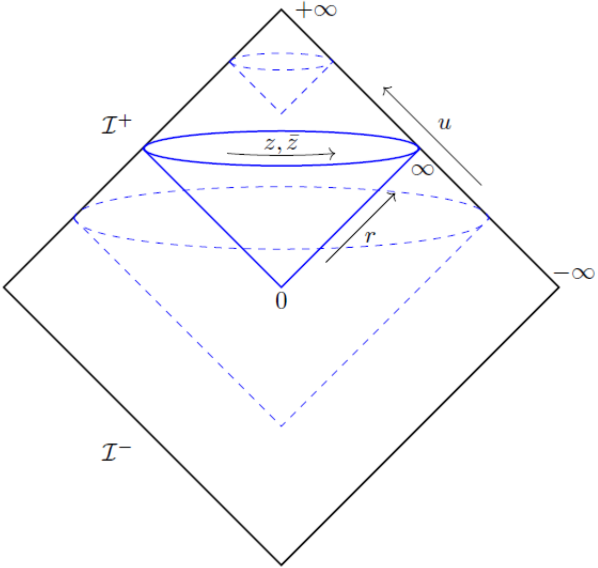
Asymptotic phase space and BMS fluxes

Null infinity: the observer's view on isolated systems



Bondi expansion

bulk metric:

$$ds^2 = \left(\frac{2M}{r} + \mathcal{O}(r^{-2})\right) du^2 - 2\left(1 + \mathcal{O}(r^{-2})\right) du dr$$
$$+ \left(r^2 \mathring{q}_{AB} + r\, C_{AB} + \mathcal{O}(r^0)\right) dx^A dx^B$$
$$+ \left(\frac{1}{2} D_B C^B_A + \frac{2}{3r}(N_A + \frac{1}{4} C^B_A D_C C^C_B) + \mathcal{O}(r^{-2})\right) du dx^A$$

metric on celestial sphere:

$$\mathring{q}_{AB}\, dx^A dx^B = 2 dz d\bar{z}$$

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BMS transformations

Asymptotic symmetries = diffeos preserving the above structure

- superrotations act as Virasoro transformations on $\mathring{q}_{AB}$

Radiative phase space

Falloffs $u \to \pm\infty$:

$$N_{zz} \equiv \partial_u C_{zz} = N_{zz}^{vac} + o(u^{-2}), \quad C_{zz} = (u + C_{\pm}) N_{zz}^{vac} - 2\partial^2 C_{\pm} + o(u^{-1})\,.$$

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Fine-grained decomposition:

$$N_{zz} = N_{zz}^{vac} + \tilde{N}_{zz}\,, \qquad C_{zz} = u N_{zz}^{vac} + C_{zz}^{(0)} + \tilde{C}_{zz}\,,$$

with

$$C_{zz}^{(0)} = -2\mathscr{D}^2 C^{(0)}\,, \qquad C^{(0)} = \frac{1}{2}(C_+ + C_-)\,.$$

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Leading and subleading soft News:

$$\mathcal{N}^{(0)}_{zz} = \int_{-\infty}^{+\infty} du\, \tilde{N}_{zz} = -4\mathscr{D}^2 N^{(0)}\,, \qquad \mathcal{N}^{(1)}_{zz} = \int_{-\infty}^{+\infty} du\, u\tilde{N}_{zz}\,.$$

Soft modes

$C^{(0)}, N^{vac}_{zz}, \mathcal{N}^{(0)}_{zz}, \mathcal{N}^{(1)}_{zz}$ are conformal fields!

Canonical structure and BMS fluxes

Symplectic form [Ashtekar-Streubel '81, Campiglia-Laddha '21]:

$$\Omega = \Omega^{hard} + \Omega^{soft}\,,$$

$$\Omega^{hard} = \int du\, d^2 z \left[\delta \tilde{N}_{zz} \wedge \delta \tilde{C}_{\bar{z}\bar{z}} + c.c.\right]\,,$$

$$\Omega^{soft} = \int d^2 z \left[\delta \mathcal{N}^{(0)}_{zz} \wedge \delta C^{(0)}_{\bar{z}\bar{z}} + \delta(\mathcal{N}^{(1)}_{zz} + C^{(0)}\mathcal{N}^{(0)}_{zz}) \wedge \delta N^{vac}_{\bar{z}\bar{z}} + c.c.\right]\,.$$

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Canonical generators:

$$\Omega^{soft}(\delta_{\mathcal{Y}}, \delta) = \delta F^{soft}_{\mathcal{Y}} \,, \qquad \Omega^{hard}(\delta_{\mathcal{Y}}, \delta) = \delta F^{hard}_{\mathcal{Y}} \,,$$

with [Donnay-Ruzziconi '21]

$$F^{hard}_{\mathcal{Y}} = \frac{1}{16\pi G} \int_{\mathscr{I}^+} du d^2 z\, \mathcal{Y} \left[\frac{3}{2}\tilde{C}_{zz}\partial\tilde{N}_{\bar{z}\bar{z}} + \frac{1}{2}\tilde{N}_{\bar{z}\bar{z}}\partial\tilde{C}_{zz} + \frac{u}{2}\partial(\tilde{N}_{zz}\tilde{N}_{\bar{z}\bar{z}})\right] \,,$$

$$F^{soft}_{\mathcal{Y}} = \frac{1}{16\pi G} \int_{\mathcal{S}} d^2 z\, \mathcal{Y} \left[-\mathscr{D}^3\mathcal{N}^{(1)}_{\bar{z}\bar{z}} + \frac{3}{2} C^{(0)}_{zz}\,\mathscr{D}\mathcal{N}^{(0)}_{\bar{z}\bar{z}} + \frac{1}{2}\mathcal{N}^{(0)}_{\bar{z}\bar{z}}\,\mathscr{D}C^{(0)}_{zz}\right] \,.$$

Tree-level soft graviton theorems

How to derive a soft theorem?

1. flux balance equation:

$$F(\mathscr{I}^+) = Q(i^+) - Q(\mathscr{I}^+_-)\,, \qquad F(\mathscr{I}^-) = Q(\mathscr{I}^-_+) - Q(i^-)$$

2. for a scattering of massless particles:

$$Q(i^+) = Q(i^-) = 0$$

3. charge conservation across i^0: [Troessaert '17, Capone-KN-Parisini '22]

$$Q(\mathscr{I}^+_-) = Q(\mathscr{I}^-_+) \equiv Q(i^0)$$

4. invariance of the $\mathcal{S}$-matrix:

$$\big[Q(i^0), \mathcal{S}\big] = 0$$

Ward identity

$$\langle \text{out}| F(\mathscr{I}^+)\mathcal{S} + \mathcal{S} F(\mathscr{I}^-) |\text{in}\rangle = 0$$

Leading soft graviton theorem

Feynman diagrams: [Weinberg '65]

$$\lim_{\omega\to 0} \omega \, \langle \mathsf{out} | a_+^{\mathsf{out}}(\omega \hat{q}) \mathcal{S} | \mathsf{in} \rangle = \hat{S}_n^{(0)+} \langle \mathsf{out} | \mathcal{S} | \mathsf{in} \rangle \,,$$

$$\hat{S}_n^{(0)\pm} = \frac{\kappa}{2} \sum_{i=1}^{n} \frac{p_i^\mu p_i^\nu \varepsilon^{\pm}_{\mu\nu}(\hat{q})}{p_i \cdot \hat{q}} \,, \qquad \kappa^2 = 32\pi G \,.$$

Supertranslation Ward identity: [Strominger et al '13 '14]

$$\langle \mathsf{out} | F_{\mathcal{T}}^{soft} \mathcal{S} | \mathsf{in} \rangle = -\langle \mathsf{out} | F_{\mathcal{T}}^{hard} \mathcal{S} | \mathsf{in} \rangle \,,$$

$$\langle \mathsf{out} | \mathcal{N}_{zz}^{(0)} \mathcal{S} | \mathsf{in} \rangle = -\frac{\kappa}{16\pi} \, \hat{S}_n^{(0)+} \langle \mathsf{out} | \mathcal{S} | \mathsf{in} \rangle \,,$$

$$\mathcal{N}_{zz}^{(0)} = -\frac{\kappa}{16\pi} \lim_{\omega\to 0} \omega \left[a_+^{\mathsf{out}}(\omega \hat{q}) + a_-^{\mathsf{out}}(\omega \hat{q})^\dagger \right] \,.$$

Tree-level subleading soft graviton theorem

Feynman diagrams: [Cachazo-Strominger '14]

$$\lim_{\omega\to 0}\,(1+\omega\partial_\omega)\,\langle \text{out}|a_-^{\text{out}}(\omega\hat{q})\mathcal{S}|\text{in}\rangle = S_n^{(1)-}\langle \text{out}|\mathcal{S}|\text{in}\rangle\,,$$

$$S_n^{(1)\pm} = -\frac{i\kappa}{2}\sum_{i=1}^{n}\frac{p_i^\mu\,\varepsilon^\pm_{\mu\nu}(\hat{q})\,q_\lambda}{p_i\cdot q}\,J_i^{\lambda\nu}\,.$$

Superrotation Ward identity: [Kapec-Lysov-Pasterski-Strominger '14]

$$\langle \text{out}|F_{\mathcal{Y}}^{soft}\mathcal{S}|\text{in}\rangle = -\langle \text{out}|F_{\mathcal{Y}}^{hard}\mathcal{S}|\text{in}\rangle\,,$$

$$\langle \text{out}|\mathcal{N}^{(1)}_{\bar{z}\bar{z}}\mathcal{S}|\text{in}\rangle = \frac{i\kappa}{16\pi}\,S_n^{(1)-}\langle \text{out}|\mathcal{S}|\text{in}\rangle\,,$$

$$\mathcal{N}^{(1)}_{\bar{z}\bar{z}} = \frac{i\kappa}{16\pi}\lim_{\omega\to 0}\,(1+\omega\partial_\omega)\left[a_-^{\text{out}}(\omega\hat{q}) - a_+^{\text{out}}(\omega\hat{q})^\dagger\right]\,.$$

Loop corrections

IR divergences

soft factorization: [Weinberg '65]

$$\mathcal{M} = \mathcal{M}_{\textsf{soft}} \, \mathcal{M}_{\textsf{finite}} \,,$$

$$\mathcal{M}_{\textsf{soft}} = \exp\left[\frac{\hbar}{\epsilon}\frac{\kappa^2}{(8\pi)^2}\sum_{i,j=1}^{n} p_i \cdot p_j \ln\frac{p_i \cdot p_j}{\mu}\right] , \qquad d = 4 - 2\epsilon \,.$$

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[Himwich et al '20, Arkani-Hamed et al '20, KN-Salzer '21]

IR divergences are controlled by the supertranslation Goldstone mode $C^{(0)}$

$$\mathcal{M}_{\text{soft}} = \exp\left[-\frac{1}{2}\sum_{i\neq j}^{n}\eta_i\eta_j\omega_i\omega_j\langle C^{(0)}(z_i,\bar{z}_i)C^{(0)}(z_j,\bar{z}_j)\rangle\right]\,,$$

$$\langle C^{(0)}(z_i,\bar{z}_i)C^{(0)}(z_j,\bar{z}_j)\rangle = \frac{\hbar}{\epsilon}\frac{\kappa^2}{(4\pi)^2}\,|z_{ij}|^2\ln|z_{ij}|^2\,.$$

Subleading soft graviton theorem

Some terms were missed in the soft flux:

$$\int_{\mathcal{S}} d^2z\, \mathcal{Y} \left[\frac{3}{2} C^{(0)}_{zz} \mathscr{D} \mathcal{N}^{(0)}_{\bar{z}\bar{z}} + \frac{1}{2} \mathcal{N}^{(0)}_{\bar{z}\bar{z}} \mathscr{D} C^{(0)}_{zz} \right] .$$

Using previous slide, we can compute the effect of Goldstone mode insertion:

$$\langle \mathsf{out} | C^{(0)}(z, \bar{z}) \mathcal{S} | \mathsf{in} \rangle = -\frac{i\kappa^2}{\epsilon}\, \hat{\sigma}'_{n+1} \langle \mathsf{out} | \mathcal{S} | \mathsf{in} \rangle \,,$$

$$\hat{\sigma}'_{n+1} \equiv \frac{\hbar}{2(4\pi)^2} \sum_{i=1}^{n} (p_i \cdot \hat{q}) \ln \frac{p_i \cdot \hat{q}}{\mu} \,.$$

Result

This reproduces the one-loop exact correction in agreement with [Bern-Davies-Nohle '14]