Higher-derivative holography with a chemical potential

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with Pablo A. Cano, Alberto Rivadulla Sánchez and Xuao Zhang

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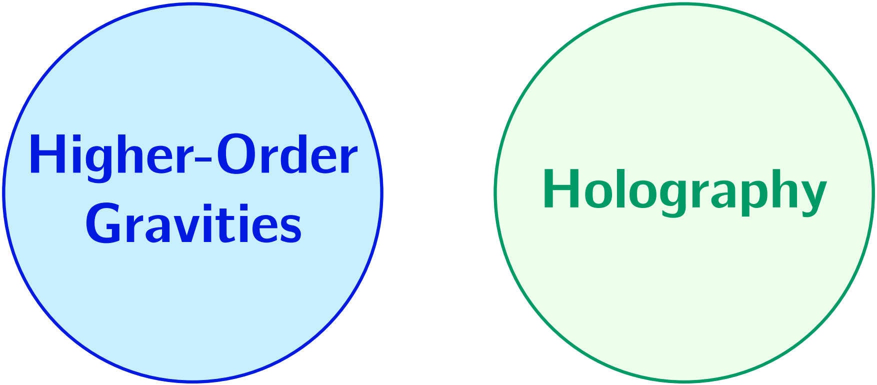

Higher-Order Gravities

Definition

A **higher-order gravity***, or equivalently higher-derivative gravity, is any theory of gravity (possibly with non-minimally coupled matter) of the form:*

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- In this talk: **metric formalism** and **Levi-Civita connection**. However, there are other intriguing and canonical possibilities, such as metric-affine theories **[*e.g.* Borunda, Janssen, Bastero-Gil '08; Olmo '11]**.

Examples of higher-order gravities

Some instances of purely-gravitational higher-order gravities:

- **Starobinsky's model [Starobinsky '80]**:

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  For $k=2$, **Gauss-Bonnet** density $\mathcal{X}_4 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$.

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Some instances of higher-order gravities with matter (for simplicity, $\mathrm{U}(1)$ gauge vector field):

- **Einstein-ModMax theory** **[Bandos, Lechner, Sorokin, Townsend '20; Flores-Alfonso, González-Morales, Linares, Maceda '20]**:

$$I = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{|g|}\left[R - \cosh\gamma F^2 + \sinh\gamma\sqrt{(F^2)^2 + (F^{\mu\nu}\star F_{\mu\nu})^2}\right].$$

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- Higher-order gravity with a **non-minimally coupled** $\mathrm{U}(1)$ vector field:

$$I = \frac{1}{16\pi G}\int \mathrm{d}^{d+1}x\sqrt{|g|}\left[R - F^2 + \ell^2(2R_{\mu}{}^{\alpha}F^{\mu\nu}F_{\alpha\nu} - R^{\alpha\beta}{}_{\rho\sigma}F^{\rho\sigma}F_{\alpha\beta})\right] .$$

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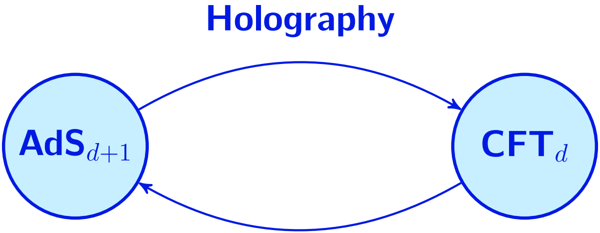

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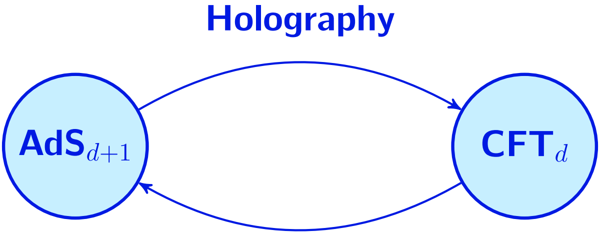
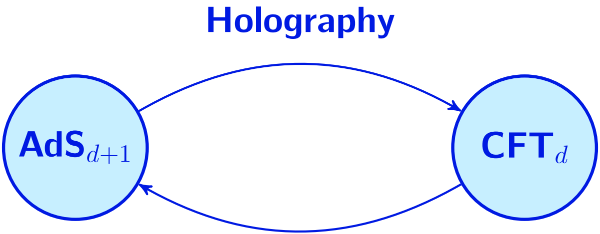


Original proposal: physical **equivalence** of type **IIB** String Theory on $\mathrm{AdS}_5 \times S^5$ with $\mathcal{N}=4$ **Super-Yang-Mills** theory. Furthermore, it states that the **strong-coupling** limit of **CFT** side may be described by **classical Supergravity**.

AdS/CFT correspondence



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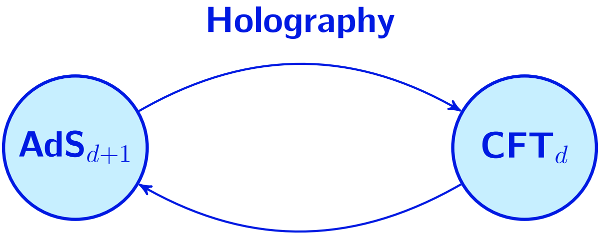

CFT interpreted to live on **boundary** of asymptotically **AdS** spacetime (**bulk**). **CFT correlators** through bulk **computations** and vice versa through the use of a **holographic dictionary**.

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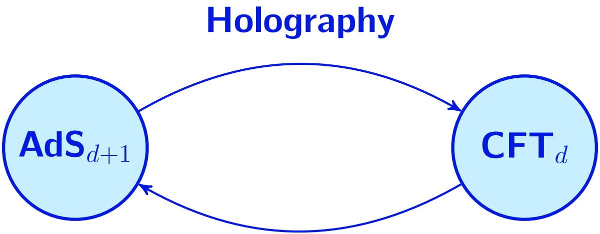

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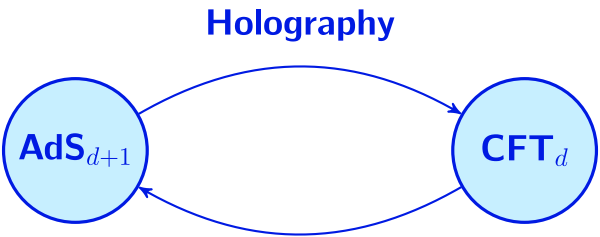

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Finite-coupling effects of **holographic CFTs** are **captured** by **higher-order terms**.

Higher-order gravities and Holography

This has motivated the study of **holographic aspects** of generic **higher-order gravities** from a **bottom-up approach**: examining arbitrary **higher-order gravities** whose **CFT** dual needs **not** to be **known**:

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This **program** has been **successfully carried out** in the literature in the recent years for **purely-gravitational** higher-order theories **[*e.g.* Cai, Nie, Zhang '10; Myers, Sinha '10; Boer, Kulaxizi, Parnachev '11; Perlmutter '13; Hung, Myers, Smolkin '14; Chu, Miao '16; Dey, Roy, Sarkar '16; Lü, Mai '18; Edelstein, Grandi, Sánchez '22]**.

Motivation for our work

Addition of **vector field**, which allows probing **CFTs** with **chemical potential**, has also been considered. However, most analyses involved perturbative approach or sticking to very particular models **[*e.g.* Liu, Szepietowski '08; Cremonini, Hanaki, Liu, Szepietowski '09, Myers, Sachdev, Singh '11; Cai, Pang '11]**.

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In this talk: **Exact** exploration of **holographic higher-order gravities** with **non-minimal couplings** to a gauge **vector** field.

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Electromagnetic Quasitopological Gravities (EQGs)

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Definition (Electromagnetic Quasitopological Gravities (EQGs))

A **theory** $\mathcal{L}(g_{\mu\nu}, R_{\mu\nu\rho\sigma}, H_{\mu_1 \ldots \mu_{d-1}})$*, with* $H = \mathrm{d}B$*, is an* **EQG** *if and only if:*

$$\frac{\delta L_{N_0,f}}{\delta f} = \frac{\partial L_{N_0,f}}{\partial f} - \frac{\mathrm{d}}{\mathrm{d}r}\frac{\partial L_{N_0,f}}{\partial f'} + \frac{\mathrm{d}^2}{\mathrm{d}r^2}\frac{\partial L_{N_0,f}}{\partial f''} + \cdots = 0\,,$$

where $N_0 = \mathrm{const.}$ *and where we defined* $L_{N,f} = \sqrt{|g|}\mathcal{L}|_{\mathrm{d}s^2_{N,f}, H_Q}$.

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How do we canonically construct an associated **bulk theory** with gauge **vector field**? $\longrightarrow$ **Dualization**.

Electromagnetic Quasitopological Gravities (EQGs)

Dualization: In $d+1$ dimensions, a map between two theories:

$$\begin{pmatrix} g_{\mu\nu} \\ \\ H_{\mu_1\ldots\mu_{d-1}} = (d-1)\partial_{[\mu_1}B_{\mu_2\ldots\mu_{d-1}]} \\ \\ \mathcal{L}(R,H) \end{pmatrix} \longrightarrow \begin{pmatrix} g_{\mu\nu} \\ \\ F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]} \\ \\ \mathcal{L}_{\text{dual}}(R,F) = \mathcal{L}(R,H(F)) \\ +\frac{4}{(d-1)!}(\star H(F))^{\mu\nu}F_{\mu\nu} \end{pmatrix},$$

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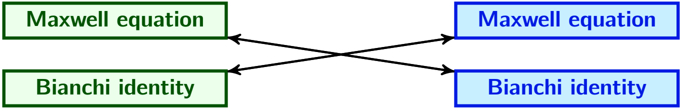

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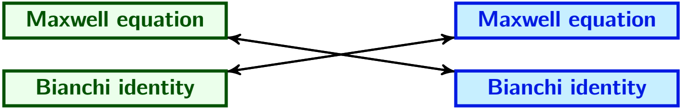

1. It maps theories with $(d-2)$-forms into theories with 1-forms.

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Dualization: In $d+1$ dimensions, a map between two theories:

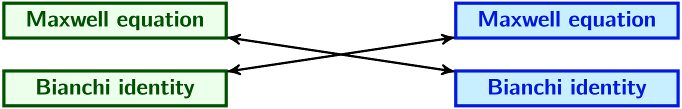
$$\begin{pmatrix} g_{\mu\nu} \\ \\ H_{\mu_1\ldots\mu_{d-1}} = (d-1)\partial_{[\mu_1}B_{\mu_2\ldots\mu_{d-1}]} \\ \\ \mathcal{L}(R,H) \end{pmatrix} \longrightarrow \begin{pmatrix} g_{\mu\nu} \\ \\ F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]} \\ \\ \mathcal{L}_{\mathrm{dual}}(R,F) = \mathcal{L}(R,H(F)) \\ +\frac{4}{(d-1)!}(\star H(F))^{\mu\nu}F_{\mu\nu} \end{pmatrix},$$

where $H_{\mu_1\ldots\mu_{d-1}}(F_{\rho\sigma})$ is obtained by inverting

$$F = \frac{(d-1)!}{4}\star\frac{\partial\mathcal{L}}{\partial H}\,.$$

This dualization map corresponds to the usual **electric-magnetic duality**:

$\mathcal{L}(R,H)$ $\mathcal{L}_{\mathrm{dual}}(R,F)$



1. It maps theories with $(d-2)$-forms into theories with 1-forms.
2. If $(g_{\mu\nu}, H_{\alpha_1\ldots\alpha_{d-1}})$ is magnetic solution $\Rightarrow$ $(g_{\mu\nu}, F_{\alpha\beta})$ is **electric** solution!

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7. **EQGs exist at all orders** and for **every** $d \geq 2$ **[Cano, ÁM '20; Bueno, Cano, Moreno, van der Velde '21; Cano, ÁM, Rivadulla, Zhang '22]**.

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where $\alpha_1, \alpha_2, \beta, \lambda$ are dimensionless couplings, L a length scale, $\mathcal{X}_4$ the Gauss-Bonnet density and $\left(H^2\right)^{\rho\sigma}{}_{\mu\nu} = H^{\rho\sigma\alpha_1\ldots\alpha_{d-3}}H_{\mu\nu\alpha_1\ldots\alpha_{d-3}}$.

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1. **Two-** and **three-point** correlators.
2. **Charged Rényi entropies**.

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Bulk vector fields in AdS become **non-dynamical** on the **boundary** and **couple** to a **current** J^a. More concretely, J^a couples to vector field with units of energy:

$$\tilde{A}_\mu = \ell_*^{-1} A_\mu \,, \quad \mu = \ell_*^{-1} A_t|_{\text{bdry}} \,,$$

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We shall be interested in the following two- and three-point correlators:

$$\langle T_{ab}(x) T_{cd}(x') \rangle = \frac{C_T}{|x-x'|^{2d}} \mathcal{I}_{ab,cd}(x-x') \,,$$

$$\langle J_a(x) J_b(x') \rangle = \frac{C_J}{|x-x'|^{2(d-1)}} I_{ab}(x-x') \,,$$

$$\langle T_{ab}(x_1) J_c(x_2) J_d(x_3) \rangle = \frac{f_{abcd}(a_2, C_J)}{|x_{12}|^d |x_{13}|^d |x_{23}|^{d-2}} \,.$$

Correlators are **fixed** up to the **central charges** C_T, C_J and the parameter a_2.

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Regarding the $\langle TT \rangle$ correlator, it is **identical** to that of **Gauss-Bonnet** gravity **[Buchel, Escobedo, Myers, Paulos, Sinha, Smolkin '10]**:

$$C_T = \frac{(1-2\lambda f_\infty)\Gamma(d+2)}{8(d-1)\Gamma(d/2)\pi^{(d+2)/2}} \frac{\tilde{L}^{d-1}}{G}\,.$$

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Regarding the $\langle JJ \rangle$ correlator:

$$C_J = \frac{C_J^{\rm EM}}{\alpha_{\rm eff}} \,, \quad C_J^{\rm EM} = \frac{\Gamma(d)}{\Gamma(d(2-1)} \frac{\ell_*^2 \tilde{L}^{d-3}}{4\pi^{d/2+1} G} \,,$$
$$\alpha_{\rm eff} = 1 - f_\infty \alpha_1 (3d^2 - 7d + 2) - f_\infty \alpha_2 (d-2) \,.$$

Two- and three-point correlators

The parameter a_2 controls the energy flux measured at infinity and is given by **[Hofman, Maldacena '08]**:

$$\langle \mathcal{E}\left(\vec{n}\right)\rangle_J = \frac{E}{\Omega_{(d-2)}}\left[1 + a_2\left(\frac{|\epsilon\cdot n|^2}{|\epsilon|^2} - \frac{1}{d-1}\right)\right],$$

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We observe it is **generically non-zero** if $\alpha_1, \alpha_2 \neq 0 \longrightarrow$ **Different** CFT **universality classes**.

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- **Energy flux** $\langle \mathcal{E}\left(\vec{n}\right) \rangle_J$ **positive** at any direction, equivalent to $-\frac{d-1}{d-2} \leq a_2 \leq d-1$. Remarkably, it turns out that this **condition** on the **boundary** is **equivalent** to **avoiding superluminal** propagation of **electromagnetic waves** in the **bulk**.

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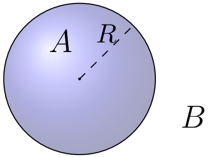

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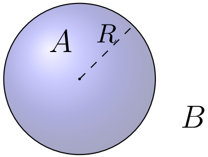

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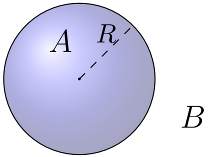

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Holographically, this implies:

$$S_n(\mu) = \frac{n}{n-1}\frac{1}{T_0}\left(\Omega(T_0/n,\mu) - \Omega(T_0,\mu)\right)\,,$$

where Ω is grand-canonical potential.

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Ω can be **computed exactly** in terms of the **horizon radius**, but we need it in terms of T. We obtain it in the **limit** of **small** μ:

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¡Muchas gracias!