EGQT gravities in three dimensions

Iberian Strings 23'

Javier Moreno
Haifa University-Technion

Based on

ArXiv: 2212.00637
Phys. Rev. D **104** (2021) 2, L021501

with Pablo Bueno, Pablo A. Cano & Guido van der Velde

January 11, 2022

Outline

Introduction

General relativity in $D = 3$ is **simpler** than its higher-dimensional version

$$S_{\text{EH}} = \frac{1}{16\pi G_{\text{N}}} \int \mathrm{d}^3x \sqrt{|g|} \left(R + \frac{2}{L^2} \right)$$

This is because

General relativity in $D = 3$ is **simpler** than its higher-dimensional version

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int \mathrm{d}^3x \sqrt{|g|} \left(R + \frac{2}{L^2} \right)$$

This is because

- The Weyl tensor **vanishes** identically $\Rightarrow$ all curvatures are Ricci

General relativity in $D = 3$ is **simpler** than its higher-dimensional version

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int \mathrm{d}^3x \sqrt{|g|} \left(R + \frac{2}{L^2} \right)$$

This is because

- The Weyl tensor **vanishes** identically $\Rightarrow$ all curvatures are Ricci
- All Einstein solutions are **locally equivalent** to maximally symmetric backgrounds

General relativity in $D = 3$ is **simpler** than its higher-dimensional version

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int \mathrm{d}^3x \sqrt{|g|} \left(R + \frac{2}{L^2} \right)$$

This is because

- The Weyl tensor **vanishes** identically $\Rightarrow$ all curvatures are Ricci
- All Einstein solutions are **locally equivalent** to maximally symmetric backgrounds
- No degrees of freedom are propagated by the metric perturbations $\Rightarrow$ **No gravitational waves**

General relativity in $D = 3$ is **simpler** than its higher-dimensional version

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int \mathrm{d}^3x \sqrt{|g|} \left(R + \frac{2}{L^2} \right)$$

This is because

- The Weyl tensor **vanishes** identically $\Rightarrow$ all curvatures are Ricci
- All Einstein solutions are **locally equivalent** to maximally symmetric backgrounds
- No degrees of freedom are propagated by the metric perturbations $\Rightarrow$ **No gravitational waves**

However, there are interesting solutions. The most famous one is the **BTZ black hole** [Bañados, Teitelboim, Zanelli]

General relativity in $D = 3$ is **simpler** than its higher-dimensional version

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int \mathrm{d}^3x \sqrt{|g|} \left(R + \frac{2}{L^2} \right)$$

This is because

- The Weyl tensor **vanishes** identically $\Rightarrow$ all curvatures are Ricci
- All Einstein solutions are **locally equivalent** to maximally symmetric backgrounds
- No degrees of freedom are propagated by the metric perturbations $\Rightarrow$ **No gravitational waves**

However, there are interesting solutions. The most famous one is the **BTZ black hole** [Bañados, Teitelboim, Zanelli]

- Locally equivalent to AdS_3, with all curvature invariants constant

General relativity in $D = 3$ is **simpler** than its higher-dimensional version

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int \mathrm{d}^3x \sqrt{|g|} \left(R + \frac{2}{L^2} \right)$$

This is because

- The Weyl tensor **vanishes** identically $\Rightarrow$ all curvatures are Ricci
- All Einstein solutions are **locally equivalent** to maximally symmetric backgrounds
- No degrees of freedom are propagated by the metric perturbations $\Rightarrow$ **No gravitational waves**

However, there are interesting solutions. The most famous one is the **BTZ black hole** [Bañados, Teitelboim, Zanelli]

- Locally equivalent to AdS_3, with all curvature invariants constant
- It is different **globally**, with a singularity in the causal structure at $r = 0$

General relativity in $D = 3$ is **simpler** than its higher-dimensional version

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int \mathrm{d}^3x \sqrt{|g|} \left(R + \frac{2}{L^2} \right)$$

This is because

- The Weyl tensor **vanishes** identically $\Rightarrow$ all curvatures are Ricci
- All Einstein solutions are **locally equivalent** to maximally symmetric backgrounds
- No degrees of freedom are propagated by the metric perturbations $\Rightarrow$ **No gravitational waves**

However, there are interesting solutions. The most famous one is the **BTZ black hole** [Bañados, Teitelboim, Zanelli]

- Locally equivalent to AdS_3, with all curvature invariants constant
- It is different **globally**, with a singularity in the causal structure at $r = 0$
- Displays event and Cauchy horizons in certain cases

General relativity in $D = 3$ is **simpler** than its higher-dimensional version

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int \mathrm{d}^3 x \sqrt{|g|} \left(R + \frac{2}{L^2} \right)$$

This is because

- The Weyl tensor **vanishes** identically $\Rightarrow$ all curvatures are Ricci
- All Einstein solutions are **locally equivalent** to maximally symmetric backgrounds
- No degrees of freedom are propagated by the metric perturbations $\Rightarrow$ **No gravitational waves**

However, there are interesting solutions. The most famous one is the **BTZ black hole** [Bañados, Teitelboim, Zanelli]

- Locally equivalent to AdS_3, with all curvature invariants constant
- It is different **globally**, with a singularity in the causal structure at $r = 0$
- Displays event and Cauchy horizons in certain cases
- Shares properties of higher-D counterparts: thermodynamics, holography, etc

While BTZ black hole solves all higher-curvature gravities, new solutions have also been constructed in certain cases.

While BTZ black hole solves all higher-curvature gravities, new solutions have also been constructed in certain cases.

- **“New massive gravity” & extensions**

Including solutions locally inequivalent to AdS_3 (dS_3, Liftshitz, etc) or curvature singularity

[Bergshoeff, Hohm, Townsend; Gullu, Sisman, Tekin; Sinha; Paulos; Oliva, Tempo, Troncoso; Ayon-Beato, Gabarz, Giribet, Hassaine, etc.]

While BTZ black hole solves all higher-curvature gravities, new solutions have also been constructed in certain cases.

- **"New massive gravity" & extensions**

Including solutions locally inequivalent to AdS_3 (dS_3, Liftshitz, etc) or curvature singularity

[Bergshoeff, Hohm, Townsend; Gullu, Sisman, Tekin; Sinha; Paulos; Oliva, Tempo, Troncoso; Ayon-Beato, Gabarz, Giribet, Hassaine, etc.]

- **Including additional fields**

Einstein-Maxwell, Einstein-Maxwell-dilaton, Maxwell-Brans-Dicke

[Clément; Kawata, Koikama; Martínez, Teitelboim, Zanelli; Hirsrchmann, Welch; Cataldo, Salgado; Dias, Lemos; Chan, Mann, etc]

While BTZ black hole solves all higher-curvature gravities, new solutions have also been constructed in certain cases.

- **"New massive gravity" & extensions**

Including solutions locally inequivalent to AdS_3 (dS_3, Liftshitz, etc) or curvature singularity [Bergshoeff, Hohm, Townsend; Gullu, Sisman, Tekin; Sinha; Paulos; Oliva, Tempo, Troncoso; Ayon-Beato, Gabarz, Giribet, Hassaine, etc.]

- **Including additional fields**

Einstein-Maxwell, Einstein-Maxwell-dilaton, Maxwell-Brans-Dicke [Clément; Kawata, Koikama; Martínez, Teitelboim, Zanelli; Hirsrchmann, Welch; Cataldo, Salgado; Dias, Lemos; Chan, Mann, etc]

Minimal and non-minimal coupling to scalar fields, including well-defined limits to Lovelock theories [Henaux, Martínez, Troncoso, Zanelli; Zhao, Xu, Zhu; Hennigar Kubiznak, Mann; Baake, Charmousis, Hassaine, San Juan, etc]

While BTZ black hole solves all higher-curvature gravities, new solutions have also been constructed in certain cases.

- **"New massive gravity" & extensions**

Including solutions locally inequivalent to AdS_3 (dS_3, Liftshitz, etc) or curvature singularity [Bergshoeff, Hohm, Townsend; Gullu, Sisman, Tekin; Sinha; Paulos; Oliva, Tempo, Troncoso; Ayon-Beato, Gabarz, Giribet, Hassaine, etc.]

- **Including additional fields**

Einstein-Maxwell, Einstein-Maxwell-dilaton, Maxwell-Brans-Dicke [Clément; Kawata, Koikama; Martínez, Teitelboim, Zanelli; Hirsrchmann, Welch; Cataldo, Salgado; Dias, Lemos; Chan, Mann, etc]

Minimal and non-minimal coupling to scalar fields, including well-defined limits to Lovelock theories [Henaux, Martínez, Troncoso, Zanelli; Zhao, Xu, Zhu; Hennigar Kubiznak, Mann; Baake, Charmousis, Hassaine, San Juan, etc]

Non-linear electrodynamics coupling. Regular black holes for certain modified Maxwell Lagrangians [Cataldo, García; Myung, Kim, Park; He, Ma; Mazharimousavi, Gurtug, Halilsoy, Unver, etc]

Generalities of (Electromagnetic) GQT gravities

GQT gravities have attracted **great interest** in recent years in higher-dimensions. What about $D = 3$?

GQT gravities have attracted **great interest** in recent years in higher-dimensions. What about $D = 3$?
Let us **define** them

Consider the solution

$$\text{SSS: } \mathrm{d}s^2 = -N^2(r)f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2_{D-2}$$

A theory $\mathcal{L}(R_{abcd})$ belongs to the GQT family if the Euler-Lagrange equation of $L_f = \sqrt{-g}\mathcal{L}\big|_{N=1,f}$ **evaluated on the single-function SSS** is satisfied [Bueno, Cano; Hennigar, Mann; et al.]

$$\frac{\partial L_f}{\partial f} - \frac{\mathrm{d}}{\mathrm{d}r}\frac{\partial L_f}{\partial f'} + \frac{\mathrm{d}^2}{\mathrm{d}r^2}\frac{\partial L_f}{\partial f''} - \ldots = 0, \quad \forall f(r)$$

GQT gravities have attracted **great interest** in recent years in higher-dimensions. What about $D = 3$?
Let us **define** them

Consider the solution

$$\text{SSS: } \mathrm{d}s^2 = -N^2(r)f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2_{D-2}$$

A theory $\mathcal{L}(R_{abcd})$ belongs to the GQT family if the Euler-Lagrange equation of $L_f = \sqrt{-g}\mathcal{L}\big|_{N=1,f}$ **evaluated on the single-function SSS** is satisfied [Bueno, Cano; Hennigar, Mann; et al.]

$$\frac{\partial L_f}{\partial f} - \frac{\mathrm{d}}{\mathrm{d}r}\frac{\partial L_f}{\partial f'} + \frac{\mathrm{d}^2}{\mathrm{d}r^2}\frac{\partial L_f}{\partial f''} - \ldots = 0, \quad \forall f(r)$$

For these theories, L_f is a **total derivative**.

GQT gravities have attracted **great interest** in recent years in higher-dimensions. What about $D = 3$?
Let us **define** them

Consider the solution

$$\text{SSS: } \mathrm{d}s^2 = -N^2(r)f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2_{D-2}$$

A theory $\mathcal{L}(R_{abcd})$ belongs to the GQT family if the Euler-Lagrange equation of $L_f = \sqrt{-g}\mathcal{L}\big|_{N=1,f}$ **evaluated on the single-function SSS** is satisfied [Bueno, Cano; Hennigar, Mann; et al.]

$$\frac{\partial L_f}{\partial f} - \frac{\mathrm{d}}{\mathrm{d}r}\frac{\partial L_f}{\partial f'} + \frac{\mathrm{d}^2}{\mathrm{d}r^2}\frac{\partial L_f}{\partial f''} - \ldots = 0, \quad \forall f(r)$$

For these theories, L_f is a **total derivative**. After integrating once, we obtain eom for $f(r)$ of order ≤ 2 for the corresponding density

GQT gravities have attracted **great interest** in recent years in higher-dimensions. What about $D=3$?
Let us **define** them

Consider the solution

$$\text{SSS: } \mathrm{d}s^2 = -N^2(r)f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2_{D-2}$$

A theory $\mathcal{L}(R_{abcd})$ belongs to the GQT family if the Euler-Lagrange equation of $L_f = \sqrt{-g}\mathcal{L}\big|_{N=1,f}$ **evaluated on the single-function SSS** is satisfied [Bueno, Cano; Hennigar, Mann; et al.]

$$\frac{\partial L_f}{\partial f} - \frac{\mathrm{d}}{\mathrm{d}r}\frac{\partial L_f}{\partial f'} + \frac{\mathrm{d}^2}{\mathrm{d}r^2}\frac{\partial L_f}{\partial f''} - \ldots = 0, \quad \forall f(r)$$

For these theories, L_f is a **total derivative**. After integrating once, we obtain eom for $f(r)$ of order ≤ 2 for the corresponding density

There are three possibilities:

- **Trivial** it does not contribute
- **Algebraic** dependence on $f(r)$ (QT [Oliva, Ray; Myers, Robinson] and Lovelock terms)
- **Second derivatives** of $f(r)$. Such as Einsteinian cubic gravity in $D=4$ [Bueno, Cano]

Some relevant properties:

- When **linearized** around any maximally symmetric background, the eom become second-order, only propagating **massless** and **traceless** gravitons $\Rightarrow$ equivalent to Einstein gravity

Some relevant properties:

- When **linearized** around any maximally symmetric background, the eom become second-order, only propagating **massless** and **traceless** gravitons $\Rightarrow$ equivalent to Einstein gravity
- They have a **continuous** and well-defined Einstein gravity limit.

Some relevant properties:

- When **linearized** around any maximally symmetric background, the eom become second-order, only propagating **massless** and **traceless** gravitons $\Rightarrow$ equivalent to Einstein gravity
- They have a **continuous** and well-defined Einstein gravity limit.
- The thermodynamic properties of black holes can be computed **analytically** [Bueno, Cano]

Some relevant properties:

- When **linearized** around any maximally symmetric background, the eom become second-order, only propagating **massless** and **traceless** gravitons ⇒ equivalent to Einstein gravity
- They have a **continuous** and well-defined Einstein gravity limit.
- The thermodynamic properties of black holes can be computed **analytically** [Bueno, Cano]
- For $D \geq 5$, there are exactly $n-1$ GQT densities at order n in curvature. Among them, 1 is a QT one [Bueno, Cano, Hennigar, Lu, JM]. Using **recurring relations** they are systematically constructed at all orders in curvature [Bueno, Cano, Hennigar]

Some relevant properties:

- When **linearized** around any maximally symmetric background, the eom become second-order, only propagating **massless** and **traceless** gravitons $\Rightarrow$ equivalent to Einstein gravity
- They have a **continuous** and well-defined Einstein gravity limit.
- The thermodynamic properties of black holes can be computed **analytically** [Bueno, Cano]
- For $D \geq 5$, there are exactly $n-1$ GQT densities at order n in curvature. Among them, 1 is a QT one [Bueno, Cano, Hennigar, Lu, JM]. Using **recurring relations** they are systematically constructed at all orders in curvature [Bueno, Cano, Hennigar]
- From EFT perspective, we can map perturbatively any $\mathcal{L}(R_{abcd})$ to some GQT theory [Bueno, Cano, JM, Murcia]

Some relevant properties:

- When **linearized** around any maximally symmetric background, the eom become second-order, only propagating **massless** and **traceless** gravitons ⇒ equivalent to Einstein gravity
- They have a **continuous** and well-defined Einstein gravity limit.
- The thermodynamic properties of black holes can be computed **analytically** [Bueno, Cano]
- For $D \geq 5$, there are exactly $n-1$ GQT densities at order n in curvature. Among them, 1 is a QT one [Bueno, Cano, Hennigar, Lu, JM]. Using **recurring relations** they are systematically constructed at all orders in curvature [Bueno, Cano, Hennigar]
- From EFT perspective, we can map perturbatively any $\mathcal{L}(R_{abcd})$ to some GQT theory [Bueno, Cano, JM, Murcia]
- They define interesting **toy models** of holographic CFTs inequivalent to Einstein gravity. It has been used to unveil new universal results valid for completely general CFTs [Bueno, Cano, Hennigar, Mann; Bueno, Cano, Murcia, Rivadulla]

In $D = 3$, all GQT gravities belong to the **trivial** family, i.e. their eom **vanish identically** on SSS ansätze [Bueno, Cano, Hennigar, Lu, JM]

In $D = 3$, all GQT gravities belong to the **trivial** family, i.e. their eom **vanish identically** on SSS ansätze [Bueno, Cano, Hennigar, Lu, JM]

However, this situation drastically changes when matter is added into the picture.

In $D = 3$, all GQT gravities belong to the **trivial** family, i.e. their eom **vanish identically** on SSS ansätze [Bueno, Cano, Hennigar, Lu, JM]

However, this situation drastically changes when matter is added into the picture. Recently, **Electromagnetic GQT gravities** were identified in higher dimensions.

In $D = 3$, all GQT gravities belong to the **trivial** family, i.e. their eom **vanish identically** on SSS ansätze [Bueno, Cano, Hennigar, Lu, JM]

However, this situation drastically changes when matter is added into the picture. Recently, **Electromagnetic GQT gravities** were identified in higher dimensions.

In $D = 4$, a theory $\mathcal{L}(R_{abcd}, F_{ab})$ belongs to the EGQT class if the on-shell action $L_{f,F^{\text{mag}}}$ evaluated in the ansatz [Cano, Murcia]

$$\text{SSSm:}\ \mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right), \quad F^{\text{mag}} = p\sin\theta\mathrm{d}\theta\wedge\mathrm{d}\varphi$$

satisfies the Euler-Lagrange equation for $f(r)$

In $D = 3$, all GQT gravities belong to the **trivial** family, i.e. their eom **vanish identically** on SSS ansätze [Bueno, Cano, Hennigar, Lu, JM]

However, this situation drastically changes when matter is added into the picture. Recently, **Electromagnetic GQT gravities** were identified in higher dimensions.

In $D = 4$, a theory $\mathcal{L}(R_{abcd}, F_{ab})$ belongs to the EGQT class if the on-shell action $L_{f,F^{\text{mag}}}$ evaluated in the ansatz [Cano, Murcia]

$$\text{SSSm: } \mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right), \quad F^{\text{mag}} = p\sin\theta\mathrm{d}\theta\wedge\mathrm{d}\varphi$$

satisfies the Euler-Lagrange equation for $f(r)$

Some remarks:

In $D = 3$, all GQT gravities belong to the **trivial** family, i.e. their eom **vanish identically** on SSS ansätze [Bueno, Cano, Hennigar, Lu, JM]

However, this situation drastically changes when matter is added into the picture. Recently, **Electromagnetic GQT gravities** were identified in higher dimensions.

In $D = 4$, a theory $\mathcal{L}(R_{abcd}, F_{ab})$ belongs to the EGQT class if the on-shell action $L_{f,F^{\text{mag}}}$ evaluated in the ansatz [Cano, Murcia]

$$\text{SSSm: } \mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right), \quad F^{\text{mag}} = p\sin\theta\mathrm{d}\theta\wedge\mathrm{d}\varphi$$

satisfies the Euler-Lagrange equation for $f(r)$

Some remarks:

- **Same classification** as GQT gravities applies for the eom

In $D=3$, all GQT gravities belong to the **trivial** family, i.e. their eom **vanish identically** on SSS ansätze [Bueno, Cano, Hennigar, Lu, JM]

However, this situation drastically changes when matter is added into the picture. Recently, **Electromagnetic GQT gravities** were identified in higher dimensions.

In $D=4$, a theory $\mathcal{L}(R_{abcd}, F_{ab})$ belongs to the EGQT class if the on-shell action $L_{f,F^{\text{mag}}}$ evaluated in the ansatz [Cano, Murcia]

$$\text{SSSm: } \mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2\right), \quad F^{\text{mag}} = p \sin\theta \mathrm{d}\theta \wedge \mathrm{d}\varphi$$

satisfies the Euler-Lagrange equation for $f(r)$

Some remarks:

- **Same classification** as GQT gravities applies for the eom
- They share some of the properties previously discussed

In $D = 3$, all GQT gravities belong to the **trivial** family, i.e. their eom **vanish identically** on SSS ansätze [Bueno, Cano, Hennigar, Lu, JM]

However, this situation drastically changes when matter is added into the picture. Recently, **Electromagnetic GQT gravities** were identified in higher dimensions.

In $D = 4$, a theory $\mathcal{L}(R_{abcd}, F_{ab})$ belongs to the EGQT class if the on-shell action $L_{f,F^{\text{mag}}}$ evaluated in the ansatz [Cano, Murcia]

$$\text{SSSm: } \mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right), \quad F^{\text{mag}} = p\sin\theta\mathrm{d}\theta\wedge\mathrm{d}\varphi$$

satisfies the Euler-Lagrange equation for $f(r)$

Some remarks:

- **Same classification** as GQT gravities applies for the eom
- They share some of the properties previously discussed
- Theories can be **dualized** and work with an electric ansatz (Normally less convenient)

In $D = 3$, all GQT gravities belong to the **trivial** family, i.e. their eom **vanish identically** on SSS ansätze [Bueno, Cano, Hennigar, Lu, JM]

However, this situation drastically changes when matter is added into the picture. Recently, **Electromagnetic GQT gravities** were identified in higher dimensions.

In $D = 4$, a theory $\mathcal{L}(R_{abcd}, F_{ab})$ belongs to the EGQT class if the on-shell action $L_{f,F^{\text{mag}}}$ evaluated in the ansatz [Cano, Murcia]

$$\text{SSSm: } \mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right), \quad F^{\text{mag}} = p\sin\theta\mathrm{d}\theta\wedge\mathrm{d}\varphi$$

satisfies the Euler-Lagrange equation for $f(r)$

Some remarks:

- **Same classification** as GQT gravities applies for the eom
- They share some of the properties previously discussed
- Theories can be **dualized** and work with an electric ansatz (Normally less convenient)
- They are shown to exist at arbitrarily high D [Cano, Murcia, Rivadulla, Zhang]

EQT gravities in $D = 3$

Based on the previous considerations, we inspect $\mathcal{L}(R_{ab}, \partial_a \phi)$ theories **linear** in curvature, that satisfiy the Euler-Lagrange equation evaluated on

$$\text{SSSm: } \mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2 \mathrm{d}\varphi^2, \quad \varphi = p\varphi$$

.

Based on the previous considerations, we inspect $\mathcal{L}(R_{ab}, \partial_a \phi)$ theories **linear** in curvature, that satisfiy the Euler-Lagrange equation evaluated on

$$\text{SSSm: } \mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2 \mathrm{d}\varphi^2, \quad \varphi = p\varphi$$

These theories are written as $\mathcal{L}_{\text{EQT}}(R_{ab}, \partial_a \phi) = \frac{1}{16\pi G_{\text{N}}} \left[R + \frac{2}{L^2} - \sum_{n=0}^{1} \mathcal{G}_n \right]$, with

$$\mathcal{G}_0 \equiv + \sum_{i=1} \beta_{0,i} L^{2(i-1)} \Phi_0^i$$
$$\mathcal{G}_1 \equiv - \sum_{j=0} \beta_{1,j} L^{2(j+1)} \Phi_0^j \left[(3+2j)\Phi_1 - \Phi_0 R \right]$$

where $\beta_{0,i}$, $\beta_{1,j}$ α_n, β_m are undetermined, dimensionless **coupling constants** and we defined $\Phi_0 \equiv g^{ab} \partial_a \phi \partial_b \phi$, $\Phi_1 \equiv R_{ab} \partial^a \phi \partial^b \phi$.

Based on the previous considerations, we inspect $\mathcal{L}(R_{ab}, \partial_a \phi)$ theories **linear** in curvature, that satisfiy the Euler-Lagrange equation evaluated on

$$\text{SSSm: } \mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2 \mathrm{d}\varphi^2, \quad \varphi = p\varphi$$

These theories are written as $\mathcal{L}_{\mathrm{EQT}}(R_{ab}, \partial_a \phi) = \frac{1}{16\pi G_{\mathrm{N}}} \left[R + \frac{2}{L^2} - \sum_{n=0}^{1} \mathcal{G}_n\right]$, with

$$\mathcal{G}_0 \equiv + \sum_{i=1} \beta_{0,i} L^{2(i-1)} \Phi_0^i$$

$$\mathcal{G}_1 \equiv - \sum_{j=0} \beta_{1,j} L^{2(j+1)} \Phi_0^j \left[(3+2j)\Phi_1 - \Phi_0 R\right]$$

where $\beta_{0,i}$, $\beta_{1,j}$ α_n, β_m are undetermined, dimensionless **coupling constants** and we defined $\Phi_0 \equiv g^{ab}\partial_a \phi \partial_b \phi$, $\Phi_1 \equiv R_{ab}\partial^a \phi \partial^b \phi$. Their eom for $f(r)$ are given by

$$\frac{r^2}{L^2} - f(r) + \mathcal{E}_{(0)} + \mathcal{E}_{(1)} = \lambda$$

where the additional terms read $\mathcal{E}_{(0)} \equiv + \sum_{i=2} \frac{\beta_{0,i} p^2}{2(i-1)} \left(\frac{pL}{r}\right)^{2(i-1)} - \beta_{0,1} p^2 \log\left(\frac{r}{L}\right)$, $\mathcal{E}_{(1)} \equiv - \sum_{j=0} \beta_{1,j} (2j+1) \left(\frac{pL}{r}\right)^{2(j+1)} f(r)$

The eom are **algebraic** in $f(r)$, so densities $\mathcal{G}_0$ and $\mathcal{G}_1$ belong to **EQT class**.

The eom are **algebraic** in $f(r)$, so densities $\mathcal{G}_0$ and $\mathcal{G}_1$ belong to **EQT class**. Explicitly, they read

$$f(r)=\frac{\left[\frac{r^2}{L^2}-\lambda-\beta_{0,1}p^2\log\left(\frac{r}{L}\right)+\sum_{i=2}\frac{\beta_{0,i}p^2}{2(i-1)}\left(\frac{pL}{r}\right)^{2(i-1)}\right]}{\left[1+\sum_{j=0}\beta_{1,j}(2j+1)\left(\frac{pL}{r}\right)^{2(j+1)}\right]}$$

.

The eom are **algebraic** in $f(r)$, so densities $\mathcal{G}_0$ and $\mathcal{G}_1$ belong to **EQT class**. Explicitly, they read

$$f(r) = \frac{\left[\frac{r^2}{L^2} - \lambda - \beta_{0,1} p^2 \log\left(\frac{r}{L}\right) + \sum_{i=2} \frac{\beta_{0,i} p^2}{2(i-1)} \left(\frac{pL}{r}\right)^{2(i-1)}\right]}{\left[1 + \sum_{j=0} \beta_{1,j} (2j+1) \left(\frac{pL}{r}\right)^{2(j+1)}\right]}$$

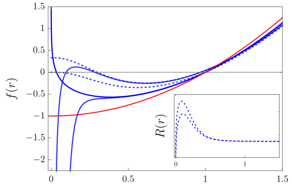
This represents **multiparametric generalizations** of BTZ controlled by $\beta_{0,i}$, $\beta_{1,j}$. In fact the charged BTZ corresponds to $\beta_{0,1} \neq 0$.

The eom are **algebraic** in $f(r)$, so densities $\mathcal{G}_0$ and $\mathcal{G}_1$ belong to **EQT class**. Explicitly, they read

$$f(r) = \frac{\left[\frac{r^2}{L^2} - \lambda - \beta_{0,1} p^2 \log\left(\frac{r}{L}\right) + \sum_{i=2} \frac{\beta_{0,i} p^2}{2(i-1)} \left(\frac{pL}{r}\right)^{2(i-1)}\right]}{\left[1 + \sum_{j=0} \beta_{1,j} (2j+1) \left(\frac{pL}{r}\right)^{2(j+1)}\right]}$$

This represents **multiparametric generalizations** of BTZ controlled by $\beta_{0,i}$, $\beta_{1,j}$. In fact the charged BTZ corresponds to $\beta_{0,1} \neq 0$. Among them we find

- Black holes with singularities whose nature depends on the value of λ.

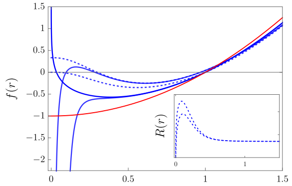


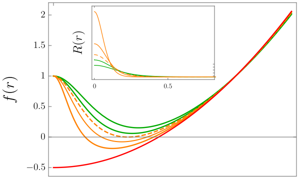
The eom are **algebraic** in $f(r)$, so densities $\mathcal{G}_0$ and $\mathcal{G}_1$ belong to **EQT class**. Explicitly, they read

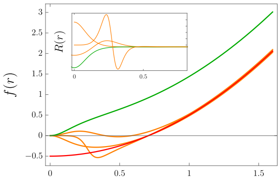
$$f(r) = \frac{\left[\frac{r^2}{L^2} - \lambda - \beta_{0,1} p^2 \log\left(\frac{r}{L}\right) + \sum_{i=2} \frac{\beta_{0,i} p^2}{2(i-1)} \left(\frac{pL}{r}\right)^{2(i-1)}\right]}{\left[1 + \sum_{j=0} \beta_{1,j} (2j+1) \left(\frac{pL}{r}\right)^{2(j+1)}\right]}$$

This represents **multiparametric generalizations** of BTZ controlled by $\beta_{0,i}$, $\beta_{1,j}$. In fact the charged BTZ corresponds to $\beta_{0,1} \neq 0$. Among them we find

- Black holes with singularities whose nature depends on the value of λ.
- **Regular** black holes, i.e., without singularity.





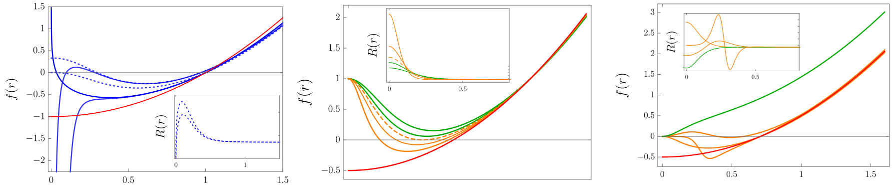


The eom are **algebraic** in $f(r)$, so densities $\mathcal{G}_0$ and $\mathcal{G}_1$ belong to **EQT class**. Explicitly, they read

$$f(r) = \frac{\left[\frac{r^2}{L^2} - \lambda - \beta_{0,1} p^2 \log\left(\frac{r}{L}\right) + \sum_{i=2} \frac{\beta_{0,i} p^2}{2(i-1)} \left(\frac{pL}{r}\right)^{2(i-1)}\right]}{\left[1 + \sum_{j=0} \beta_{1,j}(2j+1)\left(\frac{pL}{r}\right)^{2(j+1)}\right]}$$

This represents **multiparametric generalizations** of BTZ controlled by $\beta_{0,i}$, $\beta_{1,j}$. In fact the charged BTZ corresponds to $\beta_{0,1} \neq 0$. Among them we find

- Black holes with singularities whose nature depends on the value of λ.
- **Regular** black holes, i.e., without singularity.
- Horizonless spacetimes.



Electromagnetic GQT gravities in $D = 3$

We are interested in identifying what $\mathcal{L}(R_{ab}, \partial_a\phi)$ theories belong to the EGQT family **beyond** linear order in curvature.

We are interested in identifying what $\mathcal{L}(R_{ab}, \partial_a \phi)$ theories belong to the EGQT family **beyond** linear order in curvature.

First, we count what are the **possible** densities at each order in curvature n

$$
\begin{aligned}
n = 0 \quad & \Phi_0 \equiv g^{ab}\partial_a\phi\partial_b\phi \\
n = 1 \quad & R,\ \Phi_1 \equiv R_{ab}\partial^a\phi\partial_b\phi \\
n = 2 \quad & \mathcal{R}_2 \equiv R_a^b R_b^a,\ \Phi_2 \equiv R_{ac}R_b^c\partial^a\phi\partial^b\phi \\
n = 3 \quad & \mathcal{R}_3 \equiv R_a^b R_b^c R_c^a
\end{aligned}
$$

Any other invariant can be expressed in terms of these 6 “seeds” through **Schouten identities** [Paulos].

We are interested in identifying what $\mathcal{L}(R_{ab}, \partial_a \phi)$ theories belong to the EGQT family **beyond** linear order in curvature.

First, we count what are the **possible** densities at each order in curvature n

$$
\begin{aligned}
n &= 0 \quad && \Phi_0 \equiv g^{ab}\partial_a\phi\partial_b\phi \\
n &= 1 \quad && R,\ \Phi_1 \equiv R_{ab}\partial^a\phi\partial_b\phi \\
n &= 2 \quad && \mathcal{R}_2 \equiv R_a^b R_b^a,\ \Phi_2 \equiv R_{ac}R_b^c\partial^a\phi\partial^b\phi \\
n &= 3 \quad && \mathcal{R}_3 \equiv R_a^b R_b^c R_c^a
\end{aligned}
$$

Any other invariant can be expressed in terms of these 6 “seeds” through **Schouten identities** [Paulos]. Using them, the most general theory can be written as

$$\mathcal{L}(R_{ab}, \partial_a \phi) = \sum_{i,j,k,l,m,p} \alpha_{ijklmp} R^i \mathcal{R}_2^j \mathcal{R}_3^k \Phi_0^l \Phi_1^m \Phi_2^p$$

where the **curvature order** of a particular density is $n = i + 2j + 3k + m + 2p$

Among the $\#(n)$ number of monomials, what **combinations** give EGQT at fixed order n? In the case of $n = 2$ we have 5 terms

$$\mathcal{L}^{(2)}_{\text{general}} = F_{1,(2)} R^2 + F_{2,(2)} \mathcal{R}_2 + F_{3,(2)} R \Phi_1 + F_{4,(2)} \Phi_1^2 + F_{5,(2)} \Phi_2 \,.$$

where $F_{1,(2)}, \ldots, F_{5,(2)}$ are arbitrary free functions of Φ_0

Among the $\#(n)$ number of monomials, what **combinations** give EGQT at fixed order n? In the case of $n = 2$ we have 5 terms

$$\mathcal{L}^{(2)}_{\text{general}} = F_{1,(2)} R^2 + F_{2,(2)} \mathcal{R}_2 + F_{3,(2)} R\Phi_1 + F_{4,(2)} \Phi_1^2 + F_{5,(2)} \Phi_2 \,.$$

where $F_{1,(2)}, \ldots, F_{5,(2)}$ are arbitrary free functions of Φ_0

Imposing the EGQT condition **reduces** the number of independent combinations from 5 to 3

$$\tilde{\mathcal{G}}_2 \equiv F_{(2)} \Phi_0 \left(\frac{R^2}{2} - \mathcal{R}_2 \right) - \left(\Phi_0 F_{(2)}{}' + \frac{3}{2} F_{(2)} \right) \Phi_2 + G_{1,(2)} \mathcal{T}_{1,(2)} + 3 G_{2,(2)} \mathcal{T}_{2,(2)}$$

where $F_{(2)}$, $G_{1,(2)}$, $G_{2,(2)}$ are now the free functions and $\mathcal{T}_{1,(2)} \equiv \Phi_1^2 - \Phi_0 \Phi_2$, $\mathcal{T}_{2,(2)} \equiv 3\Phi_2 - \Phi_0 \left(2\mathcal{R}_2 - R^2 \right) - 2R\Phi_1$. However, $\mathcal{T}_{1,(2)}$ and $\mathcal{T}_{2,(2)}$ **vanish identically** when evaluated in the SSSm ansatz.

Among the $\#(n)$ number of monomials, what **combinations** give EGQT at fixed order n? In the case of $n=2$ we have 5 terms

$$\mathcal{L}^{(2)}_{\text{general}} = F_{1,(2)}R^2 + F_{2,(2)}\mathcal{R}_2 + F_{3,(2)}R\Phi_1 + F_{4,(2)}\Phi_1^2 + F_{5,(2)}\Phi_2\,.$$

where $F_{1,(2)},\ldots,F_{5,(2)}$ are arbitrary free functions of Φ_0

Imposing the EGQT condition **reduces** the number of independent combinations from 5 to 3

$$\tilde{\mathcal{G}}_2 \equiv F_{(2)}\Phi_0\left(\frac{R^2}{2}-\mathcal{R}_2\right)-\left(\Phi_0 F_{(2)}{}'+\frac{3}{2}F_{(2)}\right)\Phi_2 + G_{1,(2)}\mathcal{T}_{1,(2)} + 3G_{2,(2)}\mathcal{T}_{2,(2)}$$

where $F_{(2)}$, $G_{1,(2)}$, $G_{2,(2)}$ are now the free functions and $\mathcal{T}_{1,(2)} \equiv \Phi_1^2 - \Phi_0\Phi_2$, $\mathcal{T}_{2,(2)} \equiv 3\Phi_2 - \Phi_0\left(2\mathcal{R}_2 - R^2\right) - 2R\Phi_1$. However, $\mathcal{T}_{1,(2)}$ and $\mathcal{T}_{2,(2)}$ **vanish identically** when evaluated in the SSSm ansatz. Thus

There is only 1 family of EGQT gravity at $n=2$, controlled by the free function $F_{(2)}[\Phi_0]$

After illustrating the situation in $n = 2$, we move to arbitrary **higher-curvature** order. We compute $\#_{\text{nontrivial}}(n)$ densities by working with monomials constructed from the **traceless Ricci tensors** $\mathcal{S}_{ab} \equiv R_{ab} - \frac{g_{ab}}{3} R$, this is

$$\mathcal{S}_2 \equiv \mathcal{S}_{ab}\mathcal{S}^{ab}\,, \quad \mathcal{S}_3 \equiv \mathcal{S}_a^b \mathcal{S}_b^c \mathcal{S}_c^a\,, \quad \Xi_1 \equiv \mathcal{S}_{ab}\partial^a\phi\partial^b\phi\,, \quad \Xi_2 \equiv \mathcal{S}_{ac}\mathcal{S}_b^c\partial^a\phi\partial^b\phi$$

After illustrating the situation in $n = 2$, we move to arbitrary **higher-curvature** order. We compute $\#_{\text{nontrivial}}(n)$ densities by working with monomials constructed from the **traceless Ricci tensors** $\mathcal{S}_{ab} \equiv R_{ab} - \frac{g_{ab}}{3} R$, this is

$$\mathcal{S}_2 \equiv \mathcal{S}_{ab}\mathcal{S}^{ab}\,, \quad \mathcal{S}_3 \equiv \mathcal{S}_a^b \mathcal{S}_b^c \mathcal{S}_c^a\,, \quad \Xi_1 \equiv \mathcal{S}_{ab}\partial^a\phi\partial^b\phi\,, \quad \Xi_2 \equiv \mathcal{S}_{ac}\mathcal{S}_b^c\partial^a\phi\partial^b\phi$$

Defining the quantities $A \equiv -\left(2f'/r + f''\right)$ and $B \equiv \left(f'' - f'/r\right)/3$, the independent on-shell densities read

$$R|_{\text{SSSm}} = A\,, \quad \Xi_1|_{\text{SSSm}} = \frac{p^2 B}{r^2}\,, \quad \mathcal{S}_2|_{\text{SSSm}} = \frac{3B^2}{2}\,, \quad \Xi_2|_{\text{SSSm}} = \frac{p^2 B^2}{r^2}\,, \quad \mathcal{S}_3|_{\text{SSSm}} = \frac{3B^3}{4}$$

After illustrating the situation in $n = 2$, we move to arbitrary **higher-curvature** order. We compute $\#_{\text{nontrivial}}(n)$ densities by working with monomials constructed from the **traceless Ricci tensors** $\mathcal{S}_{ab} \equiv R_{ab} - \frac{g_{ab}}{3} R$, this is

$$\mathcal{S}_2 \equiv \mathcal{S}_{ab}\mathcal{S}^{ab}\,, \quad \mathcal{S}_3 \equiv \mathcal{S}_a^b \mathcal{S}_b^c \mathcal{S}_c^a\,, \quad \Xi_1 \equiv \mathcal{S}_{ab}\partial^a\phi\partial^b\phi\,, \quad \Xi_2 \equiv \mathcal{S}_{ac}\mathcal{S}_b^c\partial^a\phi\partial^b\phi$$

Defining the quantities $A \equiv -\left(2f'/r + f''\right)$ and $B \equiv \left(f'' - f'/r\right)/3$, the independent on-shell densities read

$$R|_{\text{SSSm}} = A\,, \quad \Xi_1|_{\text{SSSm}} = \frac{p^2 B}{r^2}\,, \quad \mathcal{S}_2|_{\text{SSSm}} = \frac{3B^2}{2}\,, \quad \Xi_2|_{\text{SSSm}} = \frac{p^2 B^2}{r^2}\,, \quad \mathcal{S}_3|_{\text{SSSm}} = \frac{3B^3}{4}$$

At order n, all independent densities are given by $R^i \Xi_1^{n-i}$, with $i = 0, 1, \ldots n$ times arbitrary functions of Φ_0. Thus, the number of **nontrivial densities** is

$$\#_{\text{nontrivial}}(n) = n + 1$$

After illustrating the situation in $n = 2$, we move to arbitrary **higher-curvature** order. We compute $\#_{\text{nontrivial}}(n)$ densities by working with monomials constructed from the **traceless Ricci tensors** $\mathcal{S}_{ab} \equiv R_{ab} - \frac{g_{ab}}{3} R$, this is

$$\mathcal{S}_2 \equiv \mathcal{S}_{ab}\mathcal{S}^{ab}\,, \quad \mathcal{S}_3 \equiv \mathcal{S}_a^b \mathcal{S}_b^c \mathcal{S}_c^a\,, \quad \Xi_1 \equiv \mathcal{S}_{ab}\partial^a\phi\partial^b\phi\,, \quad \Xi_2 \equiv \mathcal{S}_{ac}\mathcal{S}_b^c\partial^a\phi\partial^b\phi$$

Defining the quantities $A \equiv -\left(2f'/r + f''\right)$ and $B \equiv \left(f'' - f'/r\right)/3$, the independent on-shell densities read

$$R|_{\text{SSSm}} = A\,, \quad \Xi_1|_{\text{SSSm}} = \frac{p^2 B}{r^2}\,, \quad \mathcal{S}_2|_{\text{SSSm}} = \frac{3B^2}{2}\,, \quad \Xi_2|_{\text{SSSm}} = \frac{p^2 B^2}{r^2}\,, \quad \mathcal{S}_3|_{\text{SSSm}} = \frac{3B^3}{4}$$

At order n, all independent densities are given by $R^i \Xi_1^{n-i}$, with $i = 0, 1, \ldots n$ times arbitrary functions of Φ_0. Thus, the number of **nontrivial densities** is

$$\#_{\text{nontrivial}}(n) = n + 1$$

At this order, the EGQT condition imposes n **constraints** on the free functions, then

There is, **at most**, one family of theories of the type EGQT at each order n in curvature

Now we have to check there is **exactly** one.

Now we have to check there is **exactly** one. We notice that we are able to construct EGQT densities using lower-order ones

$$\mathcal{G}_n = \frac{(n-2)(n-1)}{n\Phi_0^2 F'[\Phi_0](n-3)} \left[\mathcal{G}_{n-2}\mathcal{G}_2 - \mathcal{G}_{n-1}\mathcal{G}_1\right], \quad n > 2$$

Now we have to check there is **exactly** one. We notice that we are able to construct EGQT densities using lower-order ones

$$\mathcal{G}_n = \frac{(n-2)(n-1)}{n\Phi_0^2 F'[\Phi_0](n-3)} \left[\mathcal{G}_{n-2}\mathcal{G}_2 - \mathcal{G}_{n-1}\mathcal{G}_1\right], \quad n > 2$$

The recurring relation **guarantees** the existence of the EGQT gravity at each order n

Now we have to check there is **exactly** one. We notice that we are able to construct EGQT densities using lower-order ones

$$\mathcal{G}_n = \frac{(n-2)(n-1)}{n\Phi_0^2 F'[\Phi_0](n-3)} \left[\mathcal{G}_{n-2}\mathcal{G}_2 - \mathcal{G}_{n-1}\mathcal{G}_1\right], \quad n > 2$$

The recurring relation **guarantees** the existence of the EGQT gravity at each order n

This allows us to write the most general EGQT gravity as

$$I_{\mathrm{EGQT}} = \frac{1}{16\pi G} \int \mathrm{d}^3x \sqrt{|g|} \left[R + \frac{2}{L^2} - \sum_{n=0} \mathcal{G}_n\right]$$

where $\mathcal{G}_0$ and $\mathcal{G}_1$ belong to the EQT subfamily, whereas

$$\mathcal{G}_n = + \sum_{k=0} \frac{(-1)^n \beta_{n,k}}{n} L^{2(k+2n-1)} \Phi_0^k \left[(2k+5n-2)\Phi_1 - n\Phi_0 R\right] \Phi_1^{n-1}, \quad n \geq 1$$

with $n > 1$ is a **genuine** EGQT (strictly second order eom for f)

We can easily compute the eom of the **general** EGQT theory, reading

$$\frac{r^2}{L^2} - f - \lambda - \beta_{0,1} p^2 \log \frac{r}{L} + \sum_{k=2} \frac{\beta_{0,k} p^2}{2(k-1)} \left(\frac{pL}{r} \right)^{2(k-1)} + \sum_{n=1} \mathcal{E}_{(n)} = 0$$

where λ is an integration constant related to the mass of the solution and $\mathcal{E}_{(n)}$ reads

$$\mathcal{E}_{(n)} \equiv \sum_{k=0} \frac{-\beta_{n,k} L^{2(2n-1+k)} p^{2(k+n)}}{n r^{3n+2k-1}} \left[n(3n+2k-2) f {f'}^{(n-1)} + (n-1) r [{f'}^{n} - n f {f'}^{n-2} f''] \right]$$

We can easily compute the eom of the **general** EGQT theory, reading

$$\frac{r^2}{L^2} - f - \lambda - \beta_{0,1} p^2 \log \frac{r}{L} + \sum_{k=2} \frac{\beta_{0,k} p^2}{2(k-1)} \left(\frac{pL}{r}\right)^{2(k-1)} + \sum_{n=1} \mathcal{E}_{(n)} = 0$$

where λ is an integration constant related to the mass of the solution and $\mathcal{E}_{(n)}$ reads

$$\mathcal{E}_{(n)} \equiv \sum_{k=0} \frac{-\beta_{n,k} L^{2(2n-1+k)} p^{2(k+n)}}{n r^{3n+2k-1}} \left[n(3n+2k-2) f f'^{(n-1)} + (n-1) r [f'^{n} - n f f'^{n-2} f''] \right]$$

The case $n = 1$ **reduces** to the EQT gravity presented before.

We can easily compute the eom of the **general** EGQT theory, reading

$$\frac{r^2}{L^2} - f - \lambda - \beta_{0,1} p^2 \log \frac{r}{L} + \sum_{k=2} \frac{\beta_{0,k} p^2}{2(k-1)} \left(\frac{pL}{r}\right)^{2(k-1)} + \sum_{n=1} \mathcal{E}_{(n)} = 0$$

where λ is an integration constant related to the mass of the solution and $\mathcal{E}_{(n)}$ reads

$$\mathcal{E}_{(n)} \equiv \sum_{k=0} \frac{-\beta_{n,k} L^{2(2n-1+k)} p^{2(k+n)}}{n r^{3n+2k-1}} \left[n(3n+2k-2) f f'^{(n-1)} + (n-1) r [f'^{n} - n f f'^{n-2} f'']\right]$$

The case $n = 1$ **reduces** to the EQT gravity presented before.

Properties of these eom

- As before, turning on different $\beta_{n,k}$ can **trigger** the existence of a variety of solutions

We can easily compute the eom of the **general** EGQT theory, reading

$$\frac{r^2}{L^2} - f - \lambda - \beta_{0,1}p^2 \log\frac{r}{L} + \sum_{k=2}\frac{\beta_{0,k}p^2}{2(k-1)}\left(\frac{pL}{r}\right)^{2(k-1)} + \sum_{n=1}\mathcal{E}_{(n)} = 0$$

where λ is an integration constant related to the mass of the solution and $\mathcal{E}_{(n)}$ reads

$$\mathcal{E}_{(n)} \equiv \sum_{k=0}\frac{-\beta_{n,k}L^{2(2n-1+k)}p^{2(k+n)}}{nr^{3n+2k-1}}\left[n(3n+2k-2)ff'^{(n-1)} + (n-1)r[f'^{n} - nff'^{n-2}f'']\right]$$

The case $n = 1$ **reduces** to the EQT gravity presented before.

Properties of these eom

- As before, turning on different $\beta_{n,k}$ can **trigger** the existence of a variety of solutions
- Now we have to deal with second order differential eom instead of algebraic ones

We can easily compute the eom of the **general** EGQT theory, reading

$$\frac{r^2}{L^2} - f - \lambda - \beta_{0,1} p^2 \log \frac{r}{L} + \sum_{k=2} \frac{\beta_{0,k} p^2}{2(k-1)} \left(\frac{pL}{r} \right)^{2(k-1)} + \sum_{n=1} \mathcal{E}_{(n)} = 0$$

where λ is an integration constant related to the mass of the solution and $\mathcal{E}_{(n)}$ reads

$$\mathcal{E}_{(n)} \equiv \sum_{k=0} \frac{-\beta_{n,k} L^{2(2n-1+k)} p^{2(k+n)}}{n r^{3n+2k-1}} \left[n(3n+2k-2) f f'^{(n-1)} + (n-1) r [f'^{n} - n f f'^{n-2} f''] \right]$$

The case $n = 1$ **reduces** to the EQT gravity presented before.

Properties of these eom

- As before, turning on different $\beta_{n,k}$ can **trigger** the existence of a variety of solutions
- Now we have to deal with second order differential eom instead of algebraic ones

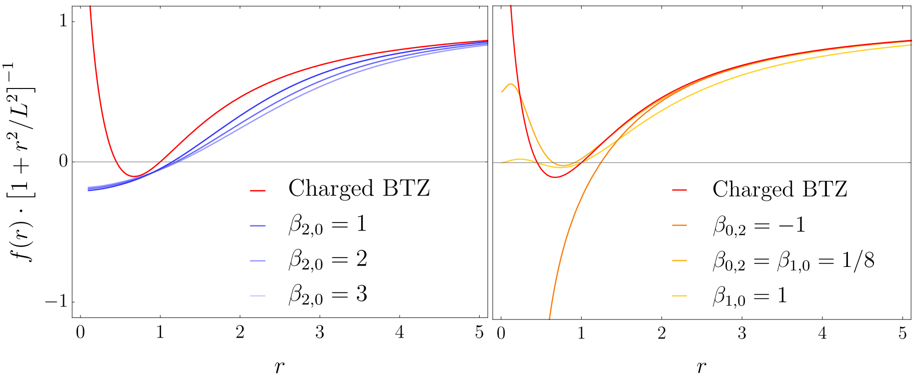
Although they cannot be solved analytically in general, it is possible to establish the **existence** of black hole solutions and construct them **numerically**

In order for the metric to describe black holes, there are two boundary conditions to be satisfied

- **Regularity** at the horizon
- Correct **asymptotic behavior**

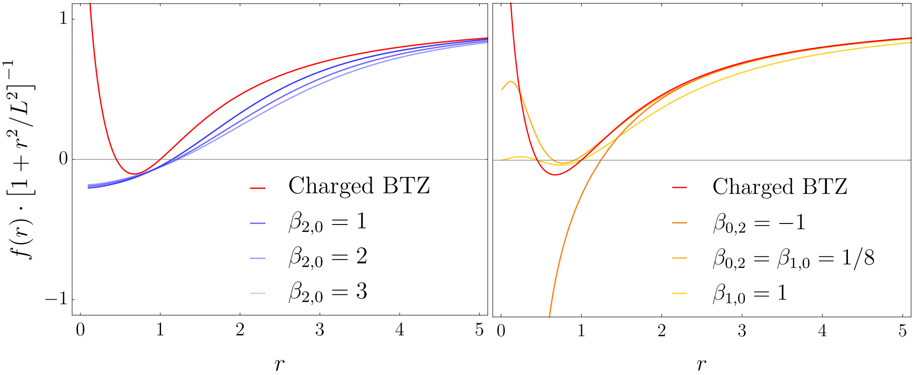
In order for the metric to describe black holes, there are two boundary conditions to be satisfied

- **Regularity** at the horizon
- Correct **asymptotic behavior**



In order for the metric to describe black holes, there are two boundary conditions to be satisfied

- **Regularity** at the horizon
- Correct **asymptotic behavior**



Higher-curvature term smooths out the charged BTZ singularity

Even though $f(r)$ can only be determined numerically, the **thermodynamics** can be computed **analytically**.

Even though $f(r)$ can only be determined numerically, the **thermodynamics** can be computed **analytically**. From $4\pi T = f'(r_{\rm h}) =$ and the condition $f(r_{\rm h}) = 0$ we obtain the constraint

$$2 - y - \sum_{k=1} \beta_{0,k} x^k - \sum_{n=1} \sum_{k=0} \beta_{n,k} \frac{(3n + 2k - 2)}{n} x^{n+k} y^n = 0$$

where

$$x \equiv \frac{L^2 p^2}{{r_{\rm h}}^2}\,, \quad \text{and} \quad y \equiv \frac{4\pi L^2 T}{r_{\rm h}}$$

The variables $\{r_{\rm h}, x, y\}$ yield more **compact** expressions than $\{r_{\rm h}, T, p\}$.

Even though $f(r)$ can only be determined numerically, the **thermodynamics** can be computed **analytically**. From $4\pi T = f'(r_\mathrm{h}) =$ and the condition $f(r_\mathrm{h}) = 0$ we obtain the constraint

$$2 - y - \sum_{k=1} \beta_{0,k} x^k - \sum_{n=1} \sum_{k=0} \beta_{n,k} \frac{(3n + 2k - 2)}{n} x^{n+k} y^n = 0$$

where

$$x \equiv \frac{L^2 p^2}{{r_\mathrm{h}}^2}, \quad \text{and} \quad y \equiv \frac{4\pi L^2 T}{r_\mathrm{h}}$$

The variables $\{r_\mathrm{h}, x, y\}$ yield more **compact** expressions than $\{r_\mathrm{h}, T, p\}$. Using them we compute

- **Entropy** using Wald's formula S
- **Free energy** F from the Euclidean on-shell action
- **Potential** associated to the charge $Q = \frac{1}{4\pi G} \int_{\mathbb{S}^1} \mathrm{d}\phi = \frac{p}{2G} = \frac{r_\mathrm{h}}{2 G_\mathrm{N} L} \sqrt{x}$

Even though $f(r)$ can only be determined numerically, the **thermodynamics** can be computed **analytically**. From $4\pi T = f'(r_{\mathrm{h}}) =$ and the condition $f(r_{\mathrm{h}}) = 0$ we obtain the constraint

$$2 - y - \sum_{k=1} \beta_{0,k} x^k - \sum_{n=1} \sum_{k=0} \beta_{n,k} \frac{(3n + 2k - 2)}{n} x^{n+k} y^n = 0$$

where

$$x \equiv \frac{L^2 p^2}{{r_{\mathrm{h}}}^2}, \quad \text{and} \quad y \equiv \frac{4\pi L^2 T}{r_{\mathrm{h}}}$$

The variables $\{r_{\mathrm{h}}, x, y\}$ yield more **compact** expressions than $\{r_{\mathrm{h}}, T, p\}$. Using them we compute

- **Entropy** using Wald's formula S
- **Free energy** F from the Euclidean on-shell action
- **Potential** associated to the charge $Q = \frac{1}{4\pi G} \int_{\mathbb{S}^1} \mathrm{d}\phi = \frac{p}{2G} = \frac{r_{\mathrm{h}}}{2G_{\mathrm{N}} L} \sqrt{x}$

With these ingredients we verify the **first law**

$$\mathrm{d}F = -S\mathrm{d}T + \Phi \mathrm{d}Q$$

Conclusions

Conclusions
and thank you

We characterized and constructed the most general EGQT gravity in $D = 3$. The key points are

Conclusions
and thank you

We characterized and constructed the most general EGQT gravity in $D = 3$. The key points are

- At **linear** curvature order, we have theories belonging to the EQT subfamily

Conclusions
and thank you

We characterized and constructed the most general EGQT gravity in $D = 3$. The key points are

- At **linear** curvature order, we have theories belonging to the EQT subfamily
- We can analytically solve the eom for EQT gravities and find a **plethora of solutions** presenting one, several or none horizons. Among them, regular black holes

Conclusions
and thank you

We characterized and constructed the most general EGQT gravity in $D = 3$. The key points are

- At **linear** curvature order, we have theories belonging to the EQT subfamily
- We can analytically solve the eom for EQT gravities and find a **plethora of solutions** presenting one, several or none horizons. Among them, regular black holes
- Beyond linear curvature order, there is **one and only one** genuine family of EGQT theories whose Lagrangian satisfies a recurring relation involving lower curvature densities

Conclusions
and thank you

We characterized and constructed the most general EGQT gravity in $D = 3$. The key points are

- At **linear** curvature order, we have theories belonging to the EQT subfamily
- We can analytically solve the eom for EQT gravities and find a **plethora of solutions** presenting one, several or none horizons. Among them, regular black holes
- Beyond linear curvature order, there is **one and only one** genuine family of EGQT theories whose Lagrangian satisfies a recurring relation involving lower curvature densities
- We presented numerical solutions to the eom that indicate the existence of black holes

Conclusions
and thank you

We characterized and constructed the most general EGQT gravity in $D = 3$. The key points are

- At **linear** curvature order, we have theories belonging to the EQT subfamily
- We can analytically solve the eom for EQT gravities and find a **plethora of solutions** presenting one, several or none horizons. Among them, regular black holes
- Beyond linear curvature order, there is **one and only one** genuine family of EGQT theories whose Lagrangian satisfies a recurring relation involving lower curvature densities
- We presented numerical solutions to the eom that indicate the existence of black holes
- Remarkably, the **thermodynamics** of the solutions in all EGQT theories can be computed **analytically**

Conclusions

and thank you

We characterized and constructed the most general EGQT gravity in $D = 3$. The key points are

- At **linear** curvature order, we have theories belonging to the EQT subfamily
- We can analytically solve the eom for EQT gravities and find a **plethora of solutions** presenting one, several or none horizons. Among them, regular black holes
- Beyond linear curvature order, there is **one and only one** genuine family of EGQT theories whose Lagrangian satisfies a recurring relation involving lower curvature densities
- We presented numerical solutions to the eom that indicate the existence of black holes
- Remarkably, the **thermodynamics** of the solutions in all EGQT theories can be computed **analytically**

Thank you