Higher-curvature Gravities from Braneworlds and the Holographic c-theorem

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Higher-curvature gravitational densities induced from braneworld holography fulfill a simple holographic c-theorem

Higher-curvature gravity

See e.g. Myers [9811042], Cano [1912.07035], Moreno [2207.12889], etc.

What is it?

- GR

$$I = \int d^d x \sqrt{-g} \, (\Lambda + R)$$

Higher-curvature gravity

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What is it?

- GR with higher-derivative operators $O(g_{ab}, R_{abcd}, \nabla_a)$ added to the action

$$I = \int d^d x \sqrt{-g} \left(\Lambda + R + \alpha_1 R^2 + \alpha_2 R_{ab} R^{ab} + \alpha_3 R_{abcd} R^{abcd} + \cdots \right)$$

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Why do we care?

- Motivated by EFT arguments and string theory.
- Respects diffeomorphism invariance.

Braneworld Holography

[Randall, Sundrum; Karch, Randall; Porrati; Verlinde; Gubser; de Haro, Skenderis, Solodukhin; Takayanagi; Neuenfeld; and many more]

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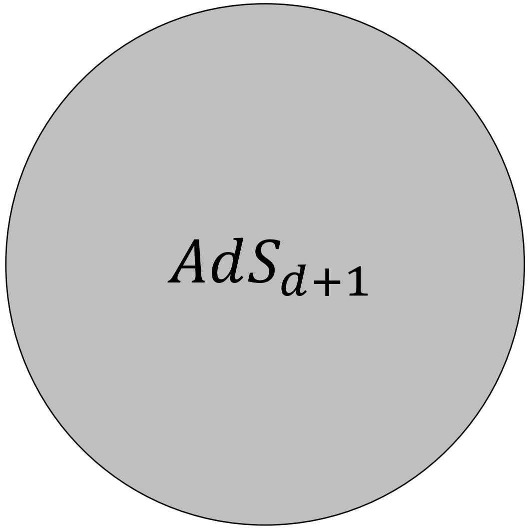
- Standard holography:



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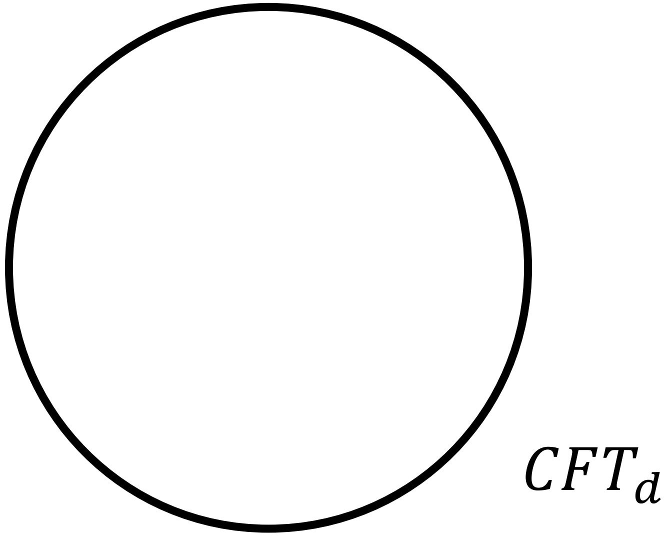

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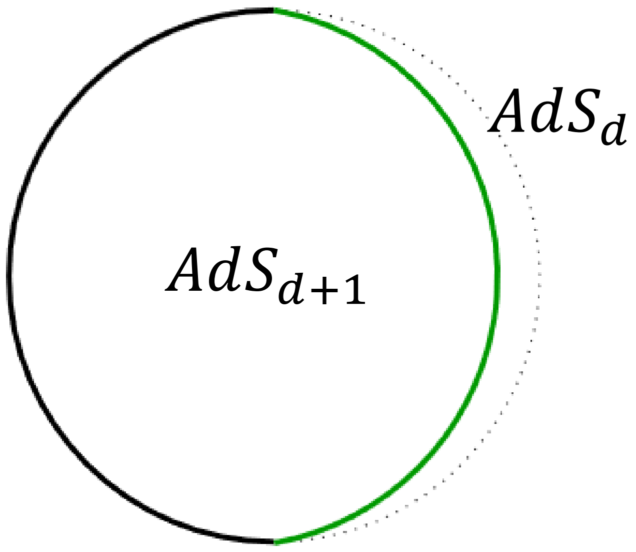

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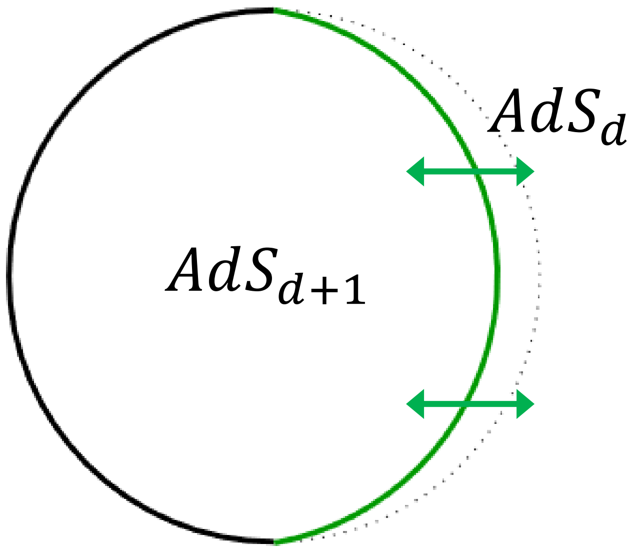
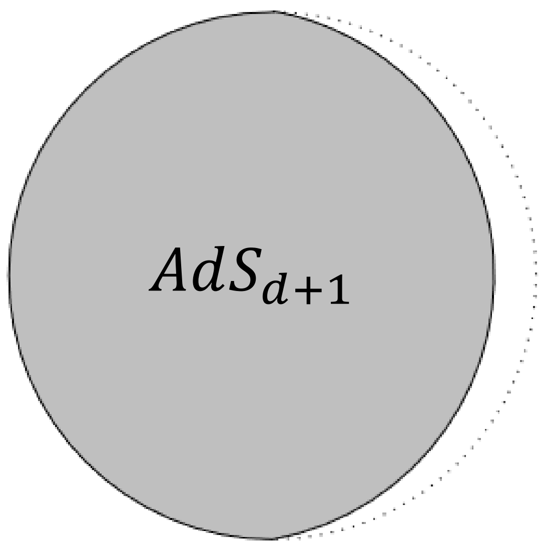
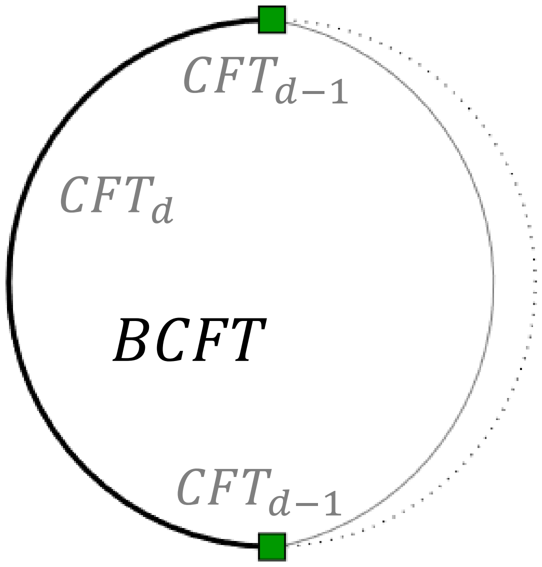


Braneworld Holography

Bulk



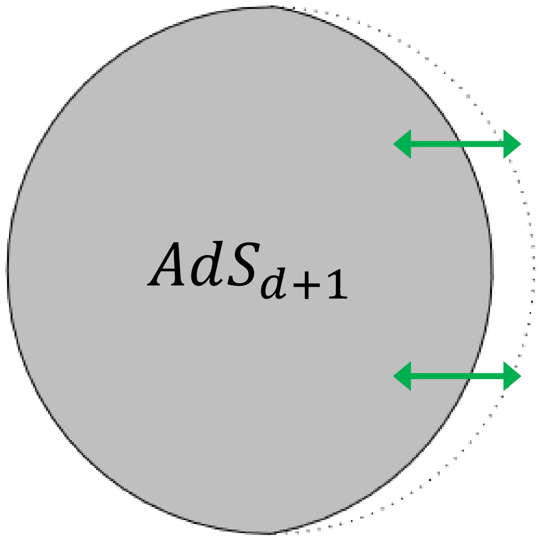
Boundary



Holographic g-theorem

[Takayanagi]

Bulk



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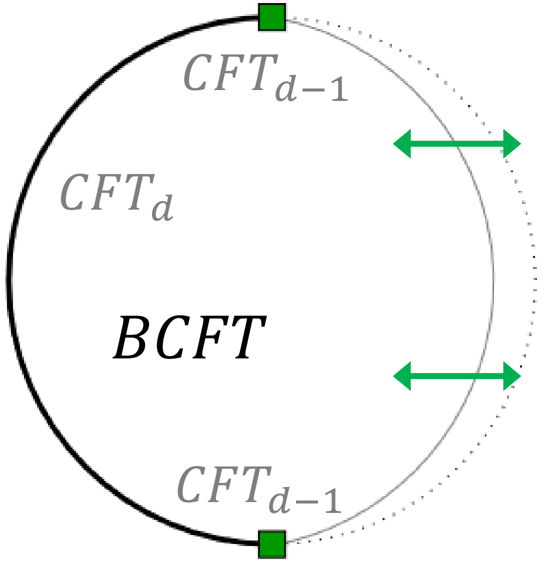

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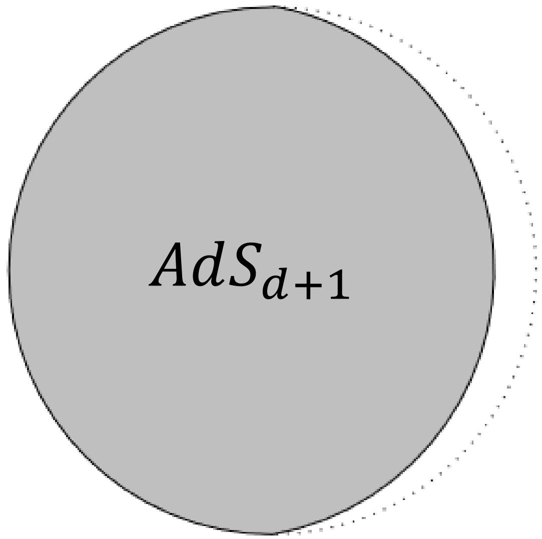

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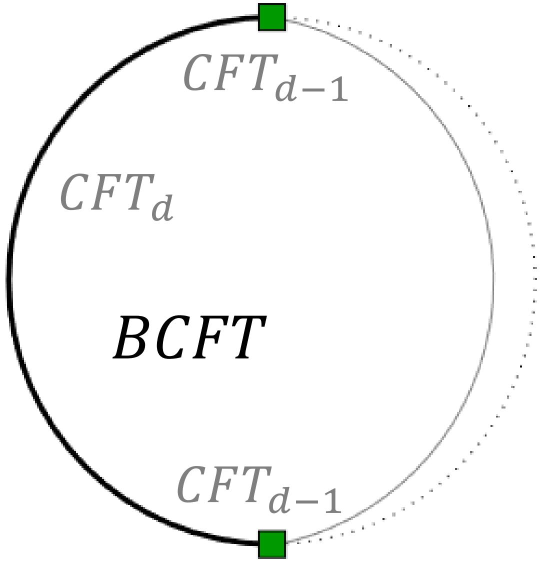

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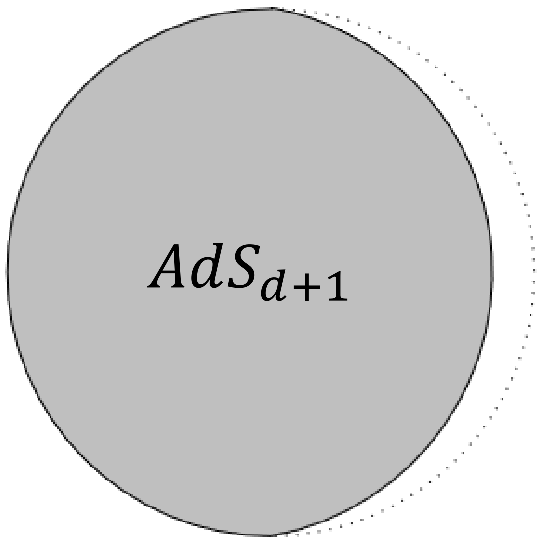
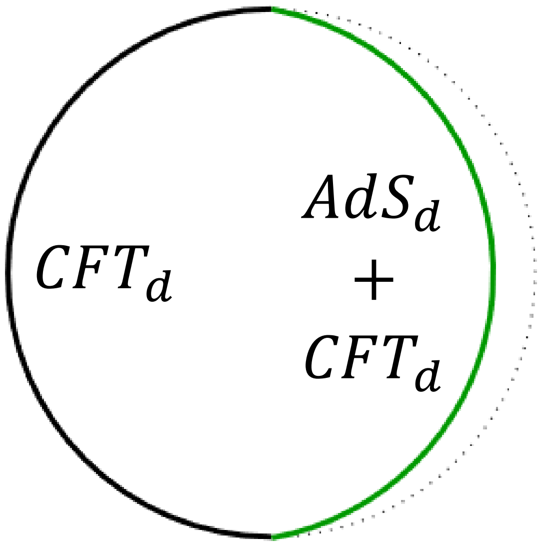
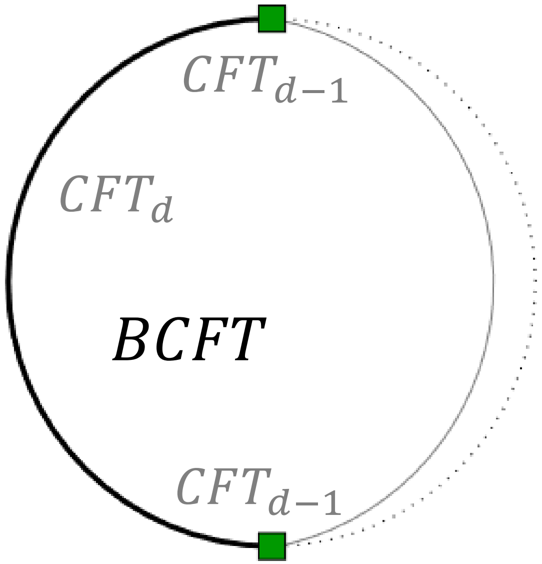


Brane



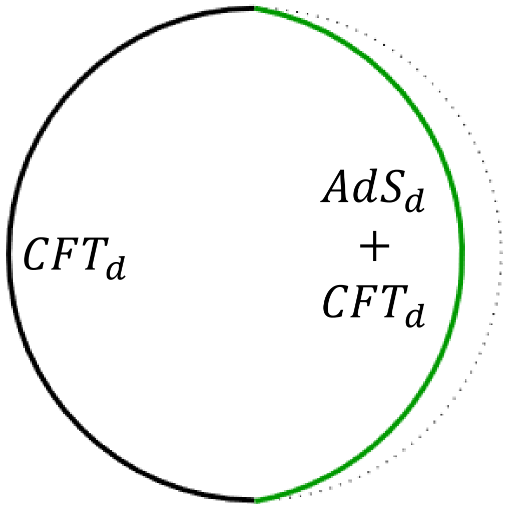
Boundary



Braneworld Holography

- We can algorithmically compute the higher-curvature effective action on the brane by integrating out the bulk.

  Holographic renormalization [Henningson, Skenderis; de Haro, Skenderis, Solodukhin; Kraus, Larsen, Siebelink; Papadimitrou; etc.]



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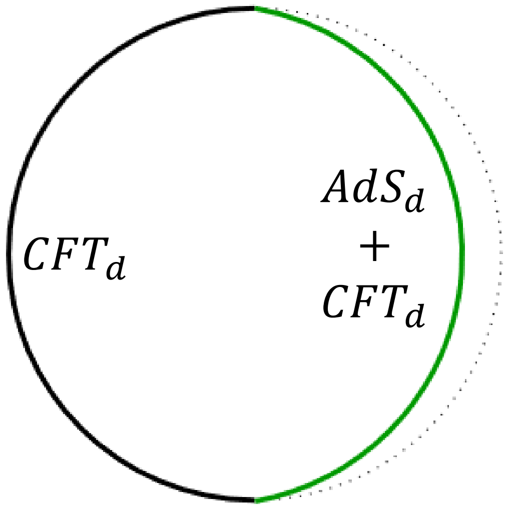
$$I_{\text{eff}} \propto \int d^d x \sqrt{-g} \left[\frac{2(d-1)(d-2)}{\ell^2} + R + \frac{\ell^2}{(d-2)(d-4)} \left(R_{ab}R^{ab} - \frac{d}{4(d-1)} R^2 \right) + \cdots \right] + I_{CFT}$$



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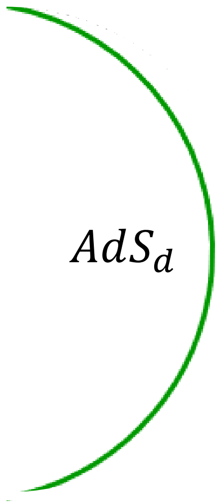
$$I \propto \int d^d x \sqrt{-g} \left[\frac{2(d-1)(d-2)}{\ell^2} + R + \frac{\ell^2}{(d-2)(d-4)} \left(R_{ab} R^{ab} - \frac{d}{4(d-1)} R^2 \right) + \cdots \right]$$



HCGs from Braneworlds

- Expect "good behaviour" for the full tower of terms.

$$I \propto \int d^d x \sqrt{-g} \left[\frac{2(d-1)(d-2)}{\ell^2} + R + \frac{\ell^2}{(d-2)(d-4)} \left(R_{ab} R^{ab} - \frac{d}{4(d-1)} R^2 \right) + \cdots \right]$$

HCGs from Braneworlds

- "Good behaviour" not necessarily expected for each order-*n* density separately.

$$\cdot\ \mathcal{L}_0 \propto \Lambda \qquad \cdot\ \mathcal{L}_1 \propto R \qquad \cdot\ \mathcal{L}_2 \propto R_{ab}R^{ab} - \frac{d}{4(d-1)}R^2$$

$$\cdot\ \mathcal{L}_3 \propto \frac{3d+2}{4(d-1)}RR_{ab}R^{ab} - \frac{d(d+2)}{16(d-1)^2}R^3 - 2R^{ab}R_{abcd}R^{cd} + \frac{d-2}{2(d-1)}R^{ab}\nabla_a\nabla_b R - R^{ab}\Box R_{ab} + \frac{1}{2(d-1)}R\Box R$$

[Balasubramanian, Kraus; Emparan, Johnson, Myers; Kraus, Larsen, Siebelink]

HCGs from Braneworlds

- Expanded computations to 5th order (and 6th order for d = 3).

  - $\mathcal{L}_4 \propto \frac{13d^2-38d-80}{8(d-1)(d-4)} R_{ab}R^{ab}R_{cd}R^{cd} + \cdots$ (32 other terms)
  - $\mathcal{L}_5 \sim 100$ terms
  - $\mathcal{L}_6^{d=3} \sim 600$ terms

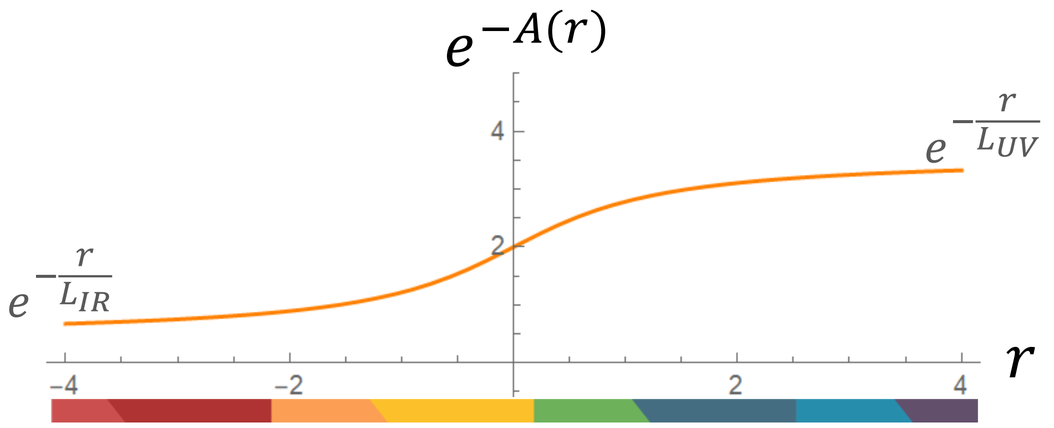
Holographic c-theorem

[Freedman, Gubser, Pilch, Warner; Myers, Sinha; Paulos; etc.]

What is it?

- A monotonicity theorem for RG flows of holographic CFTs, using a domain wall AdS bulk geometry

$$ds^2 = e^{-2A(r)}\,\eta_{ij}dx^i dx^j + dr^2$$



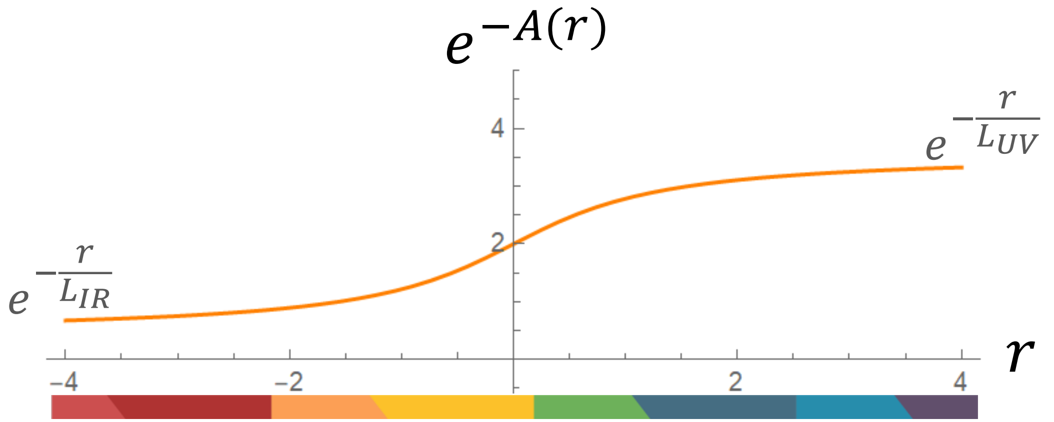
Holographic c-theorem

[Freedman, Gubser, Pilch, Warner; Myers, Sinha; Paulos; etc.]

How do we prove them?

- Showing that ∃ a monotonous function $c(r)$ using the bulk EOMs and assuming the NEC.

$$c'(r) = -\frac{\pi^{\frac{d-3}{2}}}{8\,\Gamma\left[\frac{d-1}{2}\right] G_N} \frac{T_t^t - T_r^r}{A'(r)^{d-1}}$$

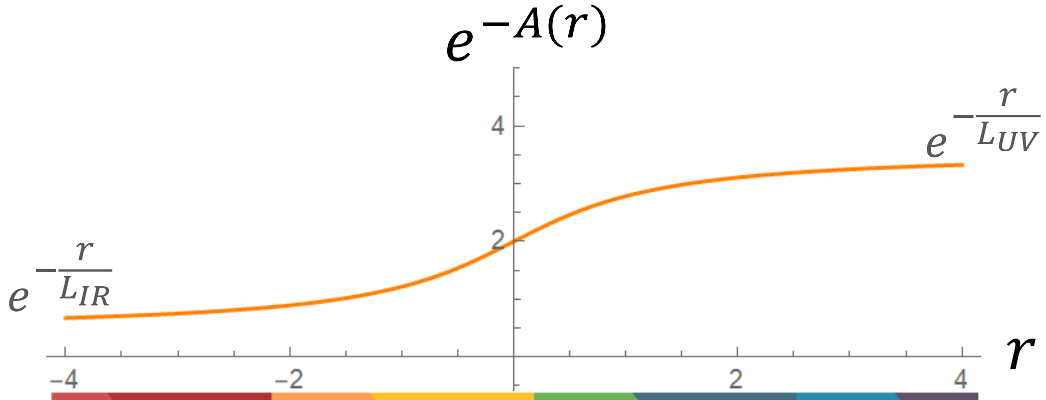


Holographic c-theorem for HCGs

How do we prove them?

- Showing that the higher-curvature density, when evaluated on the domain-wall metric, is second order in derivatives of $A(r)$ and linear in $A''(r)$.

$$c(r) = \frac{\pi^{\frac{d-1}{2}}}{2\,\Gamma\left[\frac{d-1}{2}\right]} \frac{\partial \mathcal{L}}{\partial R^{tr}{}_{tr}}\Bigg|_{A(r)}$$



Higher-curvature gravitational densities obtained from braneworld constructions fulfill a simple holographic c-theorem

$$I \propto \int d^d x \sqrt{-g} \left[\frac{2(d-1)(d-2)}{\ell^2} + R + \frac{\ell^2}{(d-2)(d-4)} \left(R_{ab}R^{ab} - \frac{d}{4(d-1)} R^2 \right) + \cdots \right]$$

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- **Proof by induction**: Run the holographic c-theorem proof through the algorithm that computes the braneworld-induced higher-curvature gravity.

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- Evidence for consistency of braneworld holographic models.
- Mechanism to build physically interesting theories of higher-curvature gravity.

Thank you!

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EXCELENCIA
MARÍA
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