Holographic BCFT Spectra from Brane Mergers

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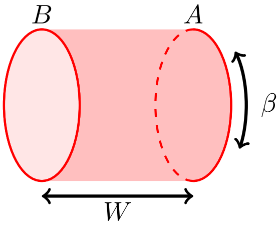
Euclidean 2D CFT on a cylinder

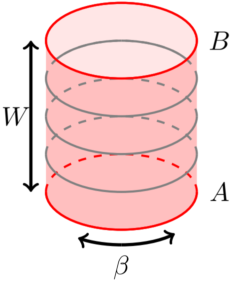
- Consider a Euclidean CFT_2 on a cylinder $(0, W) \times S^1_\beta$ with conformal boundary conditions A and B
- Object of interest: Euclidean path integral over the cylinder

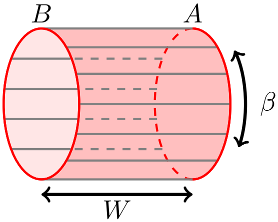
$$Z_{AB} \equiv \int_{A,B} \mathcal{D}\Psi \, e^{-I_{\mathrm{CFT}}[\Psi]} \tag{1}$$

which depends only on the modulus W/β



Slicing the path integral with circles and intervals





$$Z_{AB} = \langle A | e^{-W H^{\text{closed}}} | B \rangle$$

$$H^{\text{closed}} = \frac{2\pi}{\beta}\left(L_0 + \bar{L}_0 - \frac{c}{12}\right)$$

$$Z_{AB} = \operatorname{Tr} e^{-\beta H^{\text{open}}}$$

$$H^{\text{open}} = \frac{\pi}{W}\left(L_0 - \frac{c}{24}\right)$$

$$[L_n, L_m] = (n-m)\, L_{n+m} + \frac{c}{12} n^2 (n-1)\, \delta_{n,-m} \tag{2}$$

Closed string channel expansion

- Inserting a complete set of states of $\mathcal{H}^{\text{closed}}$ gives

$$Z_{AB} = \langle A|e^{-WH^{\text{closed}}}|B\rangle = \langle A|0\rangle\langle 0|B\rangle \exp\left(\frac{c}{6}\frac{\pi W}{\beta}\right) + \dots, \quad \frac{W}{\beta} \to \infty. \tag{3}$$

- The closed string vacuum dominates the $W/\beta \to \infty$ limit

Open string channel expansion

- Computing the trace over $\mathcal{H}^{\text{open}}$ gives

$$Z_{AB} = \operatorname{Tr} e^{-\beta H^{\text{open}}} \propto \exp\left[-\frac{\pi\beta}{2W}\left(\Delta_{\text{bcc}} - \frac{c}{12}\right)\right] + \ldots, \qquad \frac{W}{\beta} \to 0. \tag{4}$$

- If $A \neq B$, vacuum module $h = 0$ does not appear in the open string Hilbert space
- The next primary state $\Delta_{\text{bcc}} > 0$ is the boundary-condition-changing (BCC) operator

[Cardy '84, Affleck–Ludwig '94]

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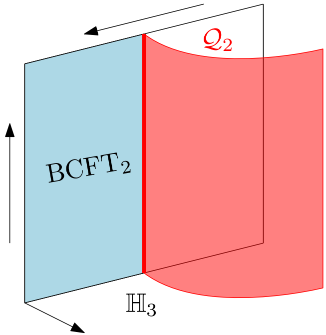
End-of-the-world branes in AdS_3 gravity

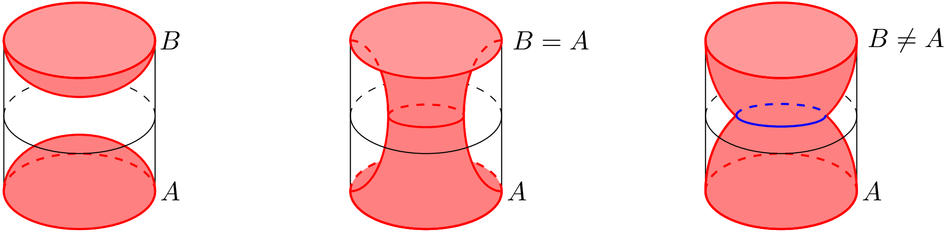
- The boundary of the CFT becomes an end-of-the-world brane in gravity
- Bulk region behind the brane is removed
- Brane tension T depends on the conformal boundary condition A, B

[Takayanagi '11]

[Fujita–Takayanagi–Tonni '11]



Holographic duals of a BCFT with two boundaries



(b) Closed string vacuum (c) Open string vacuum (d) BCC operator

- Describing BCC operators when $A \neq B$ requires non-smooth brane intersections

[Biswas–JK–Shashi–Sully '22, Miyaji–Murdia '22]

[Geng–Lüst–Mishra–Wakeham '21]

Holographic model with intersecting branes in 3D gravity

- Bulk action with a conical line defect $\mathcal{D}$ and a brane intersection $\mathcal{C} = \mathcal{Q}_A \cap \mathcal{Q}_B$:

$$I = -\frac{1}{2\kappa}\int_{\mathcal{M}} \sqrt{g}\,(R - 2\Lambda - m\,\delta_{\mathcal{D}}) - \frac{1}{\kappa}\int_{\mathcal{Q}} \sqrt{h}\,(K - T) - \frac{1}{\kappa}\int_{\mathcal{C}} \sqrt{\sigma}\,(\Theta - M), \tag{5}$$

  where Θ is the intersection angle and $T = T_{A,B}$

- Variation of the action

$$\begin{aligned}\delta I = &-\frac{1}{2\kappa}\int_{\mathcal{M}} \sqrt{g}\left(G_{ab} + \Lambda g_{ab} + \frac{1}{2} m\, g_{ab}\,\delta_{\mathcal{D}}\right)\delta g^{ab} \\ &-\frac{1}{2\kappa}\int_{\mathcal{Q}} \sqrt{h}\,\left(K_{ab} - (K - T)\,h_{ab}\right)\delta h^{ab} \\ &+\frac{1}{2\kappa}\int_{\mathcal{C}} \sqrt{\sigma}\,(\Theta - M)\,\sigma_{ab}\,\delta\sigma^{ab},\end{aligned} \tag{6}$$

[Biswas–JK–Shashi–Sully '22]

Excited states of the closed string Hamiltonian

- Conical line defect $\mathcal{D}$ at $r = 0$ in global AdS_3:

$$ds^2_{\mathbb{H}^3} = (r^2 + \alpha^2)\, d\tau^2 + \frac{dr^2}{r^2 + \alpha^2} + r^2 d\phi^2, \quad 0 < \alpha \leq 1 \tag{7}$$

  with $\tau \in \mathbb{R}$, $\phi \sim \phi + 2\pi$

- Mass of the defect

$$m = 4\pi\,(1 - \alpha) \geq 0 \tag{8}$$

- Boundary and corner Einstein's equations determine brane embeddings

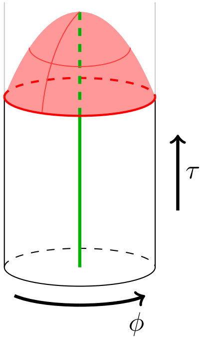
$$K_{ab} - (K - T)\, h_{ab} = 0, \quad \Theta - M = 0 \tag{9}$$

Single brane saddles

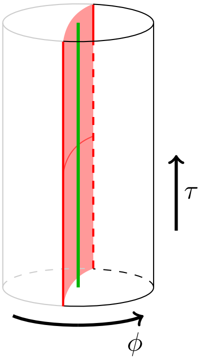
- Consider first the case of no conical defect $\alpha = 1$:

$$ds^2_{\mathbb{H}^3} = (r^2+1)\,d\tau^2 + \frac{dr^2}{r^2+1} + r^2 d\phi^2 \tag{10}$$

with $\tau \in \mathbb{R}$ and $\phi \sim \phi + 2\pi$

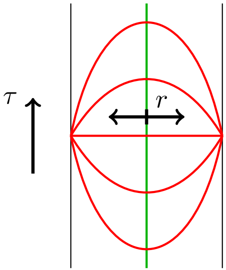


(e) Disk brane in Euclidean AdS_3

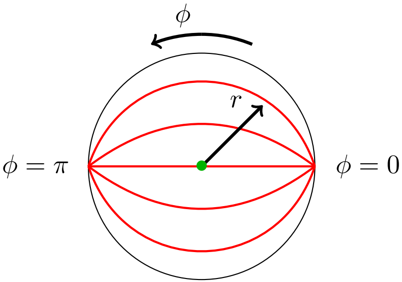


(f) Strip brane in Euclidean AdS_3

Slices of disk and strip branes



(g) Constant-ϕ slice of a disk brane



(h) Constant-τ slice of a strip brane

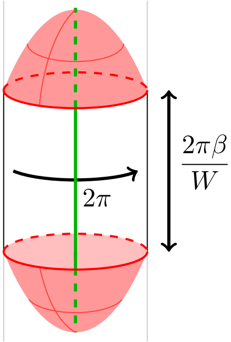
Closed string vacuum in gravity

- Renormalized on-shell gravity action:

$$e^{-I^{\text{ren}}_{\text{on-shell}}} = \left(\frac{1+T_A}{1-T_A}\right)^{\frac{c}{12}} \left(\frac{1+T_B}{1-T_B}\right)^{\frac{c}{12}} \exp\left(\frac{c}{6}\frac{\pi W}{\beta}\right)$$

- Dominates when $W/\beta \to \infty$
- Identified as the closed string vacuum for all A, B

[Takayanagi '11]



$\phi \sim \phi + 2\pi$

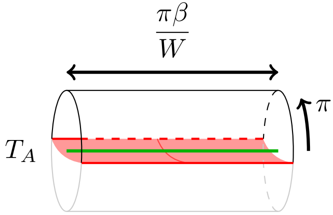
Open string vacuum in gravity

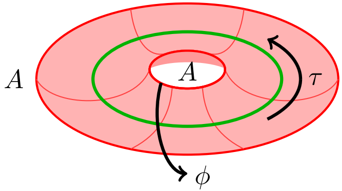
- Exists only when $A = B$
- Renormalized on-shell gravity action:

$$e^{-I^{\text{ren}}_{\text{on-shell}}} = \exp\left(\frac{c}{24}\frac{\pi\beta}{W}\right) \qquad (11)$$

- Dominates when $W/\beta \to 0$
- Identified as the open string vacuum $\Delta_{\text{bcc}} = 0$

[Takayanagi '11]





$$\tau \sim \tau + \frac{\pi\beta}{W}$$

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Strip branes with a conical line defect

- Global Euclidean AdS_3 with a conical line defect at $r = 0$:

$$ds^2_{\mathbb{H}^3} = (r^2 + \alpha^2)\, d\tau^2 + \frac{dr^2}{r^2 + \alpha^2} + r^2 d\phi^2, \quad 0 < \alpha < 1 \tag{12}$$

where $4\pi\,(1 - \alpha) = m$

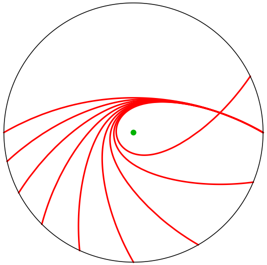
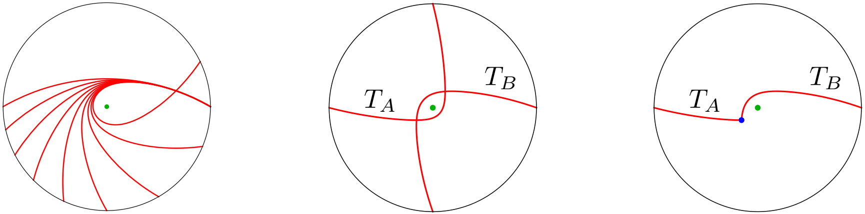


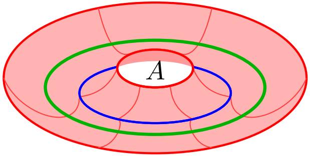
Figure: Strip brane for different α

Intersecting annulus brane saddle



(a) (b) (c)

$B \neq A$



- Requires the presence of a conical defect: $\alpha < 1$

BCC operator in gravity

- On-shell action of the intersecting saddle:

$$e^{-I^{\text{ren}}_{\text{on-shell}}} = \exp\left(\frac{c}{24}\frac{\pi\beta}{W}\alpha^2\right)$$

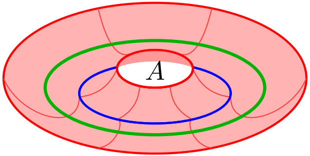
- We can identify

$$\Delta_{\text{bcc}} = \frac{c}{12}\left(1-\alpha^2\right) \tag{13}$$

- The inverted form might be more familiar

$$\alpha = \sqrt{1-\frac{12\Delta_{\text{bcc}}}{c}} \tag{14}$$

$B \neq A$



Fixing the BCC dimension Δ_{bcc}

- Intersection angle determined by geometry of brane embeddings:

$$\Theta = \Theta(\alpha) \tag{15}$$

- Corner Einstein's equation fixes α in terms of the corner mass M:

$$M = \Theta(\alpha) \quad \Rightarrow \quad \alpha = \alpha(M) \tag{16}$$

- By tuning M, we fill the gap below the BH threshold:

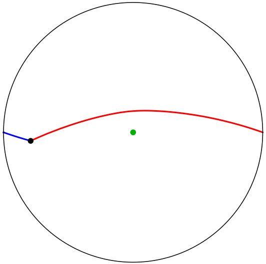
$$0 < \Delta_{\text{bcc}}(\alpha) \leq \frac{c}{12} \tag{17}$$

[Biswas–JK–Shashi–Sully '22]

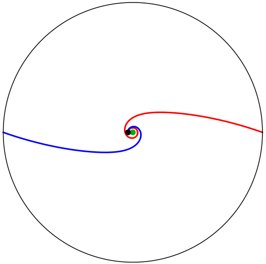
[Miyaji–Takayanagi–Ugajin '21]

Limiting cases

- No conical defect $\alpha = 1$ corresponds to the intersection running to the conformal boundary and $\Delta_{\text{bcc}} \to 0$
- Stronger conical defect $\alpha \to 0$ gives a sharper intersection and $\Delta_{\text{bcc}} \to \frac{c}{12}$



(d) $\alpha \to 1,\ \Delta_{\text{bcc}} \to 0$



(e) $\alpha \to 0,\ \Delta_{\text{bcc}} \to \frac{c}{12}$

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Closed string bra-ket wormhole saddle

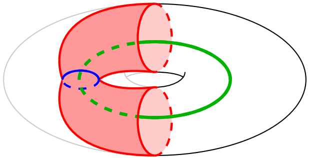

Figure: The bra-ket wormhole = intersecting positive tension disk branes

$$\overline{\langle A|0\rangle\langle 0|B\rangle} = \overline{\langle A|0\rangle}\,\overline{\langle 0|B\rangle} + (\text{wormhole}) + \dots, \tag{18}$$

[Chen–Gorbenko–Maldacena '20]

[Kusuki '22]

Scalar field coupled to two disk branes

- Scalar field of mass $m^2 = \Delta(2-\Delta)$:

$$I_{\text{bulk}} \supset \frac{1}{2\kappa}\int_{\mathcal{M}} \sqrt{g}\,\left(\nabla^a\Phi\nabla_a\Phi + m^2\Phi^2\right) - \frac{1}{\kappa}\int_{\mathcal{Q}_A}\sqrt{h}\,\lambda_A\Phi - \frac{1}{\kappa}\int_{\mathcal{Q}_B}\sqrt{h}\,\lambda_B\Phi \tag{19}$$

- Scalar field exchange between two disk branes

$$e^{-I^{\text{ren}}_{\text{on-shell}}} = \frac{2\pi\lambda_A\lambda_B}{\Delta^2(1-T_A)(1-T_B)}\left(\frac{1+T_A}{1-T_A}\right)^{\frac{c}{12}-\frac{\Delta}{2}}\left(\frac{1+T_B}{1-T_B}\right)^{\frac{c}{12}-\frac{\Delta}{2}}\frac{e^{-2\pi W/\beta\,(\Delta-c/12)}}{1-e^{-4\pi W/\beta}} \tag{20}$$

with the $SL(2,\mathbb{R})$-character appearing

Thank you

Explicit brane embeddings

- Disk brane

$$\tau = F(r; T, \tau_0) \equiv \tau_0 + \frac{1}{\alpha}\mathrm{Tanh}^{-1}\left(\frac{T\alpha}{\sqrt{f_\alpha(r) - T^2 r^2}}\right), \quad r \geq 0. \tag{21}$$

- Strip brane

$$r = p(\phi; T, \phi_0) \equiv -\frac{T\alpha}{\sqrt{1 - T^2}} \csc\left[\alpha\left(\phi - \phi_0\right)\right], \quad \phi \in \left(\phi_0, \phi_0 + \frac{\pi}{\alpha}\right), \tag{22}$$

Intersection of annulus brane

- Intersection angle

$$\cos\Theta = \frac{1}{r_*^2}\left(T_A T_B f_\alpha(r_*) + \sqrt{r_*^2 - T_A^2 f_\alpha(r_*)}\sqrt{r_*^2 - T_B^2 f_\alpha(r_*)}\right), \tag{23}$$

- Intersection depth

$$r_*^2 = \alpha^2 \csc^2(\alpha\Delta\phi)\left(\frac{T_A^2}{1-T_A^2} + \frac{T_B^2}{1-T_B^2} + \frac{2\,T_A T_B \cos(\alpha\Delta\phi)}{\sqrt{1-T_A^2}\sqrt{1-T_B^2}}\right). \tag{24}$$

Computation of the on-shell actions

- The renormalized on-shell action can be computed as boundary integral

$$I^{\text{ren}}_{\text{on-shell}} = u_0 \left(M^{\text{ren}}_{\text{ADM}} - \frac{2\pi}{\kappa} A_{\mathcal{H}} \right) \tag{25}$$