

Symmetries of $T\bar{T}$ - deformed CFTs and their holographic avatars

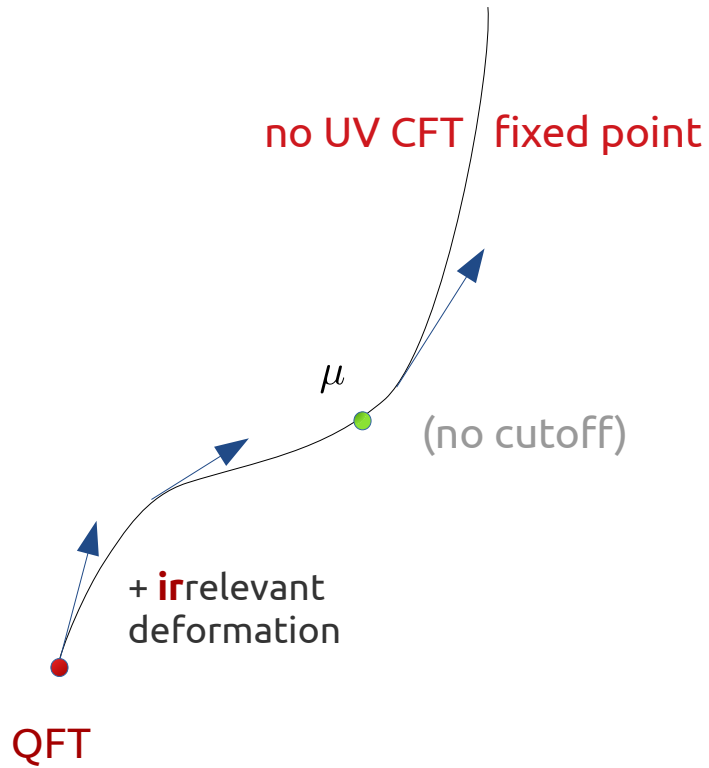
Monica Guica

IphT, CEA Saclay

based on 2212.09768 with [Silvia Georgescu](#)

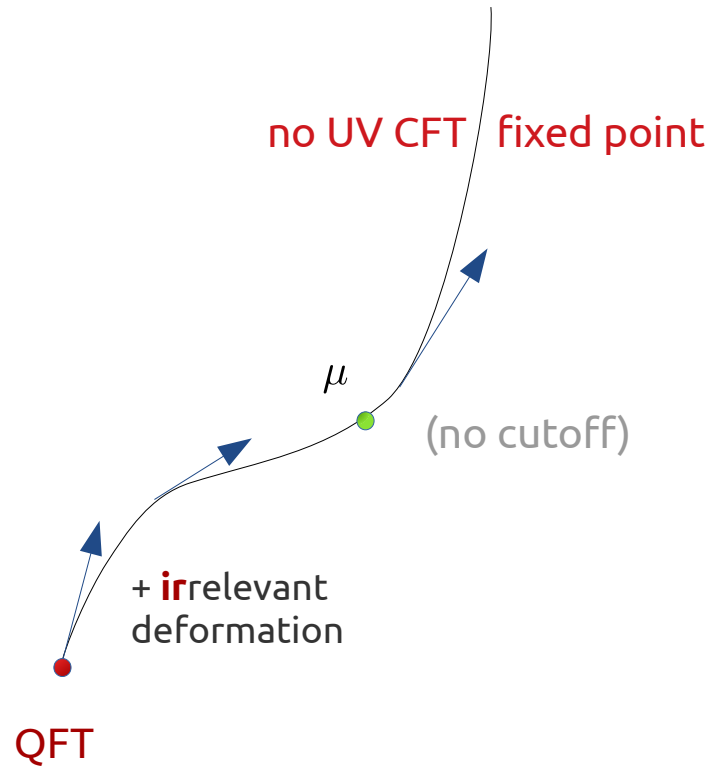
What is the $\overline{T\overline{T}}$ deformation?

- **irrelevant** deformation of 2d QFTs \rightarrow **UV complete** QFTs that are **non-local**

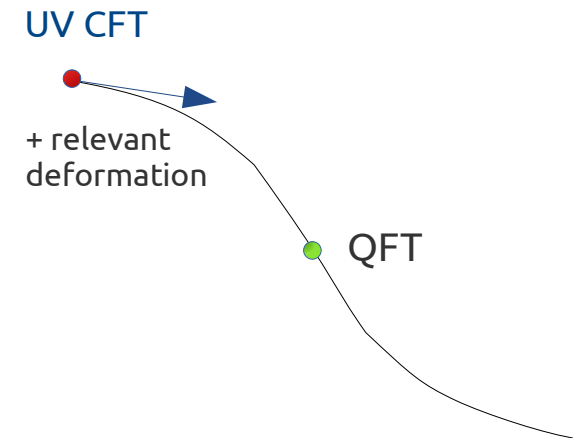


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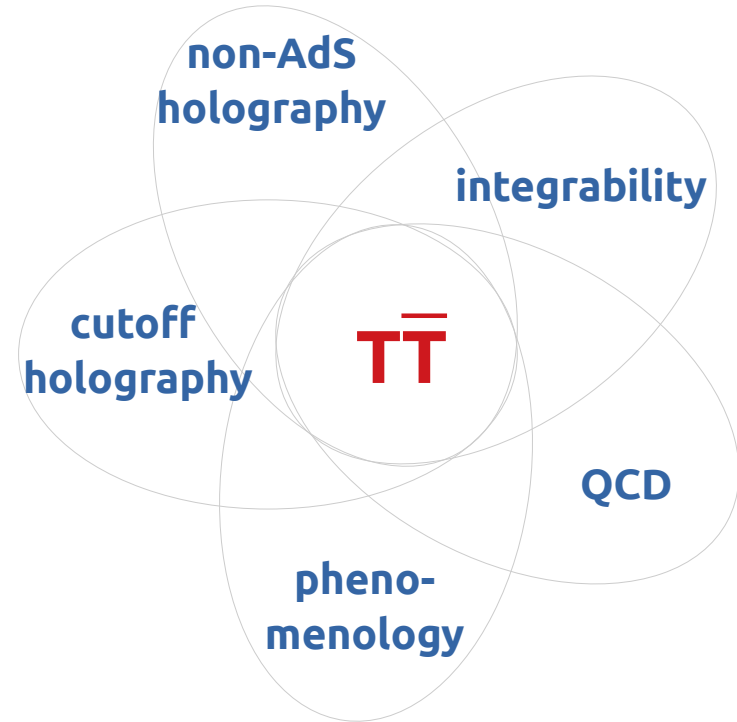
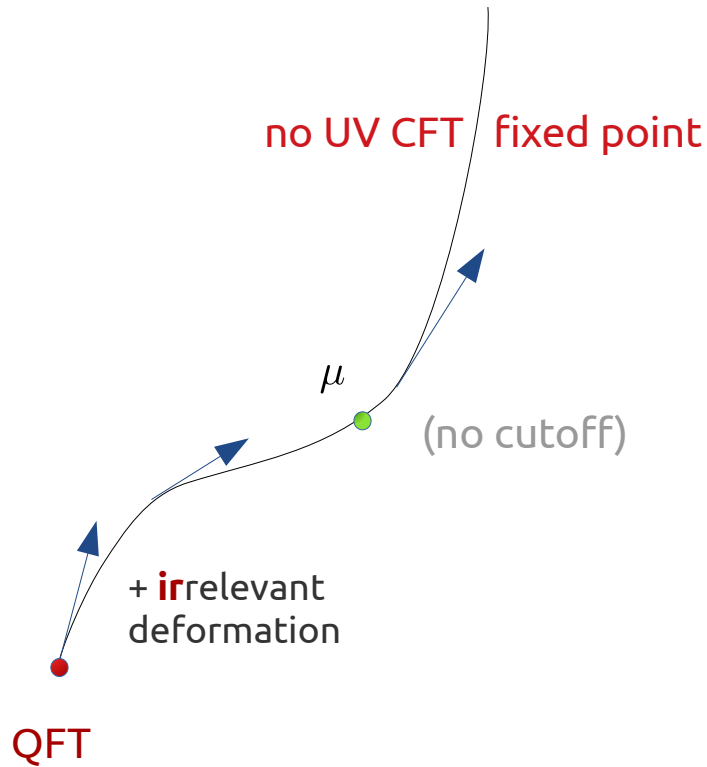


- finely tuned irrelevant flow
integrability preserved
- well-defined S-matrix \rightarrow UV completeness
- not an RG flow



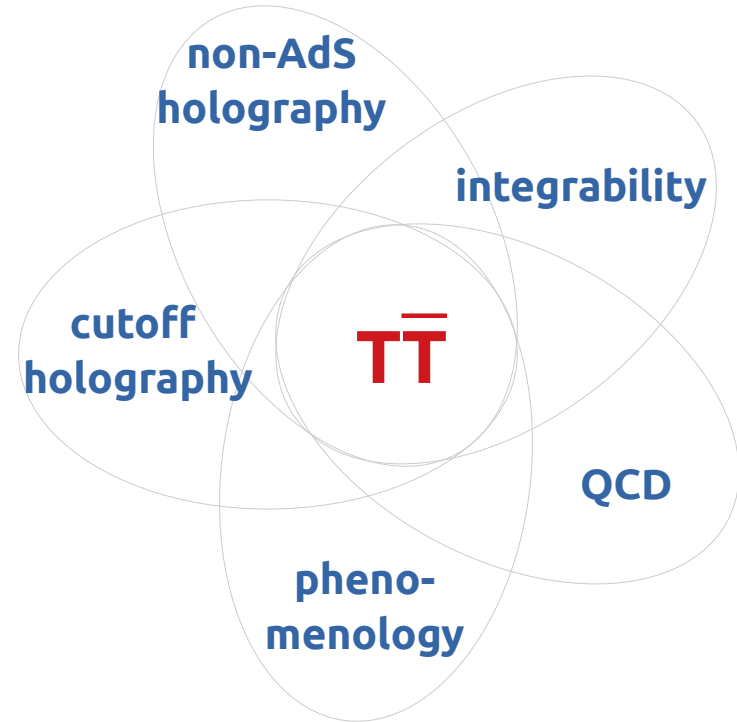
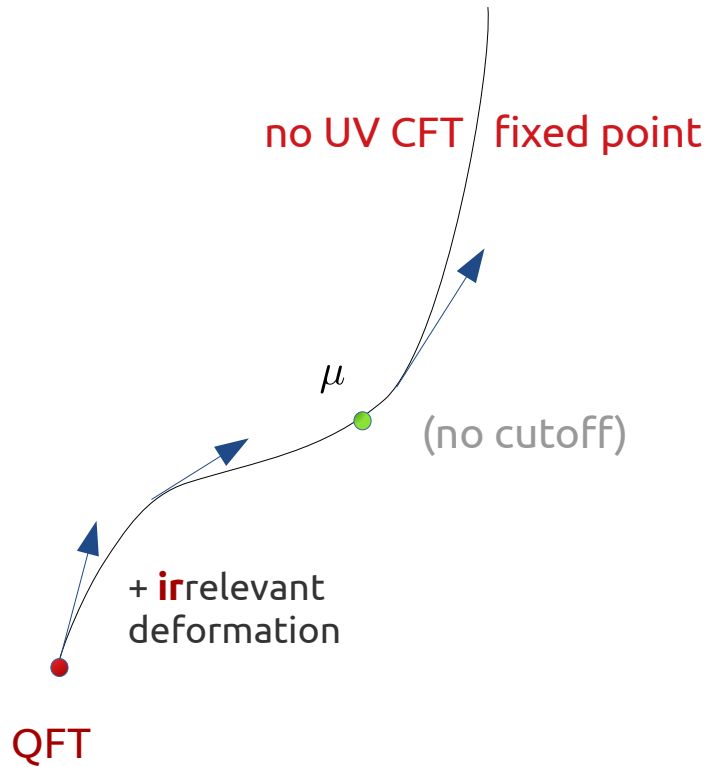
Why are they interesting?

- irrelevant deformations of 2d QFTs → UV complete QFTs that are non-local



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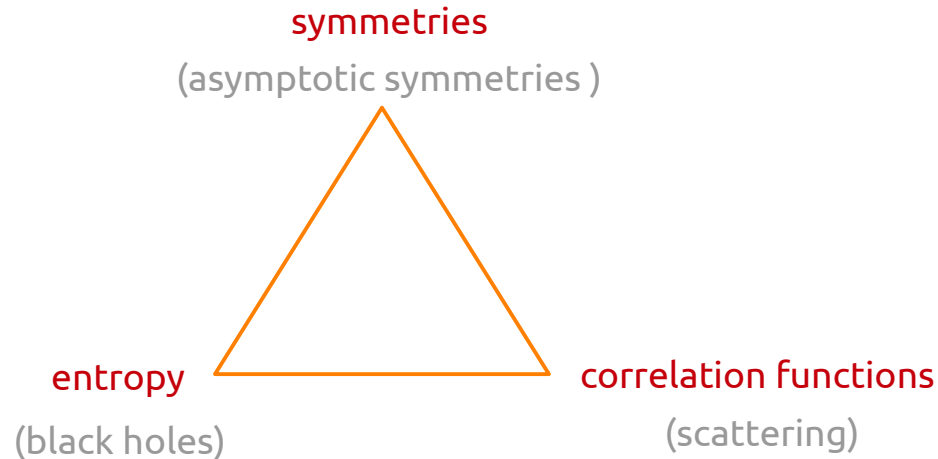
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exactly solvable

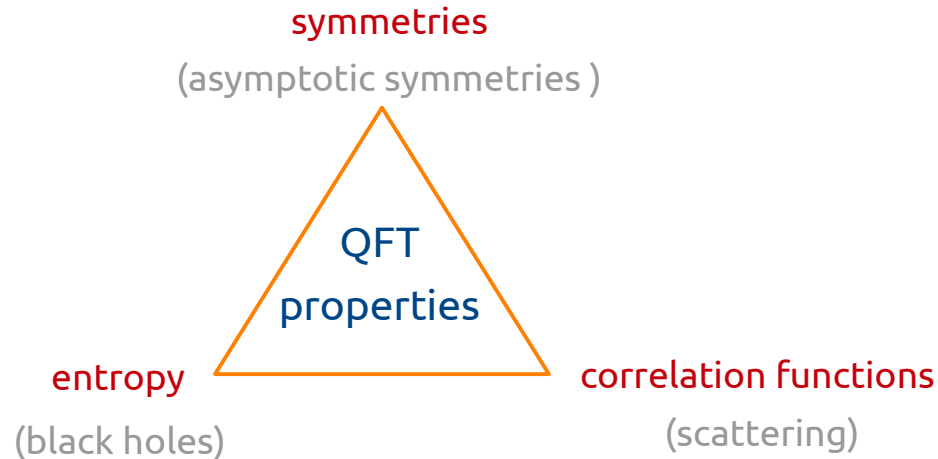
Motivation from non-AdS holography

- non-AdS holography : **hard** → no concrete examples in string theory for **asympt. flat, de Sitter**, etc.
- **infer** properties of dual QFT from spacetime: symmetries, thermodynamics, correlation functions
e.g. celestial holography programme



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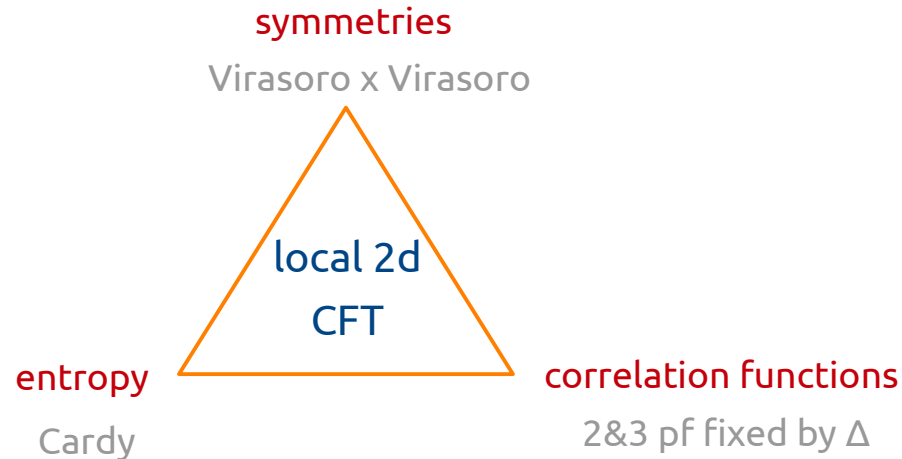
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- valuable to have an **independent QFT definition** of the dual field theory

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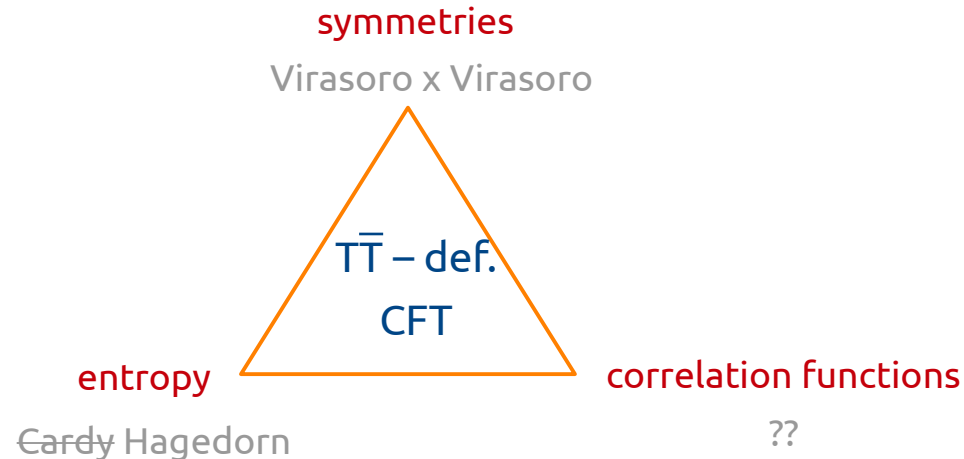
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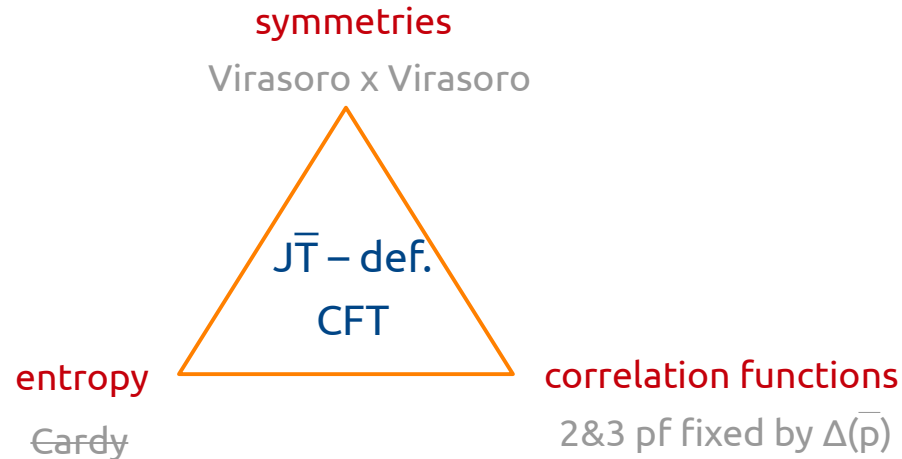
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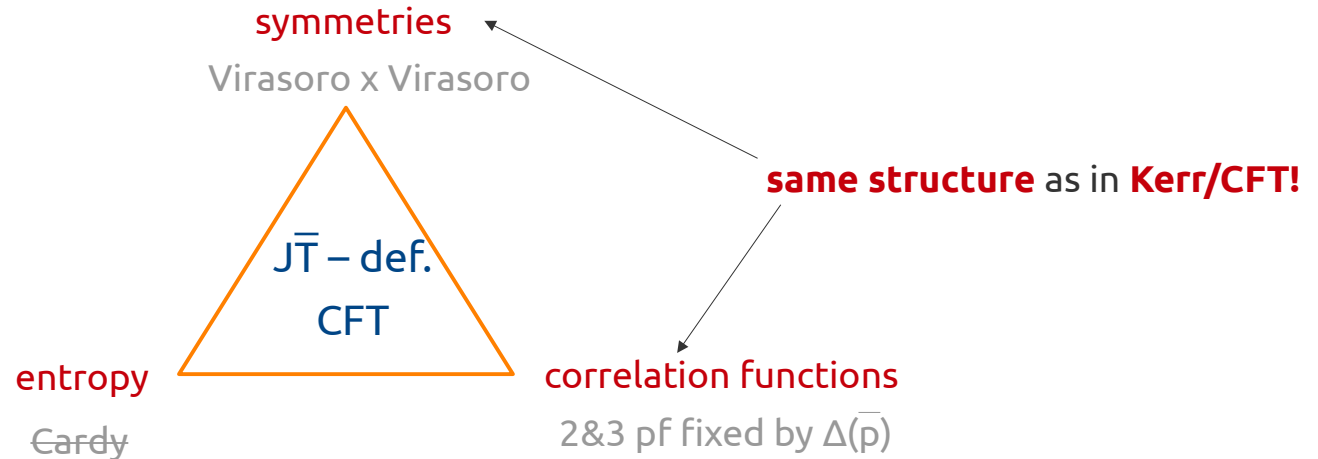
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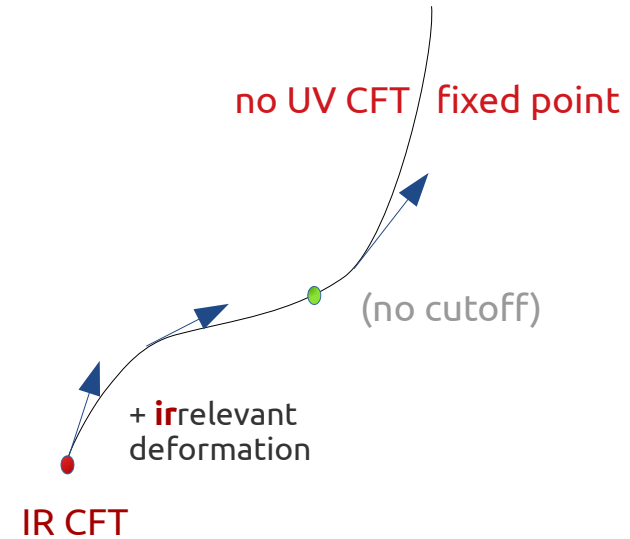
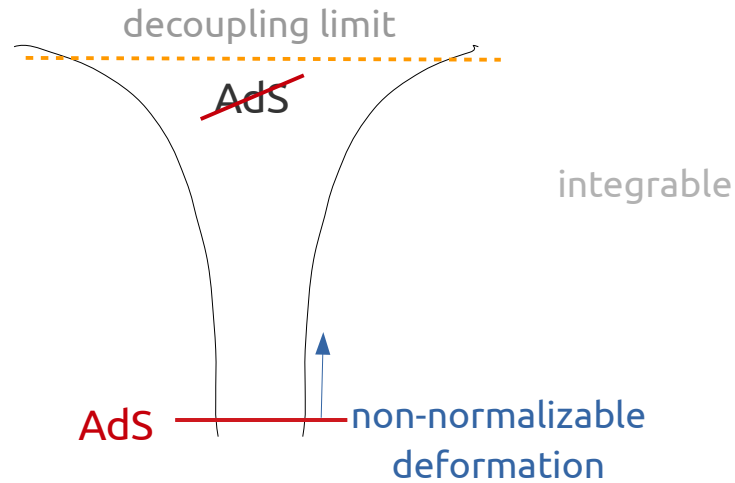
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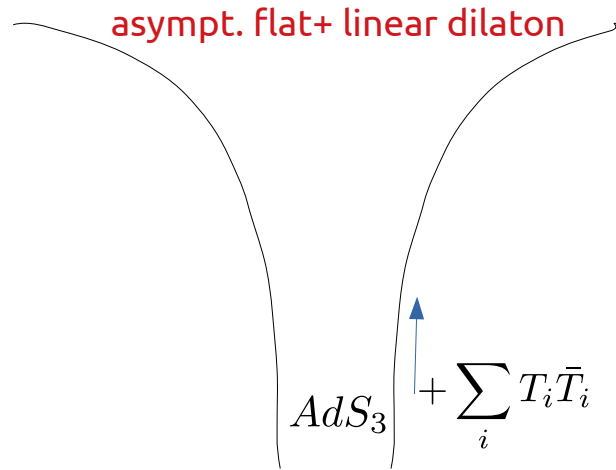
Motivation from non-AdS holography



- define holographic dual in terms of finely-tuned irrelevant flow \rightarrow (non-local) QFT
- understood in rare examples: non-commutative N=4 SYM, dipole-deformed N=4 SYM
- generically strongly-coupled & very hard to follow

Main result

Giveon, Itzhaki, Kutasov '17



= "single-trace" $T\bar{T}$ - deformed CFT

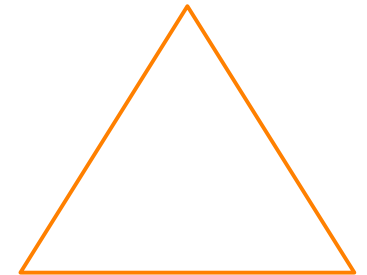
conjectured

has precisely the infinite symmetries of (single-trace) $T\bar{T}$ - deformed CFTs

→ perhaps the "QFT structure" that is relevant to this spacetime

is that of $T\bar{T}$ - deformed CFTs

Virasoro x Virasoro



Hagedorn

??

Plan

- brief **review** of $T\bar{T}$ deformations
- brief review of the **holographic** interpretation:
 - of the standard “**double-trace**” deformation \rightarrow AdS_3 with **mixed** bnd. cond. for the metric
 - of the “**single-trace**” variant \rightarrow asymptotically flat with a linear dilaton
- **infinite symmetries** of $T\bar{T}$ – deformed CFTs as **asymptotic symmetries** of the AdS_3 with mixed b.c.
- **asymptotic symmetry** group analysis of the **asymptotically linear dilaton** backgrounds
- conclusions

The $T\bar{T}$ deformation

- **universal irrelevant** deformation of 2d QFTs \rightarrow bilinear in the stress tensor components

- define $T\bar{T}$ operator

$$\lim_{y \rightarrow x} \varepsilon^{\alpha\beta} \varepsilon^{AB} T_{\alpha A}(x) T_{\beta B}(y) = \mathcal{O}_{T\bar{T}}(x) + \text{derivative terms}$$

nice factorization properties

Zamolodchikov '04

SZ '16

- the $T\bar{T}$ deformation :

$$\frac{\partial S(\mu)}{\partial \mu} = \int d^2x \mathcal{O}_{T\bar{T}}(\mu)$$

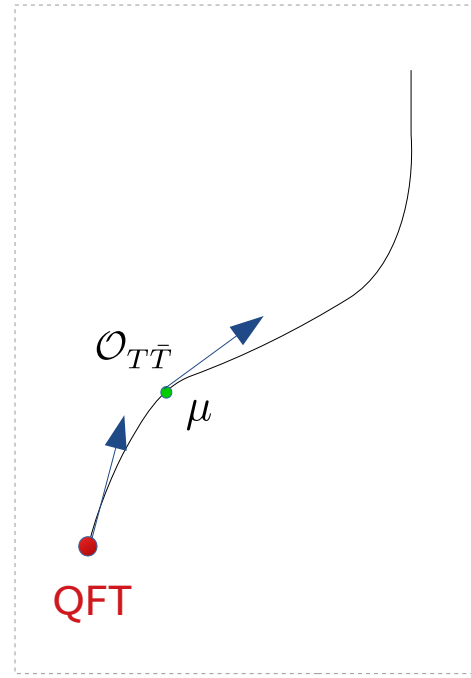
$$[\mu] = (\text{length})^2$$

Smirnov & Zamolodchikov '16; Cavaglia et al. '16

- highly **tractable** : **exact** finite -size spectrum, S-matrix, preserves integrability

- deformed theory **non-local** (scale $\sqrt{\mu}$) but argued **UV complete**

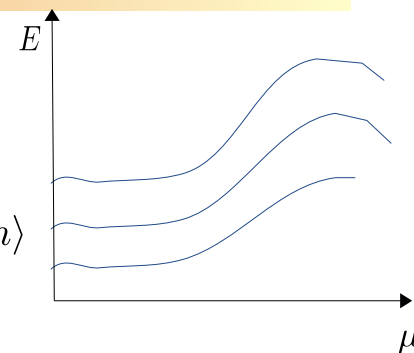
minimum length



$T\bar{T}$ - deformed CFT spectrum & thermodynamics

- in compact space (R) \rightarrow energy levels **continuously deformed**

- deformed energies** $E_\mu(R)$ determined **only by initial spectrum** $\partial_\mu E_n = \langle n | \mathcal{O}_{T\bar{T}} | n \rangle$



e.g. seed CFT

$$E_\mu(R) = \frac{R}{2\mu} \left(\sqrt{1 + \frac{4\mu E_0}{R} + \frac{4\mu^2 P^2}{R^2}} - 1 \right)$$

$R \rightarrow 0 \Rightarrow$ UV is **not** a CFT

- $\mu > 0$: ground state energy $E_0 = -\frac{c}{12R}$ becomes complex for $R < R_{min} = \# \sqrt{\mu c}$
- $\mu < 0$: all states with $E_0 > \frac{R}{4|\mu|}$ acquire imaginary energies \rightarrow no sense in compact space (CTC)

Cooper, Dubovsky, Moshen

- thermodynamics: **smoothly** deformed levels \rightarrow **unchanged** density of states

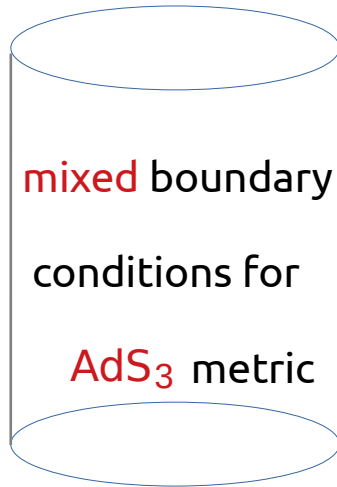
$$S(E) = S_{Cardy}(E^{(0)}(E)) = \sqrt{\frac{2\pi c}{3}(ER + \mu E^2)} \quad (P = 0)$$

- **Hagedorn** behaviour $S \propto E$ at high energy $T_H = R_{min}^{-1}$

Holographic interpretation of $T\bar{T}$ - deformed CFTs

Holographic dual of $T\bar{T}$ - deformed CFTs

- $T\bar{T}$ deformation : **double trace**
- seed CFT : large c , large gap
→ Einstein gravity + low-lying matter fields



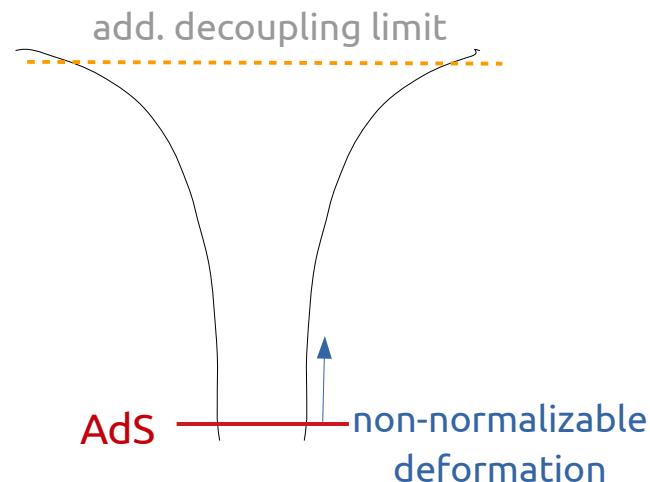
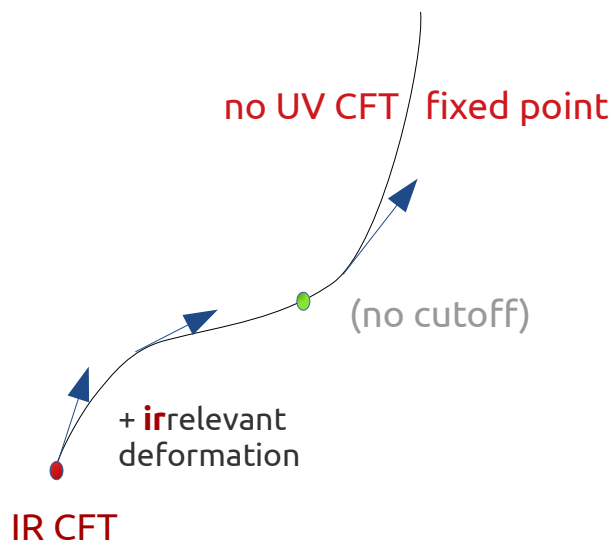
$$g_{\alpha\beta}^{(0)} - \mu g_{\alpha\beta}^{(2)} + \frac{\mu^2}{4} g_{\alpha\gamma}^{(2)} g^{(0)\gamma\delta} g_{\delta\beta}^{(2)}$$

fixed

- holographic dictionary **derived** from field theory using Hubbard-Stratonovich trick
- **1st** instance of **mixed** bnd. cond. on AdS₃ metric
→ bulk & boundary have **independent definitions**
→ contrast standard situation where properties of the boundary theory are **inferred** from the bulk
- change bnd. conditions on AdS₃ metric → **radical modification** of the bnd. theory: **local** → **non-local**
- **precision** holography
→ **perfect** match of bulk/boundary **spectrum** ✓
→ **symmetries** ✓
→ other observables?

$T\bar{T}$ & non-AdS holography

The “single-trace” $T\bar{T}$ deformation



- $T\bar{T}$ \nrightarrow non-AdS geometry because it is double-trace \rightarrow need single-trace irrelevant deformation
- AdS_3/CFT_2 gauge group: S_p (permutations) Giveon, Itzhaki, Kutasov '17

seed symmetric product orbifold CFT

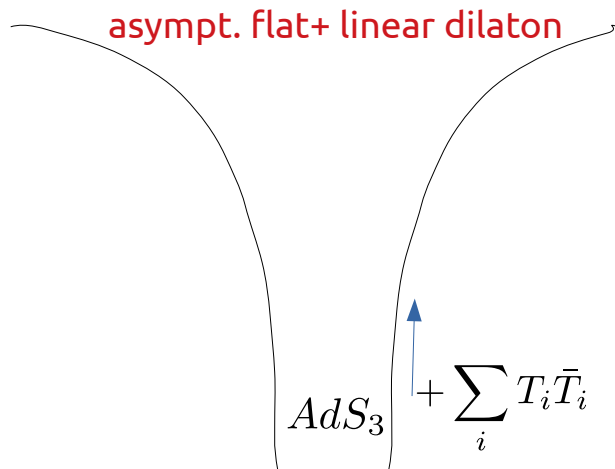


“single-trace $T\bar{T}$ ” deformation (finite μ)

$$\mathcal{M}^p/S_p$$

$$\sum_{i=1}^p T_i \bar{T}_i \Rightarrow (T\bar{T}_{def.} \mathcal{M})^p/S_p$$

The asymptotic linear dilaton background and $T\bar{T}$



N_5 NS5 and N_1 F1 strings in the NS5 decoupling limit

$$g_s \rightarrow 0, \quad \alpha' \quad \text{fixed}$$

N_1 large

UV: Little String Theory

non-gravitational, non-local theory with Hagedorn growth

IR: $AdS_3 \sim$ descr. by $(\mathcal{M}_{6N_5})^{N_1}/S_{N_1}$ symmetric orbifold CFT

- worldsheet σ -model: exactly marginal deformation of the WZW model describing AdS_3 by $J^- \bar{J}^-$

→ dual to CFT source for a $(2,2)$ single-trace operator $\sum_{i=1}^{N_1} T_i \bar{T}_i$

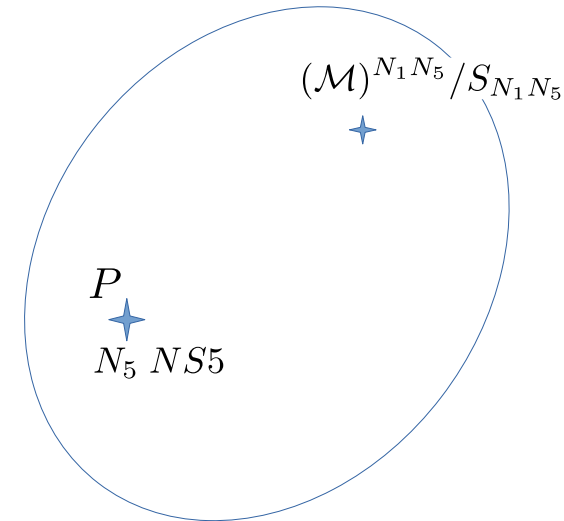
- proposed holographic relation

$$Z_{string}[\text{NS5- F1}] = Z \left[(T\bar{T} - \text{def. CFT}_{6N_5})^{N_1} / S_{N_1} \right] \quad \mu = \pi\alpha'$$

Checks of the correspondence

$$\mathcal{Z}_{string\ theory}[\text{NS5- F1}] = \mathcal{Z} [(T\bar{T}\text{- deformed } \mathcal{M}_{6N_5})^{N_1} / S_{N_1}]$$

- both sides (independently) well-defined at **finite** deformation ✓ **solvable QFT!**
- spectrum of **long string** excitations **exactly matches** single-trace $T\bar{T}$ spectrum GIK '17
- black hole entropy $S(E)$ **agrees** with $T\bar{T}$ entropy (Cardy \rightarrow Hagedorn) GIK '17
- **!** full CFT dual to AdS_3 is **not** a symmetric product orbifold
- our **perfect match** of the (infinite) symmetries brings further support for the GIK proposal



**Infinite symmetries of TT - deformed CFTs:
holographic analysis**

Asymptotic symmetries

MG, Monten '19

- holographic dual to double-trace $T\bar{T}$ – deformed CFTs: AdS_3 with **mixed bnd. cond.**

$$g_{\alpha\beta}^{(0)} - \mu g_{\alpha\beta}^{(2)} + \frac{\mu^2}{4} g_{\alpha\gamma}^{(2)} g^{(0)\gamma\delta} g_{\delta\beta}^{(2)}$$

- large diffeomorphisms** that preserve the mixed bnd. conditions **symmetries** of dual field theory

- fix $T\bar{T}$ metric to be flat \rightarrow most general allowed backgrounds param. by two functions $\mathcal{L}(u), \bar{\mathcal{L}}(v)$

$$g^{(0)} : \underbrace{\frac{(dU + \mu \bar{\mathcal{L}}(v) dV)(dV + \mu \mathcal{L}(u) dU)}{(1 - \mu^2 \mathcal{L}(u) \bar{\mathcal{L}}(v))^2}}_{dudv} \quad g^{(2)} : \frac{(1 + \mu^2 \mathcal{L}(u) \bar{\mathcal{L}}(v)) (\mathcal{L}(u) dU^2 + \bar{\mathcal{L}}(v) dV^2) + 4\mu \mathcal{L}(u) \bar{\mathcal{L}}(v) dU dV}{(1 - \mu^2 \mathcal{L}(u) \bar{\mathcal{L}}(v))^2}$$

where the **field-dependent coordinates** u, v are defined as

$$\begin{cases} U = u - \mu \int^v \bar{\mathcal{L}}(v') dv' \\ V = v - \mu \int^u \mathcal{L}(u') du' \end{cases} \quad \begin{matrix} \swarrow \\ \searrow \end{matrix} \quad \begin{matrix} \text{zero modes} \\ c_{\mathcal{L}}, c_{\bar{\mathcal{L}}} \end{matrix}$$

- diffeomorphisms that preserve the mixed bnd. cond:

$$\xi^U = f(u) + \mu \int_{c_{\mathcal{L}_f}}^v \bar{\mathcal{L}} \bar{f}' + \frac{\ell(\rho + \mu)(\rho \bar{\mathcal{L}} f''(u) - \bar{f}''(v))}{8\pi G(1 - \rho^2 \mathcal{L} \bar{\mathcal{L}})} \quad \xi^V = \bar{f}(v) + \mu \int_{c_{\bar{\mathcal{L}}_f}}^u \mathcal{L} f' + \frac{\ell(\rho + \mu)(\rho \mathcal{L} \bar{f}''(v) - f''(u))}{8\pi G(1 - \rho^2 \mathcal{L} \bar{\mathcal{L}})}$$

- $T\bar{T}$ metric \sim induced metric at $\rho = -\mu$

$$\xi^\rho = \rho(f'(u) + \bar{f}'(v))$$

Constraints on the asymptotic functions

- under these diffeomorphisms, the $\bar{T}\bar{I}$ coordinates change as

$$U \rightarrow U + f(u) + \mu \int^v \bar{\mathcal{L}} \bar{f}' \quad V \rightarrow V + \bar{f}(v) + \mu \int^u \mathcal{L} f' \leftarrow \text{winding!}$$

- the functions f, \bar{f} **must** have **winding**, in order for the periodicity of the $\bar{T}\bar{I}$ coordinates to not change

$$f(u) = f_p(u) + w_f u, \quad \bar{f}(v) = \bar{f}_p(v) + w_{\bar{f}} v$$

- this winding is **entirely fixed** by the periodic part through the conserved charges of the background

$$2\pi w_f R_u + \mu \oint dv \bar{\mathcal{L}} \bar{f}'_p(v) + 2\mu w_{\bar{f}} H_R = 0, \quad 2\pi w_{\bar{f}} R_v + \mu \oint du \mathcal{L} f'_p(u) + 2\mu w_f H_L = 0$$

where $R_{u,v}$ are the **field-dependent radii** of u, v and $H_{L,R}$ are the left/right-moving energies

$$2\pi R_u = 2\pi R + 2\mu H_R, \quad 2\pi R_v = 2\pi R + 2\mu H_L$$

Basics of the covariant phase space formalism

- this formalism computes the infinitesimal **charge difference** between two backgrounds

$$\delta Q_\xi = \oint k_\xi[\delta\Phi, \Phi]$$

where the d-2 form k_ξ is directly constructed from the action

- this charge needs to be **integrable**, checked separately ← **essential**

- charge algebra

$$\{Q_\xi, Q_\chi\} \equiv \delta_\chi Q_\xi = \oint k_\xi(\mathcal{L}_\chi \Phi, \Phi)$$

- based on the **representation theorem** $\delta_\chi Q_\xi = Q_{[\xi, \chi]^*}$

- the **modified Lie bracket** takes into account the possible field-dependence of the diffeomorphisms

Barnich, Toessaert '10

$$[\xi, \chi]^* \equiv [\xi, \chi]_{L.B.} - \delta_\xi \chi + \delta_\chi \xi$$

Asymptotic conserved charges

- we compute the charge difference between two backgrounds that differ via $\delta\mathcal{L}_{int}(u)$
- where $\delta(\mathcal{L}(u)) = \delta\mathcal{L}_{int}(u) + \mathcal{L}'(u)\delta u$ and $\delta u, \delta v$ are determined by the condition that $\delta U = \delta V = 0$
- can show winding part of the functions drops out of the final result

$$\delta Q_{f, \bar{f}} = \delta \left[\underbrace{\frac{1}{2} \oint du \mathcal{L}(u) f_p(\hat{u})}_{Q_f} - \frac{1}{2} \oint dv \bar{\mathcal{L}}(v) \bar{f}_p(\hat{v}) \right] + \pi \left(c_{\bar{\mathcal{L}}_{\bar{f}}} \delta R_v + \delta c_{\bar{\mathcal{L}}} w_{\bar{f}} R_v - c_{\mathcal{L}_f} \delta R_u - \delta c_{\mathcal{L}} w_f R_u \right)$$

- where we **assumed** that f_p, \bar{f}_p **only** depend on the fields via the combinations $\hat{u} \equiv \frac{u}{R_u}$, $\hat{v} \equiv \frac{v}{R_v}$
- **integrability** conditions: 1) set all c's to zero \rightarrow **unrescaled** (Fourier basis) generators

$$2) \quad c_{\mathcal{L}} = c_{\bar{\mathcal{L}}} = 0, \quad c_{\mathcal{L}_f} = -\frac{Q_f}{\pi R_u}, \quad c_{\bar{\mathcal{L}}_{\bar{f}}} = \frac{\bar{Q}_{\bar{f}}}{\pi R_v} \quad \text{rescaled generators}$$

labeled by functions $R_u \hat{f}_p(\hat{u}), R_v \hat{\bar{f}}_p(\hat{v})$

The charge algebra

- need to compute $\delta_\chi Q_\xi = \oint k_\xi(\mathcal{L}_\chi \Phi, \Phi)$ for some other allowed diffeomorphism $\chi_{g, \bar{g}}$
- while $\delta_\chi \mathcal{L}$ is given, $\delta_\chi \mathcal{L}_{int}$ depends on the chosen basis: integrability fixes different constants
→ **different** results for the charge **algebra**, as expected

- algebra of the **rescaled generators** is $\text{Virasoro}_L \times \text{Virasoro}_R$ with **same c** as undeformed CFT

MG, Monten '19

T \bar{T} – deformed CFTs = “non-local CFTs”

- algebra of the **unrescaled** generators (natural Fourier basis)

$$i\{Q_m, Q_n\} = \frac{1}{1 + \mu H_R/\pi} (m - n) Q_{m+n} + \frac{\mu^2 H_R}{\pi^2 (1 + \mu H/\pi) (1 + \mu H_R/\pi)} (m - n) Q_m Q_n + \mathcal{K} \delta_{m+n}$$

$$i\{Q_m, \bar{Q}_n\} = -\frac{\mu(m - n)}{\pi(1 + \mu H/\pi)} Q_m \bar{Q}_n$$

Georgescu, MG '22

- **precisely agrees** with results of **field theory** analysis

MG, Monten, Tsiaras '22

rescaled generators = original CFT generators transported along $T\bar{T}$ flow

Partial conclusions

- TT – deformed CFTs possess **Virasoro x Virasoro** symmetry, despite their **non-locality**
- confirmed by field theory analyses ← **full quantum definition** of the generators
- this Virasoro acts **differently** from standard CFTs: L_0, \bar{L}_0 are **non-linear** functions of $H_{L,R}$
- algebra in natural Fourier basis = non-linear modification of Virasoro x Virasoro
- action of these symmetries on operator has yet to be understood (field theory)
- would now like to show the **same** symmetry algebra appears for the **asymptotic linear dilaton** background, up to a factor related to the symmetric product orbifold

**Asymptotic symmetries of the
asymptotically linear dilaton background**

Setup

- as we anticipate field-dependent symmetries → turn on non-trivial background to see this
- we thus consider the asymptotically linear dilaton **black hole** backgrounds $p \leftrightarrow N_1$

$$d\bar{s}^2 = \frac{r^2}{\alpha' r^4 + \beta r^2 + \alpha' L_u L_v} \left(r^2 dU dV + L_u dU^2 + L_v dV^2 + \frac{L_u L_v}{r^2} dU dV \right) + k \frac{dr^2}{r^2}$$

$$e^{2\bar{\phi}} = \frac{kr^2}{\alpha' r^4 + \beta r^2 + \alpha' L_u L_v} \quad \beta = \sqrt{p^2 + 4\alpha'^2 L_u L_v}$$

- using a consistent truncation to 3d

$$ds^2 = ds_3^2 + \ell^2 ds_{S_3}^2, \quad H = 2\ell^2 \omega_{S^3} + b e^{2\phi} \omega_3$$

- classified linearized perturbations of this background: pure diffeos + propagating
- **allowed** diffeos: their **symplectic form** with the **allowed modes**, notably $\delta L_{u,v}$ must **vanish**
→ charge conservation $\omega[\Phi, \delta_\xi \Phi, \delta\Phi] = dk_\xi[\Phi, \delta\Phi]$

Allowed diffeomorphisms

- we choose diffeos that preserve the radial metric in Einstein frame

$$\xi_{rad}^E = \left(F_U(U, V) + \frac{k(r^2 \partial_V F_r - L_v \partial_U F_r)}{r^4 - L_u L_v} \right) \partial_U + \left(F_V(U, V) + \frac{k(r^2 \partial_U F_r - L_u \partial_V F_r)}{r^4 - L_u L_v} \right) \partial_V + \frac{r^3 F_r(U, V)}{r^4 + r^2 + L_u L_v} \partial_r$$

- symplectic form imposes:

$$F_U = f(u) + \frac{2\alpha' L_v}{p + \beta} \bar{f}(v) + c_U, \quad F_V = \bar{f}(v) + \frac{2\alpha' L_u}{p + \beta} f(u) + c_V, \quad F_r = f_r(u) + \bar{f}_r(v)$$

where

$$u \equiv \frac{(p + \beta) U + 2\alpha' L_v V}{2p}, \quad v \equiv \frac{(p + \beta) V + 2\alpha' L_u U}{2p}$$

- are **identical** with the $\bar{\text{TT}}$ field-dependent coord., upon using the TT parametrization (match energy)

$$L_u = \frac{\pi p \mathcal{L}}{1 - \mu^2 \mathcal{L} \bar{\mathcal{L}}}, \quad L_v = \frac{\pi p \bar{\mathcal{L}}}{1 - \mu^2 \mathcal{L} \bar{\mathcal{L}}}, \quad \alpha' = \frac{\mu}{\pi}$$

- integrability associated charges: choose c 's to absorb zero modes of f, \bar{f}

- boundary metric fluctuates with $\delta g_{UU} = \frac{2\alpha' L_u}{\beta} \delta g_{UV}, \quad \delta g_{VV} = \frac{2\alpha' L_v}{\beta} \delta g_{UV}$

Conserved charges and their algebra

- most charges vanish upon the constant backgrounds \rightarrow deform the background $g = \bar{g} + \epsilon \mathcal{L}_\eta \bar{g}$

$$Q_{\xi, \bar{f}}[g] = \begin{cases} \mathcal{O}(1) & \text{for } f, \bar{f} = \text{const.} \\ \mathcal{O}(\epsilon) & \text{for } f, \bar{f} \neq \text{const.} \end{cases}$$

- conserved charge is identical to TT answer up to this order in the perturbation above constant bckgnds

$$\delta_\eta Q_\xi = \frac{1}{2} \int d\sigma \partial_\sigma u \left(2p \mathcal{L} f h' + \frac{k}{\pi} h_r'' f - \frac{k}{\pi} f_r'' h \right) + RM$$

provided we choose the radial functions as $f_r(u) = -\frac{p}{4} f'(u)$, $\bar{f}_r(v) = -\frac{p}{4} \bar{f}'(v)$

(not fixed by symplectic form analysis)

- this yields a central term in the algebra that is the same as that of a symmetric product orbifold of TT

$$\{Q_m, Q_{-m}\} = \frac{2m}{R_u} Q_0 + \frac{6kp}{12} \cdot \frac{m^3}{R_u^2} I$$

- to this order, can only compute 0^{th} order term in charge algebra, c.f. the rep. theorem $\delta_\chi Q_\xi = Q_{[\xi, \chi]}$ *

The algebra at the next order

- to see the non-trivial terms in the algebra, need to compute $\delta_\chi Q_\xi$ on the **deformed** backgrounds
- need to consider the $\mathcal{O}(\epsilon)$ corrections to the diffeomorphisms χ, ξ

$$g = \bar{g} + \epsilon \mathcal{L}_\eta \bar{g}$$

- fixed by :
 - symplectic form analysis upon the **perturbed** backgrounds
 - zeroth order representation theorem
 - “minimal continuation” assumption on the functions
 - periodicity of the fixed coordinates $U, V \rightarrow$ **compensating winding** contributions

ch. algebra:

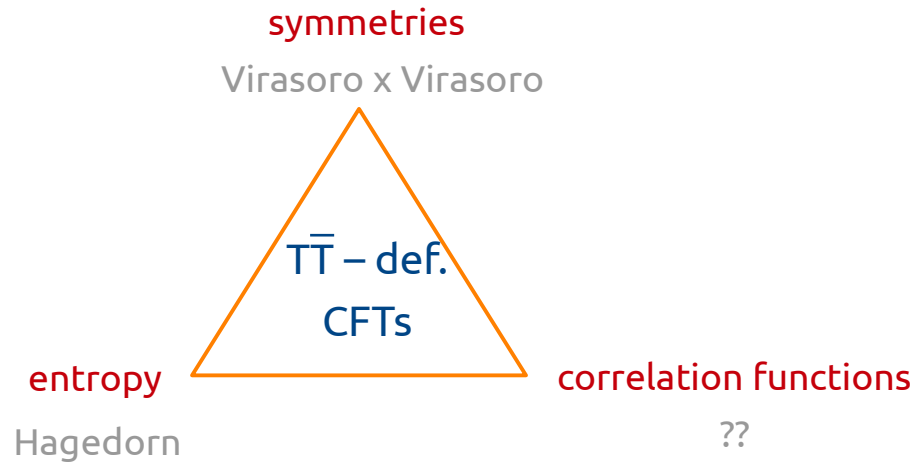
$$i\{Q_m, Q_n\} = \frac{1}{1 + \alpha' H_R/p} (m - n) Q_{m+n} + \frac{\alpha'^2 H_R}{p^2 (1 + \alpha' H/p) (1 + \alpha' H_R/p)} (m - n) Q_m Q_n + \mathcal{K} \delta_{m+n}$$

$$i\{Q_m, \bar{Q}_n\} = -\frac{\alpha' (m - n)}{p(1 + \alpha' H/p)} Q_m \bar{Q}_n \quad \text{algebra of **unrescaled** TT generators up to } \mathcal{O}(\epsilon)$$

- integrability is **essential**
- every step of the calc. is **identical** to its TT counterpart \rightarrow bckgnds & bnd. cond. are entirely **different!!**

Conclusions

- we have shown that the asymptotic symmetries of the asymptotically linear dilaton background in string theory are **precisely** those of single-trace $T\bar{T}$ – deformed CFTs
- this further suggests the relevant “QFT structure” for these bckgnds is that of $T\bar{T}$ – deformed CFTs



- a better understanding of both field theory and gravity (both **doable!**) may pave the way for **precision holography** in this background

Thank you !