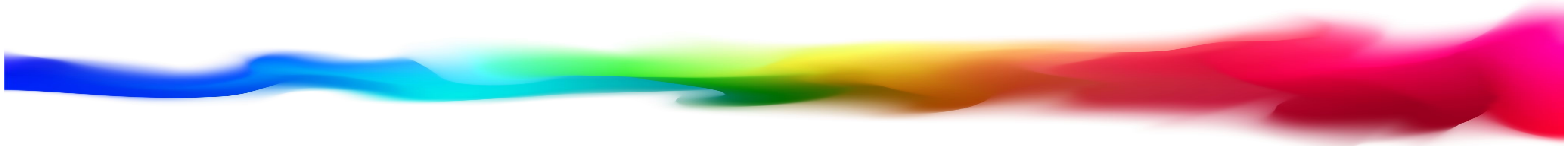
New sectors of String Theory from Supermembranes with Discrete Spectrum



María Pilar García del Moral

Work done in collaboration with: Camilo Las Heras, Pablo León and Alvaro Restuccia (U. Antofagasta, Chile)

Based on:
MPGM, Las Heras, León,Peña, Restuccia PLB19; MPGM, Las Heras, Restuccia arXiv2201.04896
MPGM, León, Restuccia JHEP2021. MPGM, León, Restuccia arXiv2301.00686; MPGM, León, Restuccia arXiv2301.xxxxx

OUTLINE OF THE TALK

- ✓ Basics on Supermembranes.
- ✓ Sectors of Supermembranes with Discrete Spectrum
- ✓ Case I: Supermembrane on a Twisted torus bundle
  - ✓ Associated String sector: N=2 Type IIB Parabolic (p,q) string in 9D
- ✓ Case II: Massive Supermembrane
  - ✓ String sector associated: N=2 Type IIA closed ’massive’ String in 10D
- ✓ Conclusions

BASICS ON SUPERMEMBRANES

- ✓ Supermembrane is a 2+1 worldvolume embedded in a 11D target space that acts a source for 11D Supergravity
- ✓ The quantization of Supermembrane theory in principle could describe at least part of the d.o.f. of M-theory. In order to consider the M2-brane as first quantized theory , analogous to strings, it needs to have a supersymmetric discrete spectrum.
- ✓ The eigenfunction of the groundstate with zero eigenvalue of 11D supermembrane was conjectured by DWHN to be expressed in terms of 11D supergravity. (1988 Open problem)
- ✓ The five string theories can be obtained as kinematical limits of Supermembrane theory
- ✓ Supermembrane compactified on toroidal backgrounds has ben shown to be U-dual invariant.

LCG M2-brane on 11D bosonic curved Backgrounds

In the Light Cone Gauge $X^+(\xi) = X^+(0) + \tau$ so that $\partial_i X^+ = \delta_{i0}$, and $\gamma^+\theta = 0$. B de Wit, Peeters, Plefka 98

$$H = \int \mathrm{d}^2\sigma \left\{ \frac{G_{+-}}{P_- - C_-} \left[\tfrac{1}{2}\Big(P_a - C_a - \frac{P_- - C_-}{G_{+-}} G_{a+}\Big)^2 + \tfrac{1}{4}(\varepsilon^{rs}\,\partial_r X^a\,\partial_s X^b)^2 \right] - \frac{P_- - C_-}{2\,G_{+-}} G_{++} - C_+ - C_{+-} + c^r\phi_r \right\}. \qquad (2.20)$$

With

$$\begin{aligned} C_a &= -\varepsilon^{rs}\partial_r X^- \partial_s X^b\, C_{-ab} + \tfrac{1}{2}\varepsilon^{rs}\partial_r X^b \partial_s X^c\, C_{abc} \\ C_\pm &= \tfrac{1}{2}\varepsilon^{rs}\partial_r X^a \partial_s X^b\, C_{\pm ab}\,, \\ C_{+-} &= \varepsilon^{rs}\partial_r X^- \partial_s X^a\, C_{+-a}\,. \end{aligned}$$

Subject to the residual DPA constraint

$$\phi_r = P_a\,\partial_r X^a + P_-\,\partial_r X^- \approx 0$$

SUPERMEMBRANES WITH DISCRETE SPECTRUM

•At present there are so far four different classes of M2-branes that has been shown to have discrete spectrum at regularized level:

- Supermembrane with central charges. *Boulton, MPGM, Restuccia NPB 03*
- Supermembrane on a pp-wave background (BMN matrix model). *Boulton, MPGM, Restuccia NPB 12*
- Supermembrane with C fluxes *MPGM, C. Las Heras, P. León, J. Peña A. Restuccia, PLB19*
- Massive Supermembrane *MPGM, P. León, A. Restuccia JHEP21*

M2-brane theory on a Constant three form background

$$S = T_d \int d^d\xi \left[\frac{1}{2}\sqrt{-g}g^{ij}\Pi_i{}^m\Pi_j{}^n\eta_{mn} + \frac{1}{2}\sqrt{-g} + \frac{1}{6}\varepsilon^{ijk}\Pi_i^M\Pi_j^N\Pi_k^L C_{LNM}\right]$$

Bergshoeff, Sezgin, Townsend'87

A flat metric and constant bosonic 3-form background

$$\begin{aligned}
&G_{\mu\nu} = \eta_{\mu\nu} \\
&\Pi_i^\mu = \partial_i X^\mu + \bar\theta\Gamma^\mu\partial_i\theta \quad ; \quad \Pi_i^\alpha = \partial_i\theta^\alpha \\
&C_{\mu\nu\alpha} = (\bar\theta\Gamma_{\mu\nu})_\alpha \\
&C_{\mu\alpha\beta} = (\bar\theta\Gamma_{\mu\nu})_{(\alpha}(\bar\theta\Gamma^\nu)_{\beta)} \ ; \ C_{\alpha\beta\gamma} = (\bar\theta\Gamma_{\mu\nu})_{(\alpha}(\bar\theta\Gamma^\mu)_\beta(\bar\theta\Gamma^\nu)_{\gamma)} \\
&C_{\mu\nu\rho} = const,
\end{aligned}$$

$$S = \int d^3\xi \left\{-\sqrt{-g} - \varepsilon^{ijk}(\bar\theta\Gamma^\rho\partial_k\theta C_{\rho\nu\mu} + \bar\theta\Gamma_{\mu\nu}\partial_k\theta)\left[\frac{1}{2}\partial_i X^\mu(\partial_j X^\nu + \bar\theta\Gamma^\nu\partial_j\theta) + \frac{1}{6}\bar\theta\Gamma^\mu\partial_i\theta\bar\theta\Gamma^\nu\partial_j\theta\right] - \frac{1}{6}\varepsilon^{ijk}\partial_i X^\mu\partial_j X^\nu\partial_k X^\rho C_{\rho\nu\mu}\right\}.$$

Case I: Supermembrane on a Twisted torus bundle

MPGM, C. Las Heras, P. León, J. Peña, A. Restuccia PLB19, JHEP 21

The Hamiltonian of a M2-brane with C fluxes in the LCG

$$\begin{aligned} H_{C_\pm} &= \int_\Sigma \sqrt{W} d^2\sigma \left\{ \frac{1}{2}\left(\frac{\widehat{P}_m}{\sqrt{W}}\right)^2 + \frac{1}{2}\left(\frac{\widehat{P}_r}{\sqrt{W}}\right)^2 + \frac{1}{4}\{X^m, X^n\}^2 + \frac{1}{2}(\mathcal{D}_r X^m)^2 , \right. \\ &+ \left. \frac{1}{2}(*\widehat{F})^2 + \frac{1}{4}(\mathbb{F}^{rs})^2 - \bar{\theta}\Gamma^-\Gamma_r \mathcal{D}_r \theta - \bar{\theta}\Gamma^-\Gamma_m \{X^m, \theta\} \right\} - \int_\Sigma d^2\sigma C_+ , \end{aligned}$$

$$\mathcal{D}_i X^m = D_i X^m + \{A_i, X^m\} , \qquad \mathcal{F}_{ij} = D_i A_j - D_j A_i + \{A_i, A_j\}$$

Subject to the local and global APD constraints

$$d(P_i dX^i + P_m dX^m + \overline{\theta}\Gamma^- d\theta) = 0 ,$$

$$\oint_{C_s} (P_i dX^i + P_m dX^m + \overline{\theta}\Gamma^- d\theta) = 0 .$$

Case I: Supermembrane on a Twisted torus bundle

MPGM, C. Las Heras, P. León, J. Peña, A. Restuccia JHEP 21

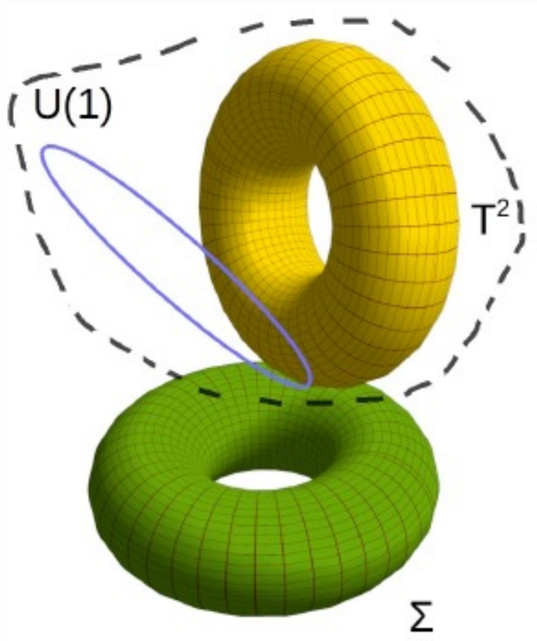
From the worldvolume picture the embedding of the M2-brane with C fluxes on the target space can be seen as a M2-brane on a twisted torus bundle with monodromy in SL(2,Z) and fluxes.

$$\mathcal{M}_G : \Pi_1(\Sigma) \rightarrow \Pi_0(Symp(T^2)) = SL(2,Z).$$

The inequivalent classes of symplectic torus bundle are classified by $H^2(\Sigma, Z^2_\rho)$
There is a 1:1 correspondence with the inequivalent classes of coinvariants

$$C_F = \left\{Q + \mathcal{M}_g\widehat{Q} - \widehat{Q}\right\}, \quad C_B = \left\{W + \mathcal{M}^*_g\widehat{W} - \widehat{W}\right\},$$

with $Q = \begin{pmatrix} p \\ q \end{pmatrix}$ and $W = \begin{pmatrix} l_1 \\ m_1 \end{pmatrix}$



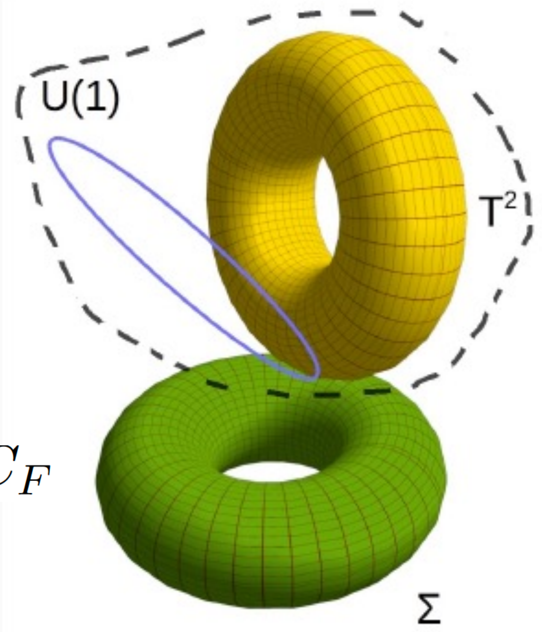
The M2-brane with C fluxes on a symplectic torus bundle with monodromy defines a M2-brane on a twisted torus bundle

$$\mathbb{T}^3_W \equiv T^2_{U(1)} \rightarrow E' \rightarrow \Sigma\,,$$

Case I: Supermembrane on a Twisted torus bundle

The Supermembrane Hamiltonian has two compatible gauge symmetries associated to the pullback of the connection defined on the structure of the twisted torus bundle a U(1) related with the induced fluxes and a symplectomorphism gauge symmetry that define a general gauge symmerty .

$$\mathbb{A} = \widehat{A} + \mathcal{A}$$



In *MPGM, Peña Restuccia JHEP12, PRD19*, we proved that the mass operator of the theory becomes defined on the orbits contained in the Coinvariants. $gQ \subset C_F$

Recently in *MPGM, Las Heras, Restuccia '22* we obtain the invariance of the mass operator M2-brane on the complete coinvariant when the monodromy is restricted to be parabolic. New symmetries appear.

Case I: Supermembrane on a Twisted torus bundle

MPGM, C Las Heras, A. Restuccia Arxiv 2201.04896

Symmetries

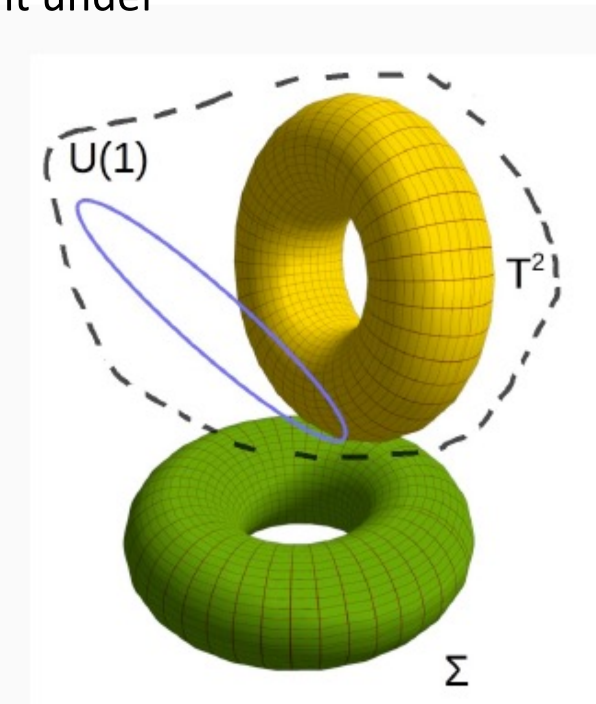
For the case of parabolic monodromy, the M2-brane mass operator is invariant under

$$Q' = \Lambda Q, \quad \mathbb{W}' = \Lambda^* \mathbb{W}, \quad \tau' = \tau + \frac{\mathbb{Z}}{q},$$

with

$$\Lambda = \begin{pmatrix} 1 & \frac{\mathbb{Z}}{q} \\ 0 & 1 \end{pmatrix}$$

$$\Lambda^* = \Omega^{-1}\Lambda\Omega = \begin{pmatrix} 1 & -\frac{\mathbb{Z}}{q} \\ 0 & 1 \end{pmatrix}$$



Hence the charges are mapped onto the associated coinvariant. They are classified by only one integer q. The symmetry is a parabolic subgroup of $SL(2,\mathbb{Q})$

Physicallly, all of the M2-brane bundles of the same coinvariant become identified.

$$Q \xrightarrow{\Lambda} C_F = \begin{pmatrix} \widetilde{\mathbb{Z}} \\ q \end{pmatrix}$$

$$W \xrightarrow{\Lambda^*} C_B = \begin{pmatrix} l_1 - \mathbb{Z} \\ m_1 \end{pmatrix}$$

Case I: Supermembrane on a Twisted torus bundle

MPGM, C Las Heras, A. Restuccia Arxiv 2201.04896

Symmetries

Inequivalent classes of M2-brane coinvariants leave invariant the mass operator under

$$C_{q_1} \xrightarrow{\widetilde{\Lambda}} C_{q_2}, \quad \tau \to \frac{\left(1+\frac{\mathbb{Z}_2}{q_2}\beta\right)\tau + \left(-\frac{\mathbb{Z}_1}{q_1}+\frac{\mathbb{Z}_2}{q_2}\left(1-\frac{\mathbb{Z}_1}{q_1}\beta\right)\right)}{\beta\tau + 1 - \frac{\mathbb{Z}_1}{q_1}\beta}, \quad \mathbb{W} \to \widetilde{\Lambda}^* \mathbb{W},$$

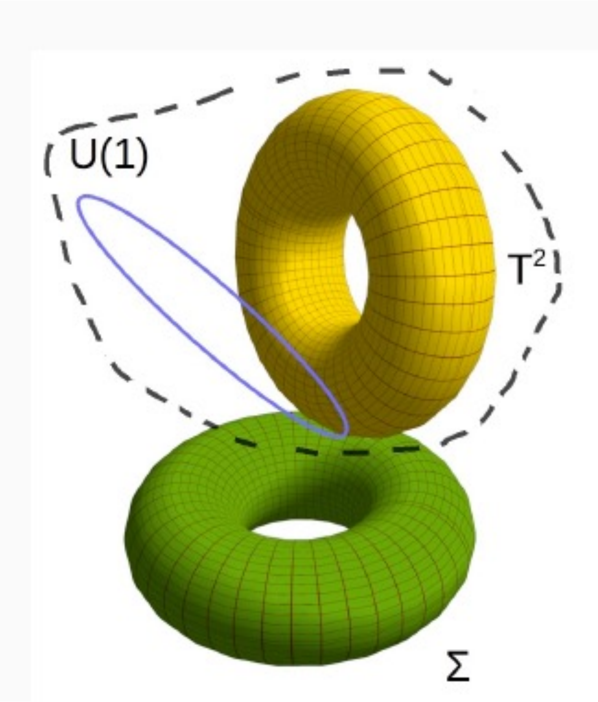
$$R \to R\left|\beta\tau + 1 - \frac{\mathbb{Z}_1}{q_1}\beta\right|, \quad A \to A e^{i\varphi_\tau}, \quad \Gamma \to \Gamma e^{i\varphi_\tau},$$

with

$$\widetilde{\Lambda} = \Lambda_{q_2}\mathcal{M}_\beta\Lambda_{q_1}^{-1} = \begin{pmatrix} 1+\frac{\mathbb{Z}_2}{q_2}\beta & -\frac{\mathbb{Z}_1}{q_1}+\frac{\mathbb{Z}_2}{q_2}\left(1-\frac{\mathbb{Z}_1}{q_1}\beta\right) \\ \beta & 1-\frac{\mathbb{Z}_1}{q_1}\beta \end{pmatrix}$$

$$\Lambda^* = \Omega^{-1}\Lambda\Omega$$

The symmetry group corresponds to a different parabolic subgroup of $SL(2,\mathbb{Q})$



Parabolic (p,q) string in 9D

MPGM, C Las Heras, A. Restuccia Arxiv 2201.04896

Schwarz '95 obtained the formulation of (p,q)-strings. He conjectured they had its origin in the M2-brane on M9xT2. In *MPGM, Martin, Restuccia '08*, the bosonic M2-brane excitations were obtained.

Now we have extended these results to the supersymemtric case of the M2-brane with C fluxes. We analize two cases: Trivial monodromy and nontrivial monodromy

$$M^2 = \beta^2 M^2_{(p,q)} \qquad T_{(p,q)} = \frac{|q\lambda_0 - p|}{(\mathrm{Im}(\lambda_0))^{1/2}} T_c \qquad \tau = \lambda_0, \quad \beta^2 = \frac{TA^{1/2}_{T^2}}{T_c}, \quad R_B^2 = (TA^{3/2}_{T^2} T_c)^{-1}$$

Term inherited from flux background

Trivial Monodromy case: By double dimensional reduction of the supersymmetric case

$$M^2_{(p,q)} = \left(\frac{n}{R_B}\right)^2 + (2\pi R_B m T_{(p,q)})^2 + 4\pi T_{(p,q)}(N_L + N_R) - \boxed{2P^0_- T_c^{1/6} R_B^{-2/3} k_+}$$

- **Important point**: We demosntrate that only the supermembrane with C fluxes (with nontrivial central charge) is able to generate string bound states (p,q). Vanishing central charge implies fundamental strings only.

Parabolic (p,q) string in 9D

MPGM, C Las Heras, A. Restuccia Arxiv 2201.04896

NonTrivial Monodromy case: By double dimensional reduction of the supersymmetric case the parabolic string mass operator is obtained

$$M^2_{C_q} = \left(\frac{n}{R_B}\right)^2 + (2\pi R_B \widehat{m}_8 T_{C_q})^2 + 4\pi T_{C_q}(N_L + N_R) - \frac{2P^0_{-} T^{1/6}_{C_q} R_B^{-2/3} n k_+}{|\widehat{\lambda}^T C_q|^{1/6}},$$

where the tension becomes modified

$$T_{C_q} \equiv |\widehat{\lambda}^T C_q| T_c, \quad \widehat{\lambda}^T = \frac{m_1}{(\mathrm{Im}(\lambda_0))^{1/2}} \begin{pmatrix} -1 & \lambda_0 \end{pmatrix}$$

Symmetries

The coinvariant Cq contains different p-charge. Inherited from the M2-brane with monodromy, there is a residual parabolic symmetry in SL(2,Q) that moves the charges Q inside the same equivalence class, leaving invariant the mass operator. We conjecture that this is the origin of parabolic gauge symmetry at effective level, in analogy with gauge invariance.

$$Q' = \Lambda Q, \qquad \lambda_0' = \lambda_0 + \frac{\mathbb{Z}}{q},$$

Parabolic (p,q) string in 9D

MPGM, C Las Heras, A. Restuccia Arxiv 2201.04896

Symmetries

Symmetries between different class of N=2 Parabolic strings become restricted to a parabolic subgroup of SL(2,Q). They are inherited from the supermembrane with parabolic monodromy

$$C_{q_2} = \widetilde{\Lambda} C_{q_1} \quad \Rightarrow \quad \begin{aligned} Q' &= \widetilde{\Lambda} Q, \\ \lambda_0' &= \frac{\left(1 + \frac{\mathbb{Z}_2}{q_2}\beta\right)\lambda_0 + \left(-\frac{\mathbb{Z}_1}{q_1} + \frac{\mathbb{Z}_2}{q_2}\left(1 - \frac{\mathbb{Z}_1}{q_1}\beta\right)\right)}{\beta\lambda_0 + 1 - \frac{\mathbb{Z}_1}{q_1}\beta} \end{aligned}$$

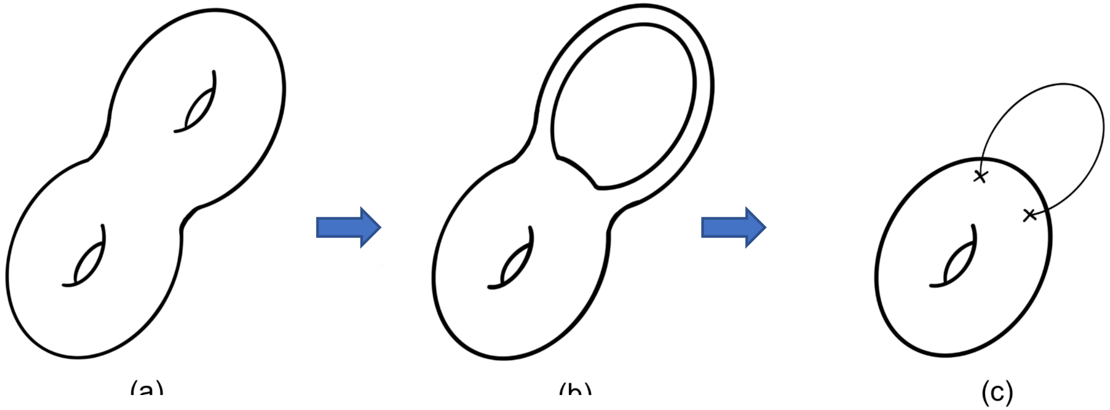
They leave invariant the Mass operator. This symmetry is the analogue symmetry to the SL(2,Z) in the Schwarz (p,q) string .

We claim this represents the string worldsheet description associated with the type II parabolic gauged supergravity in 9D.

Case II: Massive Supermembrane

MPGM, P. León, A. Restuccia:, JHEP21, Arxiv 230100686

Based on previous results (MPGM+Restuccia'15) we have developped a formulation of the 11D Massive M2-brane in a space with ten noncompact dimensions. To this end we have considered the worldvolume of a 11D supermembrane with genus 2 in a particular limit.



Membrane worldvolume

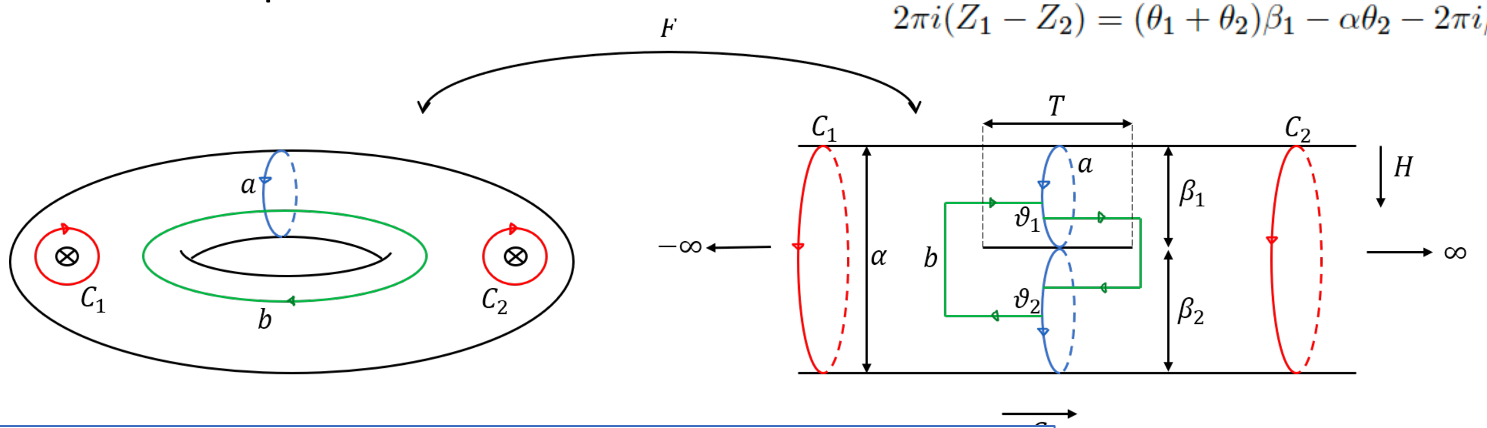
$$H = \int_{\Sigma_2 \to \tilde{\Sigma}_{1,2}} \mathcal{H} = \int_{\Sigma_{1,2}} \mathcal{H}.$$

Case II: Massive Supermembrane

MPGM, P. León, A. Restuccia:, JHEP21, Arxiv 230100686

The target space considered is a twice puntured torus described by Light Cone diagram via a Mandelstam map

$$2\pi i(Z_1 - Z_2) = (\theta_1 + \theta_2)\beta_1 - \alpha\theta_2 - 2\pi i\beta_1\tau.$$



S.B Giddings, S.A. Wolpert (1987).

$$F(z) = \alpha \ln\left[\frac{\Theta_1(z - Z_1|\tau)}{\Theta_1(z - Z_2|\tau)}\right] - 2\pi i\alpha\frac{Im(Z_1 - Z_2)}{Im\tau}(z - z_0),$$

The Mandelstam map

For simplicity the number of pucntures has been taken 2, which is the mínimum necessary, But it can be generalized. In String theory LC diagrams corresponds to one loop interaction Diagrams. Here, this is not the interpretation.

Case II: Massive Supermembrane

MPGM, P. León, A. Restuccia:, JHEP21, Arxiv 230100686

$$F = G + iH.$$

G is a single valued function but dG is harmonic due to its poles
H s a multivalued functin and dH is harmonic.

Behaviour
Near the punctures

$$\begin{cases} G \sim (-1)^{r+1}\alpha \ln|z - Z_r|, \\ H \sim (-1)^{r+1}\alpha\varphi, \quad \text{with} \quad \varphi \in (0, 2\pi) \quad (r = 1, 2). \end{cases}$$

Behaviour
Near the zeros

$$\begin{cases} G(z) - G(P_a) \sim \frac{1}{2} Re(D(P_a)(z - P_a)^2), \\ H(z) - H(P_a) \sim \frac{1}{2} Im(D(P_a)(z - P_a)^2), \end{cases}$$

with

$$D(P_a) = \sum_{r=1}^{2} (-1)^{r+1} \left[\frac{\partial_z^2 \Theta_1(P_a - z_r, \tau)}{\Theta_1(P_a - z_r, \tau)} - \left(\frac{\partial_z \Theta_1(P_a - z_r, \tau)}{\Theta_1(P_a - z_r, \tau)} \right)^2 \right].$$

Case II: Massive Supermembrane

MPGM, P. León, A. Restuccia:, JHEP21, Arxiv 230100686

The target space is considered a twice puntured torus described by Light Cone diagram via a Mandelstam map

$$\tilde{X}^m = \begin{cases} X^m(t,z,\bar{z}) & \text{over} \quad \Sigma_{1,2} \\ Y^m(t,u) & \text{over} \quad \gamma_2 \end{cases}, \qquad \tilde{\Psi} = \begin{cases} \Psi(t,z,\bar{z}) & \text{over} \quad \Sigma_{1,2} \\ \Theta(t,u) & \text{over} \quad \gamma_2 \end{cases},$$

$$\tilde{X}^r = \begin{cases} X^K(t,z,\bar{z})\delta_1^r + X^H(t,z,\bar{z})\delta_2^r & \text{over} \quad \Sigma_{1,2} \\ Y^m(t,u) & \text{over} \quad \gamma_2 \end{cases}.$$

The special embedding maps are decomposed as follows

$$X^K = K + A^K, \quad X^H = H + A^H$$

Case II: Massive Supermembrane

MPGM, P. León, A. Restuccia:, JHEP21, Arxiv 230100686

The Hamiltonian of the massive supermembrane is defined on ten non compact dimensions contains a topological term* defined in terms of the parameters of the singularities, non-vanishing new mass terms

$$H=\frac{(l\alpha T_{M2}m)^2}{2P_0^+}+\frac{1}{2P_0^+}\lim_{\epsilon\to 0}\int_{\Sigma'}d\sigma^2\sqrt{W}\Bigg[\left(\frac{P_m}{\sqrt{W}}\right)^2+\left(\frac{P_K}{\sqrt{W}}\right)^2+\left(\frac{P_H}{\sqrt{W}}\right)^2$$
$$+T_{M2}^2\Big(\frac{1}{2}\{X^m,X^n\}^2+m^2\{X^m,H\}^2+\{X^m,K\}^2+2\{X^m,K\}\{X^m,A^K\}$$
$$+\{X^m,A^K\}^2+2m\{X^m,H\}\{X^m,A^H\}+\{X^m,A^H\}^2+2m\{H,A^K\}\{A^H,A^K\}$$
$$+m^2\{H,A^K\}^2+2\{A^H,K\}\{A^H,A^K\}+\{A^H,A^K\}^2+\{K,A^H\}^2+\{K,A^K\}^2$$
$$+\{H,A^H\}^2\Big)-2P_0^+T_{M2}(\bar{\Psi}\Gamma^-\Gamma_m\{X^m,\Psi\}+\bar{\Psi}\Gamma^-\Gamma_K\{A^K,\Psi\}+\bar{\Psi}\Gamma^-\Gamma_H\{A^H,\Psi\}$$
$$+\bar{\Psi}\Gamma^-\Gamma_K\{K,\Psi\}+\bar{\Psi}\Gamma^-\Gamma_H\{H,\Psi\})\Bigg]. \tag{3.23}$$

It can be shown that it satisfies the suficiency criteria for discreteness of the spectrum

Case II: Massive Supermembrane

MPGM, P. León, A. Restuccia:, JHEP21, Arxiv 230100686

It is subject to a local APD constraint and four global APD constraints, the new ones are associated with curves between the singular points. At String theory level they generate the level matching constraint and they restrict the configurations values at the singularities.

Subject to local and global APD

$$df = 0. \qquad \zeta_1 = \int_a f = 0, \quad \zeta_2 = \int_b f = 0, \qquad \zeta_3 = \int_{C_1} f = 0 \quad \zeta_4 = \int_{\gamma_1} f = 0,$$

With

$$f \equiv \left(\frac{P_K}{\sqrt{W}} dX^K + \frac{P_H}{\sqrt{W}} dX^H + \frac{P_m}{\sqrt{W}} dX^m + \bar{\Psi}\Gamma^- d\Psi \right),$$

The massive supermembrane breaks ½ of supersymmetry and it is N=1

N=2 Type IIA massive String

MPGM, P. León, A. Restuccia, Arxiv 2301.xxxxx to Appear.

Performing a very particular double dimensional reduction, we obtain a type IIA N=2 String in ten non compact dimensions that we denote as massive String.

$$H_s = l^2\tilde{T}_s + \frac{1}{2\tilde{T}_s}\int_0^\pi d\theta\left([P_{\hat{m}}^2 + \tilde{T}_s^2(\partial_\theta \hat{X}^{\hat{m}})^2 - \frac{i}{\pi}\tilde{T}_s\chi^\dagger\rho^0\rho^1\partial_\theta\chi]\right),$$

with

$$\tilde{T}_s = m\Omega T_s.$$

It contains a topological term that encodes the singularities and the zeros topology. This term is related to the presence of a cosmological constant at effective level. It also contains a modified tension that also depends explicity on the non trivial shape of the LCD.

It satisfies the following level matching constraint

$$\int_0^\pi \left(P_{\hat{m}} d\hat{X}^{\hat{m}} + \frac{i\Omega}{2\tilde{T}_s}\chi^\dagger d\chi\right) = 2P_0^K l$$

TO APPEAR

Massive Supergravity

E. Bergshoeff, Y. Lozano, T. Ortin 98

The 11D uplift of massive supergravity is

$$S = \frac{1}{\kappa}\int d^{11}x\sqrt(-g)\left[R - \frac{1}{2\times 4!}F(4)^2 - \frac{1}{8}m^2|k^2|^2\right] + \frac{1}{(144)^2}\frac{\epsilon}{\sqrt{-g}}\left(2^4\partial C\partial CC + 3^2 m\partial CC(i_kC)^2 + \frac{9}{20}m^2C(i_kC)^4\right),$$

It admits as a source a wrapped M9 brane in 11D which under dimensional reduction along the isometry it becomes a D8-branes which are the source of 10D Romans supergravity. To couple a ten form potential associated with the M9-brane it is necessary to promote the mass parameter m into a field M(x)

$$\tilde{S} = \frac{1}{11!k}\int d^{11}x\epsilon^{\mu_1..\mu_11}\partial_{[\mu_1}A^{(10)}_{\mu_2...\mu_{11}]}.$$

where

$$\mathcal{L}_k M = \mathcal{L}_k A^{(10)} = 0,$$

Connection with Romans Supergravity

MPGM, P. León, A. Restuccia:, JHEP21, Arxiv 230100686, Arxiv 2301.xxxxx to Appear.

The dimensions of the singularities are consistent with the presence of wrapped M9-branes. Furthermore at the level of the algebra we obtain a Z0m central charge that was conjectured to be dual to the M9-branes.

The metric describing the target space M9x LCD that we are considering is

$$ds^2 = ds^2_{M_9} + l^2 d\hat{G}^2 + \alpha^2 d\hat{H}^2$$

The metric of the LCD around the singularities

$$ds^2 \approx ds^2_{M_9} + \frac{l^2}{r^2} dr^2 + \alpha^2 d\varphi^2$$

One of the solutions to Romans supergravity explored by Bergshoeff at'99 and Sato'2000 for the following ansatz

$$X^i = \xi^i, \quad i = 0, 1, .., 8. \quad A^{(10)}_{0..,8\theta} = A^{(10)}_{0..,8\theta}(r),$$

Connection with Romans Supergravity

MPGM, P. León, A. Restuccia:, JHEP21, P. León PHD thesis, Arxiv 2301.xxxxx to Appear.

Equations of motion with respect to Ar lead $\partial_r M = \kappa T_{M9}\delta(r) = \kappa T_{M9} r\hat{\delta}(r),$

$$M = \begin{cases} 0 & \text{si} \quad r = 0 \\ \bar{m} & \text{si} \quad r > 0 \end{cases} .$$

with $\boxed{\bar{m} = \kappa T_{M9}}$ Mass term of the cosmological constant

Massive supergravity metric coupled to a M9-brane is

$$ds^2_{M9-brane} = -H(y)^{-p/3}(dt^2 - dx_8^2) + H(y)^{-10p/3-2}dy^2 + H(y)^{5p/3}dz^2 \qquad H(y) = c \pm \frac{m}{p}|y|$$

An approximation to this metric occurs when p is small and $\beta = 10p - 2 \text{ y } c = 0,$

Both metrics agree when $ds^2_{M9-brane} \approx ds^2_{M_9} + \frac{p^2}{m^2y^2}dy^2 + dz^2 \Rightarrow l = \frac{p}{m}$

The topological term of the Massive supermembrane formualtion is directly related in this approximation to the cosmological term of the 11D uplift of Romans supergravity.

Conclusions

- We have obtained two new sectors of the supermembrane with discrete spectrum.

M2-brane on M9xT2 with C fluxes and monodromy sector:

- Case: trivial monodromy
  - By dimensional reduction we obtain (p,q) strings. In 9D all (p,q) strings are associated to this sector. If the C fluxes vanishes -or equivalently the central charge is zero-, they can only describe fundamental strings. The flux contributions generates a new term.

- Case: parabolic monodromy
  - The physics of the of M2-brane bundles with parabolic monodromy become identified by the inequivalent classes of coinvariants.
  - The elements of each class are related by a parabolic SL(2,Q) symmetry and we conjecture it as the origin of parabolic gauged symmetry in tyoe II gauged supergravities.
  - Different classes of M2-brane bundles can be mapped by a different parabolic SL(2,Q) symmetry that leaves invariant the mass operator.

Conclusions

- By perfoming a dimensional reduction we obtain parabolic strings. They are classified by a single integer. Their mass operator contain a topological term and it acquires a modified tension. The flux contributions add a new term.

- Their mass operator is invariant under two distinct parabolic SL(2,Q) symetries at inherited but reduced from the ones of the M2-brane. The first one corresponds to all of the strings with origin in the same coinvariant at M-theory level. The second one relates different classes of parabolic (p,q) strings, and it is the analogue of the SL(2,Z) symmetry of standar (p,q) strings.

Conclusions

Massive M2-brane

- We obtain the 11D M2-brane on a twice punctured torus or equivalently on M9x LCD. It is formuolated in ten non compact dimensions. It contains mass terms for all of the fields and its Hamiltonian has a discrete spectrum.

- It also contains a topological term that at low energies can be associated with the presence of a cosmological term. It is associated with an approximate solution of Romans supergravity

- By a very particular double dimensional reduction, we obtain the worldsheet description of new sector of string denote it as a massive string: a type IIA N=2, closed string that contains a topological term due to the nontrivial background, a modified tension that contains information about this moduli, and a different level matching condition from the standar one.

Conclusions

Connection with Romans Supergravity:

- The E.O.M of supergravity background lead to curvature terms with singularities that are compatible with the presence of wrapped M9's.

- In the superalgebra of the M2-brane it contains a central charge Z0m with a temporal and spatial dependence The presence of this type of central charges has been associated with the presence of M9-branes.

- The massive supermembrane is a source of a background which is an approximation to one of the massive supergravity solutions with M9-branes explored in the literature. The cosmological constant is related to the topological term of the M2-brane in this approximate background.

- We expect that the M2-brane formulated in more complicated punctured backgrounds with three forms will be sources of exact massive supergravity backgrounds

Thanks!

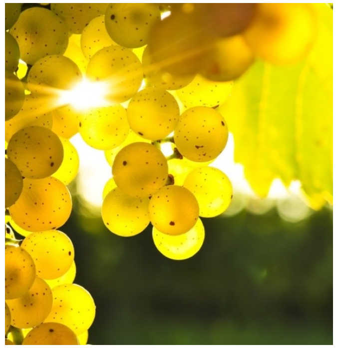


Foto tomada del sitio web https://www.spain.info/es/region/la-rioja/