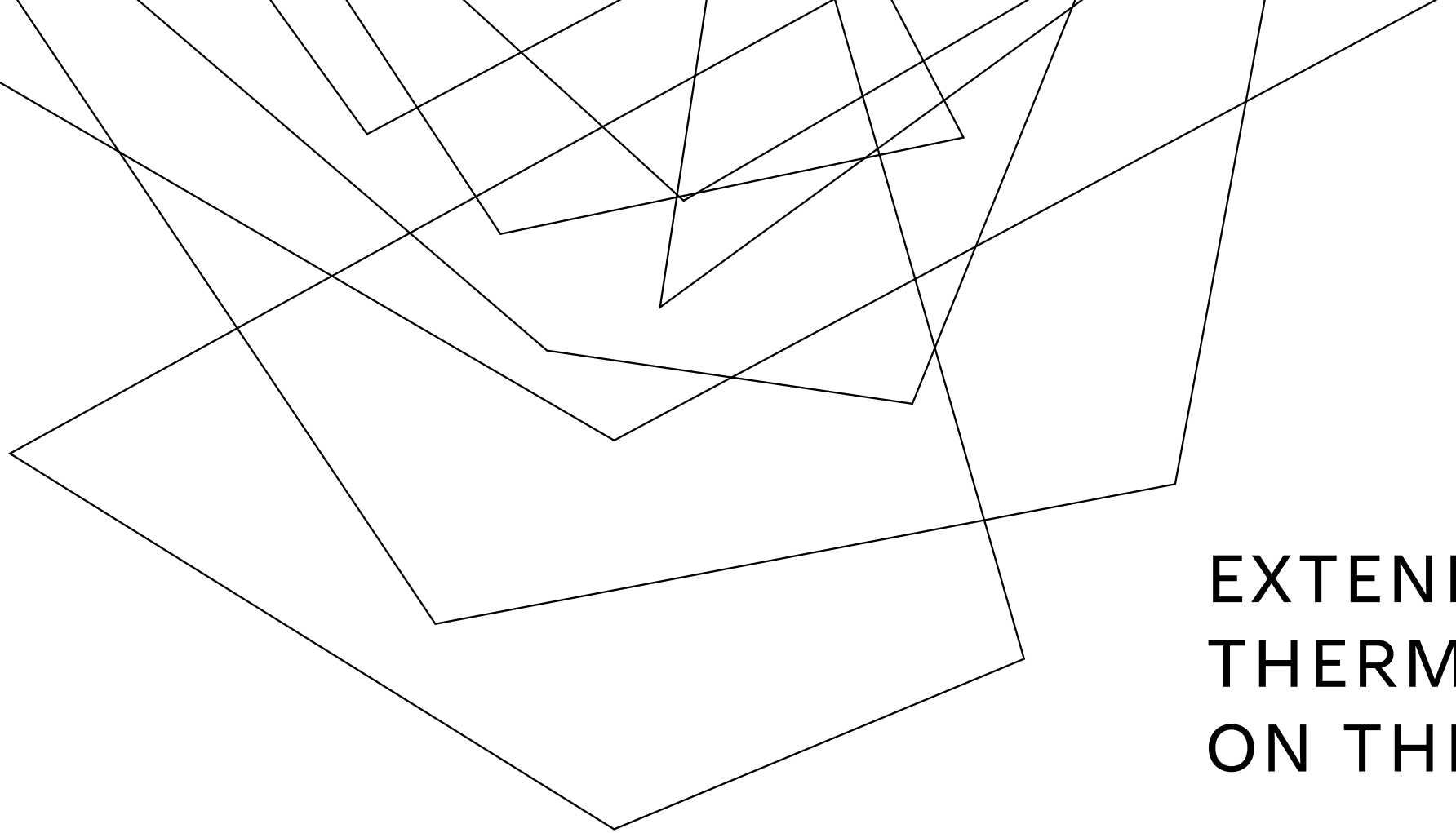

Iberian Strings '23
Murcia

EXTENDED THERMODYNAMICS ON THE BRANE

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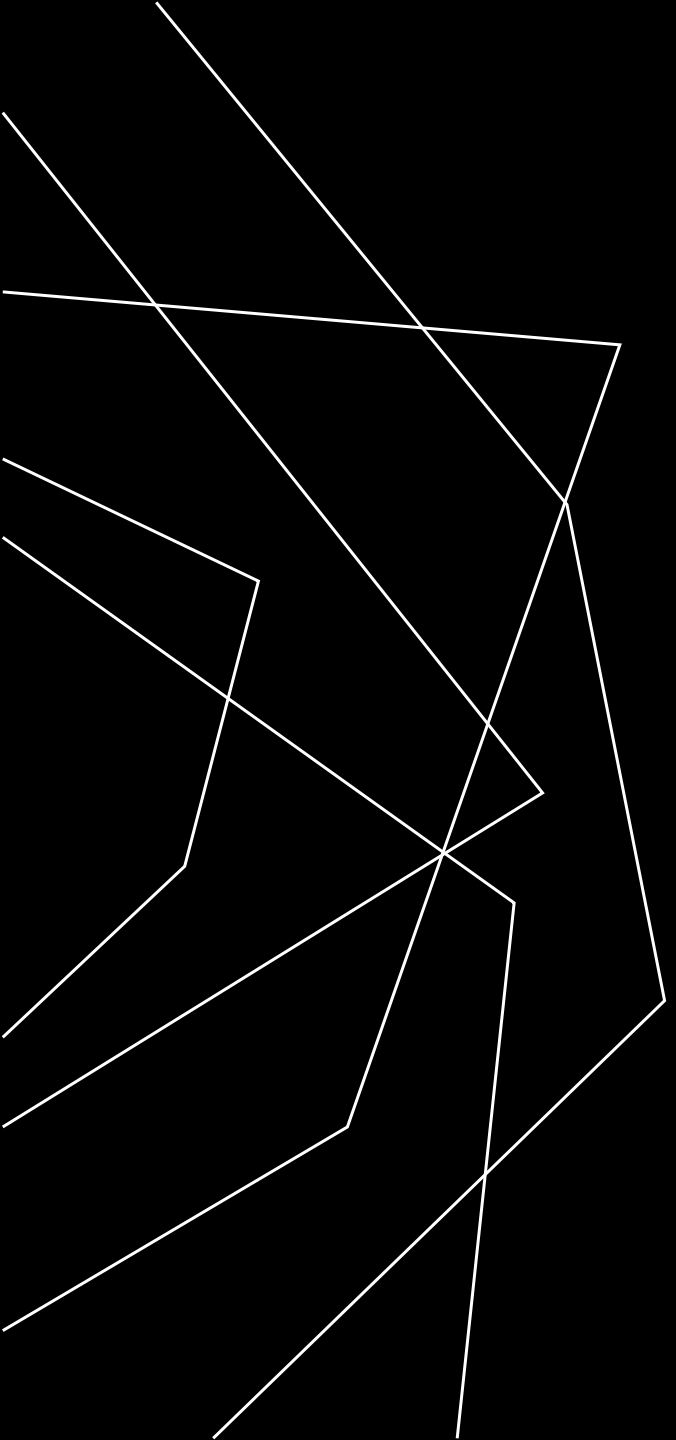


INTRODUCTION

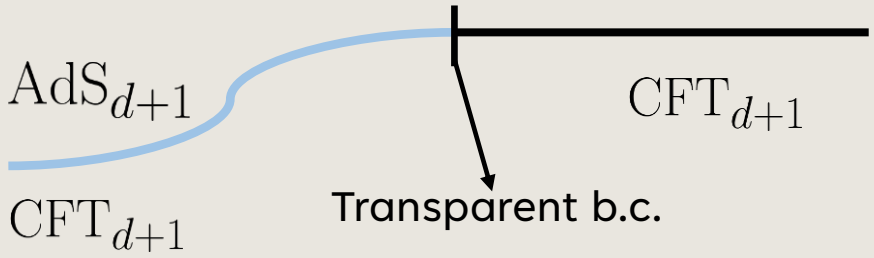
Doubly-holographic models, also known as KR braneworlds, have shown to be very useful for understanding recent developments in quantum gravity (AdS/CFT) and string theory

One application of considering warped geometries with branes is that it provides a generalization of AdS/CFT to spaces with boundaries

DOUBLY HOLOGRAPHIC INTERPRETATION

1. UV Perspective

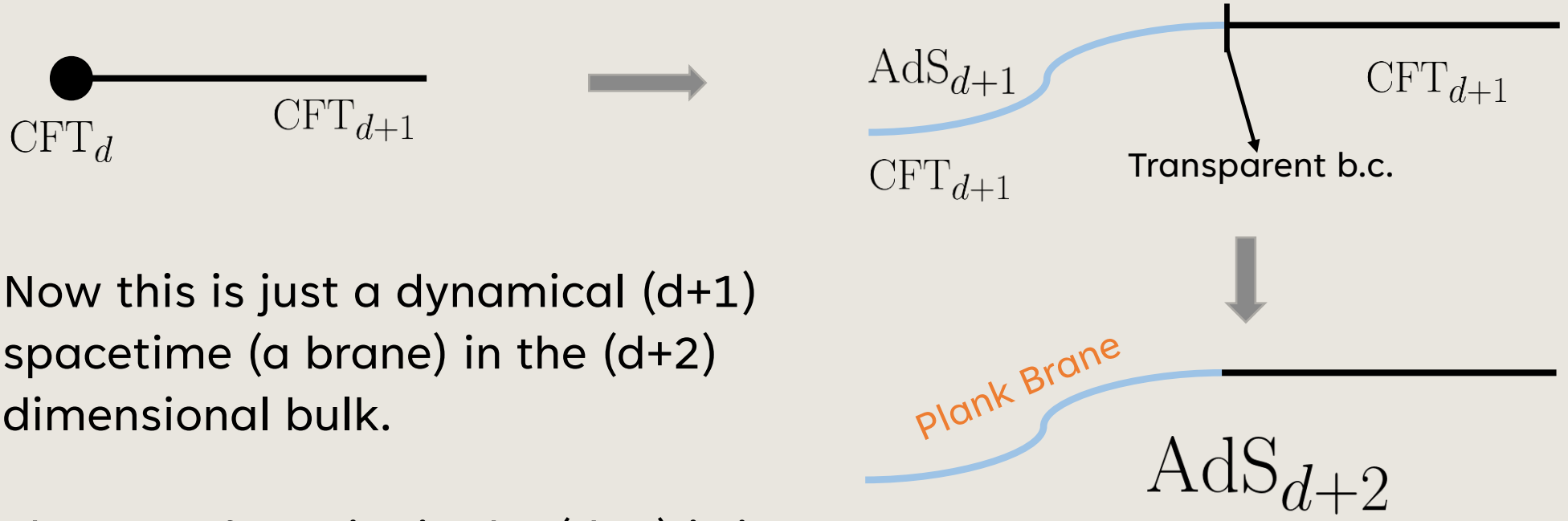
2. Brane Perspective



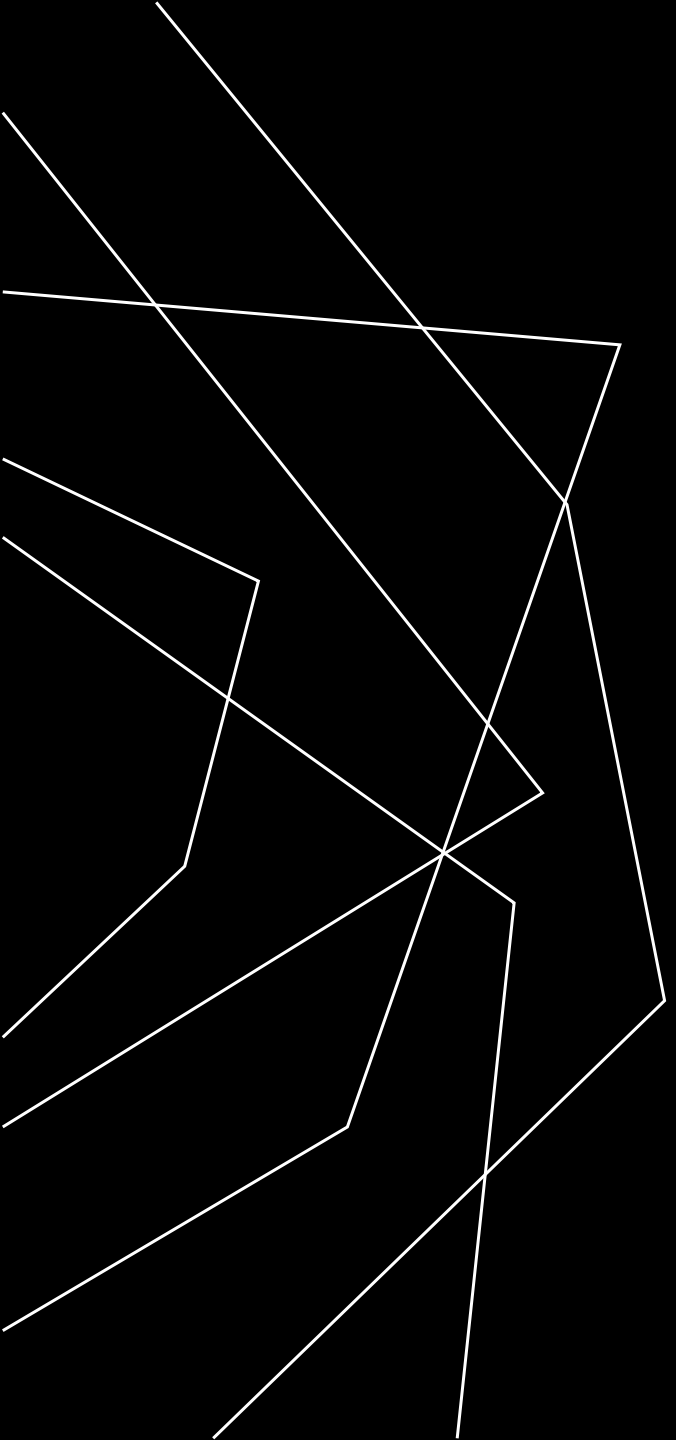
3. Gravitational Perspective

View the whole setup as a
(d+2) AdS without any CFT
- Classical EE with a brane source

DOUBLY HOLOGRAPHIC INTERPRETATION



- Now this is just a dynamical (d+1) spacetime (a brane) in the (d+2) dimensional bulk.
- The EFT of gravity in the (d+1) is just a localization of this (d+2) gravity on the brane (like in the RS setup)



BRANEWORLD MODEL

d-dimensional bulk

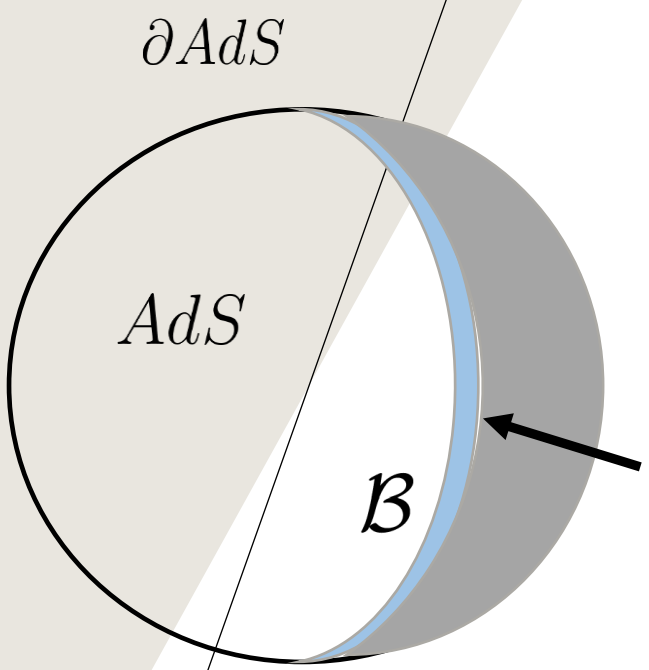
and

a (d-1)-dimensional brane

Randall, Sundrum (1999)
Karch, Randall (2001)

BRANEWORLD HOLOGRAPHY

Basic idea of BRANEWORLD gravity is that you want to recover gravity localized on some lower dimensional surface of some higher dimensional bulk spacetime



Brane

Cutting the bulk with a (Plank) brane

- Introduces a (d-1)-dimensional graviton massive mode localized on the brane
- The CFT is also cutoff in the UV

⟹ get dynamics on the brane

Emparan, Horowitz, Myers (1999)

INTRODUCE A BLACK HOLE IN THE BULK

It is possible to construct exact 4D solutions describing localized black holes bound to a brane

To obtain these solution, notice that a black hole on a brane in AdS is accelerating.

There is a solution to Einstein's equation that describes accelerating black holes: the C-metric

- This solution can be extended to include a cosmological constant

ADS4 C-METRIC

$$ds^2 = \frac{\ell^2}{(\ell + xr)^2}\left[-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2\left(\frac{dx^2}{G(x)} + G(x)d\phi^2\right)\right]$$

$$H(r) = \frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r} \qquad G(x) = 1 - \kappa x^2 - \mu x^3$$

ℓ_3 Brane curvature radius

ℓ Brane position, tension^(-1)

μ Quantum corrections on the brane

Note: the black hole is NOT in the center of AdS_4 – it's accelerating

ROTATING ADS4 C-METRIC

$$ds^2 = \frac{\ell^2}{(\ell + xr)^2}\left[-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2\left(\frac{dx^2}{G(x)} + G(x)d\phi^2\right)\right]$$

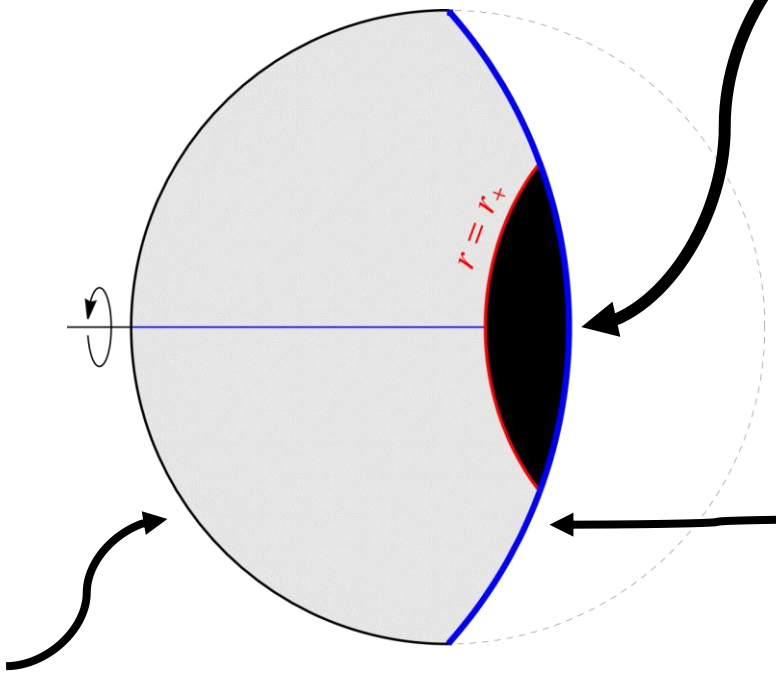
The rotating AdS C-metric:

- Rotation parameter a
- Bulk structure similar to Kerr-AdS4
  (inner and outer horizons, ring singularity)

Very interesting features!

Emparan, Tomašević (2020)
Emparan, AMF, Way (2020)

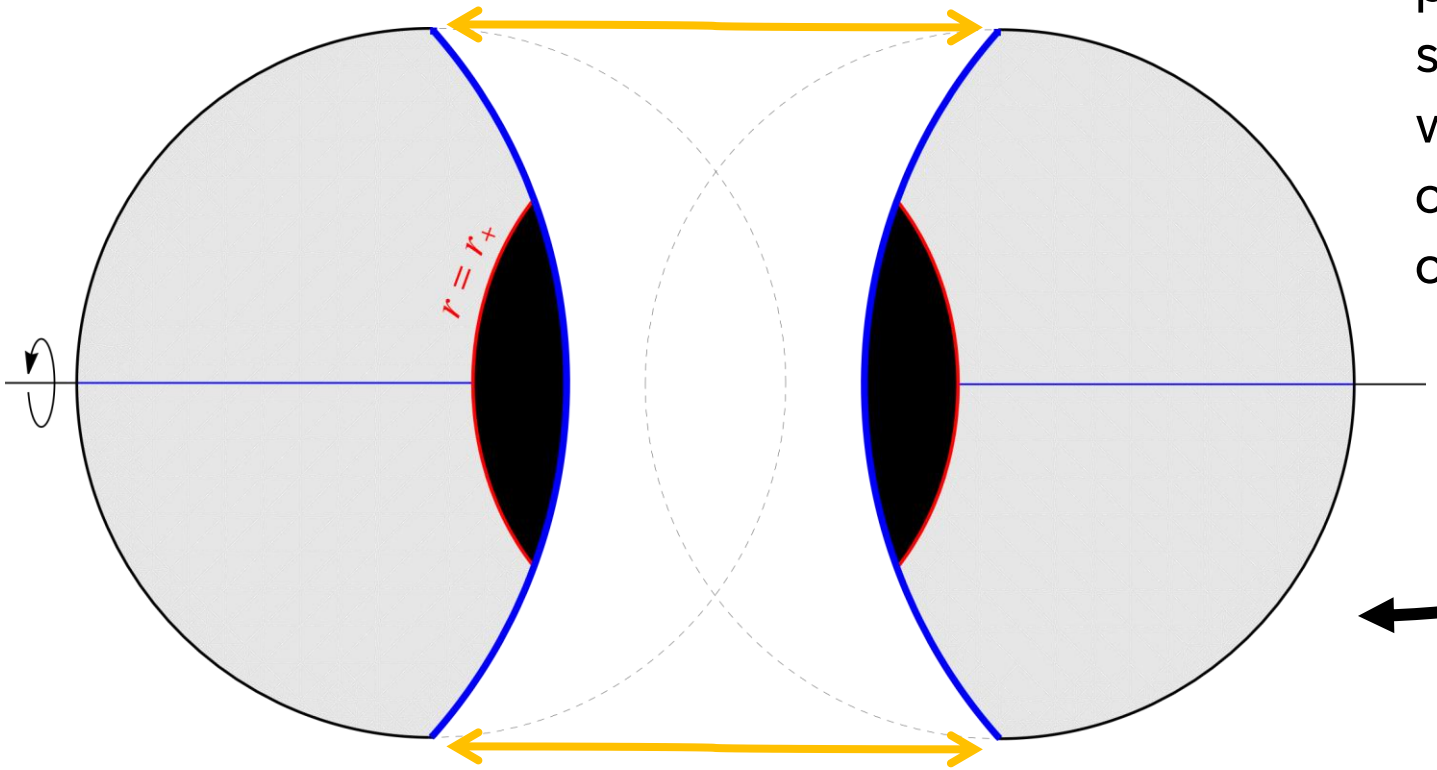
BRANEWORLD WITH A BLACK HOLE



The BH in the bulk is intersecting the brane

CFT_3 on the brane

The 4D theory has an asymptotic boundary where the CFT_3 lives



When the brane is placed in the spacetime,
we erase the other part of the spacetime and consider another copy

The gluing between the two parts of the spacetime is done using Israel gluing conditions along the brane

QUESTION: CAN WE RECOVER LOWER DIMENSIONAL GRAVITY ON THE BRANE?

- we do recover 2 + 1 gravity at large distances along the brane
- Deviations from 2+1 gravity arise at order 1/r, reflecting the 4D nature of the black hole.

- The quantum thermal radiation does, have energy, and therefore, will affect the spacetime geometry (back-reaction)

$$G_{\mu\nu}(g_{\alpha\beta}) = 8\pi G_N \langle T_{\mu\nu}(g_{\alpha\beta}) \rangle$$

- Classical geometry is modified by the effects of quantum fields

  classical black holes map to “quantum black holes”

  accounting for all order in backreaction

Emparan, Horowitz, Myers (2000)
Emparan, Fabbri, Kaloper (2002)

METRIC quBTZ

The 3D metric induced on the brane at x=0
(satisfies the Israel junction conditions)

$$ds^2 = \frac{\ell^2}{(\ell + x/r)^2}\left[-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2\left(\frac{dx^2}{G(x)} + G(x)d\phi^2\right)\right]$$

$$H(r) = \frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r} \qquad G(x) = 1 - \kappa x^2 - \mu x^3$$

METRIC quBTZ

Emparan, Horowitz, Myers (2000)
Emparan, Fabbri, Kaloper (2002)
Emparan, AMF, Way (2020)

3D metric induced on the brane at x=0

$$ds^2 = -\left(\frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}\right) dt^2 + \frac{1}{\frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}} dr^2 + r^2 d\phi^2$$

Classical limits $\mu = 0$

- $\kappa = -1$ BTZ
- $\kappa = +1$ Global or Conical AdS3

$\mu \neq 0$ $\kappa = -1$ quBTZ
different properties of the horizon,
has curvature singularity

METRIC quBTZ

3D metric induced on the brane at x=0

$$ds^2 = -\left(\frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}\right) dt^2 + \frac{1}{\frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}} dr^2 + r^2 d\phi^2$$

Classical limits $\mu = 0$

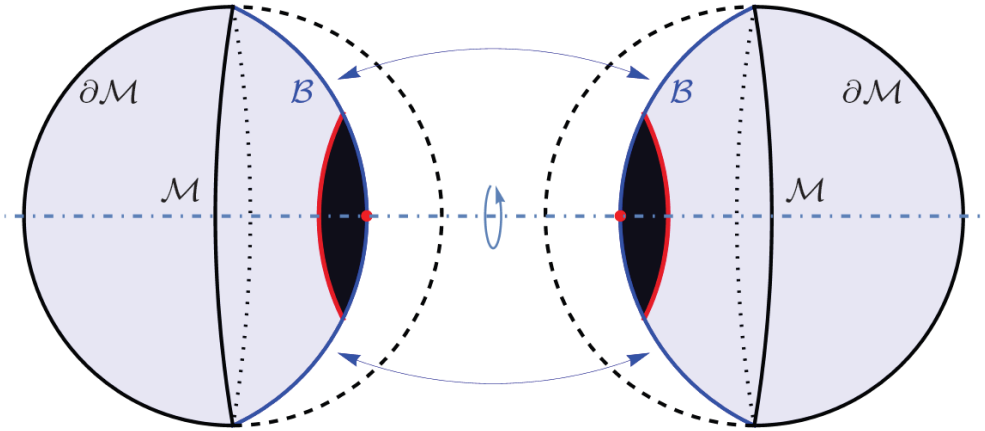
- $\kappa = -1$ BTZ
- $\kappa = +1$ Global or Conical AdS_3

Interpretation: is as a solution of a theory of 3D gravity, with higher curvature terms, coupled to a large number of quantum conformal fields

EFFECTIVE ACTION ON THE BRANE

Action characterizing the bulk: $I_{\rm Bulk}[\mathcal{M}] + I_{\rm GHY}[\partial\mathcal{M}] + I_{\rm Brane}[\mathcal{B}]$

$$
\begin{aligned}
I_{\rm Bulk} &= \frac{1}{16\pi G_{d+1}} \int_{\mathcal{M}} d^{d+1}x\sqrt{-g}\left(\hat{R} - 2\Lambda_{d+1}\right) \\
I_{\rm GHY} &= \frac{1}{8\pi G_{d+1}} \int_{\partial\mathcal{M}} d^{d}x\sqrt{-h}K\,, \\
I_{\rm Brane} &= -\tau \int_{\mathcal{B}} d^{d}x\sqrt{-h}\,.
\end{aligned}
$$



- Integrating out the bulk between the boundary up to Brane (typically taken to be near the boundary) amounts to removing CFT dof above the UV cutoff, leading to the brane effective action

- The CFT action arises from integrating out normalizable modes, which accounts for the dual CFT state.

$$I = I_{\rm Bgrav}[\mathcal{B}] + I_{\rm CFT}[\mathcal{B}]$$

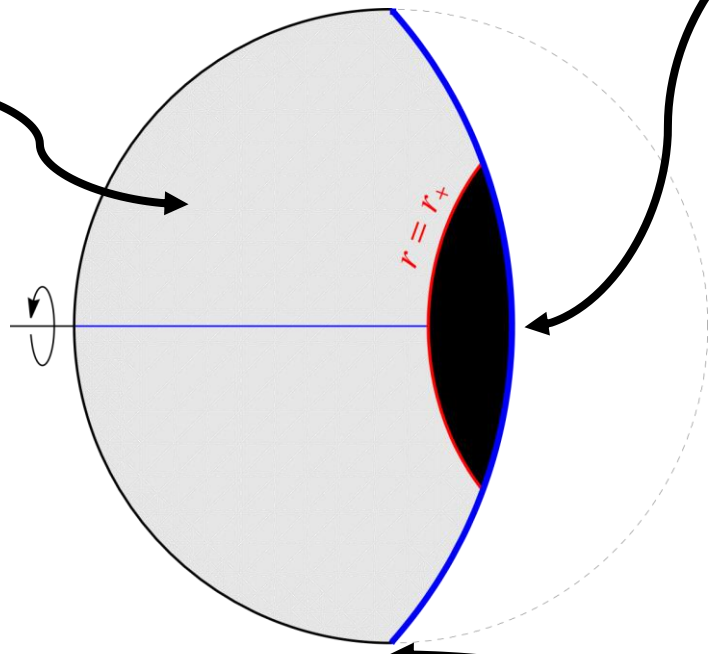
$$
\begin{aligned}
I_{\rm Bgrav} = \frac{1}{16\pi G_d}\int_{\mathcal{B}} d^{d}x\sqrt{-h}\Big[R - 2\Lambda_d \\
+ \frac{L_{d+1}^2}{(d-4)(d-2)}(R^2\text{-terms}) + \cdots\Big]
\end{aligned}
$$

MORE ABOUT THE BLACK HOLE ON THE BRANE

The bulk spacetime is a solution of the vacuum EE with negative CC in 4D

3D CC obtained from:
4D CC & brane tension

$$\frac{1}{\ell_4^2} = \frac{1}{\ell^2} + \frac{1}{\ell_3^2}$$



The BH in the bulk is intersecting the brane

Transparent boundary conditions for the CFT btw the boundary and the brane (we consider the dCFT)

GENERALIZED ENTROPY IN BW HOLOGRAPHY

Emparan, AMF, Way (2020)

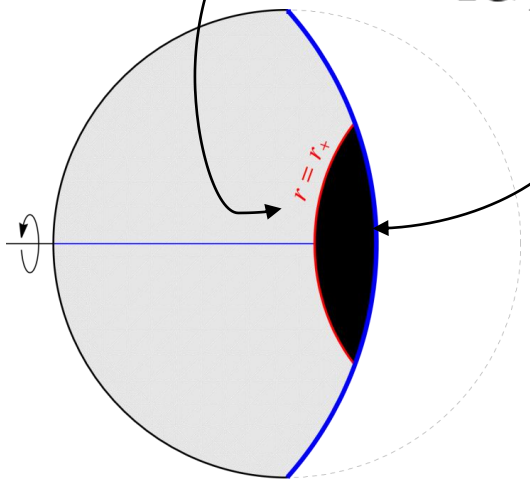
$$S_{\rm gen} = S_{\rm Wald} + S_{\rm outside}$$

Bulk BH entropy

$$S_{gen} = \frac{A_{D+1}}{4G_{D+1}}$$

Brane BH entropy

$$S_{\rm Wald} = \frac{A_D}{4G_D} + \dots$$



Entropy of the CFT that leaves in the presence of the BH

→ Obtained by subtraction:

$$S_{\rm ent} = S_{\rm gen} - S_{\rm Wald}$$

Use this result in BH thermodynamics

QUANTUM ENTROPY: 1ST AND 2ND LAW

If the holographic interpretation of braneworld is consistent, then:

$$T dS_{\rm gen} = dM - \Omega dJ$$

1st law: Not trivial

In D+1 dim bulk

On D dim brane w/ higher curvature gravity

Holds exactly to all orders in the backreaction and higher-curvature corrections.

$$\Delta S_{\rm gen} \geqslant 0$$

2nd law: Trivial

Thermodynamics with fixed tension

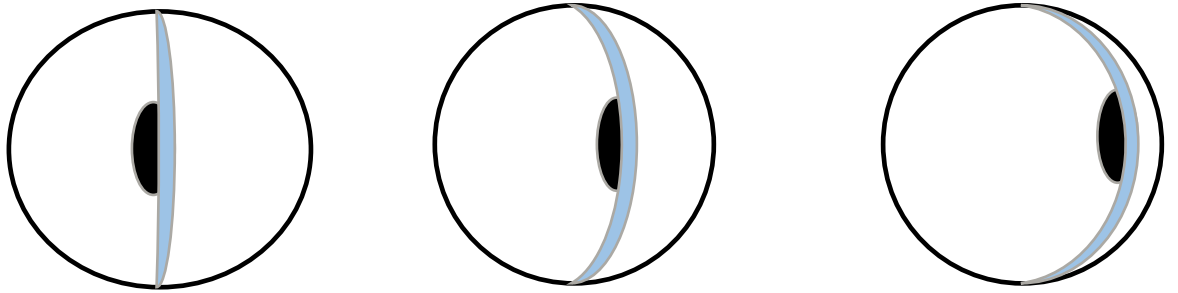
$S_{\rm Wald}$ alone will not satisfy these relations!

VARIATION OF THE TENSION

From the bulk perspective, it is very natural to tune the tension of the brane since it is a physical parameter of the system.

The tension determines the position of the brane via Israel junction conditions, thence varying the tension changes the position.

$\ell \to \infty$ $\quad 0 < \ell < \infty$ $\quad \ell \to 0$



$$\tau \propto \frac{1}{G_4 \ell}$$

4D AdS BH (only Z2 symmetry)

BH+CFT

$\kappa = -1$ bd geometry has a BTZ

Accordingly, varying the tension alone (keeping the other bulk parameter fixed) corresponds to varying the CC on the brane:

$$\delta\tau = \frac{\delta\Lambda_d}{8\pi G_d}$$

EXTENDED THERMODYNAMICS FRAMEWORK

Where is the PdV term?

1. Consider an asymptotically AdS black hole spacetime
2. Identify the cosmological constant with a thermodynamic pressure

$$P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi L^2}$$

3. Allow this to be a (thermo)dynamical quantity

- Consistency between 1st law and Smarr formula in AdS
- Concept of the thermodynamic Volume (conjugate quantity)
  $\rightarrow$ Isoperimetric inequalities conjecture for AdS black hole

[Doland, Creighton, Mann, Kastor, Traschen, Padmanabhan, Cvetic Gibbons, Kubiznak, Pope, Gregory, Fischler, Nguyen, Johnson, Karch, Ortin,.....]

INDUCED EXTENDED THERMODYNAMICS

The bulk black hole thermodynamics gives the thermodynamics of the quantum black hole, including the extension with a variable pressure on the brane

Consider the case when the brane tension is treated as a thermodynamic variable, similar to the surface tension of liquids.

The effect is that the brane will do work on the black hole system, such that the bulk first law becomes

$$dM = TdS + A_\tau d\tau$$

From the brane perspective, the variation of the tension induces extended thermodynamics on the brane.

<u>The bulk first law maps to the brane first law</u>

$$dM = TdS_{\rm gen} + V_3 dP_3$$

generalizing the quantum first law at fixed tension!

DOUBLY HOLOGRAPHIC INTERPRETATION

Since the brane geometry is asymptotically AdS3, a third description of the bulk-brane system emerges

> we may replace the induced gravity theory on the brane with a defect CFT2 coupled to the CFT3, hence, our construction exhibits “double holography”

We can now ascribe a dual field theory interpretation to quBTZ thermodynamics:

$$dE = TdS_2 - P_2 dV_2 + \mu_2 dc_2$$

For fixed L4 and G4, we see variations in the brane tension cascades down to variations in c2 due to the proportionality with L3

CONCLUSIONS

- Braneworld holography offers a natural framework to explore extended black hole thermodynamics:

  varying brane tension induces a variable pressure of a quantum black hole on the brane to all orders in 1/c

- The crucial point is that the variation of the pressure on the brane follows directly from variation of the tension.
- As proof of concept, we focused on a specific type of black hole, the qBTZ, exhibiting markedly different behavior from the classical BTZ

THANK YOU

REF: arXiv:2212.14055

''Higher-dimensional origin of extended black hole thermodynamics''

Authors: Antonia M. Frassino, Juan F. Pedraza, Andrew Svesko, Manus R. Visser