Towards a non-relativistic AdS/CFT duality

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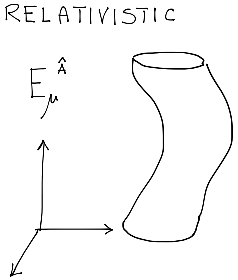
Iberian Strings 2023

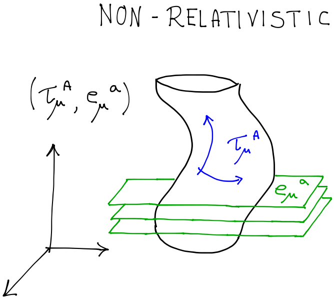
Based on:

"Light-Cone Gauge in Non-Relativistic $AdS_5 \times S^5$ String Theory"
with J.M. Nieto and A. Torrielli arXiv:2102.00008,
"Classical string solutions in non-relativistic $AdS_5 \times S^5$: closed and twisted sectors" with J.M. Nieto arXiv:2109.13240,
"Coset space actions for nonrelativistic strings"
with S. van Tongeren arXiv:2203.07386,
"Extending the non-relativistic string AdS coset"
with J.M. Nieto arXiv:2208.02295,
"Non-relativistic string monodromies"
with J.M. Nieto and O. Ohlsson Sax arXiv:2211.04479

Motivations

- Interested in non-AdS holography
- Strings with: 1) NR target space, 2) relativistic world-sheet





Foliation “2+8” $\hat{A} = (A, a)$

$A = 0, 1 \qquad a = 2, ..., 8$

- NR (bosonic) string is Weyl anomaly free in $d = 26$

  [Gomis, J. Oh, Z. Yan, 2019][Gallegos, Gursoy, Zinnato, 2019]
- Target space geometry is string Newton-Cartan

  [Harmark, Hartong, Obers, 2017][Bergshoeff, Gomis, Yan, 2018]
- First considered in flat space [Gomis, Ooguri, 2000][Danielsson, Guijosa, Kruczenski, 2000] then in $\mathrm{AdS}_5 \times S^5$ [Gomis, Gomis, Kamimura, 2005]

Holography....

- Relativistic strings in $\mathrm{AdS}_5 \times S^5$ / $\mathcal{N} = 4$ SYM. Holography has been extensively studied. One needs observables to match:

  "string excitations = dimensions of gauge-invariant operators"

$$E(\sqrt{\lambda}, m, ...) = \Delta(\lambda, m, ...)$$

- Integrability: key property to determine the spectrum exactly.

Questions:

- Can we determine the spectrum of NR strings in $\mathrm{AdS}_5 \times S^5$?
- is the theory integrable?

Outline

1. NR strings in $AdS_5 \times S^5$
2. Classical string solutions (BMN-like, GKP-like)
3. Semiclassical expansion of the NR action
4. Coset formulation + Lax pair
5. Spectral curve

NR strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$

Flat space

$$S = \int \partial X^\mu \partial X^\nu \eta_{\mu\nu}$$

NR limit [Gomis, Ooguri, 2000]

$$X^0 \to c X^0 \,, \qquad X^1 \to c X^1 \,, \qquad c \to \infty$$

Curved spacetime

$$S = \int \partial X^\mu \partial X^\nu g_{\mu\nu}$$

rescaling of X^μ?

Homogeneous spaces G/H (e.g. $\mathrm{AdS}_5 \times \mathrm{S}^5$) [Metsaev, Tseytlin, 1998]

$$S = \int \langle A, PA \rangle \ , \qquad A = g^{-1}\mathrm{d}g \ , \qquad g \in G$$

choice of g = choice of coordinates

(e.g. $g = e^{X \cdot T}$)

NR limit = IW contr.: $\mathfrak{so}(2,4) \oplus \mathfrak{so}(6) \longrightarrow$ s.Newton-Hooke$_5 \oplus$Eucl$_5$

translations = "2 + 8" $P_0 \to \frac{P_0}{c}, \ P_1 \to \frac{P_1}{c}$ (+ boost)

rescaling of generators $\longleftrightarrow$ rescaling of coords

$E_\mu{}^{\hat{A}}$: long. $E_\mu{}^A = c\tau_\mu{}^A + \frac{1}{c} m_\mu{}^A$ $A = 0, 1$

transv. $E_\mu{}^a = e_\mu{}^a$ $a = 2, ..., 8$

metric: $g_{\mu\nu} = -c^2 \tau_\mu{}^A \tau_\nu{}^B \eta_{AB} + \text{finite}$

add closed B-field $B_{\mu\nu} = c^2 \varepsilon_{AB} \tau_\mu{}^A \tau_\nu{}^B$

$$\text{Divergent}(g_{\mu\nu} + B_{\mu\nu}) = \lambda_A F^A + \frac{1}{c^2}\lambda_A \lambda^A = \text{finite} + \text{subleading}$$

λ_A are 2 non-propagating worldsheet scalars (Lagrange multipliers)

Now we can take $c \to \infty$

$$S^{\mathrm{NR}} = \int \mathrm{d}^2\sigma \left(\gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu H_{\mu\nu} + \varepsilon^{\alpha\beta} (\lambda_+ \theta_\alpha{}^+ \tau_\mu{}^+ + \lambda_- \theta_\alpha{}^- \tau_\mu{}^-) \partial_\beta X^\mu \right)$$

$\theta_\alpha{}^\pm$ zweibein of w.s. metric, $H_{\mu\nu}$ is the finite piece from $g_{\mu\nu}$
Coordinates:

$$AdS_5 \;\; (t,\; z,\; z_2,\; z_3,\; z_4) \quad \times \quad S^5 \;\; (\phi,\; y_1,\; y_2,\; y_3,\; y_4)$$

String Newton-Cartan vielbeine

$\tau_\mu{}^A \;:\; AdS_2 \; (t, z) \qquad e_\mu{}^a \;:\; f(z)\, \mathbb{R}^3 \; (z_2, z_3, z_4) \times \mathbb{R}^5 \; (\phi, y_1, ..., y_4)$

Solving equations of motion for $\lambda_\pm$ (fix conformal gauge)

$$t = \kappa\tau \qquad z = -2\tan(\kappa\sigma/2) \qquad \kappa \in \mathbb{Z}$$

The string must have winding!

Relativistic string in $\text{AdS}_5 \times S^5$

BMN solution (point-like) [Berenstein, Maldacena, Nastase, 2002]

$$t = \phi = \kappa\tau$$

Dispersion relation $E = J$

In light-cone gauge ($X_\pm = t \pm \phi$), and in large string tension $T \gg 1$, action expands about strings in pp-wave

NR string in $\text{AdS}_5 \times S^5$

BMN-like (extended string) [AF, Nieto, 2021]

$$t = \kappa\tau \qquad \phi = \omega\tau \qquad z = -2\tan(\kappa\sigma/2) \qquad \lambda_\pm \sim \cos(\kappa\sigma)$$

Dispersion relation $E \sim J^2$

In light-cone gauge ($X_\pm = t \pm \phi$), in $T \gg 1$ and $R \gg 1$, action expands about free fields + corrections σ-dependent

Reason: 1) z is not isometry $z = z_{\text{cl}} + z_{\text{fl}}$ 2) z is not in the lightcone $X_\pm$

Classical Integrability

- Goal: find a Lax pair for the NR action in $\mathrm{AdS}_5 \times S^5$
- Strategy for relativistic string: capture the metric (vielbein) inside a Maurer-Cartan form (coset action) [Metsaev, Tseytlin, 1998]

$$\mathrm{AdS}_5 \times S^5 = \frac{SO(2,4)\times SO(6)}{SO(1,4)\times SO(5)} = \frac{\text{isometry}}{\text{isotropy}}$$

$$S = \int \mathrm{d}^2\sigma \gamma^{\alpha\beta} \langle A_\alpha, P A_\beta \rangle$$

A_α is the Maurer-Cartan form. ($PA = A^{(1)}$)

- E.o.m. $\partial_\alpha(\gamma^{\alpha\beta} A_\beta^{(1)}) + \gamma^{\alpha\beta}[A_\alpha^{(0)}, A_\beta^{(1)}] = 0$
- Lax pair [Bena, Polchinski, Roiban, 2003]

$$\mathscr{L} = A^{(0)} + \ell_1 A^{(1)} - \ell_2 \star A^{(1)} \qquad \ell_1^2 - \ell_2^2 = 1$$

Equivalence: $\mathrm{d}\mathscr{L} + \mathscr{L} \wedge \mathscr{L} = 0 \iff$ E.o.m.

- NR string: isometry of $\mathrm{AdS}_5 \times S^5$ is ∞-dim, but isotropy finite.
- Truncate the NR isometry to capture the NC vielbeine (suggested by Lie algebra expansion)

[AF, van Tongeren 2022][AF, Nieto, 2022]

$$S^{\mathrm{NR}} = \int \mathrm{d}^2\sigma \gamma^{\alpha\beta} \langle J_\alpha, P J_\beta \rangle \qquad J_\alpha \equiv A_\alpha - (\star\Lambda)_\alpha$$

A_α is the Maurer-Cartan form. Λ_α external current, depends on $\lambda_\pm$

- Equations of motion

$$\partial_\alpha(\gamma^{\alpha\beta} J_\beta^{(1)}) + \gamma^{\alpha\beta}[J_\alpha^{(0)}, J_\beta^{(1)}] = 0 \qquad \mathcal{E}^{\lambda_\pm} = \varepsilon^{\alpha\beta} \theta_\alpha{}^\pm A_\beta{}^{H_\pm} = 0$$

- Lax pair

$$\mathscr{L}^{\mathrm{NR}} = A^{(0)} + \ell_1 A^{(1)} - \ell_2 \star J^{(1)} \qquad \ell_1^2 - \ell_2^2 = 1$$

on solutions of $\mathcal{E}^{\lambda_\pm} = 0$

Spectral curve

- Alternative route to the spectrum, it captures TBA equations
- Compute the eigenvalues of monodromy matrix

$$\mathcal{M} = \mathrm{P}\exp\left(\int_0^{2\pi} \mathrm{d}^2\sigma \ \mathscr{L}_\sigma^{\mathrm{NR}}(\xi)\right)$$

ξ = spectral parameter

- <u>Theorem</u>: On solutions of $\mathcal{E}^{\lambda_\pm} = 0$ all eigenvalues are ξ-independent

[AF, Nieto, Ohlsson Sax 2022]

- $\mathcal{M}$ evaluated on BMN-like sol. is **non-diagonalisable**

$$\mathcal{M}|_{\text{BMN-like}} = S \begin{pmatrix} {\color{blue}\times} & {\color{red}\times} & {\color{red}\times} & & & \\ & {\color{blue}\times} & {\color{red}\times} & & & \\ & & {\color{blue}\times} & & & \\ & & & {\color{blue}\times} & {\color{red}\times} & \\ & & & & {\color{blue}\times} & \\ & & & & & {\color{blue}\times} \end{pmatrix} S^{-1}$$

${\color{blue}\times}$ no ξ-dep. ${\color{red}\times}$ yes ξ-dep. $\Longrightarrow$ spectral curve defined by "${\color{red}\times}$"

- Reason of non-diagonalisability:

  $\mathfrak{so}(2,4) \oplus \mathfrak{so}(6)$ is semi-simple, but s.Newton-Hooke$_5$$\oplus$Eucl$_5$ is not

- semi-simple part of s.Newton-Hooke$_5$$\oplus$Eucl$_5$ is diagonalisable

Same apply for **relativistic** string in flat space.

- Poincaré algebra is not semi-simple.
- Eigenvalues of monodromy on any solution do not depend on ξ

Diagonalisable: $\mathcal{M} = S e^{p_i(\xi) C_i} S^{-1}$ $\quad C_i \in \text{Cartan}$

Non-diagonalisable: $\mathcal{M} = S e^{q_i(\xi) W_i} S^{-1}$ $\quad W_i \in \text{MAS}$

(MAS = maximal abelian subalgebra)

[AF, Nieto, Ohlsson Sax 2022]

Summary of NR strings in $\mathrm{AdS}_5 \times S^5$

- NR string needs winding (consequence of $\mathcal{E}^{\lambda_\pm} = 0$)
- having winding spoils semiclassical expansion of the action (z is not an isometry)
- found a coset formulation + Lax pair
- Monodromy is non-diagonalisable, its eigenvalues are ξ-independent
- proposed an alternative definition of spectral curve

Future directions

- Relativistic solution that flows to BMN-like?
  Take the NR limit directly on the relativistic spectrum.
- generalised spectral curve, Y-system
- SUSY coset action
- Deformations

$$S^{\mathrm{NR}} = \int \mathrm{d}^2\sigma \gamma^{\alpha\beta} \langle J_\alpha, \mathcal{O} J_\beta \rangle$$

- Identify the "dual" limit on $\mathcal{N} = 4$ SYM
- Carroll strings: boring or interesting?

Thank you for your attention!

More questions?

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