On action and properties of 10D multiple D0-brane system.

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Outline

Dirichlet p-branes (or Dp-branes) are supersymmetric extended objects

- On which the fundamental $D = 10$ superstrings can have its ends attached.
- In 10D, there exist supersymmetric Dp-branes:
  - $p = 0,\ 2,\ 4,\ 6,\ 8$ in type IIA superspace.
  - $p = 1,\ 3,\ 5,\ 7,\ 9$ in type IIB superspace.
- Its worldvolume action is given by the sum of the nonlinear Dirac-Born-Infeld (DBI) term and Wess-Zumino (WZ) term [Cederwall, von Gussich, Nilsson, Westerberg, 1996; Cederwall, von Gussich, Nilsson, Sundell, Westerberg 1996; Aganagic, Popescu, Schwarz 1996; Bergshoeff, Townseng 1996; Bandos, Sorokin, Tonin 1997].

Systems of multiple branes

- In 1995, E. Witten argued that the system of N nearly coincident Dp-branes
  - carries non-Abelian gauge fields on center of energy worldvolume.
  - Its gauge fixed description at very low energy limit is given by the action of non-Abelian U(N) SUSY Yang-Mills (SYM) theory at low energy.
- In it, the $N = 1$ case gives the action for Abelian U(1) SYM which can be identified as a weak field limit of gauge fixed version of the single Dp-brane.

Problem statement

- Despite a number of very interesting results obtained during the past 27 years [Tseytlin 1997; Emparan 1998; Myers 1999; Sorokin 2000; Panda 2003; Lozano, Janssen et al 2002-2005; Howe, Lindstrom, Wulff 2005, 2007; Bandos 2008, 2012, 2018]
  - The complete supersymmetric action for mDp-branes was not known even for the simplest case of $p = 0$.
  - It is widely believed that the **bosonic limit** of this system is given by the Myers's "dielectric brane" action [Myers 1999]. However, despite extensive study, it still resists the supersymmetric generalization.
- A candidate to complete doubly supersymmetric (spacetime supersymmetry + worldline supersymmetry) action for the mD0-brane system was constructed by Igor Bandos in 2018 [JHEP 2018] but
  - an attempt do obtain it by dimensional reduction from mM0 action failed, which was a bit surprising. This was one of the original motivations of our study.

The dimensional reduction of multiple Mp-brane action

- For $p = 0$, it is expected that mD0 action
  - be invariant under rigid spacetime supersymmetry,
  - be invariant under local worldline supersymmetry which
    - acts on the center of energy sector like κ-symmetry of single D0-brane,
    - acts on the physical fields as a local version of the SUSY of SU(N) SYM or as its generalization.

Dynamical variables describing the mD0 system

- The set of center of energy variables contains coordinate functions

$$Z^M(\tau) = \big(x^\mu(\tau), \theta^{\alpha 1}(\tau), \theta^2_\alpha(\tau)\big) \ , \qquad \mu = 0, ..., 9 \ , \quad \alpha = 1, ..., 16 \ ,$$

  given by bosonic 10-vector and two fermionic Majorana-Weyl spinors, describing the embedding of the center of energy worldline $\mathcal{W}^1$ in flat type IIA target superspace,

$$\mathcal{W}^1 \subset \Sigma^{(16|32)} : \quad Z^M = Z^M(\tau) \ .$$

- The relative motion of the mD0 constituents are described by matrix fields of the d $= 1$ $\mathcal{N} = 16$ SU(N) SYM multiplet.
- We use some auxiliary fields. In particular, these are spinor moving frame variables.

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Moving frame in 10D

- To write the candidate to multiple D0-branes action (mD0) we need to introduce the moving frame fields described by

$$(u^0_\mu, u^i_\mu) \in \mathsf{SO}(1,9)\ ,$$

where $i = 1, ..., 9$ are vector indices of $\mathsf{SO}(9)$ group. This implies that $u^0_\mu = u^0_\mu(\tau)$ and $u^i_\mu = u^i_\mu(\tau)$ satisfy

$$u^0_\mu u^{\mu 0} = 1\ , \quad u^0_\mu u^{\mu i} = 0\ , \quad u^i_\mu u^{\mu j} = -\delta^{ij}\ .$$

Spinor moving frame = $\sqrt{\text{moving frame}}$

- Moving frame fields are related to spinor moving frame Spin$(1,9)$ valued matrix

$$v_\alpha^q \in \text{Spin}(1,9) \ ,$$

by the conditions of Lorentz invariance of σ-matrices

$$u_\mu^{(\nu)} \sigma^\mu_{\alpha\beta} = v_\alpha^q \sigma^{(\nu)}_{qp} v_\beta^p \ ,$$

where $q = 1, ..., 16$ are spinor indices of SO(9) group.

- With a suitable representation of σ-matrices, the latter constraints can be split into

$$u_\mu^0 \sigma^\mu_{\alpha\beta} = v_\alpha^q v_\beta^q \ , \quad u_\mu^i \sigma^\mu_{\alpha\beta} = v_\alpha^q \gamma^i_{\alpha\beta} v_\beta^p \ , \quad v_\alpha^q \tilde{\sigma}_\mu^{\alpha\beta} v_\beta^p = u_\mu^0 \delta_{qp} + u_\mu^i \gamma^i_{qp} \ ,$$

This can be used to define the inverse spinor moving frame matrix

$$v_\alpha^q \tilde{\sigma}^{\mu\alpha\beta} u_\mu^0 = v_q^\beta \ , \qquad v_\alpha^q v_q^\beta = \delta_\alpha^\beta \ .$$

Outline

The complete nonlinear action for a supersymmetric multiple D0-brane system

$$S_{\mathrm{mD0}} = \int_{\mathcal{W}^1} mE^0 - im\int_{\mathcal{W}^1}(\mathrm{d}\theta^1\theta^2 - \theta^1\mathrm{d}\theta^2) - \frac{1}{\mu^6}\int_{\mathcal{W}^1}\frac{\mathrm{d}\mathcal{M}}{\mathcal{M}}\mathrm{tr}(\mathbb{P}^i\mathbb{X}^i) +$$

$$+\frac{1}{\mu^6}\int_{\mathcal{W}^1}\left(\mathrm{tr}\left(\mathbb{P}^i\mathrm{D}\mathbb{X}^i + 4i\mathbf{\Psi}_q\mathrm{D}\mathbf{\Psi}_q\right) + \frac{2}{\mathcal{M}}E^0\mathcal{H}\right) + \frac{1}{\mu^6}\int_{\mathcal{W}^1}\frac{1}{\sqrt{2\mathcal{M}}}(E^{1q} - E^2_q)\times$$

$$\times\,\mathrm{tr}\left(-4i(\gamma^i\mathbf{\Psi})_q\mathbb{P}^i + \frac{1}{2}(\gamma^{ij}\mathbf{\Psi})_q[\mathbb{X}^i,\mathbb{X}^j]\right)\ ,$$

$$\text{where } \mathcal{H} = \frac{1}{2}\mathrm{tr}\left(\mathbb{P}^i\mathbb{P}^i\right) - \frac{1}{64}\mathrm{tr}\left[\mathbb{X}^i,\mathbb{X}^j\right]^2 - 2\,\mathrm{tr}\left(\mathbb{X}^i\,\mathbf{\Psi}\gamma^i\mathbf{\Psi}\right)\ .$$

- It is written in terms of the variables used for the single D0-brane (now the center of mass variables) and
- SU(N) SYM fields: $9+9$ bosonic and 16 fermionic Hermitean traceless $N\times N$ matrix matter fields $\mathbb{X}^i = \mathbb{X}^i(\tau)$, $\mathbb{P}^i = \mathbb{P}^i(\tau)$ and $\mathbf{\Psi}_q = \mathbf{\Psi}_q(\tau)$ and the bosonic anti-Hermitean worldline gauge field $\mathbb{A} = \mathrm{d}\tau\mathbb{A}_\tau(\tau)$.

- The latter enters in the action from the covariant derivatives of matrix matter fields

$$\mathrm{D}\mathbb{X}^i := \mathrm{d}\mathbb{X}^i - \Omega^{ij}\mathbb{X}^j + [\mathbb{A}, \mathbb{X}^i] \; , \qquad \mathrm{D}\boldsymbol{\Psi}_q := \mathrm{d}\boldsymbol{\Psi}_q - \frac{1}{4}\Omega^{ij}\boldsymbol{\Psi}_p + [\mathbb{A}, \boldsymbol{\Psi}_q] \; ,$$

which also include the composite $\mathrm{SO}(9)$ connection

$$\Omega^{ij} := u^{\mu i}\mathrm{d}u^j_\mu \; .$$

- Moving frame vector u^0_μ and spinor frame matrix field v_α^q and its inverse v_q^α are used to construct bosonic and fermionic forms

$$E^0 = \Pi^\mu u^0_\mu \ , \qquad E^{1q} = \mathrm{d}\theta^{1\alpha} v_\alpha^q \ , \qquad E^2_q = \mathrm{d}\theta^2_\alpha v_q^\alpha \ .$$

where

$$\Pi^\mu = \mathrm{d}x^\mu - i\mathrm{d}\theta^1\sigma^\mu\theta^1 - i\mathrm{d}\theta^2\tilde{\sigma}^\mu\theta^2 = \mathrm{d}\tau\Pi^\mu_\tau$$

is the 10D Volkov-Akulov 1-form in type IIA superspace.

- $\mathcal{M} = \mathcal{M}\left(\mathcal{H}/\mu^6\right)$ is an arbitrary positively definite function.
- The particular case of this action with

$$\mathcal{M} = \frac{m}{2} + \sqrt{\frac{m^2}{4} + \frac{\mathcal{H}}{\mu^6}}$$

can be obtained by dimensional reduction of the 11D multiple M-wave (multiple M0-branes) action [I. Bandos and U.D.M. Sarraga arXiv:2212.14829].

- Another representative of this system with $\mathcal{M} = m$ was constructed in [I. Bandos JHEP 2018].

Invariance under (target IIA superspace) rigid supersymmetry

- By construction, the mD0-brane action is invariant under SUSY

$$\delta_\epsilon \theta^{1\alpha} = \epsilon^{1\alpha} , \qquad \delta_\epsilon \theta^2_\alpha = \epsilon_\alpha{}^2 , \qquad \delta_\epsilon v^q_\alpha = 0 ,$$

$$\delta_\epsilon x^\mu = i\theta^1 \sigma^\mu \epsilon^1 + i\theta^2 \tilde{\sigma}^\mu \epsilon^2$$

with constant fermionic parameters $\epsilon^{1\alpha}$ and ϵ^2_α which acts nontrivially on the center of mass variables only.

Invariance under local worldline supersymmetry which

- acts on the center of mass variables as κ-symmetry of single D0-brane

$$\delta_\kappa \theta^{1\alpha} = \kappa^q v^\alpha_q / \sqrt{2} , \qquad \delta_\kappa \theta^2_\alpha = -\kappa^q v_\alpha{}^q / \sqrt{2} ,$$

$$\delta_\kappa x^\mu = i\delta_\kappa \theta^1 \sigma^\mu \theta^1 + i\delta_\kappa \theta^2 \tilde{\sigma}^\mu \theta^2 ,$$

$$\delta_\kappa v^q_\alpha = 0 \;\Rightarrow\; \delta_\kappa u^0_\mu = 0 = \delta_\kappa u^i_\mu .$$

The local worldline supersymmetry of the matrix matter fields

$$\delta_\kappa \mathbb{X}^i = \frac{4i}{\sqrt{\mathcal{M}}}\,\kappa\gamma^i \boldsymbol{\Psi} + \frac{1}{\mu^6}\,\frac{\mathcal{M}'}{\mathcal{M}}\,\delta_\kappa \mathcal{H}\,\mathbb{X}^i - \frac{1}{\mu^6}\,\frac{\mathcal{M}'}{\mathcal{M}}\,\Delta_\kappa \mathcal{K}\,\mathbb{P}^i\ ,$$

$$\delta_\kappa \mathbb{P}^i = -\frac{1}{\sqrt{\mathcal{M}}}\,\left[\kappa\gamma^{ij}\boldsymbol{\Psi}, \mathbb{X}^j\right] - \frac{1}{\mu^6}\,\frac{\mathcal{M}'}{\mathcal{M}}\,\delta_\kappa \mathcal{H}\,\mathbb{P}^i + \frac{1}{\mu^6}\,\frac{\mathcal{M}'}{\mathcal{M}}\Delta_\kappa \mathcal{K}\left(\tfrac{1}{16}\left[\left[\mathbb{X}^i, \mathbb{X}^j\right], \mathbb{X}^j\right] - \right.$$
$$\left. -\gamma^i_{pq}\{\boldsymbol{\Psi}_p, \boldsymbol{\Psi}_q\}\right),$$

$$\delta_\kappa \boldsymbol{\Psi}_q = -\frac{1}{2\sqrt{\mathcal{M}}}\,(\kappa\gamma^i)_q \mathbb{P}^i - \frac{i}{16\sqrt{\mathcal{M}}}\,(\kappa\gamma^{ij})_q\,\left[\mathbb{X}^i, \mathbb{X}^j\right] - \frac{i}{4\mu^6}\,\frac{\mathcal{M}'}{\mathcal{M}}\,\Delta_\kappa \mathcal{K}\,\left[(\gamma^i\boldsymbol{\Psi})_q, \mathbb{X}^i\right]\ ,$$

where

$$\delta_\kappa \mathcal{H} = \frac{1}{\sqrt{\mathcal{M}}}\,\frac{\mathrm{tr}\left(\kappa^q \boldsymbol{\Psi}_q\left(\left[\mathbb{X}^i, \mathbb{P}^i\right] - 4i\{\boldsymbol{\Psi}_q, \boldsymbol{\Psi}_q\}\right)\right)}{1 + \frac{1}{\mu^6}\,\frac{\mathcal{M}'}{\mathcal{M}}\,\mathfrak{H}}$$

and

$$\Delta_\kappa \mathcal{K} = \frac{1}{2\sqrt{\mathcal{M}}}\,\frac{\mathrm{tr}\left(4i(\kappa\gamma^i\boldsymbol{\Psi})\mathbb{P}^i + \frac{5}{2}(\kappa\gamma^{ij}\boldsymbol{\Psi})[\mathbb{X}^i, \mathbb{X}^j]\right)}{1 + \frac{1}{\mu^6}\,\frac{\mathcal{M}'}{\mathcal{M}}\,\mathfrak{H}}$$

with

$$\mathfrak{H} := \mathrm{tr}\left(\mathbb{P}^i\mathbb{P}^i\right) + \frac{1}{16}\mathrm{tr}\left[\mathbb{X}^i, \mathbb{X}^j\right]^2 + 2\,\mathrm{tr}\left(\mathbb{X}^i\,\boldsymbol{\Psi}\gamma^i\boldsymbol{\Psi}\right)\ .$$

- From variation with respect to coordinate functions

$$u^0_\mu \mathrm{d}u^{i\mu}\left(m+\frac{2}{\mu^6}\frac{\mathcal{H}}{\mathcal{M}}\right)=0 \implies \boxed{\Omega^i := u^0_\mu \mathrm{d}u^{i\mu}=0} \text{ when } \mathcal{M}\neq -\frac{2}{m}\frac{\mathcal{H}}{\mu^6}\ ,$$

$$\left(m+\frac{1}{\mu^6}\frac{\mathcal{H}}{\mathcal{M}}\right)\left(E^{1q}+E^2_q\right)=\frac{-i}{4\sqrt{2\mathcal{M}}\mu^6}\gamma^i_{qp} i\nu_p\Omega^i\ .$$

The first result implies that the latter gives $\boxed{E^{1q}+E^2_q=0\ .}$

- From the (spinor) moving frame variation

$$\mu^6 E^i\left(m+\frac{2}{\mu^6}\frac{\mathcal{H}}{\mathcal{M}}\right)-\frac{1}{2\sqrt{2\mathcal{M}}}\left(E^{1q}+E^2_q\right)\gamma^i_{qp} i\nu_p - 2\mathrm{tr}\left(\mathbb{P}^{[j}\,\mathbb{X}^{k]}+i\Psi\gamma^{jk}\Psi\right)\Omega^i=0$$

and with the above results, we obtain

$$\boxed{E^i=\Pi^\mu u^i_\mu=0\ .}$$

- These are the same e.o.m. as for the single D0-brane.
- Equations of motion for the matrix fields are complicated, but can be simplified by gauge fixing.

SO(9) and SU(N) gauge fixing

- As we are dealing with d $= 1$ field theory, gauge fields can be always gauged away. So, we can fix the local SO(9) gauge symmetry by

$$\Omega^{ij} = u^i_\mu \mathrm{d}u^{j\mu} = 0$$

which (together with $\Omega^i := u^0_\mu u^{i\mu} = 0$) implies

$$\mathrm{d}u^0_\mu = 0 \ , \qquad \mathrm{d}u^i_\mu = 0 \ , \qquad \mathrm{d}v^q_\alpha = 0 \ , \qquad \mathrm{d}v_q^\alpha = 0 \ .$$

- Similarly, using the SU(N) gauge symmetry we can set

$$\mathbb{A} = 0 \ .$$

- In this gauge the covariant derivatives $\mathrm{D} = \mathrm{d}\tau \mathrm{D}_\tau$ reduces to $\mathrm{d} = \mathrm{d}\tau \mathrm{d}_\tau = \mathrm{d}\tau \frac{\mathrm{d}}{\mathrm{d}\tau}$.

Gauge fixing of κ-symmetry

- Under rigid supersymmetry and worldline supersymmetry

$$\delta\theta^{1q} = \epsilon^{1q} + \kappa^q/\sqrt{2} \ , \qquad \delta\theta^2_q = \epsilon^2_q - \kappa^q/\sqrt{2} \ .$$

- The gauge worldline SUSY (κ-symmetry) can be used to fix the gauge

$$\theta^2_q = 0 \quad \Longrightarrow \quad \epsilon^2_q = \kappa^q/\sqrt{2} \ .$$

- Then, from $E^{1q} + E^2_q = 0$ (with $\mathrm{d}v^q_\alpha = 0 = \mathrm{d}v^q_\alpha$), we obtain $\mathrm{d}\theta^{1q} = 0$ and

$$E^0 = \mathrm{d}x^\mu u^0_\mu = \mathrm{d}x^0 \ , \qquad E^i = \mathrm{d}x^\mu u^i_\mu = \mathrm{d}x^i \ .$$

- E^0 and E^i are supersymmetric since now

$$\delta x^0 = i(\epsilon^{1q} + \epsilon^2_q)\theta^{1q} \ , \qquad \delta x^i = i(\epsilon^{1q} + \epsilon^2_q)\gamma_{qp}\theta^{1p} \ ,$$

and these terms are constants due to $\mathrm{d}v^q_\alpha = 0$, $\mathrm{d}\theta^{1q} = 0$ and $\mathrm{d}\epsilon = 0$.

Based on all above...

- We can fix the gauge with respect to the reparametrization invariance by setting

$$E^0 = \mathrm{d}x^0 = \mathrm{d}\tau \quad \Longrightarrow \quad E^0_\tau = \dot{x}^0 = 1$$ and it still preserves supersymmetry.

Gauged fixed form of the field equations

- Taking into account center of mass equations and the gauge fixed conditions

$$\Omega^i = 0 \ , \quad E^i = 0 \ , \quad \mathrm{d}u^0_\mu = 0 \ , \quad \mathrm{d}u^i_\mu = 0 \ , \quad \mathrm{d}v^q_\alpha = 0 \ , \quad \theta^2 = 0.$$

the equations for the bosonic matrix fields read

$$\dot{\mathbb{X}}^i = -\frac{2}{\mathcal{M}} \frac{\left(1 - \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathcal{H}\right)}{\left(1 + \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathfrak{H}\right)} \dot{x}^0 \mathbb{P}^i \ , \quad \dot{\mathbb{P}}^i = \frac{2}{\mathcal{M}} \frac{\left(1 - \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathcal{H}\right)}{\left(1 + \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathfrak{H}\right)} \dot{x}^0 \left(\frac{1}{16}[[\mathbb{X}^i, \mathbb{X}^j], \mathbb{X}^j] - \gamma^i_{pr}\{\mathbf{\Psi}_p, \mathbf{\Psi}_r\} \right),$$

and for the fermionic matrix field

$$\dot{\mathbf{\Psi}}_q = -\frac{i}{2\mathcal{M}} \frac{\left(1 - \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathcal{H}\right)}{\left(1 + \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathfrak{H}\right)} \dot{x}^0 [(\gamma^i \mathbf{\Psi})_q, \mathbb{X}^i] \ .$$

- An important observation is that if we formally define new (field dependent) time variable by

$$\mathsf{d}t = \mathsf{d}x^0 \, \frac{2}{\mathcal{M}} \frac{\left(1 - \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathcal{H}\right)}{\left(1 + \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathfrak{H}\right)} \, ,$$

  the above equations acquire the form

$$\frac{\mathsf{d}}{\mathsf{d}t}\mathbb{X}^i = -\mathbb{P}^i \, , \quad \frac{\mathsf{d}}{\mathsf{d}t}\mathbb{P}^i = \frac{1}{16}[[\mathbb{X}^i, \mathbb{X}^j], \mathbb{X}^j] - \gamma^i_{pr}\{\mathbf{\Psi}_p, \mathbf{\Psi}_r\} \, , \quad \frac{\mathsf{d}}{\mathsf{d}t}\mathbf{\Psi}_q = -\frac{i}{4}\,[(\gamma^i\mathbf{\Psi})_q, \mathbb{X}^i]$$

  which are exactly 1d $\mathcal{N} = 16$ SU(N) SYM equations.

- Let us stress that

$$\mathrm{d}t = \mathrm{d}x^0 \, \frac{2}{\mathcal{M}} \frac{\left(1 - \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathcal{H}\right)}{\left(1 + \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathfrak{H}\right)} \; ,$$

cannot be considered as 1d general coordinate transformation of proper time τ.

The simplest way to be convinced is to notice that

- If it was the gauge symmetry, it would connect the model with any invertible $\mathcal{M}(\mathcal{H})$ to the model with $\mathcal{M}(\mathcal{H}) = 1$ (or $\mathcal{M}(\mathcal{H}) = m$ as in [I. Bandos JHEP 2018].)

$$\dot{\mathbb{X}}^i = -\frac{1}{m}\mathbb{P}^i \; , \qquad \dot{\mathbb{P}}^i = \frac{1}{m}\left(\frac{1}{16}[[\mathbb{X}^i, \mathbb{X}^j], \mathbb{X}^j] - \gamma^i_{pr}\{\mathbf{\Psi}_p, \mathbf{\Psi}_r\}\right) \; .$$

- But, calculating the canonical momentum $p_\mu = \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu}$, we find that $p_\mu p^\mu = \mathfrak{M}^2$ with

$$\mathfrak{M} = m + \frac{2}{\mu^6}\frac{\mathcal{H}}{\mathcal{M}}$$

giving thus the mass of the mD0-system. This is clearly invariant under all gauge symmetries.

- So, this $\mathcal{M}(\mathcal{H})$ determines the physical characteristic (mass) of the system and cannot be changed by any gauge symmetry.
- Thus what we have found is an interesting correspondence between equations of mD0 with different $\mathcal{M}(\mathcal{H})$ but this correspondence does not imply gauge equivalence.

Outline

Conclusions

- The main result of this work is the set of complete nonlinear candidate actions for 10D supersymmetric multiple D0-brane system that includes an arbitrary nonvanishing function $\mathcal{M}(\mathcal{H})$.
- These actions are doubly supersymmetric i.e. it posses spacetime supersymmetry and worldline supersymmetry, the counterpart of κ-symmetry of single D0-brane. Notice that the form of the latter depends on the choice of $\mathcal{M}(\mathcal{H})$ function.
- The presence of an arbitrary positively definite $\mathcal{M}(\mathcal{H})$ in our candidate mD0 action is to be understood better.
- It contains physical information as it enters the expression for the mass of mD0 system.
- A particular model with $\mathcal{M}(\mathcal{H}) = \frac{m}{2} + \sqrt{\frac{m^2}{4} + \frac{\mathcal{H}}{\mu^6}}$ is obtained by dimensional reduction of mM0 system.
- We have found an interesting formal correspondence of the equations of mD0 system with any $\mathcal{M}(\mathcal{H})$ with SYM equations.

- It does not imply gauge equivalence with SYM (since $\mathcal{M}(\mathcal{H})$ defines mass of mD0 system).
- However it can be used to relate (some) solutions of mD0 and SYM equations.
- In particular, all the SUSY solutions of mD0 equations are SUSY solutions of SYM.

Outlook

- One of the ways to clarify what member of our family of candidate action is preferable for description of mD0 (if it is unique) is to check whether T-duality relates it to mD1 action.
- Such a check requires to construct mD1 action, the problem which we are studying presently.

The end!

Thank you for your attention!

Appendix I: single D0-brane and its κ-symmetry

10D D0-brane in flat type IIA superspace in the moving frame formulation

$$S_{\text{D0}} = m \int_{\mathcal{W}^1} E^0 - im \int_{\mathcal{W}^1} \left(\mathrm{d}\theta^{1\alpha} \theta^2_\alpha - \theta^{1\alpha} \mathrm{d}\theta^2_\alpha \right) \ ,$$

where $\mathrm{d} = \mathrm{d}\tau \partial_\tau$ and E^0 is the contraction

$$E^0 = \Pi^\mu u^0_\mu \ , \qquad \left[E^i = \Pi^\mu u^i_\mu \right]$$

of the pull-back to the worldline of the 10D Volkov-Akulov 1-form

$$\Pi^\mu = \mathrm{d}x^\mu - i\mathrm{d}\theta^1 \sigma^\mu \theta^1 - i\mathrm{d}\theta^2 \tilde{\sigma}^\mu \theta^2 = \mathrm{d}\tau \Pi^\mu_\tau$$

This is the first order form of the 10D massive superparticle action [de Azcarraga-Lukierski 1982 for $D = 4$]

$$S = m \int_{\mathcal{W}^1} \sqrt{\Pi^\mu \Pi_\mu} - im \int_{\mathcal{W}^1} (\mathrm{d}\theta^1 \theta^2 - \theta^1 \mathrm{d}\theta^2)$$

but we strongly need spinor frame to write the action for mD0.

κ-symmetry of single D0-brane

$$\delta_\kappa \theta^{1\alpha} = \kappa^q v_q^\alpha \ , \qquad \delta_\kappa \theta^2_\alpha = -\kappa^q v_\alpha^q \ ,$$

$$\delta_\kappa v_\alpha^q = 0 \Longrightarrow \delta_\kappa u_\mu^i = \delta_\kappa u_\mu^0 = 0 \ ,$$

$$\delta_\kappa x^\mu = i\delta_\kappa \theta^{1\alpha} \sigma^\mu_{\alpha\beta} \theta^{1\alpha} + i\delta_\kappa \theta^2 \tilde{\sigma}^{\mu\alpha\beta} \theta^2_\beta \ ,$$

which is parametrized by fermionic function $\kappa^q = \kappa^q(\tau)$ carrying spinor index of SO(9).

The local fermionic κ-symmetry implies

- that the ground state is invariant under a part $(1/2)$ of the spacetime supersymmetry.
- that one of the two spinor fermionic coordinate functions can be removed (gauge fixing).

Appendix II: Covariant derivatives and variations

- Derivatives of the moving frame are given by

$$\mathrm{D}u_\mu^0 := \mathrm{d}u_\mu^0 = u_\mu^i \Omega^i \ , \qquad \mathrm{D}u_\mu^i := \mathrm{d}u_\mu^i + u_\mu^j \Omega^{ji} = u_\mu^0 \Omega^i \ .$$

- Derivatives of the spinor moving frame are given by

$$\mathrm{D}v_\alpha^q := \mathrm{d}v_\alpha^q + \frac{1}{4}\Omega^{ij} v_\alpha^p \gamma_{pq}^{ij} = \frac{1}{2}\gamma_{qp}^i v_\alpha^p \Omega^i \ ,$$

$$\mathrm{D}v_q^\alpha := \mathrm{d}v_q^\alpha + \frac{1}{4}\Omega^{ij} v_p^\alpha \gamma_{pq}^{ij} = -\frac{1}{2}\gamma_{qp}^i v_p^\alpha \Omega^i \ .$$

- They are expressed in terms of Cartan forms

$$\Omega^i = u_\mu^0 \mathrm{d}u^{\mu i} \ , \qquad \Omega^{ij} = u_\mu^i \mathrm{d}u^{\mu j} \ ,$$

whose derivatives have the forms (Maurer-Cartan equations)

$$\mathrm{D}\Omega^i = \mathrm{d}\Omega^i + \Omega^j \wedge \Omega^{ji} = 0 \ , \qquad \mathrm{d}\Omega^{ij} + \Omega^{ik} \wedge \Omega^{kj} = -\Omega^i \wedge \Omega^j \ .$$

- The variation of the moving frame and spinor moving frame are given by

$$\delta u_\mu^0 = u_\mu^i i_\delta \Omega^i \ , \qquad \delta u_\mu^i = u_\mu^0 i_\delta \Omega^i \ ,$$

$$\delta v_\alpha^q = \frac{1}{2}\gamma_{qp}^i v_\alpha^p i_\delta \Omega^i \ , \qquad \delta v_q^\alpha = -\frac{1}{2}\gamma_{qp}^i v_p^\alpha i_\delta \Omega^i \ .$$

Appendix III: e.o.m from matrix matter fields variation

$$\mathsf{D}\mathbb{X}^i = -\frac{2}{\mathcal{M}}\left(1-\frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathcal{H}\right)E^0\mathbb{P}^i + \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\left(\mathbb{X}^i\mathsf{d}\mathcal{H} - \mathbb{P}^i\mathsf{d}\mathcal{K}\right) + \\ + \frac{1}{\sqrt{2\mathcal{M}}}\left(E^{1q}-E^2_q\right)\left(4i(\gamma^i\mathbf{\Psi})_q - \frac{1}{2\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}i\nu_q\mathbb{P}^i\right) \ ,$$

$$\mathsf{D}\mathbb{P}^i = \frac{2}{\mathcal{M}}\left[\left(1-\frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathcal{H}\right)E^0 + \frac{1}{\mu^6}\frac{\mathcal{M}'}{4\sqrt{2\mathcal{M}}}\left(E^{1q}-E^2_q\right)i\nu_q\right]\times \\ \times\left(\frac{1}{16}\left[[\mathbb{X}^i,\mathbb{X}^j],\mathbb{X}^j\right] - \gamma^i_{pr}\left\{\mathbf{\Psi}_p,\mathbf{\Psi}_r\right\}\right)\frac{1}{\sqrt{2\mathcal{M}}}\left(E^{1q}-E^2_q\right)[(\gamma^{ij}\mathbf{\Psi})_q,\mathbb{X}^j] + \\ + \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathsf{d}\mathcal{K}\left(\frac{1}{16}\left[[\mathbb{X}^i,\mathbb{X}^j],\mathbb{X}^j\right] - \gamma^i_{pr}\left\{\mathbf{\Psi}_p,\mathbf{\Psi}_r\right\}\right) - \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathbb{P}^i\mathsf{d}\mathcal{H} \ ,$$

$$\mathsf{D}\mathbf{\Psi}_q = -\frac{i}{2\mathcal{M}}\left[\left(1-\frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathcal{H}\right)E^0 + \frac{1}{4\mu^6}\frac{\mathcal{M}'}{\sqrt{2\mathcal{M}}}\left(E^{1p}-E^2_p\right)i\nu_p\right][(\gamma^i\mathbf{\Psi})_q,\mathbb{X}^i] - \\ - \frac{i}{4\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}[(\gamma^i\mathbf{\Psi})_q,\mathbb{X}^i]\mathsf{d}\mathcal{K} - \frac{1}{2\sqrt{2\mathcal{M}}}\left(E^{1p}-E^2_p\right)\left(\gamma^i_{pq}\mathbb{P}^i + \frac{i}{8}\gamma^{ij}_{pq}[\mathbb{X}^i,\mathbb{X}^j]\right) \ .$$

- Equations of motion for the matrix fields are complicated, but can be simplified by gauge fixing.
- Before describing this, let us notice that they imply

$$\boxed{\mathrm{d}\mathcal{H} = 0} \quad \text{and} \quad \boxed{i\mathrm{D}\nu_q = \frac{2\sqrt{2}}{\sqrt{\mathcal{M}}}(E^{1q} - E^2_q)\mathcal{H}\ .}$$

where

$$i\nu_q := \mathrm{tr}\left(-4i(\gamma^i\boldsymbol{\Psi})_q\mathbb{P}^i + \frac{1}{2}(\gamma^{ij}\boldsymbol{\Psi})_q[\mathbb{X}^i, \mathbb{X}^j]\right)$$

- These are Noether identities for gauge symmetries: reparametrization invariance (1d general coordinate invariance) and local worldline SUSY.

Appendix IV: other expressions

- The term $\mathcal{K}$ express the combination

$$\mathcal{K} = \mathsf{tr}\left(\mathbb{X}^i \mathbb{P}^i\right) \ .$$

- Its differential form reads

$$\mathrm{d}\mathcal{K} = -\frac{2}{\mathcal{M}} \frac{\mathfrak{H}}{\left(1 + \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathfrak{H}\right)} \left(1 - \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathcal{H}\right) E^0 +$$

$$+ \frac{(E^{1q} - E^2_q)}{\sqrt{2\mathcal{M}}} \frac{\mathrm{tr}\left(4i(\gamma^i \mathbf{\Psi})_q \mathbb{P}^i + (\gamma^{ij}\mathbf{\Psi})_q [\mathbb{X}^i, \mathbb{X}^j]\right) - \frac{1}{2\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathfrak{H}\, i\nu_q}{\left(1 + \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathfrak{H}\right)} .$$

- Its worldline supersymmetry variation of $\mathcal{K} = \mathsf{tr}\left(\mathbb{X}^i \mathbb{P}^i\right)$ by

$$\Delta_\kappa \mathcal{K} = \delta_\kappa(\mathrm{tr}(\mathbb{X}^i\, \mathbb{P}^i)) + \frac{1}{2\sqrt{\mathcal{M}}} i\kappa^q \nu_q \ .$$