T-duality building blocks in stringy corrections

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Iberian Strings
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based on
2210.16593, 2108.04370 [MD, James Liu]

Outline

- Introduction and Motivation
- Cosmological and D-dimensional Reduction
- Setup of T-duality building blocks
- Example - H^2R^3 scattering amplitudes
- Concluding Remarks

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  - learn about five point functions with the new building blocks

Torus reduction - a review

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- closed string NSNS fields and the tree-level effective action
- massless fields: (G_{AB}, B_{AB}, ϕ)

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- reduction on T^d proceeds by making the ansatz

$$\begin{aligned}
ds_{10}^2 &= g_{\alpha\beta}dx^\alpha dx^\beta + g_{ij}(dy^i + A^i_\alpha dx^\alpha)(dy^j + A^j_\beta dx^\beta),\\
B &= \tfrac{1}{2}\hat{B}_{\alpha\beta}dx^\alpha \wedge dx^\beta + B_{\alpha i}dx^\alpha \wedge (dy^i + A^i_\beta dx^\beta)\\
&\quad + \tfrac{1}{2}b_{ij}(dy^i + A^i_\alpha dx^\alpha) \wedge (dy^j + A^j_\beta dx^\beta),\\
\phi &= \tfrac{1}{2}\Phi + \tfrac{1}{4}\log\det g_{ij},
\end{aligned}$$

Torus Reduction - A Review

- Torus-reduced 2∂ Lagrangian

$$\mathcal{L}_{10-d} = e^{-\Phi}\left(R + \partial_\alpha\Phi\partial^\alpha\Phi - \frac{1}{12}\hat{H}_{\alpha\beta\gamma}\hat{H}^{\alpha\beta\gamma} + \frac{1}{8}\,\mathrm{Tr}(\partial_\alpha\mathcal{H}\eta\partial^\alpha\mathcal{H}\eta)\right)$$

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- $\mathcal{H}$ is the $(2d)\times(2d)$ scalar matrix

$$\mathcal{H} = \begin{pmatrix} g^{-1} & -g^{-1}b \\ bg^{-1} & g - bg^{-1}b \end{pmatrix}, \qquad \eta = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}, \qquad \mathcal{S} \equiv \eta\mathcal{H}$$

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- $\mathcal{H}$ is the generalized metric taking values in $O(d,d)$ and η is the $O(d,d)$ invariant metric
- 2∂ Lagrangian is invariant under $O(d,d)$ T-duality transformations of the form

$$\mathcal{S} \to \mathfrak{g}^{-1}\mathcal{S}\mathfrak{g}, \qquad \text{for} \quad \mathfrak{g}^T\eta\mathfrak{g} = \eta$$

Torsionful connection

- torsionful connection

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$$\Omega_\pm = \omega \pm \tfrac{1}{2} H = \omega^{AB} \pm \tfrac{1}{2} H_M{}^{AB} dx^M,$$

- Riemann tensor

$$R_{MN}{}^{PQ}\,(\Omega_\pm) = R_{MN}{}^{PQ}(\Omega) \pm \nabla_{[M} H_{N]}{}^{PQ} + \frac{1}{2} H_{[M}{}^{PR} H_{N]R}{}^{Q}.$$

The T-duality building blocks

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- One advantage of introducing $N_{\pm}$ is that the T-duality invariant traces can be compactly written as

$$\mathrm{Tr}(\dot{\mathcal{S}}^{2n}) = 2(-1)^n\,\mathrm{Tr}((N_+N_-)^n), \qquad (n \geq 0).$$

- sufficient to check if the action can be written in this form

The T-duality building blocks

- invariants are formed by taking traces of products of $\mathcal{S}$ and its derivatives

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$$\mathcal{S}^2 = 1,$$

- there can be at most one $\mathcal{S}$ separating derivative terms inside the trace
- form the projections

$$P_{\pm} = \frac{1}{2}(1 \pm \mathcal{S}).$$

- n derivatives acting on $\mathcal{S}$

Single Derivatives

- characterize single trace invariants formed out of $\mathcal{S}$ and $\partial_\mu \mathcal{S}$.
- single derivatives: only two inequivalent possibilities

$$\begin{aligned}\mathrm{Tr}(\partial_{\mu_1}\mathcal{S}\partial_{\mu_2}\mathcal{S}\cdots\partial_{\mu_{2n}}\mathcal{S}) = (-1)^n[&\mathrm{Tr}(N_{\mu_1 -}N_{\mu_2 +}\cdots N_{\mu_{2n}+}) \\ &+ \mathrm{Tr}(N_{\mu_1 +}N_{\mu_2 -}\cdots N_{\mu_{2n}-})].\end{aligned}$$

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Second Derivatives

second derivatives on $\mathcal{S}$

- the invariants explicitly break the alternating signature of N [MD, Liu, 2210.16593]

$$\mathcal{S} = W \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} W^{-1},$$

$$\partial_\mu \mathcal{S} = W \begin{pmatrix} 0 & N_{\mu-} \\ -N_{\mu+} & 0 \end{pmatrix} W^{-1},$$

$$\nabla_\mu \nabla_\nu \mathcal{S} = W \begin{pmatrix} N_{(\mu-} N_{\nu)+} & \nabla_{(\mu} N_{\nu)-} + Y_{\mu\nu-} \\ -\nabla_{(\mu} N_{\nu)+} - Y_{\mu\nu+} & -N_{(\mu+} N_{\nu)-} \end{pmatrix} W^{-1},$$

$$Y_{\mu\nu\pm} = \frac{1}{2}(N_{(\mu\mp} N_{\nu)\pm} - N_{(\mu\pm} N_{\nu)\pm})$$

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- build invariants by multiplying these quantities together and taking the trace
- the diagonal terms follow the alternating pattern
- $Y_{\mu\nu\pm}$ breaks the even/odd of diagonal/off-diagonal entries

The Algorithm

- breaking of the alternating pattern can also be seen from the Riemann tensor

$$R_{\rho\sigma}{}^{\mu\nu}(\Omega_\pm) = R_{\rho\sigma}{}^{\mu\nu}(\omega_\pm),$$
$$R_{ij}{}^{\mu\nu}(\Omega_\pm) = -\tfrac{1}{4}(g_{ik}N_{\pm}^{\mu k}{}_l N_{\mp}^{\nu l}{}_j - g_{jk}N_{\pm}^{\mu k}{}_l N_{\mp}^{\nu l}{}_i),$$
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- our steps to ensure T-duality

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5. find constraints on the higher derivative couplings

Some Examples:
H^2R^3 Scattering Amplitudes

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- reexamine tree-level 8∂ terms in the type II effective action
  - reformulate via our building blocks
  - consider each order of H separately (order H^{2n} does not affect the counterterms introduced at $\mathcal{O}(H^{2n-2})$)
- [Liu, Minasian, 1912.10974]

$$\mathcal{L}_{\text{tree}} \sim \underbrace{R^4}_{\mathcal{O}(H^0)} + \underbrace{H^2R^3}_{\mathcal{O}(H^2)} + \cdots$$

- focus on the cosmological reduction and utilize the algorithm

T-duality at $\mathcal{O}(H^0)$

- the cosmological reduction [Meissner, 9610131, Godazgar, Godazgar, 1306.4918, Hohm, Zwiebach, 1510.00005]
- pure gravity sector

$$\mathcal{L}_{R^4} \sim t_8 t_8 R^4 - \frac{1}{4}\epsilon_8\epsilon_8 R^4$$

$$\mathcal{L}_{R^4} \to 3\,\mathrm{Tr}(\dot{\mathcal{S}})^4 - \tfrac{9}{2}\,\mathrm{Tr}(\dot{\mathcal{S}}^8) - \tfrac{1}{64}(\mathrm{Tr}\,L)^8 - \mathrm{Tr}\left(L^3\right)(\mathrm{Tr}\,L)^5 + \tfrac{45}{16}\,\mathrm{Tr}\left(L^4\right)(\mathrm{Tr}\,L)^4$$
$$- 6\,\mathrm{Tr}\left(L^5\right)(\mathrm{Tr}\,L)^3 - \tfrac{5}{2}\left(\left(\mathrm{Tr}\left(L^3\right)\right)^2 - 3\,\mathrm{Tr}\left(L^6\right)\right)(\mathrm{Tr}\,L)^2$$

- remaining terms can be rewritten in terms of the dilaton

T-duality at $\mathcal{O}(H^2)$

- by matching with the tree-level string five-point amplitude [Liu, Minasian, 1912.10974]:

$$\mathcal{L}_{H^2R(\Omega_+)^3} \sim -2t_8t_8H^2R\left(\Omega_+\right)^3 - \frac{1}{6}\epsilon_9\epsilon_9H^2R\left(\Omega_+\right)^3 + 8\cdot 4!\sum_i^8 d_i H^{\mu\nu\lambda}H^{\rho\sigma\zeta}\tilde{Q}^i_{\mu\nu\lambda\rho\sigma\zeta} + \cdots ,$$

- recover four-dimensional supersymmetry based on Calabi-Yau compactification [Grimm, Mayer, Weissenbacher, 1702.08404]

$$t_8t_8H^2R^3 \equiv t_{8\mu_1\cdots\mu_8}t_8^{\nu_1\cdots\nu_8}H^{\mu_1\mu_2\alpha}H_{\nu_1\nu_2\alpha}R^{\mu_3\mu_4}{}_{\nu_3\nu_4}R^{\mu_5\mu_6}{}_{\nu_5\nu_6}R^{\mu_7\mu_8}{}_{\nu_7\nu_8}$$

- six-index combinations

$$\begin{aligned}
\tilde{Q}^1_{\mu\nu\lambda\alpha\beta\gamma} &= R_{\mu\alpha a}{}^{b}R_{\nu\beta b}{}^{c}R_{\lambda c\gamma}{}^{a}, & \tilde{Q}^5_{\mu\nu\lambda\alpha\beta\gamma} &= R_{\mu abc}R_{\nu\alpha}{}^{bc}R_{\lambda\beta\gamma}{}^{a},\\
\tilde{Q}^2_{\mu\nu\lambda\alpha\beta\gamma} &= R_{\mu\nu a}{}^{b}R_{\alpha\beta b}{}^{c}R_{\lambda c\gamma}{}^{a}, & \tilde{Q}^6_{\mu\nu\lambda\alpha\beta\gamma} &= R_{\mu abc}R_{\alpha\beta}{}^{bc}R_{\nu\lambda\gamma}{}^{a},\\
\tilde{Q}^3_{\mu\nu\lambda\alpha\beta\gamma} &= R_{\mu\nu a}{}^{b}R_{\lambda\alpha b}{}^{c}R_{\beta c\gamma}{}^{a}, & \tilde{Q}^7_{\mu\nu\lambda\alpha\beta\gamma} &= R_{\mu abc}R_{\nu}{}^{a}{}_{\alpha}{}^{c}R_{\lambda\beta\gamma}{}^{b},\\
\tilde{Q}^4_{\mu\nu\lambda\alpha\beta\gamma} &= R_{\mu a\alpha}{}^{b}R_{\nu b\beta}{}^{c}R_{\lambda c\gamma}{}^{a}, & \tilde{Q}^8_{\mu\nu\lambda\alpha\beta\gamma} &= R_{\mu\nu\alpha\beta}R_{\lambda abc}R_{\gamma}{}^{abc}.
\end{aligned}$$

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- follow a cosmological reduction and utilize $N_\pm$
- only introduce complete basis up to $\mathcal{O}(H^2)$ and leave the rest undetermined with terms fixed via the five-point scattering amplitudes
- the couplings are found to be [Liu, Minasian, 1912.10974]

$$\{d_i\} = k\left(1, -\frac{1}{4}, 0, \frac{1}{3}, 1, \frac{1}{4}, -2, \frac{1}{8}\right), \qquad k = 1$$

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$$d_3 = \frac{1}{2}\left(4 - d_1\right),\ d_4 = \frac{4}{3},\ d_6 = \frac{1}{8}\left(-2d_1 - 4d_2 - 2d_5 + 20\right),\ d_7 = -8,\ d_8 = \frac{1}{2}.$$

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- T-duality does not uniquely fix all 8 coefficients $d_i \to$ not sufficient in completely determining the couplings for the five-point function.
- coefficients agree provided $k = 4$ which corrects a normalization error in [Liu, Minasian, 1912.10974]

Conclusion and Outlook

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  - torsionful connection allows us to write $O(d,d)$ invariants as $N_{\pm}$

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