Holographic Floquet states in low dimensions

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Based on:

- A. Garbayo, J. Mas, A. V. Ramallo (2020). [JHEP10(2020)013]
- MB, A. Garbayo, J. Mas, A. V. Ramallo (2022) [JHEP12(2022)020]

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  - Light-induced superconductivity [Fausti et al 2011, Mitrano et al 2016]
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  - Artificial Weyl semimetals [Zhang et al 2016, Hübener et al 2017, Bucciantini et al 2017]

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  - Artificial Weyl semimetals [Zhang et al 2016, Hübener et al 2017, Bucciantini et al 2017]
- We study: Strongly-coupled (2+1)-dimensional gauge theory + external rotating electric field:

$$\mathcal{E} = \mathcal{E}_x + i\mathcal{E}_y = E e^{i\Omega t}$$

- For the 3+1 case, see: [Hashimoto et al 2017, Kinoshita et al 2017]

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Intersection of N_c D3 branes and N_f D5 branes along (2+1) dimensions:

	1	2	3	4	5	6	7	8	9
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D5:	x	x	-	x	x	x	-	-	-

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We choose $N_f \ll N_c \quad \Rightarrow$ No backreaction. The D5-brane is treated as a probe in the $AdS_5 \times \mathbb{S}^5$ geometry
DBI action:

$$S = -N_f T_5 \int d^6\xi \sqrt{-\det\left(g_6 + 2\pi\alpha' \mathcal{F}\right)} \qquad\qquad \mathcal{F} = d\mathcal{A}$$

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- Worldvolume gauge field with one-form potential

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- Lagrangian density:

$$\mathcal{L} = \sqrt{(1-\psi^2)\left[(\rho^4 g^2 - \Omega^2 b^2)\left((1-\psi^2)b'^2 + h(1-\psi^2+\rho^2\psi'^2)\right) + (1-\psi^2)\rho^4 g^2 b^2 \chi'^2\right]}$$

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- UV boundary behavior

$$c(\rho) = b(\rho) e^{i\chi(\rho)} = \frac{iE}{\Omega} + \frac{j}{\rho^2} + \dots, \qquad \psi(\rho) = \frac{m}{\rho} + \frac{\mathcal{C}}{\rho^2} + \dots$$

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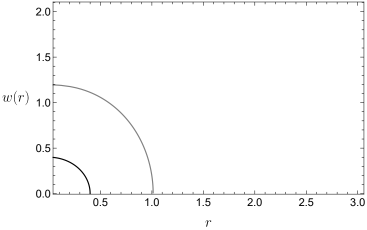
$$b_0 = \frac{\rho_c^4 - \rho_h^4}{\Omega \rho_c^2} \qquad b_0 = b(\rho = \rho_c)$$

- $\rho = \rho_c \rightarrow$ pseudohorizon (of the effective metric). Boundary conditions for the integration imposed here [A. Karch, A. O'Bannon 2007]

Types of embeddings

Three possibilities:



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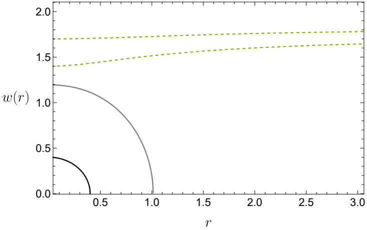

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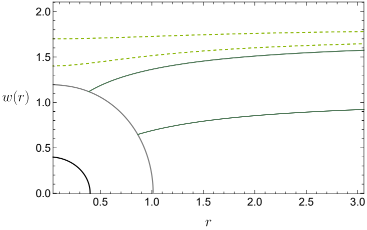

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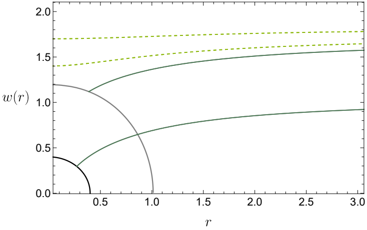

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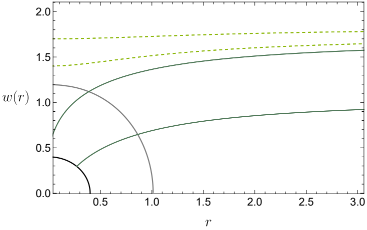

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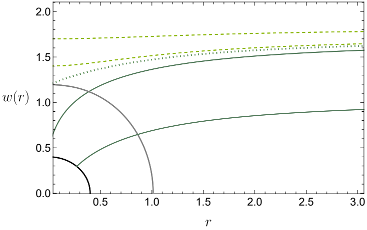

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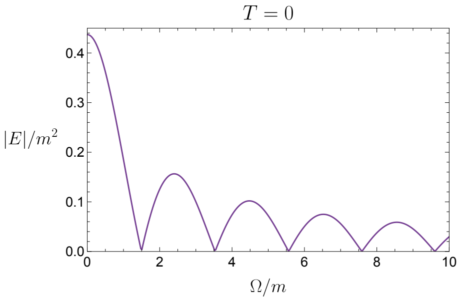
- The Joule heating plays the role of an order parameter for the transition.

Phase diagram (critical embeddings)



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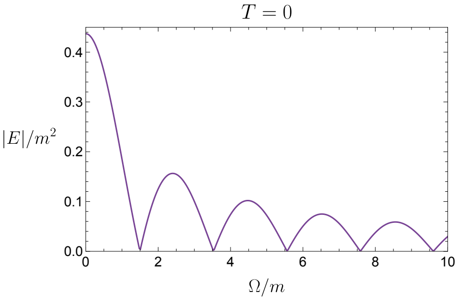

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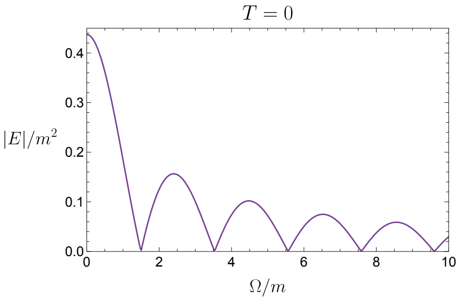

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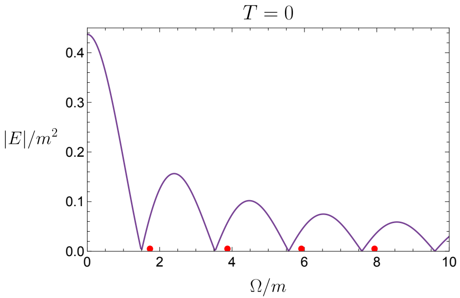

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- $\Omega_{meson}/m = 2\sqrt{\left(n+\frac{1}{2}\right)\left(n+\frac{3}{2}\right)} = 1.7320,\ 3.8730,\ 5.9161,\ 7.9372$
  [D. Arean, A. V. Ramallo 2006] For the 3+1 case see [Mateos et al 2003]

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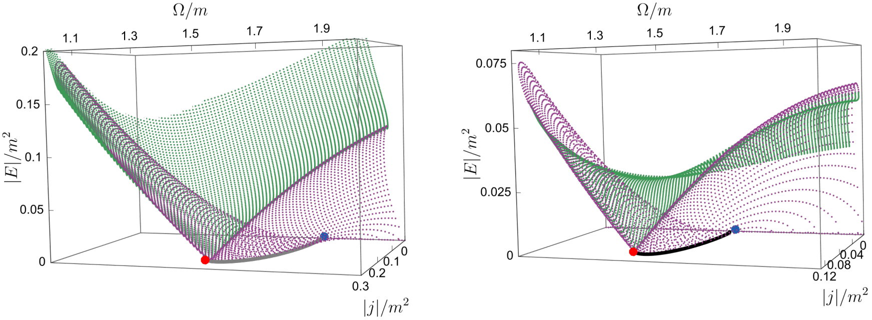

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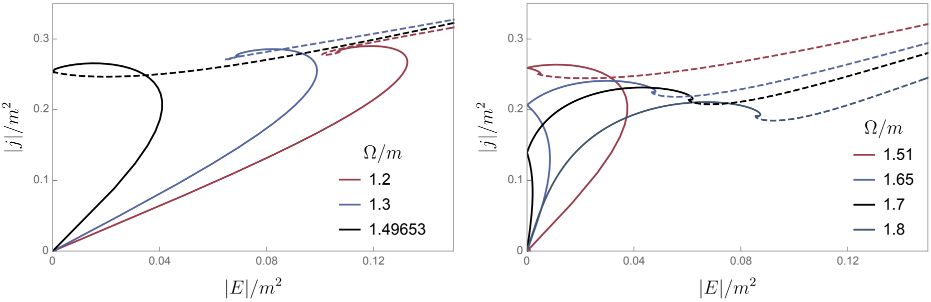
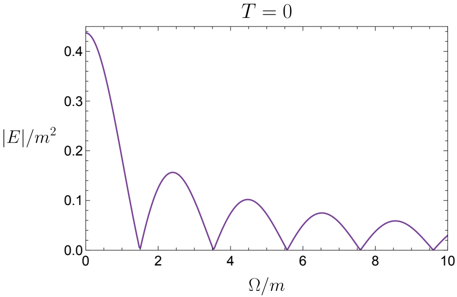
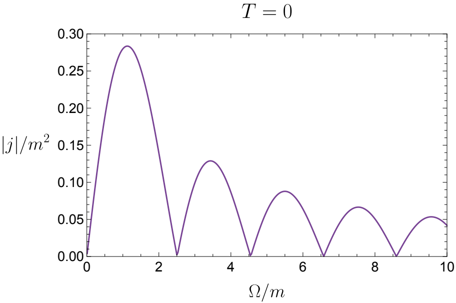


Floquet supression points





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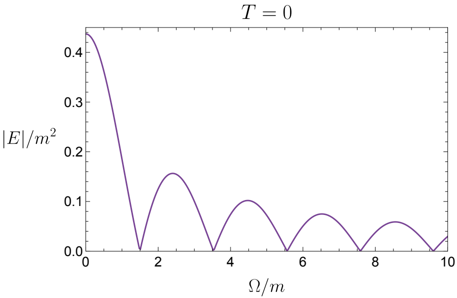
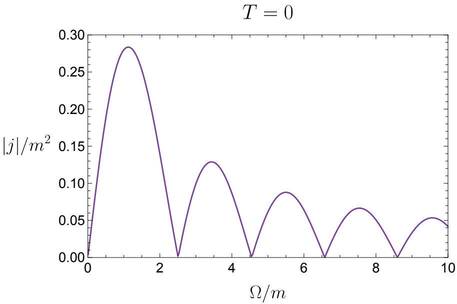
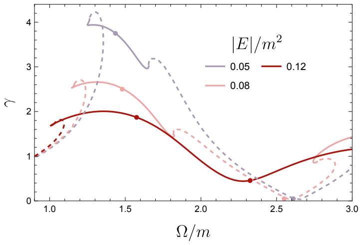


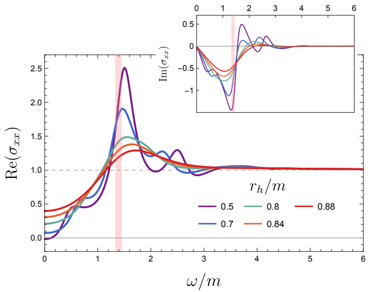


$\sim$ Dynamical localization in driven lattices

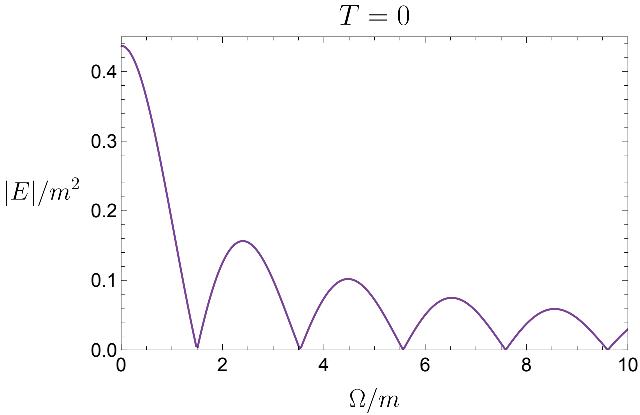
Other results

- Analytic solutions:
  - Massless case ($T = 0$ and $T \neq 0$)
  - Small mass solutions ($T = 0$ and $T \neq 0$)
  - Large frequency ($T = 0$)
- Angle between $\vec{j}$ and $\vec{E}$ (non-linear conductivity)
- Photovoltaic optical conductivity





Effect of the temperature



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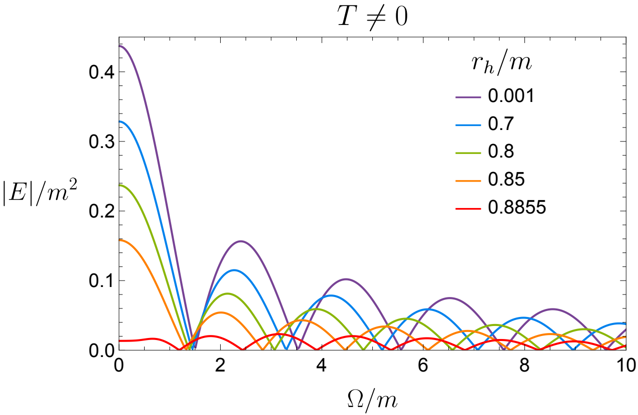

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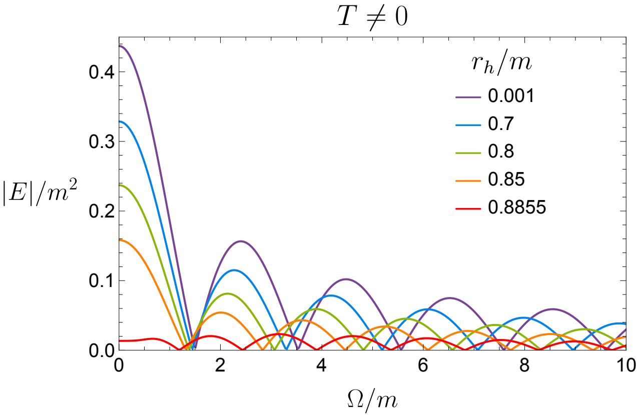
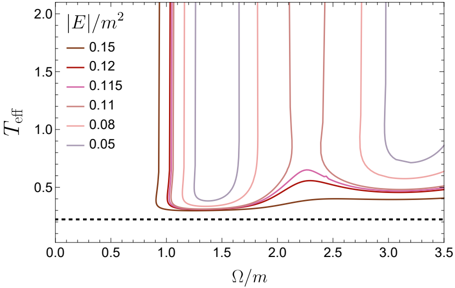
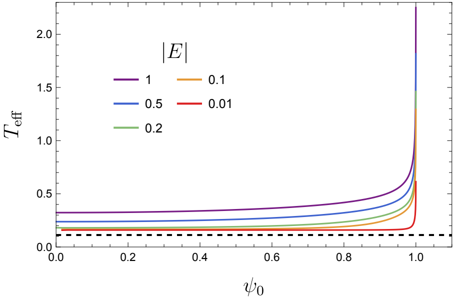


→ Smaller lobes as we increase T

Effective temperature

$$T_{eff} = \frac{2\rho_c h(\rho_c) - \Omega b'(\rho_c)}{2\pi b(\rho_c)\chi'(\rho_c)}$$





Future directions

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- Clarify the nature of the phase transition (Schwinger-Keldysh approach for open systems?)

Thank you for your attention!

Holographic setup

- Background 10d metric. $\mathrm{AdS}_5 \times \mathbb{S}^5$

$$ds^2 = \frac{r^2}{R^2}\Big(-f(r)\ dt^2 + d\vec{x}_3^2\Big) + \frac{R^2}{r^2}\frac{dr^2}{f(r)} + R^2\left(d\theta^2 + \cos^2\theta d\Omega_2^2 + \sin^2\theta d\Omega_2^2\right)$$

with

$$f(r) = 1 - \left(\frac{r_H}{r}\right)^4, \qquad T = \frac{r_H}{\pi R^2}, \quad T: \text{Hawking temperature}$$

- With the change of coordinates

$$\rho^2 = \frac{r^2}{2}\left(1 + \sqrt{1 - \frac{r_h^4}{r^4}}\right)$$

the induced metric on the D5-brane is

$$ds^2 = g_{tt}dt^2 + g_{ii}(dx^2 + dy^2) + g_{\rho\rho}d\rho^2 + g_{\Omega\Omega}d\Omega_2^2$$

where

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