The Analytic Wavefunction

(with Mang Hei Gordon Lee, Scott Melville, Enrico Pajer)
arxiv: 2212:08009

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9th January 2023

Introduction

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Can we use the amplitudes technology to study cosmological correlators?

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How do we compute $\Psi[\phi]$?

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$\psi^{(n)}(\mathbf{k})$ are the wavefunction coefficients. They can be computed in perturbation theory using Feynman diagrams. Its analytic structure is deeply related to that of cosmological correlators.

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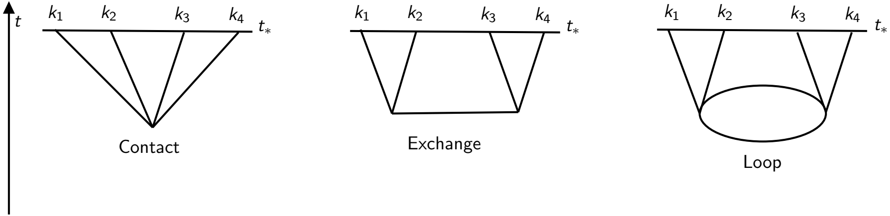

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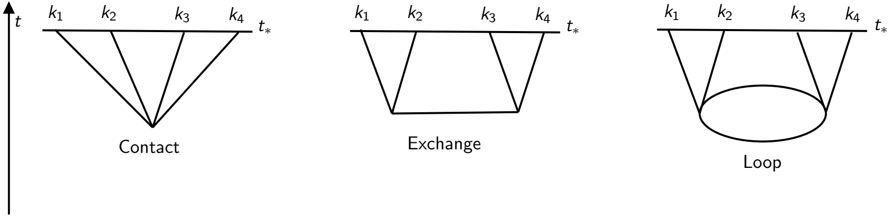

External lines are bulk-to-boundary propagators. They obey the free theory equations of motion and Dirichlet boundary conditions:

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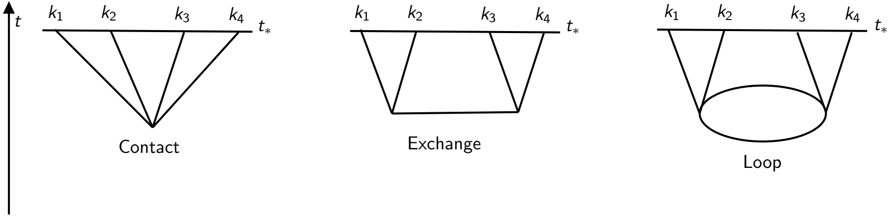

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Internal lines are bulk-to-bulk propagators.

$$(\partial_{t_1}^2 + \Omega_{\mathbf{k}}^2)G_k(t_1, t_2) = -\delta(t_1 - t_2)\ ,\ G_k(t_*, t_2) = G_k(t_1, t_*) = 0 \tag{9}$$

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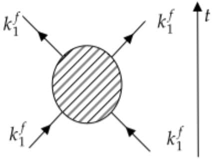

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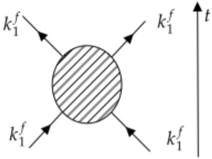

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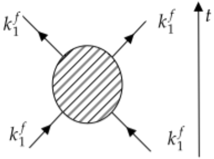

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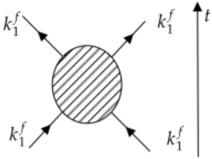

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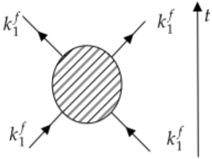

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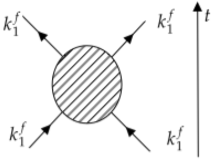

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α_n are the Wilson coefficients of the EFT and are fixed by UV physics. The EFT interactions are constrained by the IR symmetries and the boundary conditions.

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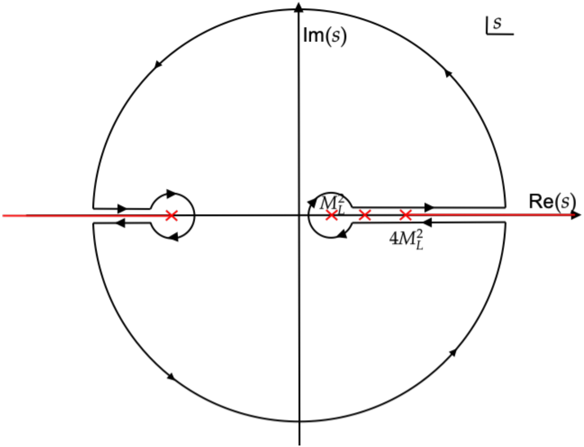

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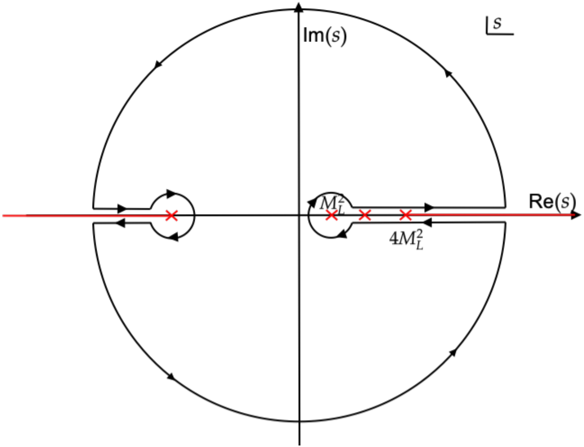

$\bar{A}(s,t)$ satisfies a dispersion relation:

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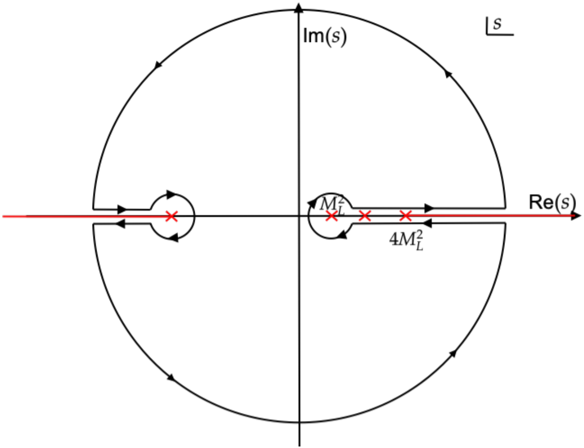

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The residue at $s' = \infty$ is related to UV subtraction. We have included the isolated poles into the discontinuity along the real axis.

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Therefore, we can derive sum rules for the Wilson coefficients of the EFT evaluating the RHS integrals at $s=0$ and taking derivatives on the LHS at $s=0$:

$$\bar{A}(0,0) = \alpha_0 = \underset{\infty}{\text{Res}}\left(\frac{\bar{A}_{\text{UV}}(s',0)}{s'}\right) + \int \frac{ds'}{2\pi i}\frac{\text{disc}(\bar{A}_{\text{UV}}(s',0))}{s'} \quad (13)$$

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Going off-shell

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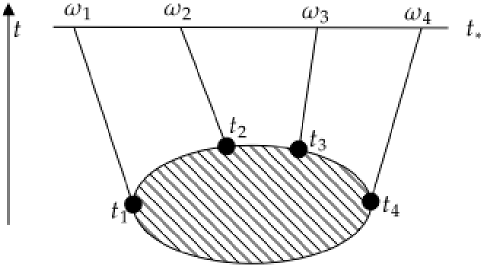
$$\tilde{\psi}^{(n)}(\omega, \mathbf{k}) = \prod_a^n \left(\int_{-\infty}^{t_*} dt_a e^{i\omega_a(t-t_*)} \right) \underbrace{\langle \phi = 0 | \mathcal{T} \left(\prod_a^n \hat{\Phi}(t_a) \right) | \mathrm{BD} \rangle}_{\text{Loops + Amputated vertex}}$$



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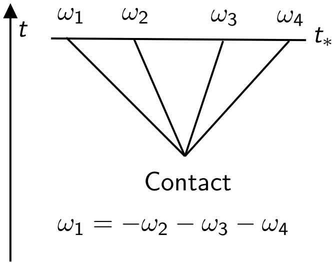
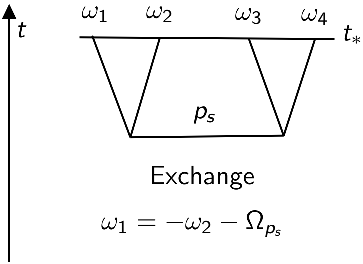


Contact

$$\omega_1 = -\omega_2 - \omega_3 - \omega_4$$



Exchange

$$\omega_1 = -\omega_2 - \Omega_{p_s}$$

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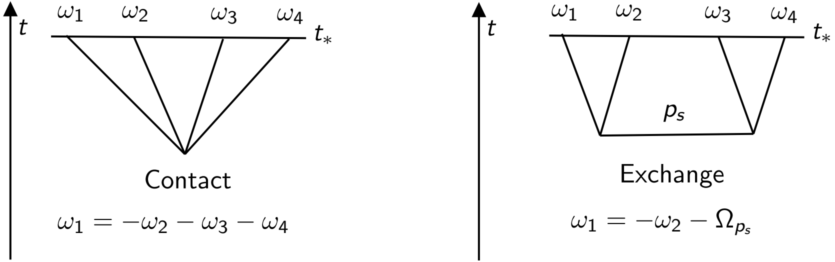
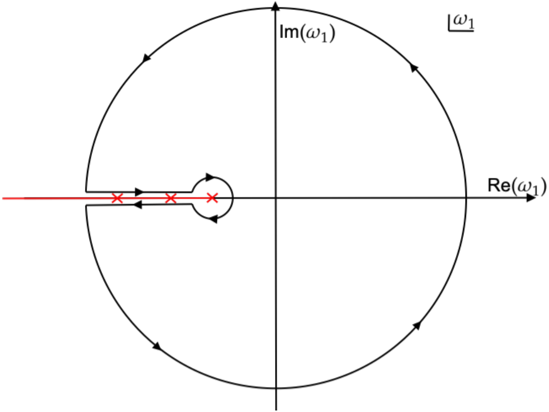


All singularities are located on the ω_1 negative real axis.

Analytic structure II



- $\omega_T \tilde{\psi}^{(n)}(\omega, \mathbf{k})$ is analytic in the lower half of the ω_1 complex plane.
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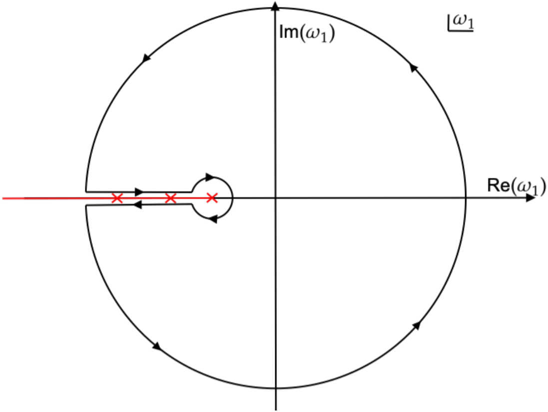

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$\omega_T \tilde{\psi}^{(n)}(\omega, \mathbf{k})$ also satisfies a dispersion relation:

$$\omega_T \tilde{\psi}^{(n)}(\omega, \mathbf{k}) = \operatorname*{Res}_{\infty} \left(\frac{\omega_T \tilde{\psi}^{(n)}(\omega', \mathbf{k})}{\omega' - \omega_1} \right) + \int \frac{d\omega'}{2\pi i} \frac{\mathrm{disc}(\omega_T \tilde{\psi}^{(n)}(\omega', \mathbf{k}))}{\omega' - \omega_1}$$

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$$\mathcal{L}_{\rm EFT}^{\rm bulk} \supset \frac{\alpha_0}{4!}\Phi^4 + \frac{\alpha_2}{4}\Phi^2\Box\Phi^2 + \frac{\alpha_4}{4}\Phi^2\Box^2\Phi^2$$

$$\mathcal{L}_{\rm EFT}^{\rm bdy} \supset \frac{\beta_{00}}{4!}\Phi^4(t_*) + \frac{\beta_{20}}{4}\Phi^2(t_*)\Box\Phi^2(t_*) - \frac{\beta_{11}}{4}\Phi^2(t_*)\partial_t\Phi^2(t_*) - \frac{\beta_{22}}{4}\Phi^2(t_*)\partial_i^2\Phi^2(t_*) - \frac{\beta_{31}}{4}\Phi^2(t_*)\partial_t\Box\Phi^2(t_*) - \frac{\beta_{31}^{'}}{4}\Box\Phi^2(t_*)\partial_t\Phi^2(t_*)$$

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$$\text{IR} \Rightarrow \omega_T \tilde{\psi}^{(n)}_{\text{IR}}(\omega, \mathbf{k}) = \int \frac{d\omega'}{2\pi i} \frac{\text{disc}(\omega_T \tilde{\psi}^{(n)}(\omega', \mathbf{k}))}{\omega' - \omega_1} \Leftarrow \text{UV} \tag{16}$$

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Like for amplitudes, we have to take derivatives with respect to $\omega_{a\neq 1}$ and the internal momenta p_s, p_t, p_u to obtain a complete list of the sum rules.

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The future ahead:

- How is UV bulk unitarity encoded in the boundary Wilson coefficients?
- Can we build positivity bounds with these sum rules?
- How do we import more amplitudes technology to $\tilde{\psi}^{(n)}(\omega, \mathbf{k})$?